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Effective lengths of braced frame columns

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Abstract. In several design codes and specifications, simplified formulae and charts are given for determining the effective lengths of frame columns. It is shown that these formulae may yield rather erroneous results in certain cases. This is due to the fact that, the code formulae utilise only local stiffness distributions. In this paper, a simplified procedure for determining approximate values for the buckling loads of braced frames is developed. The procedure utilises a fictitious load analysis of frames and yields errors less than 10%, which may be considered suitable for design purposes. The proposed procedure is applied to several numerical examples and it is shown that all the errors are in the acceptable range.

Keywords: buckling load; buckling length; effective length; non-sway mode; braced frames; isolated subassembly; multi-storey frames; design codes.

1. Introduction

Determining the effective (buckling) lengths of frame columns is one of the significant phases of frame design. Theoretically, effective length of an individual column is determined by calculating the system-buckling load of the frame. Since a full system instability analysis, may be quite involved for frames met in practical applications, simplified formulae and charts are given for determining the effective lengths of frame columns in most of the design codes and specifications, (AISC 1988, ACI 1989). The "Isolated subassembly approach" of specifications has been originally developed by Galambos (1968). In AISC, 2005 it is stated that, "For braced frames, K for compression members shall be taken as 1.0, unless structural analysis indicates a smaller value may be used." In the relevant commentary, extensive details ranging from the most rigorous second-order theory formulae to the charts of the isolated subassembly approach are provided. Similar formulae and charts exist in other widely applied specifications such as Eurocode 3 (1992) and DIN 18800 (1990).

A major limitation of the methods based on isolated subassembly approach is that they do not properly recognize the interaction effects of adjacent elements other than the ones at immediate neighbourhood of the joints. Hellesland and Bjorhovde (1996) have showed that this approach may result in significant errors in certain cases. Efforts to improve the applicability of subassembly approach include modifications proposed by Duan and Chen (1988, 1989) and an iterative procedure developed by Bridge and Fraser (1987). Another method of improvement for unbraced

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frames is the "Storey buckling approach" which accounts for the horizontal interaction between columns in a storey, (Yura 1971, LeMessurier 1977). White and Hajjar (1997) have showed that this approach may result in significant errors in unsymmetrical cases. The majority of the studies to improve the results of subassembly approach are devoted to "unbraced" frames.

On the other hand, for the case of "braced" frames a limited number of studies exist. Aristizabal-Ochoa (1997) and Cheong-Siat-Moy (1999) have developed methods including both braced, unbraced and "partially braced" structures. Another interesting improvement approach is proposed by Hellesland and Bjorhovde (1997) which involves a post processing procedure using weighted mean values of effective lengths. Mahini and Seyyedian (2006) have proposed another post processing approach depending on determining the critical elements of the structure.

In this study, a practical method is developed for determining the effective lengths of columns in unbraced frames. The method is based on computing an approximate value for system buckling load by using the results of a fictitious loading.

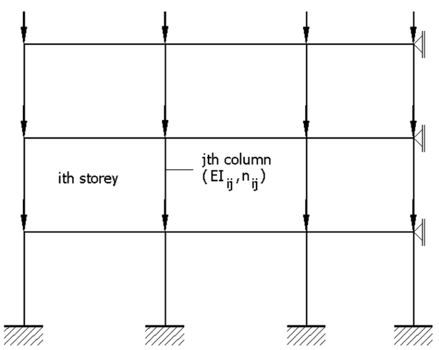
2. System buckling load of braced frames

A multi-storey braced frame which is composed of beams and columns made of linear elastic material is under the effect of vertical loads as shown in Fig. 1(a).

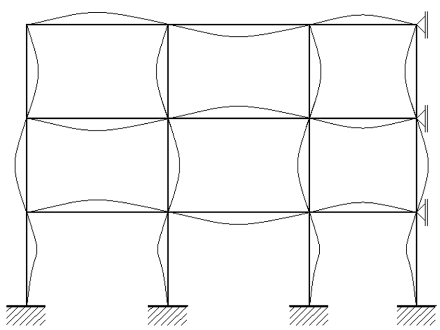
Each axial load may be expressed as

$$N_{ij} = n_{ij}P \tag{1}$$

where n_{ij} is a dimensionless coefficient and P is an arbitrarily chosen load parameter. The frame is in the state of "Stabile Equilibrium" and if the axial deformations are neglected, all the displacements and deformations are zero. Internal forces of the frame columns consist of only axial forces $N_{i,j}$ while all the internal forces of beams are zero. However, when the load parameter reaches to a critical P_{cr} value, another state of "Unstable Equilibrium" may exist. The displacement diagram corresponding to this new state, which is shown schematically in Fig. 1(b), is called the "Buckling Mode" of the structure (Horne and Merchant 1965). Once the buckling load parameter P_{cr} is determined, the effective length s_{ij} of an individual column can be computed by



a) System and loading



b) Buckling mode

Fig. 1 Multi storey frame and buckling mode

$$s_{ij} = \pi \sqrt{\frac{EI_{ij}}{n_{ij} P_{cr}}} \tag{2}$$

where EI_{ij} is the bending stiffness of the column.

