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Soil interaction effects on the performance of compliant liquid column damper for seismic vibration control of short period structures

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Abstract. The paper presents a study on the effects of soil-structure-interaction (SSI) on the performance of the compliant liquid column damper (CLCD) for the seismic vibration control of short period structures. The frequency-domain formulation for the input-output relation of a flexible-base structure with CLCD has been derived. The superstructure has been modeled as a linear, single degree-of-freedom (SDOF) system. The foundation has been considered to be attached to the underlying soil medium through linear springs and viscous dashpots, the properties of which have been represented by complex valued impedance functions. By using a standard equivalent linearization technique, the nonlinear orifice damping of the CLCD has been replaced by equivalent linear viscous damping. A numerical stochastic study has been carried out to study the functioning of the CLCD for varying degrees of SSI. Comparison of the damper performance when it is tuned to the fixed-base structural frequency and when tuned to the flexible-base structural frequency has been made. The effects of SSI on the optimal value of the orifice damping coefficient of the damper has also been studied. A more convenient approach for designing the damper while considering SSI, by using an established model of a replacement oscillator for the structure-soil system has also been presented. Finally, a simulation study, using a recorded accelerogram, has been carried out on the CLCD performance for the flexible-base structure.

Keywords: CLCD; SSI; complex valued impedance functions; short period structures; seismic vibration; equivalent SDOF oscillator; time history analysis.

1. Introduction

The emergence of passive/active control devices in the last two decades has been a remarkable development in the field of structural protection from environmental loads. The non-requirement of an external supply of power in the case of the passive control devices has made them very popular and different types of passive dampers like the friction dampers, viscoelastic dampers, tuned mass dampers (TMDs) and tuned liquid dampers (TLDs) have been successfully implemented worldwide (Soong and Dargush (1997)). The liquid dampers, in particular, provide some unique benefits, such as low cost, simple implementation, especially in existing structures, effectiveness even for low

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vibrational amplitudes, etc. The liquid column damper (LCD) is a type of liquid damper in which the liquid oscillates in a tube-like container containing orifice(s). It offers some further advantages as it has a high volumetric efficiency for a given volume of liquid, consistent behaviour across a wide range of excitation levels and a very definite damping mechanism.

The LCD, as in the case of some other passive dampers like the TMD, operates on the principle of transference of the vibrational energy of the structure to the mass of the damper when the natural frequency of the structure and damper are near equal (i.e., tuned). The conventional LCD is rigidly attached to the structure and is a long period system as the liquid column oscillates at a frequency which is inversely proportional to the square root of the length of the liquid column. Thus, from the requirement of tuning, the LCD is chiefly applicable to flexible structures such as tall buildings, cable-stayed bridges and towers. It has been well researched by Sakai *et al*. (1991), Xu *et al*. (1992), Balendra *et al*. (1995, 1999), Won *et al*. (1996) and Yalla and Kareem (2000), among others, for the mitigation of wind and earthquake induced vibrations of flexible structures. Recently, it has also been studied for the reduction of wave-induced vibrations of the floating offshore platform by Lee *et al*. (2006).

However, in case of short period structures, which are greatly susceptible to earthquake-induced vibrations, tuning the frequency of the LCD to the natural frequency of the structure may result in a practically infeasible length of the liquid column. In order to overcome this difficulty, a compliant model of the LCD (CLCD) has been developed by Ghosh and Basu (2004). In the CLCD, the frequency of the damper system consisting of the container and liquid, acting as a mass damper, is tuned to the primary structure. It allows an additional degree of freedom which removes the necessity of tuning the structural frequency to that of the frequency of the oscillating liquid column. The damping, however, is provided by the motion of the liquid through the orifice(s) of the U-shaped container.

Since proper tuning of the frequency of any damper system to the natural frequency of the structure is a very important aspect for the effective performance of a damper, the design of the control device requires reasonably accurate information of the properties of the structure to which it is to be attached. Generally, the damper is designed for the fixed-base structure. However, if the structure is founded on compliant soil, the soil structure interaction (SSI) effects may cause a reduction of the fundamental frequency, an increase in the overall damping of the system and result in a modification of the actual foundation motion from the free-field ground motion (Dey and Gupta (1999) and Dutta *et al*. (2004)). A significant change in the structural properties will cause a considerable effect on the performance of the damper which needs careful attention. In the work by Ghosh and Basu (2004) the study of the CLCD was carried out for fixed-base structures only. In this paper, the issue of the effects of SSI on the performance of the CLCD as a seismic vibration control device has been taken up.

Previous work on the functioning of passive dampers while considering SSI is very limited. Xu and Kwok (1992) examined the response of soil-structure-tuned mass damper (TMD) system in the frequency domain for the case of wind excitations. They reported that the tuning of the TMD should be done to the fundamental frequency of the soil-structure system. This was found to be effective for moderate to stiff soils but for soft soil the performance of the properly tuned mass damper remained poor. Takewaki (2000) has developed a method for the response reduction of structures by a combination of viscous damper and TMD considering SSI effects. Wang and Lin (2005) studied the vibration control of irregular buildings modeled as torsionally coupled structures due to base motions considering the SSI effect by multiple tuned mass dampers (MTMD). They observed that the decrease in relative stiffness of soil to structure generally amplifies both SSI and MTMD detuning effect, especially for a building with highly torsionally coupled effect. By appropriately enlarging the frequency spacing of the optimal MTMD, the detuning effect can be reduced. Their investigations also showed that the MTMD is more effective than the single TMD when the SSI effect is significant. Ghosh and Basu (2005) studied the performance of the conventional LCD model for seismic applications considering SSI effects. The study indicated that for medium to soft soil, the effect of SSI on the damper performance is not significant. However, for very soft soils, the effectiveness of the conventionally tuned LCD is greatly diminished. For such cases, it is essential to tune the damper to the natural frequency of the structure-foundation system.

In this work, a transfer function formulation for the flexible-base structure, with CLCD has been developed. A linear structure-soil model has been adopted. The rocking component of the free-field ground motion has been neglected. However, both translational as well as rocking motions of the foundation have been considered. The effect of the foundation motions has been considered through suitable modification of the actual input to the superstructure via the sub-structure approach (e.g., Dey and Gupta 1999). The foundation stiffness and damping, which are frequency-dependent, have been expressed by complex-valued impedance functions evaluated by researchers such as Veletsos and Wei (1971), Wong and Luco (1978) etc. Varying degrees of SSI have been studied by considering different values of the shear wave velocity of the soil. The outcome of designing the CLCD using the properties of the flexible-base structure and the effect of SSI on the optimal orifice damping coefficient of the damper have also been examined.

A simplified and useful design of the CLCD considering SSI effects using the equivalent single degree-of-freedom (SDOF) oscillator model for structure-foundation system developed by Wolf (1985) has also been presented. The results obtained from Wolf's model which uses the approximate frequency independent parameters of soil have been compared with those obtained from the proposed transfer function formulation of the flexible-base structure. The Wolf's model has also been used for a simulation study, using a recorded earthquake excitation, on the behaviour of the CLCD for a flexible-base structure.