In certain simple cases, buckling load parameter may be determined by using the stability functions (Horne and Merchant 1965). For general cases however, it is necessary to utilise specially prepared software. In this paper, a practical method will be explained and applied to the numerical examples. The method, which is developed by using the procedure given by Çakiroglu (1977) is applied, by using a simple quotient based on the results of a fictitious load analysis.

3. Effective lengths according to design codes

In several design codes and specifications, simplified formulae and charts are given for calculating the buckling lengths of individual columns. These simple formulae have the advantage of enabling the designer to obtain the effective lengths, without applying the tedious computations (or special software) which are necessary for the calculation of the overall-buckling load.

- In Eurocode 3 (1992) first the "distribution factors" at both ends of the columns are calculated by means of simple quotients. Then the charts given for both braced and unbraced frames are used to determine "effective length multiplier" K. The effective length s of a column with height h_c is computed by

$$s = Kh_c \tag{3}$$

- In AISC (1988) the distribution factors are calculated by using somewhat different quotients and are then used in determining the effective lengths multiplier K by means of nomographs.
- Formerly the same nomographs were also used in ACI codes. Recently, they are replaced by simple formulae for both braced and unbraced frames (ACI 1989).

Application of code formulae on several numerical examples have shown that erroneous results may be encountered for both braced and unbraced frames. This is mainly because, only local stiffness distributions are considered in these formulae, while the general behaviour of the frame is not taken into account. Recently, in AISC (1999), the isolated subassembly approach has been abandoned and it has been stated that "…the effective length factor K of compression members shall be determined by structural analysis". However in several widely used codes (such as Eurocode 3 1992 and ACI 1989) the subassembly approach and related charts and formulae are still being used.

Discussion of effective lengths of unbraced frames is left out of the scope of this study which was given in a previous paper (Özmen and Girgin 2005). The erroneous results encountered for braced frames will presently be demonstrated on several numerical examples.

3.1 Typical frames

With the purpose of testing the charts and formulae, eight "Typical frames" shown in Fig. 2 are chosen. Using special software prepared by Girgin (1996), the exact values of the buckling loads for the typical frames are determined. All the buckling loads may be expressed as