2. Modeling of the CLCD-structure-foundation system

The CLCD-structure-foundation model investigated is shown in Fig. 1. The tube-like container of the LCD has cross-sectional area, A, and horizontal dimension, B. It contains liquid of mass density, ρ and column length, L. The head loss coefficient, controlled by the opening ratio of the orifice(s) installed in the damper tube, is denoted by ξ. The mass, stiffness and damping of the superstructure, modeled as a SDOF system, are denoted by M_1, K_1 and C_1 respectively. The stiffness and damping of the member connecting the LCD to the structure are given by K_2 and C_2 respectively. The mass of the container of the LCD, exclusive of the liquid mass, is denoted by M_C. Thus the total mass of the damper system is expressed as $(M_C + \rho AL)$. The foundation of the structure is considered to be a rigid slab of mass, M_0, anchored to the surface of a homogeneous, viscoelastic half-space through linear springs and viscous damping elements. It is assumed that both the mass of the structure, M_1, and the mass of the damper are concentrated at a height, h, from the foundation.

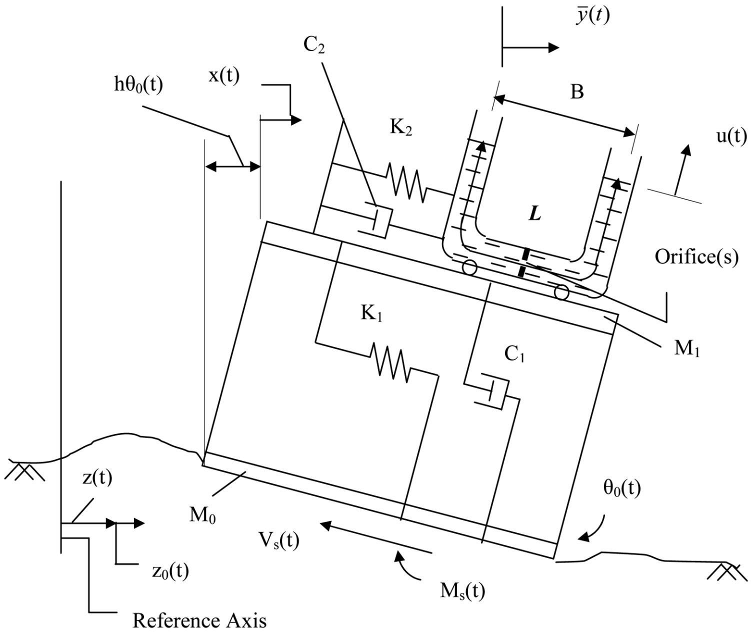

Fig. 1 CLCD-structure-foundation model

3. Formulation of transfer function

By neglecting the rocking component of the free-field ground excitation and the effects of kinematic interaction, the foundation input motion is assumed to be same as the free field ground translation, represented by $z(t)$. Let the foundation undergo translation, $z_0(t)$ and the rotation, $\theta_0(t)$, relative to the soil medium. Let $x(t)$ denote the horizontal displacement of the structure relative to the foundation and $u(t)$ be the change in elevation of the liquid column in the damper. $\bar{y}(t)$ represents the horizontal motion of the container relative to the SDOF system. Assuming small displacements, the total horizontal displacement of the structure is given by $\{x(t) + z(t) + z_0(t)$ $h\theta_0(t)\}$. The interaction forces between the foundation and the underlying soil interface are represented by $V_s(t)$, the base shear and $M_0(t)$, the base moment.

The equation of motion of the liquid column from equilibrium may be written as (Saoka *et al.* 1988)

$$\rho AL\ddot{u}(t) + \frac{1}{2}\rho A\xi|\dot{u}(t)|\dot{u}(t) + 2\rho Agu(t) = -\rho AB\{\ddot{\bar{y}}(t) + \ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t) + h\ddot{\theta}_0(t)\} \quad (1)$$

The following equivalent linear equation may be written to represent the nonlinear system in Eq. (1)

$$\rho AL\ddot{u}(t) + 2\rho AC_P\dot{u}(t) + 2\rho Agu(t) = -\rho AB\{\ddot{\bar{y}}(t) + \ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t) + h\ddot{\theta}_0(t)\} \quad (2)$$

where, C_P represents the equivalent linear damping co-efficient, and is expressed by Xu *et al.* (1992)

$$C_P = \frac{\sigma_{\dot{u}}}{\sqrt{2\pi}}\xi \tag{3}$$

C_P may be obtained by minimizing the mean square value of the error between Eqs. (1) and (2). In Eq. (3), $\sigma_{\dot{u}}$ is the standard deviation of the liquid velocity, $\dot{u}(t)$. As is evident from Eq. (3), the dependence of C_P on the response, $\sigma_{\dot{u}}$, of the liquid column calls for an iterative solution procedure for C_P.

Normalizing Eq. (2) with respect to the mass of liquid in the container, ρAL, leads to

$$\ddot{u}(t) + 2\frac{C_P}{L}\dot{u}(t) + \omega_l^2 u(t) = -\alpha\{\ddot{\bar{y}}(t) + \ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t) + h\ddot{\theta}_0(t)\} \tag{4}$$

where, $\omega_l = [=\sqrt{2g/L}]$ is defined as the frequency of the liquid oscillating in the LCD and $\alpha[=(B/L)]$ is the ratio of the horizontal portion of the liquid column to its total length.

The equation of motion for the mass, M_1, of the SDOF system, is given by

$$M_1\{\ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t) + h\ddot{\theta}_0(t)\} + C_1\dot{x}(t) + K_1 x(t) = C_2\dot{\bar{y}}(t) + K_2\bar{y}(t) \tag{5}$$

where, $\{C_2\dot{\bar{y}}(t) + K_2\bar{y}(t)\}$ denotes the interaction between the structure and the damper.

The dynamic equilibrium of the whole damper system, comprising of the liquid column mass and the container mass, leads to the following equation

$$(\rho AL + M_C)\{\ddot{\bar{y}}(t) + \ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t) + h\ddot{\theta}_0(t)\} + \rho AB\ddot{u}(t) + C_2\dot{\bar{y}}(t) + K_2\bar{y}(t) = 0 \tag{6}$$

On normalizing Eq. (6) with respect to $(\rho AL + M_C)$, the following is obtained,

$$\{\ddot{\bar{y}}(t) + \ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t) + h\ddot{\theta}_0(t)\} + \frac{\alpha}{(1+\tau)}\ddot{u}(t) + 2\zeta_2\omega_2\dot{\bar{y}}(t) + \omega_2^2\bar{y}(t) = 0 \tag{7}$$

In Eq. (7), $\omega_2[=\{k_2/(\rho AL + M_C)^{1/2}\}]$ and $\zeta_2[=\{C_2/2\omega_2(\rho AL + M_C)\}]$ denote the natural frequency and the damping ratio of the whole damper system, considering the liquid in the container to be relatively still. Also in Eq. (7), the term, $\tau[=\{M_C/\rho AL\}]$, represents the ratio of container mass to the liquid mass.

On normalizing Eq. (5) with respect to the mass, M_1, of the structure, leads to

$$\{\ddot{x}(t) + \ddot{z}(t)\} + 2\zeta_1\omega_1\dot{x}(t) + \omega_1^2 x(t) = \hat{\mu}\{2\zeta_2\omega_2\dot{\bar{y}}(t) + \omega_2^2\bar{y}(t)\} - \ddot{z}_0(t) - h\ddot{\theta}_0(t) \tag{8}$$

where, $\zeta_1 \& \omega_1$ represent the damping ratio and natural frequency of the SDOF system respectively. The ratio of the total mass of the damper (including the mass of the liquid as well as of the container) to that of the structure is denoted by $\hat{\mu}[=\{(\rho AL + M_C)/M_1\}]$.

The equation of equilibrium for the complete structure-foundation system in translation can be written as

$$V_S(t) + M_1\ddot{x}(t) + M_T(\ddot{z}(t) + \ddot{z}_0(t)) + M_1 h\ddot{\theta}_0(t) = C_2\dot{\bar{y}}(t) + K_2\bar{y}(t) \tag{9}$$

where, M_T denotes the total mass of the structure-foundation system and is equal to $(M_1 + M_0)$.

The equation of equilibrium for the complete structure-foundation system in rotation, ignoring the contributions of the gravitational forces, can be written as

$$M_s(t) + M_1 h\{\ddot{x}(t) + \ddot{z}(t) + \ddot{z}_0(t)\} + I_T\ddot{\theta}_0(t) = h[C_2\dot{\bar{y}}(t) + \bar{y}(t)] \tag{10}$$

where, I_T denotes the mass moment of inertia of the structure-foundation system about a horizontal axis at the foundation level and is equal to $(I_0 + I + M_1 h^2)$. Here, I_0 is the mass moment of inertia of the foundation and I represents the mass moment of inertia of the mass, M_1, about a horizontal axis through its center.

The interaction forces between the foundation and underlying soil are related to the foundation displacements in the frequency domain by the complex valued impedance functions $K_{VV}(\omega)$, $K_{VM}(\omega)$ $(=K_{MV}(\omega)$ by reciprocity theorems) and $K_{MM}(\omega)$ (Veletsos and Wei 1971, Wong and Luco 1978). The relationship between the base shear and overturning moment acting at the foundation soil interface and the corresponding foundation translation and rotation are expressed as (Chopra and Gutierrez 1974, Dey and Gupta 1999)

$$\begin{Bmatrix} V_S(\omega) \\ M_S(\omega) \end{Bmatrix} = \begin{bmatrix} K_{VV}(\omega) & K_{VM}(\omega) \\ K_{MV}(\omega) & K_{MM}(\omega) \end{bmatrix} \begin{Bmatrix} z_0(\omega) \\ \theta_0(\omega) \end{Bmatrix} \tag{11}$$

Here, $V_S(\omega)$ and $M_S(\omega)$ denote the Fourier transforms of $V_S(\omega)$ and $M_S(\omega)$ respectively. $K_{VV}(\omega)$ and $K_{MM}(\omega)$ represent the translational and rocking impedance functions respectively, while $K_{VM}(\omega)$ $(=K_{MV}(\omega))$ is the coupling impedance function of the foundation.

Due to the frequency dependence of the foundation impedance functions, it is necessary to transform the time domain equations into the frequency domain. The Fourier transformation of Eqs. (4), (7), (8), (9) and (10), leads respectively to the following equations

$$U(\omega) = -H_1(\omega)\alpha(\{\ddot{Z}(\omega) + \ddot{Z}_0(\omega) + h\ddot{\theta}_0(\omega)\} - \omega^2\{X(\omega) + \bar{Y}(\omega)\}) \tag{12}$$

$$\bar{Y}(\omega) = -H_2(\omega)\left[\{\ddot{Z}(\omega) + \ddot{Z}_0(\omega) + h\ddot{\theta}_0(\omega)\} - \omega^2\left\{X(\omega) + \frac{\alpha}{1+\tau}U(\omega)\right\}\right] \tag{13}$$

$$X(\omega) = -H_3(\omega)\left[\{\ddot{Z}(\omega) + \ddot{Z}_0(\omega) + h\ddot{\theta}_0(\omega)\} - \hat{\mu}\{2i\zeta_2\omega_2\omega + \omega_2^2\}\bar{Y}(\omega)\right] \tag{14}$$

$$V_S(\omega) = -H_T(\ddot{Z}(\omega) + \ddot{Z}_0(\omega)) - M_1 h\ddot{\theta}_0(\omega) + M_1\omega^2 X(\omega) + (K_2 + i\omega C_2)\bar{Y}(\omega) \tag{15}$$

$$M_S(\omega) = -M_1 h(\ddot{Z}(\omega) + \ddot{Z}_0(\omega)) - I_T\ddot{\theta}_0(\omega) + M_1\omega^2 hX(\omega) + h(K_2 + i\omega C_2)\bar{Y}(\omega) \tag{16}$$

In Eqs. (12) to (16), $X(\omega), \ddot{Z}(\omega), \ddot{Z}_0(\omega), \ddot{\theta}_0(\omega), U(\omega), \bar{Y}(\omega), V_S(\omega)$ and $M_S(\omega)$ are the Fourier Transforms of the corresponding time-dependent variables. Further in Eq. (12), $H_1(\omega)$ is the transfer function relating the displacement of the LCD modeled as a spring-dashpot SDOF oscillator to the free field ground acceleration and is given by

$$H_1(\omega) = \left[\frac{1}{\omega_l^2 - \omega^2 + 2i\frac{C_P}{L}\omega}\right] \tag{17}$$

In Eq. (13), the expression for $H_2(\omega)$ is the transfer function relating the horizontal motion of the container relative to the SDOF system and is given by,

$$H_2(\omega) = \left[\frac{1}{\omega_2^2 - \omega^2 + 2i\zeta_2\omega_2\omega}\right] \tag{18}$$

In Eq. (14) the expression for $H_3(\omega)$ represents the transfer function relating the horizontal displacement of the structure relative to the foundation and is given by

$$H_3(\omega) = \left[\frac{1}{\omega_1^2 - \omega^2 + 2i\zeta_1\omega_1\omega}\right] \tag{19}$$

Solving Eqs. (12), (13) and (14) leads to the following input-output relation in frequency domain

$$H(\omega) = H_X^1(\omega)\{\ddot{Z}(\omega) + \ddot{Z}_0(\omega) + h\ddot{\theta}_0(\omega)\} \tag{20}$$

where,

$$H_X^1(\omega) = \frac{-H_3(\omega)\left[H_2(\omega)\,\widehat{\mu}(\omega_2^2 + 2i\zeta_2\omega_2\omega)\{1 + \beta(\omega)\} + \{1 - \omega^2\beta(\omega)H_2(\omega)\}\right]}{\{1 - \omega^2\beta(\omega)H_2(\omega)\} - \omega^2 H_2(\omega)H_3(\omega)\,\widehat{\mu}(\omega_2^2 + 2i\zeta_2\omega_2\omega)\{1 + \beta(\omega)\}} \tag{21}$$

In Eq. (21) the expression for $\beta(\omega)$ is given by,

$$\beta(\omega) = \frac{\alpha^2\omega^2}{(1 + \tau)}H_1(\omega) \tag{22}$$

In Eq. (21) $H_X^1(\omega)$ denotes the transfer function relating the displacement, relative to the foundation, of a SDOF system CLCD, to the total input acceleration to the SDOF system.