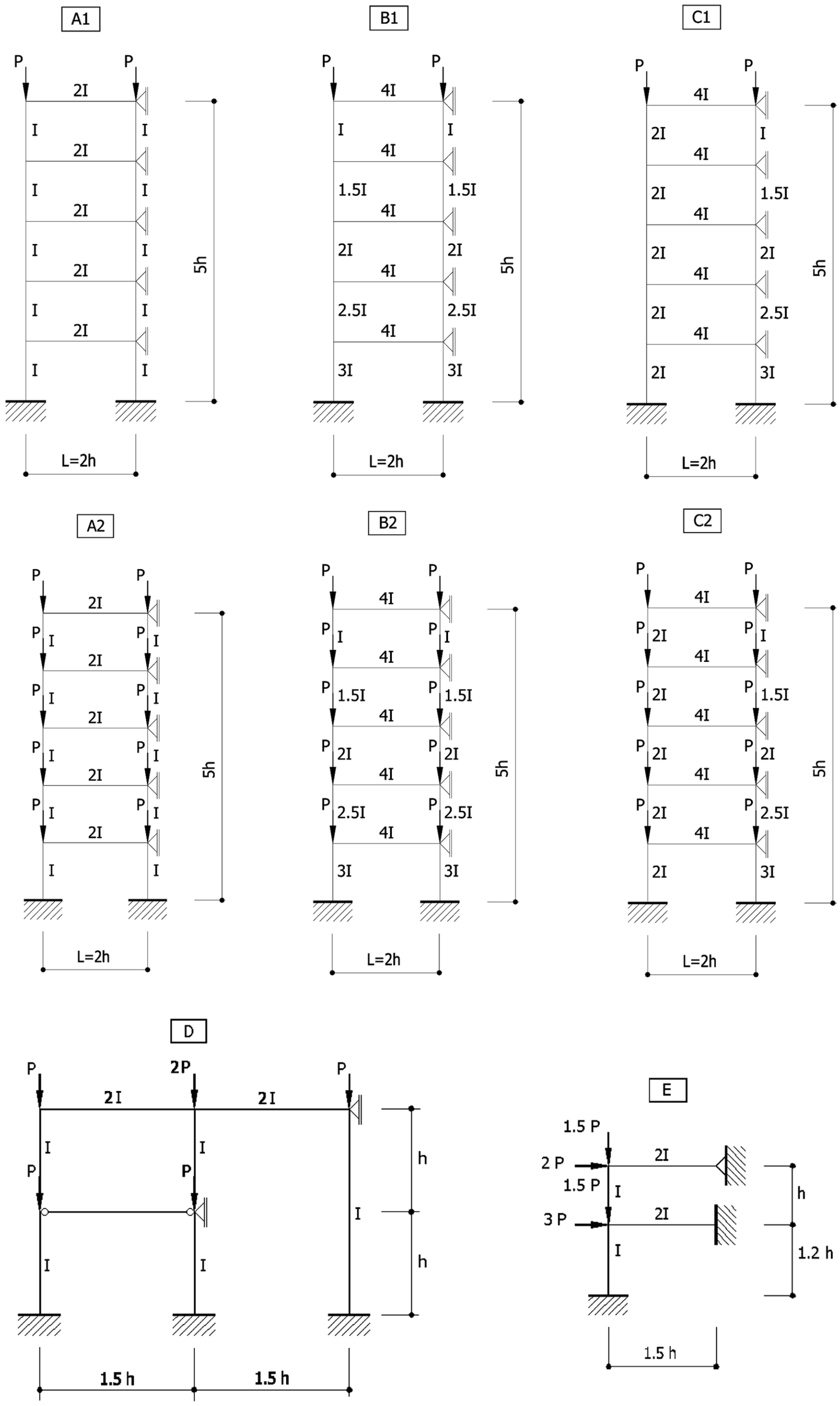


Fig. 2 Schematic elevations and loadings of typical frames

Table 1 Buckling load multipliers for typical frames

Type	C_{cr}
A1	14.39
B1	21.34
C1	23.64
A2	4.22
B2	9.92
C2	8.98
D	7.30
E	5.12

$$P_{cr} = C_{cr}\frac{EI}{h^2} \tag{4}$$

Buckling load multipliers C_{cr} are shown in Table 1.

In calculating the above values, axial and shear deformations are neglected. Considering Eqs. (2) and (3), the exact value for the effective length multiplier K for any column can be computed by

$$K = \frac{\pi}{h_c}\sqrt{\frac{EI_c}{nP_{cr}}} \tag{5}$$

where the bending stiffness of the column is denoted by EI_c.

3.2 Calculations according to various design codes

In this section, the "K-factor approach" used in several design codes will be applied to typical frames and the results will be discussed. First, frame type A1 shown in Fig. 2 will be considered. Calculating the distribution factors at both ends of the columns and using the chart given for non-sway frames in Eurocode 3 (1992) reveals K factors, which are shown in column 2 of Table 2.

On the other hand, the exact value of the buckling load for this frame is found to be

$$P_{cr} = 14.39\frac{EI}{h^2} \tag{6}$$

as shown in Table 1. Using this value, the exact effective length multipliers are calculated through Eq. (5) and shown in column 3 of the table. Since for this example all columns are identical and axial forces are equal, all the exact K factors are identical as expected. Errors corresponding to the

Table 2 Effective length multipliers for frame type A1

Storey	K (Eurocode 3)	K (Exact)	Relative error (%)
5	0.764	0.828	−7.7
4	0.810	0.828	−2.2
3	0.810	0.828	−2.2
2	0.810	0.828	−2.2
1	0.639	0.828	−22.8

Table 3 Error ranges for typical frames (%)

Type	Code		
	Eurocode 3	AISC (1988)	ACI (1989)
A1	–22.8 ~ –2.2	–20.8 ~ 3.3	–3.4 ~ 2.6
B1	–44.0 ~ 1.9	–43.5 ~ 9.0	–28.9 ~ 15.8
C1	–41.7 ~ 7.3	–40.5 ~ 14.7	–25.2 ~ 21.9
A2	–50.0 ~ 5.9	–35.0 ~ 36.7	–32.1 ~ 43.8
B2	–30.5 ~ 5.9	–25.7 ~ 4.9	–21.0 ~ 20.5
C2	–48.4 ~ 9.2	–45.1 ~ 36.7	–42.7 ~ 21.4
D	–29.45 ~ 4.16	–26.69 ~ 4.15	–14.03 ~ 26.64
Limits	–50.0 ~ 9.2	–45.1 ~ 41.1	–42.7 ~ 43.8

K factors of Eurocode 3 are shown in the last column of Table 2.

It is seen that, the range of relative errors on *K* factors vary between –22.8% and –2.2%, which may be considered as not representing a serious inaccuracy from the designer's point of view. However, it is interesting to encounter an error as high as –22.8% at the lowermost storey. Because the frame under consideration has been chosen as being as regular as possible. Hence it satisfies all the assumptions in deriving the isolated subassembly equations which are the basis of code charts and formulae.

The *K* factors for all the typical frames are found in the same manner and the encountered error ranges are shown in Table 3. The error ranges found for other codes (AISC 1988, ACI 1989) are also shown in the table.

Frame type E is not included in the table, since the beams of this frame are also under the effect of axial forces. Hence it may not be appropriate to apply code approaches to this particular frame.

It is clearly seen that all the considered codes yield errors, which are almost of the same order reaching as high as –50%. Moreover, more detailed examinations on *K* factors have shown that 86%, 71% and 49% of the *K* factors for respectively, Eurocode 3, AISC and ACI, are negative i.e. on the unsafe side.