Now, on substituting Eq. (20) in Eq. (14) the following equation is obtained.

$$\overline{Y}(\omega) = H_2(\omega)\left[\frac{\omega^2 H_X^1(\omega) + \beta(\omega)(\omega^2 H_X^1(\omega) - 1) - 1}{1 - \omega^2\beta(\omega)H_2(\omega)}\right](\ddot{Z}(\omega) + \ddot{Z}_0(\omega) + h\ddot{\theta}_0(\omega)) \tag{23}$$

For the flexible-base structure, the transfer function relating the structural displacement to the free-field ground acceleration will have to account for the accelerations of the foundation. The treatment for the same is given as follows.

On substituting Eqs. (11), (20) and (23) in Eqs. (15) and (16) and using $\ddot{Z}_0(\omega) = -\omega^2\ddot{Z}_0(\omega)$ and $\ddot{\theta}_0(\omega) = -\omega^2\theta_0(\omega)$, the following simultaneous equations in $Z_0(\omega)$ and $\theta_0(\omega)$ may be obtained

$$\begin{bmatrix} K_{VV}(\omega) - \omega^2\lambda_5(\omega) & K_{VM}(\omega) - \omega^2\lambda_6(\omega) \\ K_{VM}(\omega) - \omega^2\lambda_6(\omega) & K_{MM}(\omega) - \omega^2\{\lambda_4(\omega) + h^2\lambda_2(\omega)\} \end{bmatrix}\begin{Bmatrix} Z_0(\omega) \\ \theta_0(\omega) \end{Bmatrix} = \begin{Bmatrix} -\lambda_5(\omega) \\ -\lambda_6(\omega) \end{Bmatrix}\ddot{Z}(\omega) \tag{24}$$

where,

$$\lambda_1(\omega) = 1 - \omega^2 H_X^1(\omega) \tag{25}$$

$$\lambda_2(\omega) = -H_2(\omega)\left[\frac{\omega^2 H_X^1(\omega) + \beta(\omega)(\omega^2 H_X^1(\omega) - 1) - 1}{1 - \omega^2\beta(\omega)H_2(\omega)}\right](K_2 + i\omega C_2) \tag{26}$$

$$\lambda_3(\omega) = M_T - M_1\omega^2 H_X^1(\omega) \tag{27}$$

$$\lambda_4(\omega) = -M_1 h^2\omega^2 H_X^1(\omega) + I_T \tag{28}$$

$$\lambda_5(\omega) = \lambda_3(\omega) + \lambda_2(\omega) \tag{29}$$

$$\lambda_6(\omega) = h(M_1\lambda_1(\omega) + \lambda_2(\omega)) \tag{30}$$

Now, by solving Eq. (24) and using $\ddot{Z}_0(\omega) = -\omega^2 Z_0(\omega)$ and $\ddot{\theta}_0(\omega) = -\omega^2\theta_0(\omega)$, the following expressions are obtained.

$$\ddot{Z}_0(\omega) = H_{ZZ}^f(\omega)\ddot{Z}(\omega) \tag{31}$$

$$\ddot{\theta}_0(\omega) = H_{\theta Z}^f(\omega)\ddot{Z}(\omega) \tag{32}$$

Here, it is observed that $H_{ZZ}^f(\omega)$ and $H_{\theta Z}^f(\omega)$ respectively denote the transfer functions relating the translational and rocking accelerations of the foundation to the free-field ground acceleration. These are given by,

$$H_{ZZ}^f(\omega) = -\omega^2\frac{[K_{VM}(\omega)-\omega^2\lambda_6(\omega)]\lambda_6(\omega)-[K_{MM}(\omega)-\omega^2\{\lambda_4(\omega)+h^2\lambda_2(\omega)\}]\lambda_5(\omega)}{[K_{VV}(\omega)-\omega^2\lambda_5(\omega)][K_{MM}(\omega)-\omega^2\{\lambda_4(\omega)+h^2\lambda_2(\omega)\}]-[K_{VM}(\omega)-\omega^2\lambda_6(\omega)]^2} \tag{33}$$

$$H_{\theta Z}^f(\omega) = -\omega^2\frac{[K_{VM}(\omega)-\omega^2\lambda_6(\omega)]\lambda_5(\omega)-[K_{VV}(\omega)-\omega^2\lambda_5(\omega)]\lambda_6(\omega)}{[K_{VV}(\omega)-\omega^2\lambda_5(\omega)][K_{MM}(\omega)-\omega^2\{\lambda_4(\omega)+h^2\lambda_2(\omega)\}]-[K_{VM}(\omega)-\omega^2\lambda_6(\omega)]^2} \tag{34}$$

By substituting Eqs. (31) and (32) in Eq. (20), the transfer function relating the displacement of the structure with CLCD, relative to the foundation, to the free field ground acceleration is obtained. The resulting equation is given as

$$X(\omega) = H_X(\omega)\ddot{Z}(\omega) \tag{35}$$

where,

$$H_X(\omega) = H_X^1(\omega)\{1 + H_{ZZ}^f(\omega) + hH_{\theta Z}^f(\omega)\} \tag{36}$$

When the soil stiffness is very high, the movement of the foundation, relative to the surrounding soil medium is of insignificant value. This would imply that the effects of SSI are negligible and the structure may be assumed to have a "fixed" base. As observed earlier, in the context of Eqs. (20), (21) and (22), the transfer function relating the displacement relative to ground acceleration is given by $H_X^1(\omega)$. This also follows directly from Eq. (36), when the transfer function relating the foundation accelerations to the free-field ground acceleration, i.e., $H_{ZZ}^f(\omega)$ and $H_{\theta Z}^f(\omega)$ are negligibly small.

The evaluation of the equivalent damping parameter, C_P, requires the transfer function, $H_U(\omega)$, relating the displacement of the liquid column in the damper to the free- field ground acceleration. This is obtained by substituting Eqs. (31), (32), (35), (23) and (36) in Eq. (12) which leads to the following expression

$$U(\omega) = H_U(\omega)\ddot{Z}(\omega) \tag{37}$$

where,

$$H_U(\omega) = H_1(\omega)\alpha\left[\frac{\omega^2 H_2(\omega)\{1+\beta(\omega)\}\{\omega^2 H_X^1(\omega)-1\}}{\{1-\omega^2\beta(\omega)H_2(\omega)\}} + \omega^2 H_X^1(\omega) - 1\right]\{1 + H_{ZZ}^f(\omega) + hH_{\theta Z}^f(\omega)\} \tag{38}$$

If the ground acceleration is characterized by a white noise power spectral density function (PSDF) of intensity S_0, then the PSDF of the displacement response of the structure, denoted by $S_X(\omega)$, is expressed by Newland (1993)

$$S_X(\omega) = |H_X(\omega)|^2 S_0 \tag{39}$$

Also, the PSDF of the liquid velocity, $\dot{u}(t)$, represented by $S_{\dot{u}}(\omega)$, is evaluated from the following expression

$$S_{\dot{u}}(\omega) = \omega^2 |H_u(\omega)|^2 S_0 \tag{40}$$

The root mean square (r.m.s.) value of the displacement response of the structure, relative to the ground, and the r.m.s. value of the velocity response of the liquid column, equal to the standard deviation, $\sigma_{\dot{u}}$, may be numerically evaluated by computing the square root of the area under the corresponding PSDF curve as given by Eqs. (39) and (40) respectively.

4. Numerical study

An example short period structure with $\omega_1 = 20.94$ rad/s (0.3s) and $\zeta_1 = 1\%$ is considered. It is subjected to a white noise PSDF input with $S_0 = 100$ cm^2/s^3. The values of M_1 and h are considered to be 3×10^5 kg and 5.0 m respectively while M_0 and I_0 are assumed to be negligible. The values of the impedance functions are taken from the results by Wong and Luco (1978) for the following soil parameters: mass density, $(\bar{\rho}) = 1500$ kg/m^3; Poisson's ratio $(\lambda) = 0.3$; hysteretic damping ratio $(\varsigma) = 0.02$; characteristic length of the rigid square foundation = 3.0 m. As in the work of Ghosh and Basu (2004), $\zeta_2 = 0$ and $\tau = 1$ are considered. A commonly used value of 0.9 is assumed for α and a feasible value of $\hat{\mu} = 0.03$ is considered. The tuning ratio of the CLCD is evaluated from the expression given by Den Hartog (1956) and is equal to 0.9709. The CLCD offers the advantage of choosing practically feasible values of ξ and L without compromising on damper efficiency by choosing an optimal value of the ratio (ξ/L). In the present case, $L = 2.0$ m is considered. The optimal value of (ξ/L) corresponding to the minimum r.m.s. value of the displacement response of the fixed-base structure is evaluated numerically and is obtained as 426 m^{-1}.

The effect of different soil conditions is demonstrated by choosing different shear wave velocities, $\upsilon_s[=\sqrt{G/\bar{\rho}}]$ where, G is the shear modulus of soil. Fig. 2 shows the varying effects of SSI on the displacement transfer function of the structure without damper. As the soil becomes softer, as

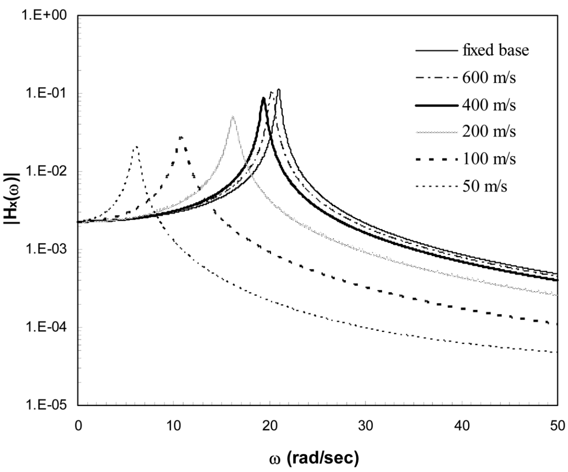

Fig. 2 Displacement transfer function ($|H_x(\omega)|$) of the example SDOF system alone for different soil conditions

Table 1 Comparison of response reduction for different soil and tuning conditions

Condition of soil		Fixed base	υ_s = 600 m/s	υ_s = 400 m/s	υ_s = 100m/s
Natural freq. of structure-foundation system (rad/s)		20.94	20.20	19.35	10.80
Response reduction (%)	Tuned to natural frequency of fixed-base structure	54.96	50.87	42.99	2.00
	Tuned to natural frequency of structure-foundation system	54.96	52.89	49.67	23.82

represented by decreasing values of υ_s, there is a significant shift in the natural frequency of the system from that of the fixed-base case, a quantitative estimate of which is presented in Table 1. Also, there is a considerable reduction in the amplitude of the transfer function peak indicating the increased damping in the system due to SSI.

To compare the performance of the CLCD between the fixed-base and flexible-base cases, the transfer function curves for the fixed base case with and without CLCD are plotted in Fig. 3. The curve with CLCD exhibits two peaks of greatly reduced magnitude, indicating the effect of tuning. The peaks are nearly equal as the optimum value of ξ is being used. The reduction in the r.m.s value of the displacement response of the structure is about 55%. Next, in Figs. 4 to 6, the transfer function curves for the structure (a) without damper, (b) with CLCD tuned to the natural frequency of structure alone and (c) with CLCD tuned to fundamental frequency of the structure-foundation system are compared for different soil conditions. The corresponding reductions in the r.m.s. values of the displacement response of the structure are shown in Table 1. The results demonstrate that even for moderately stiff soil it would be advantageous to tune the CLCD to the fundamental frequency of the structure-foundation system rather than to the fixed-base frequency. In the case of very soft soil (υ_s = 100 m/s), ignoring the effects of SSI in tuning the CLCD can render the damper to be practically ineffective. Here, it is possible to realize some amount of response reduction by tuning the CLCD to the flexible-base frequency of the system. However, the control achieved is considerably less than that in the fixed-base case. This is because, in case of significant

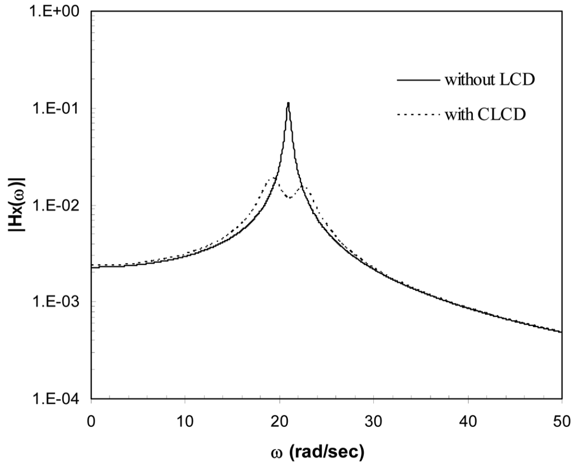

Fig. 3 Displacement Transfer Function of example SDOF system for fixed-base case

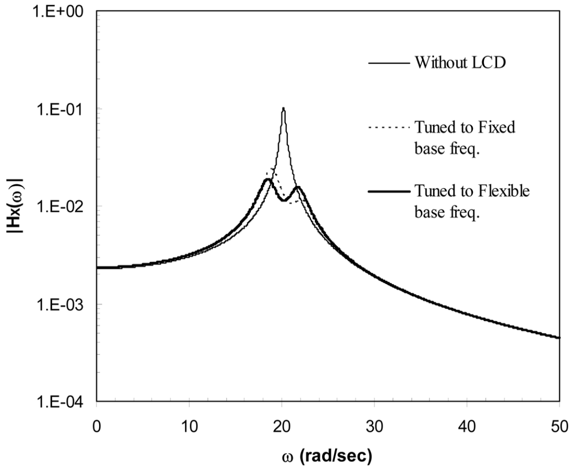

Fig. 4 Displacement Transfer Function of example SDOF system ($\upsilon_s = 600$ m/s) for different tuning conditions

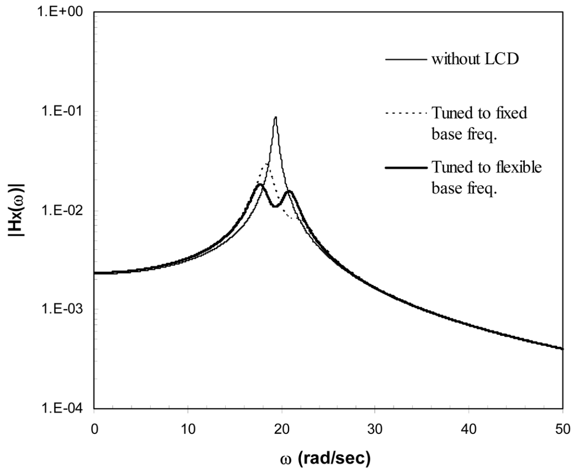

Fig. 5 Displacement Transfer function of structure ($\upsilon_s = 400$ m/s) for different tuning conditions