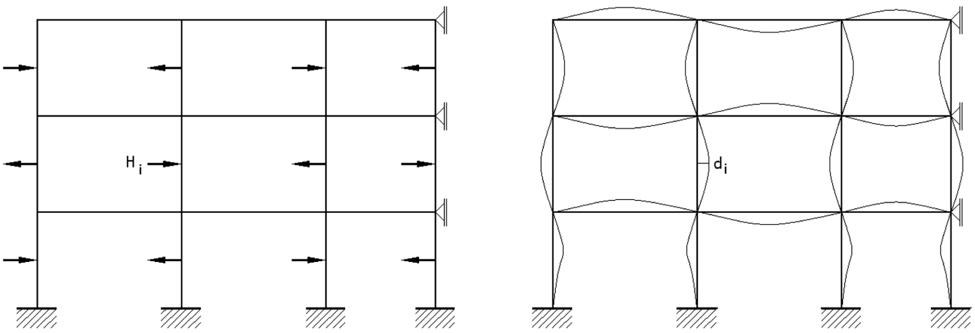
It is clear that these results represent a serious degree of inaccuracy from the designer's point of view. This is due to the fact that all codes use similar formulae, which consider only the local (isolated) stiffness distributions. However, investigations carried on a number of numerical examples have shown that, buckling length multipliers are dependent on

- Overall axial force distribution,
- Overall stiffness distribution,
- Location of the individual element

together with local stiffness distributions. It is concluded that, the buckling length multipliers should be determined by taking into account all these factors i.e., considering not only the local stiffness distributions, but also the overall characteristics of the structure.

4. A simplified procedure for determining the buckling load

In the following, a practical method will be explained and applied to the numerical examples. The method, which is developed by using the procedure given by Çakiroglu (1977), is applied by using



a) Fictitious lateral loading b) Displacements

Fig. 3 Multi storey frame and fictitious lateral loading

a simple quotient based on the results obtained by standard frame analysis software.

Consider the fictitious lateral loading shown in Fig. 3 applied to the frame shown in Fig. 1. The lateral loads are applied to the midpoints of the columns. It is assumed that this loading provides displacements identical to (or proportional with) those corresponding to the buckling mode.

The buckling load parameter can be determined by using Betti's Reciprocal Theorem applied to the states shown in Figs. 1 and 3. According to this theorem, it may be written that

$$W_1 = W_2 \tag{7}$$

where W_1 is the virtual work of the force system in Fig. 1(a) in conjunction with the displacements in Fig. 3(b), and W_2 is the virtual work of the force system in Fig. 3(a) in conjunction with the displacements in Fig. 1(b), (Neal 1964). Since the displacements of Figs. 1(b) and 3(b) are assumed as being the same, the displacements and deformations corresponding to the lateral fictitious loading will be used in the following.

4.1 Determination of W_1

According to the Principle of Virtual Works, W_1 can be computed as the work done by the internal forces of the loading shown in Fig. 1, in conjunction with the deformations induced by the fictitious lateral loading. The displacement diagram of an infinitely small portion of one of the columns together with the internal forces is shown in Fig. 4.

If the axial deformations are neglected the virtual work in this small portion can be computed by the product of the couple Ndv and the rotation dv/dx. Hence, the virtual work on any column can be obtained by

$$w = \int_{x=0}^{h_c} N dv \frac{dv}{dx} \tag{8}$$

or substituting N with the expression given by Eq. (1)

$$w = nP \int_{x=0}^{h_c} \left(\frac{dv}{dx}\right)^2 dx \tag{9}$$

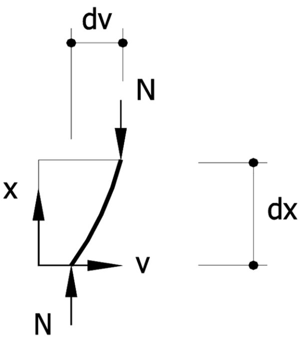


Fig. 4 Displacement diagram of a column portion

where h_c denotes the height of the individual column. It must be noted that, the indices are omitted for the sake of simplicity. The integral at the right hand side may more easily be handled by considering the upper and lower parts separately. Thus

$$w = w_L + w_U \tag{10}$$

may be written. Here w_L and w_U are given respectively by

$$w_L = nP \int_{x=0}^{c} \left(\frac{dv}{dx}\right)^2 dx \tag{11}$$

and

$$w_U = nP \int_{x=c}^{h_c} \left(\frac{dv}{dx}\right)^2 dx \tag{12}$$

where $c = h_c/2$ as shown in Fig. 5(b).

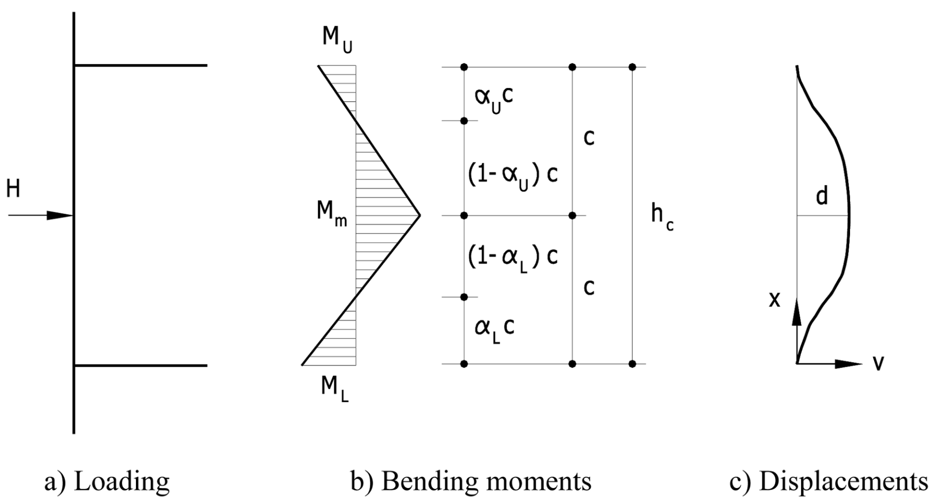


Fig. 5 Bending moment and displacement diagrams for braced column

The bending moment and relative displacement diagrams of an individual column are shown in Fig. 5. α_L and α_U are dimensionless coefficients designating the locations of the point of contraflexure for lower and upper parts, respectively. d denotes the midpoint displacement of the column.

The deformation expression for the lower part of the column is

$$\frac{d^2 v}{dx^2} = -\frac{M(x)}{EI} \tag{13}$$

where the bending moment function $M(x)$ may be expressed as

$$M(x) = M_L\left(1 - \frac{x}{\alpha_L c}\right) \tag{14}$$

Substituting $M(x)$ into Eq. (13) and integrating twice with the boundary conditions

$$\begin{array}{lll} v = 0 & \text{for} & x = 0 \quad \text{and} \\ v = d & \text{for} & x = c \end{array}$$

yields

$$v(x) = \left(\frac{d}{c} + \frac{M_L c(3\alpha_L - 1)}{6EI\alpha_L}\right)x + \frac{M_L}{12EI\alpha_L c}x^2(x - 3\alpha_L c) \tag{15}$$

After substituting the derivative of $v(x)$ into Eq. (15) and carrying out the integral

$$w_L = n\frac{d^2}{c}\chi_L = 2n\frac{d^2}{h_c}\chi_L \tag{16}$$

is obtained. Here χ_L denotes a dimensionless coefficient given by

$$\chi_L = 1 + \left(\frac{M_L c^2}{EId}\right)^2\left(\frac{1}{12} - \frac{1}{12\alpha_L} + \frac{1}{45\alpha_L^2}\right) \tag{17}$$

Similarly

$$w_U = 2n\frac{d^2}{h_c}\chi_U \tag{18}$$

and

$$\chi_U = 1 + \left(\frac{M_U c^2}{EId}\right)^2\left(\frac{1}{12} - \frac{1}{12\alpha_U} + \frac{1}{45\alpha_U^2}\right) \tag{19}$$

is found for the upper part of the column. χ_L and χ_U are dimensionless coefficients, which will be discussed in the following sections.

The total virtual work can be expressed as

$$W_1 = \sum w_L + w_U = 2P\sum n\frac{d^2}{h_c}(\chi_L + \chi_U) \tag{20}$$

Here the summation will be carried out for all the columns.

4.2 Determination of W_2

The virtual work of the force system in Fig. 5(a) in conjunction with the displacements in Fig. 1(b) (Fig. 5(c)) can simply be written as

$$W_2 = \sum Hd \tag{21}$$

where H and d represent the lateral load and column midpoint displacement, respectively. The summation will be carried out for all loaded points.

4.3 Simplified buckling load formula

Substituting the expressions for W_1 and W_2 given respectively by Eqs. (20) and (21) into Eq. (7) and solving for P (P_{cr}), the buckling load is obtained as

$$P_{cr} = \frac{\sum\limits_{\substack{Loaded \\ points}} Hd}{2 \sum\limits_{Columns} n \frac{d^2}{h_c} (\chi_L + \chi_U)} \tag{22}$$

It must be noted that, this formula is approximate since the lateral loading corresponding to the buckling load displacements, are not known initially. However, application on several numerical examples has shown that, the value of P_{cr} is not strongly dependent to the initial choice of lateral loads. It may be recommended that, lateral load at each column midpoint should be selected as proportional to the axial force coefficient n of the particular column.

4.4 The χ coefficients

It is seen that when applying Eq. (22), it is necessary to compute χ_L and χ_U coefficients for each individual column. As can be seen in Eqs. (17) and (19), these coefficients are dependent on the bending moments; hence, a tedious amount of computation is required. However, it can be shown that, χ values vary in a rather narrow range and can easily be simplified.

Let us consider the basic equation used in the approximate methods of lateral load analysis, which may be expressed as

$$\delta = \frac{Q}{k \frac{12EI}{h_c^2}} \tag{23}$$

Here Q denotes the shear force of the individual column and k is a dimensionless coefficient varying between 0 and 1, which depends on the stiffness of beams at each end of the column, (Muto 1964).