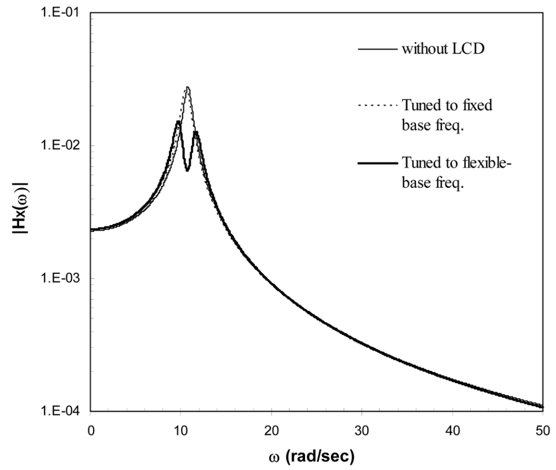

Fig. 6 Displacement Transfer Function of the structure ($\upsilon_s = 100$ m/s) for different tuning conditions

SSI, the response of the structure without damper itself is far less as compared to the fixed-base case and there is a limit to the maximum energy dissipation by a passive control device. In all the above analysis, for different soil conditions, the optimum (ξ/L) is assumed to be the same as that for the fixed-base case. Fig. 7 compares the variation in the response reduction with (ξ/L) for the fixed-base case with that for two typical flexible-base cases representing medium stiff and very soft soils ($\upsilon_s = 400$ m/s and $\upsilon_s = 100$ m/s respectively). It is observed that the effects of SSI on the optimal value of (ξ/L) are negligible. Hence in the design of the CLCD for the flexible-base case, the values of the optimum (ξ/L) as evaluated for the fixed-base case may be used.

It must be noted that the study in this paper is based on the linear behaviour of the structure and soil underlying the foundation. Trifunac *et al.* (2001) have shown that nonlinearity in the response

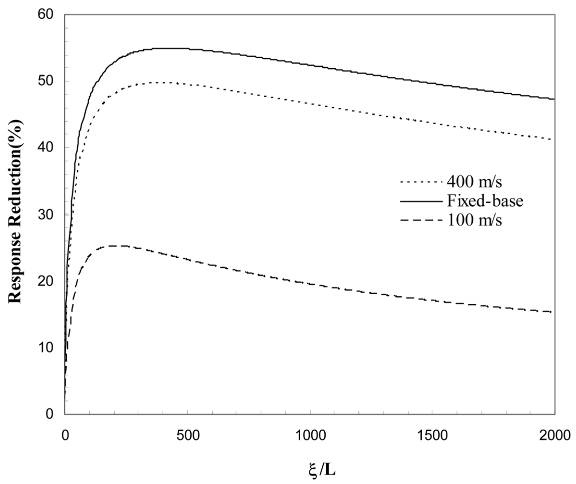

Fig. 7 Variation in response reduction with ξ/L for fixed-base and flexible-base conditions ($\upsilon_s = 400$ m/s and $\upsilon_s = 100$ m/s)

of the foundation soil may cause the structure-foundation system frequency to change substantially from one earthquake to another as well as during a particular earthquake. Under such conditions, the effectiveness of the CLCD will have to be studied by proper modelling of the soil, which caters for its nonlinear behaviour.

5. Design of CLCD for SSI using Wolf's Equivalent SDOF Oscillator

From the foregoing discussion it is evident that for the effective performance of the CLCD, tuning the damper to the flexible-base frequency of the structure is advantageous in case of moderate SSI effects and essential in case of significant SSI effects. Hence, estimation of the natural frequency of the flexible-base structure is one of the major inputs for the design of the CLCD for SSI. A simple approach for this would be to use the model of the equivalent SDOF oscillator as proposed by Wolf (1985) which represents the soil-structure system considered in Fig. 1. The closed form expressions for the natural frequency and damping ratio of the equivalent SDOF system given by Wolf (1985) have been developed by considering approximate frequency independent soil coefficients. Thus, by tuning the CLCD frequency to the natural frequency of the equivalent oscillator, it is possible to determine the control achieved by the damper by using the displacement transfer function of the fixed-base structure given in Eq. (21). In the approach using Wolf's model, while calculating the r.m.s. value of the displacement response to white noise input, the spectral intensity has to be scaled by the ratio of the square of the natural frequency of the replacement oscillator to the square of the natural frequency of the original fixed-base structure.

An illustrative study on the approach using Wolf's model is carried out by considering four example cases with the following combinations of natural period and damping ratio: 0.3*s*, 0.01; 0.3*s*, 0.03; 0.7*s*, 0.01 and 0.7*s*, 0.03. The unmodified spectral intensity of the base white noise input is assumed to be the same as before, i.e., $S_0 = 100\ \text{cm}^2/\text{s}^3$. Three different shear wave velocities, namely 100 m/s, 200 m/s and 400 m/s, representing very soft, soft and moderately stiff soil conditions are considered. As before, the values of the orifice damping coefficient (ξ) are those

Table 2 Comparison of results from proposed formulation and from using Wolf's model

Natural freq. of fixed base structure (rad/s)	Damping ratio of fixed base structure	Optimum ξ	Shear wave vel. (m/s)	Natural freq. of str.-fdn. system (rad/s)		Percent of response redn. by CLCD tuned to flexible base freq.		Damping ratio of equivalent SDOF system (by Wolf's Model)
				Results from proposed Formulation	Results from Wolf's Model	Results from proposed Formulation	Results from Wolf's Model	
20.944	0.01	852	100	10.80	10.81	23.82	22.65	0.047
			200	16.10	16.12	37.24	34.78	0.027
			400	19.35	19.35	49.67	48.13	0.014
	0.03	1052	100	10.80	10.81	21.78	20.39	0.052
			200	16.10	16.12	29.77	26.71	0.038
			400	19.35	19.35	32.62	31.00	0.031
8.976	0.01	232	100	7.30	7.32	40.86	38.44	0.022
			200	8.45	8.46	51.38	50.13	0.013
			400	8.85	8.84	54.38	54.05	0.011
	0.03	282	100	7.30	7.32	31.25	28.33	0.036
			200	8.45	8.46	32.84	31.55	0.031
			400	8.85	8.84	32.34	32.03	0.030

obtained from the optimization of (ξ/L) for the fixed-base case and are indicated in Table 2. The other structural and damper parameters, along with the soil parameters, are assumed to be same as those considered in the previous numerical study.

For each structure-foundation system the equivalent parameters are evaluated using Wolf's expression and presented in Table 2. It is observed that in all cases the equivalent natural frequency of the SDOF system as per Wolf's model matches very closely with the natural frequency of the structure-foundation system obtained from the peak of the transfer function evaluated from the formulation given in previous section. The reductions in the r.m.s. value of the displacement response of the structure using the equivalent SDOF oscillator with tuned CLCD are presented in Table 2. The results indicate that the control predicted using Wolf's model is very close to that obtained from the transfer function formulations. The maximum error in using the replacement oscillator is about 10% and is on the conservative side. Table 2 also shows the damping ratio of Wolf's equivalent SDOF system model which indicates the increased value of damping due to SSI effects from that of the fixed-base structure. As expected, softer soil associated with greater SSI effects results in greater damping in the structure. It is also observed that the increase in damping is greater for the stiffer structure and for the case of lower initial damping in the structure.