In the case of braced frames loaded at midpoints of the columns, Eq. (23) takes the form

$$d = \frac{Q_L}{k_L \frac{12EI}{c^3}} \tag{24}$$

where Q_L and k_L denote shear force and k coefficient for the lower part of the column, respectively. Eq. (24) can alternatively be written as

$$d = \frac{\dfrac{M_L}{\alpha_L c}}{k_L \dfrac{12EI}{c^3}} = \frac{M_L c^2}{12 k_L \alpha_L EI} \tag{25}$$

from which

$$\frac{M_L c^2}{EId} = 12 k_L \alpha_L \tag{26}$$

is obtained. Substituting this into Eq. (17)

$$\chi_L = 1 + 144 k_L^2 \alpha_L^2 \left(\frac{1}{12} - \frac{1}{12\alpha_L^2} + \frac{1}{45\alpha_L^2} \right) \tag{27}$$

is found. Similarly, for the upper part of the column

$$\chi_U = 1 + 144 k_U^2 \alpha_U^2 \left(\frac{1}{12} - \frac{1}{12\alpha_U^2} + \frac{1}{45\alpha_U^2} \right) \tag{28}$$

can be obtained. Eqs. (27) and (28) can be expressed in the general form of

$$\chi = 1 + 144 k^2 \alpha^2 \left(\frac{1}{12} - \frac{1}{12\alpha} + \frac{1}{45\alpha^2} \right) \tag{29}$$

It is seen that this expression is dependent only to the two dimensionless variables, namely k and α. The variation of χ is shown on the diagrams in Fig. 6.

On the other hand, calculations carried out on the columns of several numerical examples, have yielded the results shown as dots on Fig. 7.

Considering the relatively narrow range for values of χ and the practical upper bound of 1.40, it is reasonable to assume a constant and conservative value of

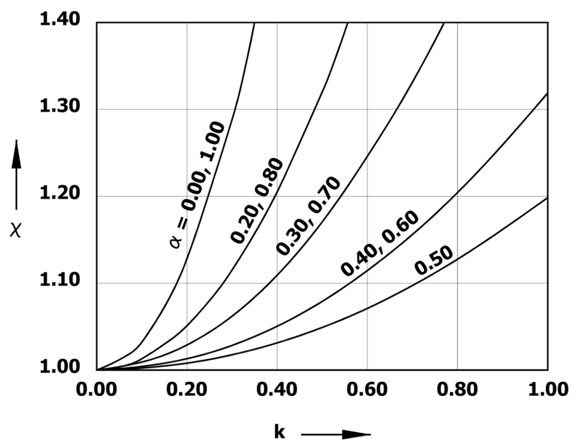


Fig. 6 Theoretical variation of χ values

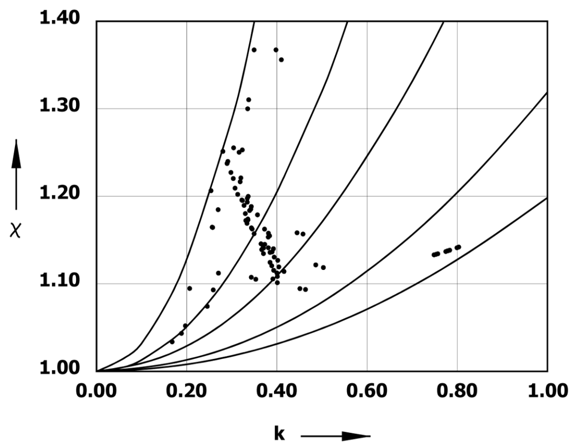


Fig. 7 Variation of χ values for numerical examples

$$\chi = \chi_L = \chi_u = 1.40$$

for practical purposes. Thus, Eq. (22) takes the rather practical form of

$$P_{cr} = \frac{\sum\limits_{\substack{Loaded\\points}} Hd}{5.60 \sum\limits_{Columns} n\dfrac{d^2}{h_c}} \tag{30}$$

4.5 Analysis procedure

Buckling lengths of frame columns can be determined as follows:

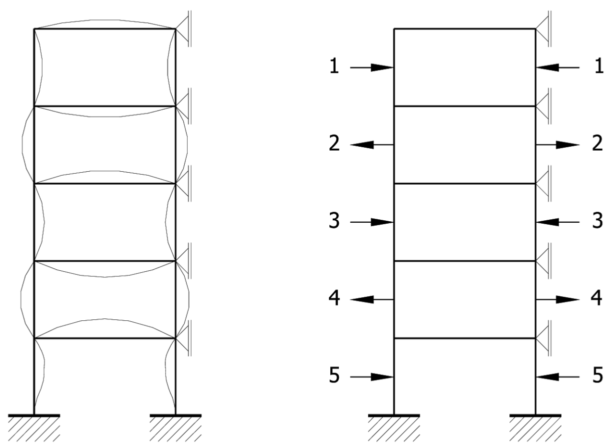
- Apply lateral forces proportional to the axial loads at midpoints of the columns considering the signs of assumed buckling mode shape,
- Compute midpoint displacements of the columns using any existing software,
- Compute the critical load P_{cr} by using Eq. (30),
- Determine the effective length factors of columns by using Eq. (5).

5. Numerical examples

In the following, the procedure outlined above will be applied to numerical examples and the results will be discussed.

5.1 Example 1: Frame Type B2

Dimensions and loading of the first example is the same as typical frame type B2 which is shown in Fig. 2 of Section 3.1. The shape of the buckling mode is shown in Fig. 8(a).



a) Buckling mode shape b) Fictitious loading

Fig. 8 Buckling mode and fictitious loading for Example 1

Table 4 Buckling load calculations for Example 1

Storey	H	$10^2\frac{EI}{h^3}d$	$10^2\frac{EI}{h^3}Hd$	n	h_c	$10^4\frac{(EI)^2}{h^5}n\frac{d^2}{h_c}$
5	1.00	1.380	1.380	1.00	h	1.90
4	-2.00	-1.922	3.844	2.00	h	7.39
3	3.00	2.258	6.774	3.00	h	15.30
2	-4.00	-2.305	9.220	4.00	h	21.25
1	5.00	1.559	7.795	5.00	h	12.15
Sum			29.013			57.99

Table 5 Effective length calculations for Example 1

Storey	h_c	I_c	n	K (Proposed)	K (Exact)	Error (%)
5	h	I	1.00	1.051	0.997	5.4
4	h	1.5I	2.00	0.910	0.864	5.4
3	h	2I	3.00	0.858	0.814	5.4
2	h	2.5I	4.00	0.831	0.789	5.4
1	h	3I	5.00	0.814	0.773	5.4

The directions of the fictitious lateral loads are chosen as being compatible with the buckling mode shape displacements as shown in Fig. 8(b). Their magnitudes are equal to the axial force coefficients for each column. After carrying out frame analysis for the fictitious loading, column midpoint displacements d are obtained. The terms used for the application of Eq. (30) is shown in Table 4.

Only the left half of the frame is considered due to symmetry. Applying Eq. (30) yields

$$P_{cr} = \frac{10^{-2} \times 29.013}{5.60 \times 10^{-4} \times 57.99} \frac{EI}{h^2} = 8.93\frac{EI}{h^2}$$

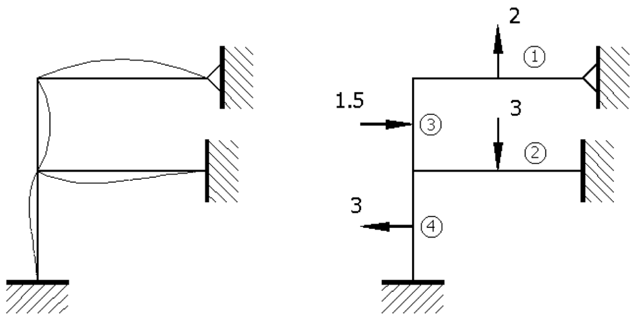
which has an error of −9.9%. Calculations of the effective length multipliers by using Eq. (5) are shown in Table 5.

The exact values of K factors together with the errors involved are also shown in the table. It must be noted that the errors for all the columns are the same as expected. This is due to the fact that they are computed by using the same equation used for exact calculations, namely Eq. (5).

5.2 Example 2: Frame Type E

As the second numerical example, typical frame type E which is shown in Fig. 2 of Section 3.1 is selected. This frame represents a wharf structure, which was first introduced by Bridge and Fraser, (1987), represents a special case in which beams are also subjected to axial forces. The shape of the buckling mode is shown in Fig. 9(a).

Fictitious loading and element numbers are shown in Fig. 9(b). It must be noted that, for this example, fictitious loads are also applied to the beams, since they are also under the effect of axial loads. The beams are included also in the buckling load calculations. The terms used for the application of Eq. (30) is shown in Table 6.



a) Buckling mode shape b) Fictitious loading

Fig. 9 Buckling mode and fictitious loading for Example 2

Table 6 Buckling load calculations for Example 2

Element No.	H	$10^2\frac{EI}{h^3}d$	$10^2\frac{EI}{h^3}Hd$	n	h_c	$10^4\frac{(EI)^2}{h^5}n\frac{d^2}{h_c}$
1	2.00	6.515	13.030	2.00	1.5h	56.59
2	−3.00	−4.775	14.325	3.00	1.5h	45.60
3	1.50	3.735	5.603	1.50	h	20.93
4	−3.00	−4.411	13.233	3.00	1.2h	48.64
Sum			46.191			171.76

Table 7 Effective length calculations for Example 2

Element No.	h_c	I_c	n	K (Proposed)	K (Exact)	Error (%)
1	1.5h	2I	2.00	0.956	0.926	3.3
2	1.5h	2I	3.00	0.781	0.756	3.3
3	h	I	1.50	1.171	1.134	3.3
4	1.2h	I	3.00	0.690	0.668	3.3

Applying Eq. (30) yields

$$P_{cr} = \frac{10^{-2} \times 46.191}{5.60 \times 10^{-4} \times 171.76}\frac{EI}{h^2} = 4.80\frac{EI}{h^2}$$

which has an error of −6.2%. Calculations of the effective length multipliers by using Eq. (5) are shown in Table 7.

The exact values of K factors together with the errors involved are also shown in the table. Here again the errors for all the elements (3.3%) are the same as expected.

5.3 Typical frames

The K factors for all the typical frames are found in the same manner and the encountered errors are shown in Table 8. The errors found for the two most notable methods, namely the methods

Table 8 Errors on effective length multipliers (%)

Frame type	Proposed method	Hellesland-Bjorhovde	Mahini-Seyyedian
A1	6.7	−2.1	9.8
A2	4.3	−10.4	7.4
B1	0.9	−11.3	15.4
B2	5.4	−4.8	2.1
C1	1.4	−8.9	21.5
C2	4.0	−8.4	10.8
D	−2.59	−7.64	2.62
E	3.25	−16.74	−3.44

developed by Hellesland and Bjorhovde (1997) and by Mahini and Seyyedian (2006) are also shown in the table.