6. Simulation study on CLCD performance in case of SSI effects

A time history analysis is carried out to examine the functioning of the CLCD by subjecting the example structure as in Figs. 2-7 to the recorded S00E component of the 1940 Imperial Valley earthquake at the El Centro site, which is a fairly broad-banded excitation in its frequency content.

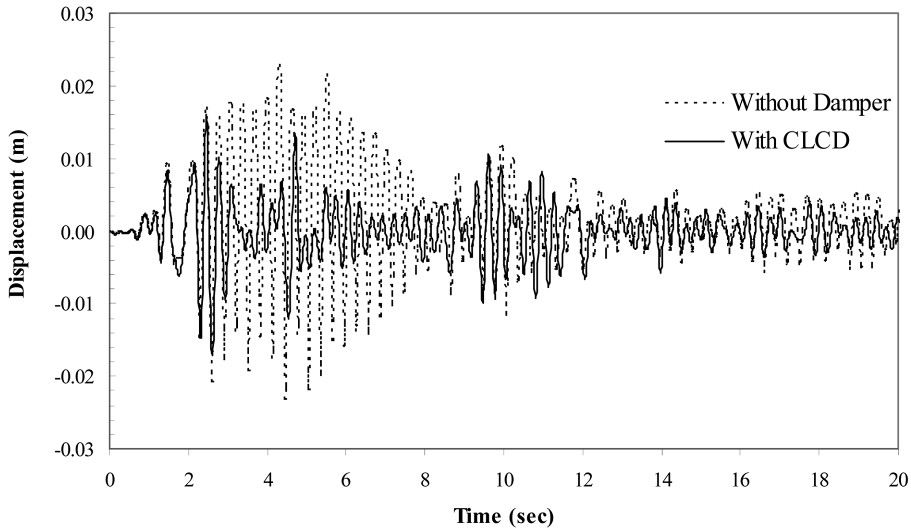

Fig. 8 Displacement Time History of fixed-base structure (0-20s), without and with CLCD, for the El Centro excitation

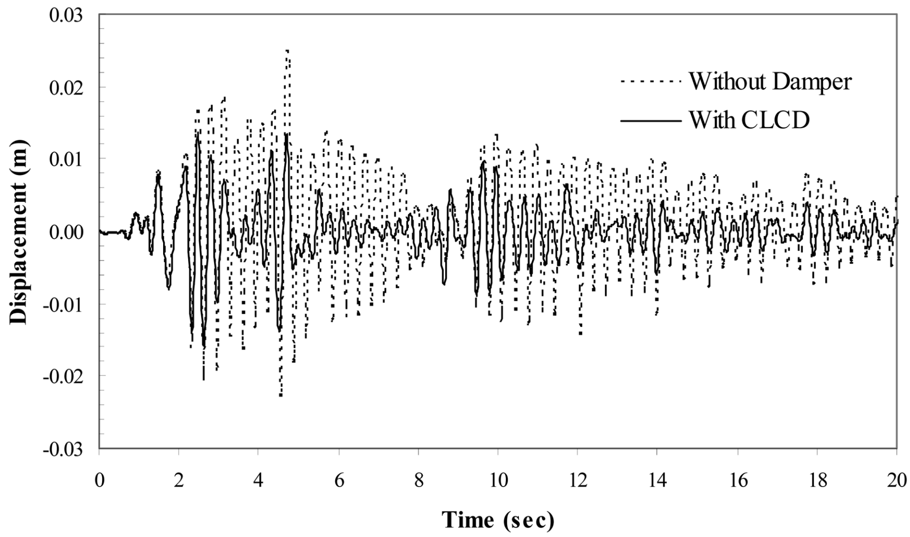

Fig. 9 Displacement Time History of flexible-base structure (0-20s), without and with CLCD, ($\upsilon_s = 400$ m/s), for the El Centro excitation

The effects of SSI on the CLCD performance are studied by using the Wolf's equivalent SDOF oscillator to model the soil-structure system. As outlined in the previous section, the CLCD frequency is tuned to the natural frequency of the equivalent oscillator. The fourth-order Runge-Kutta method is employed for the time integration of the response of the structure-damper system. As required by the approach using Wolf's model, while evaluating the response, the input acceleration has been scaled by the ratio of the square of the natural frequency of the replacement oscillator to the square of the natural frequency of the original fixed-base structure.

Different soil effects are demonstrated by varying the shear wave velocity of the soil. The

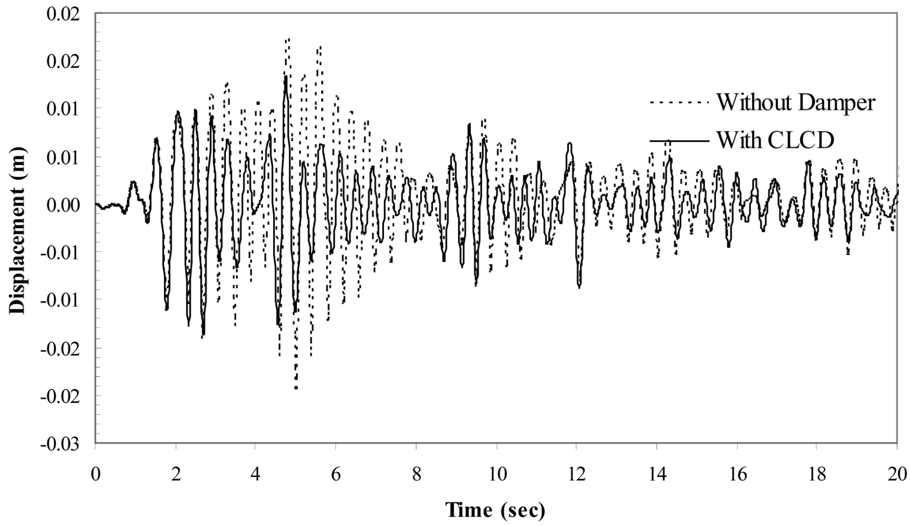

Fig. 10 Displacement Time History of flexible-base structure (0-20s), without and with CLCD, ($\upsilon_s = 200$ m/s), for the El Centro excitation