The results will be compared and discussed presently. For the time being it suffices to mention that the two methods considered herein also produce errors which are the same for all the columns of a particular frame.

5.4 Parametric investigation

In order to test the validity of the method presented herein, the typical frames A1, A2, B1, B2, C1 and C2 have been augmented by taking their storey numbers as 2, 4, 6, 8 and 10, consecutively. The errors encountered for this parametric study is shown in Table 9.

The K factors for the typical frames of parametric study are also found by the methods of Hellesland and Bjorhovde (1997) and Mahini and Seyyedian (2006). The error ranges for the methods under consideration are summarised in Table 10.

Inspection of the results of various methods have revealed the following facts:

- Almost all of the results obtained by the method of Mahini and Seyyedian (2006) have positive errors on buckling length multipliers with very few exceptions. Hence they are on the safe side for the great majority of the cases. However, this method may provide results with errors near to or greater than 20% for several cases.
- All the results obtained by the method of Hellesland and Bjorhovde (1997) have negative errors i.e., they are at the unsafe side. Absolute values of error magnitudes are greater than 10% for several cases.

Table 9 Errors for the frames of parametric study (%)

Type	Number of stories				
	2	4	6	8	10
A1	5.6	6.4	6.8	7.1	7.3
A2	5.8	4.8	3.9	3.1	2.3
B1	6.9	5.3	2.7	0.0	−2.5
B2	6.5	6.0	5.2	4.7	4.5
C1	3.2	2.9	1.7	0.3	−1.1
C2	0.9	4.0	4.6	3.7	2.1

Table 10 Error ranges for the frames of parametric study (%)

	Method		
	Proposed	Hellesland-Bjorhovde	Mahini-Seyyedian
Minimum	−2.5	−17.2	-2.8
Maximum	7.3	−0.9	26.3
Average	4.1	−7.7	11.6

- Almost all of the results obtained by the proposed method have positive errors with very few exceptions. Hence they are on the safe side for the great majority of the cases. Maximum error of all the inspected cases is 7.3%, while the overall average of the errors is a mere 4.1%.

6. Conclusions

In this paper, determination of effective lengths of braced frame columns is investigated. The main conclusions derived, may be summarised as follows:

1. It is shown that, simplified formulae and charts, which are given in several design codes and specifications, may yield rather erroneous results for effective lengths of the columns. This is due to the fact that the code formulae refer only to local stiffness distributions, instead of the overall behaviour of the structures.
2. A simplified procedure for determining the approximate value for the buckling load of braced frames is developed. Effective lengths of columns may then be calculated by means of a simple formula.
3. The procedure, which utilises a fictitious load analysis of the frames, yields errors less than 10%. This error order may be considered acceptable from the designer's point of view.
4. The proposed procedure is applied to several numerical examples and it is seen that all the errors are in the acceptable range and the great majority of them are on the safe side.

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Appendix A - Exact values of the buckling loads

In the numerical examples presented above, all the results are compared with the "exact" values of buckling loads and the corresponding errors are determined. In view of the characteristics of existing software, determining the exact values seems to be a somewhat delicate matter, which will be discussed herein.

Since the most widely used contemporary structural program is SAP2000 (version 8), the buckling loads of the exact values of above examples are determined by a special application of this software. However, standard application of SAP2000 i.e., application without dividing the columns into pieces have produced quite meaningless results for all the typical frames. This is due to the fact that, SAP2000 uses the "geometric stiffness" formulae for the members with P-Δ effects, (Wilson 2002). Formerly, Horne and Merchant (1965) have described P-Δ (or second-order) effects by using the "stability functions". However, the geometric stiffness formulae recognise these effects only approximately. These formulae are reasonably accurate when the member lengths are sufficiently small. Moreover, when the joint displacements are zero, (which is the case for braced frames) the results become meaningless. Hence when using SAP2000, it is necessary to divide the compression members into proper number of pieces. When the columns of typical frames are divided into two pieces, the errors on the buckling load multipliers are found as shown in Table A1.

These rather large (and unsafe) errors are again due to the approximations introduced by geometric stiffness formulae. However, the errors diminish quite swiftly, when the columns are divided into pieces and the number of pieces increased. Parametric investigations on the typical frames has revealed that, in order to achieve reasonably accurate results i.e., results with an error order of less than 5%, compression members should be divided into at least 8 pieces. It is clear that, this operation increases the degree of freedom numbers considerably.

Table A1 Buckling load multipliers for typical frames

Type	C_{cr} (SAP2000)	C_{cr} (Exact)	Error (%)
A1	17.14	14.39	19.1
B1	24.92	21.34	16.8
C1	27.56	23.64	16.6
A2	5.21	4.22	23.5
B2	12.05	9.92	21.5
C2	11.00	8.98	22.5
D	9.36	7.30	28.2
E	6.44	5.12	25.8

In the above numerical examples, all the exact values are determined through SAP2000 solutions by dividing the compression members into 50 pieces. The results are then checked by using a special-purpose software developed by Girgin (1996), which computes the system-buckling load utilising the stability functions.