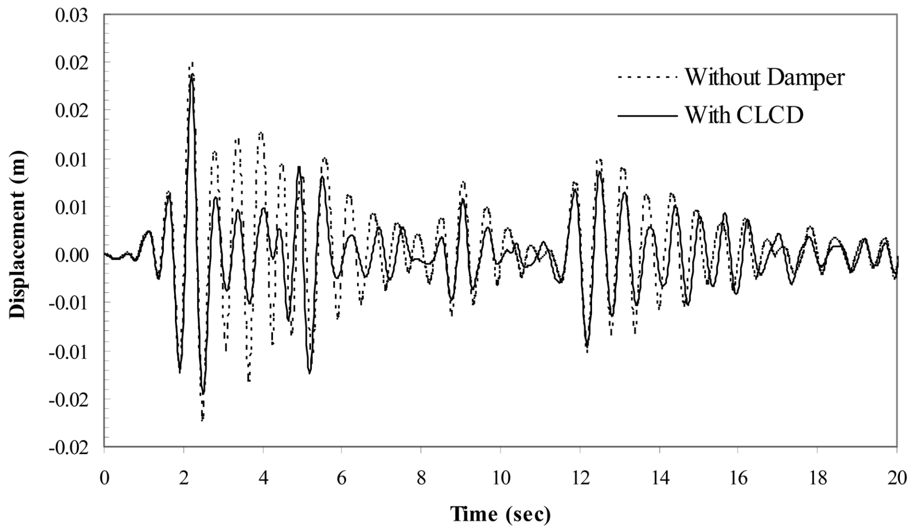

Fig. 11 Displacement Time History of flexible-base structure (0-20s), without and with CLCD, ($\upsilon_s = 100$ m/s), for the El Centro excitation

optimum value of (ξ/L) for different soil conditions are obtained by minimizing the r.m.s. value of displacement of the soil-structure-foundation system. For the sake of comparision, the results for the fixed-base case are presented in Fig. 8 while Figs. 9-11 indicate the displacement response of the structure, without damper and with CLCD, when subjected to the El Centro excitation for varying soil conditions. The maximum reductions in the r.m.s displacement obtained for the four cases in Figs. 8-11 are 50.96%, 49.72%, 27.45% and 22.63% respectively. These results are in good agreement with those in Table 1, thereby reinforcing the observations made from Table 1.

7. Conclusions

The displacement transfer function of a structure, modeled as a SDOF system and founded on compliant soil, with an attached CLCD has been formulated. Using this formulation, a numerical study on a typical short period structure-CLCD system considering different soil stiffness has revealed that even in case of medium soft soil tuning the CLCD to the fundamental frequency of the structure-foundation system would considerably improve the damper performance. This is significant as past studies have shown that the SSI effects are trivial for the performance of the conventional LCD attached to structures founded on similar soil conditions. In case of soft soils the CLCD tuned to the fixed-base frequency of the structure would be useless and it would be essential to consider the fundamental frequency of the structure-foundation system while designing the damper. Even with proper tuning, the performance of the CLCD in case of soft soils is considerably less as compared to the fixed-base case. The equivalent SDOF oscillator for the structure-soil system as proposed by Wolf has also been considered for the assessment of the CLCD performance in case of SSI effects. Results obtained from this approach have compared very well with those from the transfer function formulation, thereby indicating that it may be conveniently used in the design of the damper for structures with compliant base. This has allowed the application of the Wolf's model to a simulation study using recorded accelerogram to observe the performance of the CLCD when tuned to the fundamental frequency of the structure-foundation system under different soil conditions. The observations corroborate those obtained from the earlier frequency domain study.

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References

Balendra, T., Wang, C.M. and Cheong, H.F. (1995), "Effectiveness of tuned liquid column dampers for vibration control of towers", *Eng. Strut*., **17**(9), 668-675.

Balendra, T., Wang, C.M. and Rakesh, G. (1999), "Vibration control of various types of buildings using TLCD", *J. Wind Eng. Ind. Aerod.,* **83**(1-3), 197-208.

Chopra, A.K. and Gutierrez, J.A. (1974), "Earthquake response analysis of multistorey buildings including foundation interaction"*, Earthq. Eng. Struct. D.,* **3**, 65-77.

Den Hartog, J.P. (1956), *Mechanical Vibrations*, McGraw-Hill, New York.

Dey, A. and Gupta, V.K. (1999), "Stochastic seismic response of multiply – supported secondary system in flexible-base structures", *Earthq. Eng. Struct. D.,* **26**, 351-369.

Dutta, S.C., Bhattacharya, K. and Roy, R. (2004), "Response of low-rise building under seismic ground excitation incorporating soil-structure interaction", *Soil Dyn. Earthq. Eng.,* **24**(12), 893-914.

Ghosh, A. and Basu, B. (2004), "Seismic vibration control of short period structures using the liquid column damper", *Eng. Struct*., **26**, 1905-1913.

Ghosh, A. and Basu, B. (2005), "Effects of soil interaction on the performance of the liquid column damper for seismic applications", *Earthq. Eng. Struct. D*., **34**, 1375-1389.

Lee, H.H., Wong, S.-H. and Lee, R.-S. (2006), "Response mitigation on the offshore floating platform system with tuned liquid column damper", *Ocean Eng.,* **33**, 1118-1142.

Newland, D.E. (1993), "An introduction to random vibrations, spectral and wavelet analysis", Longman, **73**.

Sakai, F., Takaeda, S. and Tamaki, T. (1991), "Tuned Liquid Column Dampers (TLCD) for cable-stayed bridges", *Proc. Specialty Conf. Innovation in Cable-Stayed Bridges*, *Fukuoka, JSCE*, Japan, 197-205.

Saoka, Y., Sakai, F., Takaeda, S. and Tamaki, T. (1988), "On the suppression of vibrations by tuned liquid column dampers", *Annual Meeting of JSCE*, *JSCE*, Tokyo.

Soong, T.T. and Dargush, G.F. (1997), "Passive energy dissipation systems in structural engineering", John Wiley & Sons.

Takewaki, I. (2000), "Soil-structure random response reduction via TMD-VD simultaneous Use", *Comput. Meth. Appl. Mech. Eng.,* **190**, 677-690.

Trifunac, M.D., Ivanovic, S.S. and Todorovska, M.I. (2001), "Apparent periods of a building. I: Fourier analysis", *J. Struct. Eng.*, **127**(5), 517-526.

Trifunac, M.D., Ivanovic, S.S. and Todorovska, M.I. (2001), "Apparent periods of a building. II: Time-frequency analysis", *J. Struct. Eng.*, **127**(5), 527-537.

Veletsos, A.S. and Wei, Y.T. (1971), "Lateral and rocking vibration of footings", *J. Soil Mech.,* ASCE, **97**, 1227-1248.

Wang, J.-F. and Lin, C.-C. (2005), "Seismic performance of multiple tuned mass dampers for soil-irregular building interaction systems", *Int. J. Solid Struct.,* **42**(20), 5536-5554.

Wolf, J.P. (1985), "Dynamic soil-structure interaction", *Int. Series in Civil Eng. Mech.*, Prentice-Hall: Englewood Cliffs, NJ.

Won, A.Y.J., Pires, J.A. and Haroun, M.A. (1996), "Stochastic seismic evaluation of tuned liquid column dampers"*, Earthq. Eng. Struct. D*., **25**, 1259-1274.

Wong, H.L. and Luco, J.E. (1978), "Tables of impedance functions and input motions for rectangular foundations", Ref. CE 78-15, University of Southern California, Los Angles, C.A.

Xu, Y.L. and Kwok, K.C.S. (1992), "Wind-induced response of soil-structure-damper system", *8th Int. Conf. Wind Eng.*, Canada, 2057-2068.

Xu, Y.L., Samali, B. and Kwok, K.C.S. (1992), "Control of along – wind response of the structures by mass and liquid dampers", *J. Eng. Mech.*, ASCE, **118**(1), 20-39.

Yalla, S.K. and Kareem, A. (2000), "Optimum absorber parameters for tuned liquid column dampers", *J. Struct. Eng.,* **126**(8), 906-915.