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Dynamical behavior of generalized thermoelastic diffusion with two relaxation times in frequency domain

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Abstract. A general solution to the field equations of homogeneous isotropic generalized thermoelastic diffusion with two relaxation times (Green and Lindsay theory) has been obtained using the Fourier transform. Assuming the disturbances to be harmonically time–dependent, the transformed solution is obtained in the frequency domain. The application of a time harmonic concentrated and distributed loads have been considered to show the utility of the solution obtained. The transformed components of displacement, stress, temperature distribution and chemical potential distribution are inverted numerically, using a numerical inversion technique. Effect of diffusion on the resulting expressions have been depicted graphically for Green and Lindsay (G-L) and coupled (C-T) theories of thermoelasticity.

Keywords: generalized thermoelastic diffusion; time harmonic; concentrated and distributed loads; fourier transform.

1. Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observation. First, the equation of heat conduction of this theory doesn't contain any elastic terms; second, the heat equation is of a parabolic type, predicting infinite speeds of propagation of heat waves. Biot (1956) introduced the theory of coupled thermoelasticity to overcome the first short coming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming. Since the heat equation for the coupled theory is also parabolic.

Two generalizations to the coupled theory were introduced. This first is due to Lord and Shulman

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(1967), who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. Since the heat equation of this theory is of the wave type if automatically ensures finite speeds of propagation of heat of elastic waves. The remaining governing for this theory namely, the equation of motion and constitutive relations remain the same as those for the coupled and uncoupled theories.

The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature- rate -dependent themoelasticity. Muller (1971), in a review of the thermodynamics of thermoelastic solids proposed an entropy production inequality with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay (1972) obtained independently by Suhubi (1975). This theory contains two constants that act as two relation times and modify all the equations of the coupled theory not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry.

Diffusion can be defined as the random walk, of an ensemble of particles from regions of high concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in Metal Oxide Semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. Study of the phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from oil deposits.

Nowacki (1974a,b,c,d, 1976) developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. Recently Sherief *et al.* (2004) developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speeds of propagation of waves. Olesiak and Pyryev (1995) discussed a coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. Sherief and Saleh (2005) investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh (2005, 2006) discussed the reflection phenomena of waves from free surface of an elastic solid with generalized thermodiffusion. Recently, Aouadi (2006) studied the thermoelastic-diffusion interactions in an infinitely long solid cylinder subjected to thermal shock on its surface with a permeating substance. Aouadi (2006) investigated the problem of thermoelastic half-space with a permeating substance in contact with the bounding plane in context of the theory of generalized thermoelastic diffusion with one relaxation time and with variable electrical and thermal conductivity. The formulation and solution of the problems in frequency domain are simpler than in the time-domain. This is, off course, due to the absence of the time variable in the frequency domain formulation and hence, the transformation of the dynamic problem into the static like problem. Many researchers have dealt with the dynamic problems in the frequency domain. Sato (1969) gave theoretical expressions both in time and frequency domains for seismic waves due to a double couple in an infinite elastic medium in Cartesian, cylindrical and spherical-polar coordinates. Schiavone and Tait (1995) examined the bending of a Mindlin type thermoelastic plate due to time harmonic source. Allam, Elasibai and Abou Elregal (2002) obtained the thermal stress distribution in a harmonic field for a homogeneous isotropic infinite body with a cylindrical bob.

Sharma *et al*. (2000) discussed time harmonic sources in a generalized thermoelastic continuum. Kumar and Rani (2005) investigated the dynamic response of a homogeneous, isotropic thermoelastic half-space with voids subjected to time harmonic mechanical and thermal sources.

The present investigation is to determine the components of displacement, stress, temperature distribution and chemical potential distribution in an isotropic homogeneous elastic solid with generalized thermoelastic diffusion subjected to concentrated and distributed loads.

2. Basic equations

Following, Green and Lindsay (1972) and Sherief *et al.* (2004), the governing equations for isotropic homogeneous elastic solid with generalized thermoelastic diffusion in the absence of body forces and heat sources are

The constitutive relations

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}\left[\lambda e_{kk} - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta - \beta_2\left(1 + \tau^1\frac{\partial}{\partial t}\right)C\right] \tag{1}$$

$$P = -\beta_2 e_{kk} + b\left(1 + \tau^1\frac{\partial}{\partial t}\right)C - a\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta \tag{2}$$

The equation of motion

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta_{,i} - \beta_2\left(1 + \tau^1\frac{\partial}{\partial t}\right)C_{,i} = \rho\frac{\partial^2 u_i}{\partial t^2} \tag{3}$$

The equation of heat conduction

$$\rho C_E\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\theta + \beta_1 T_0\frac{\partial e}{\partial t} + aT_0\left(\frac{\partial}{\partial t} + \tau^0\frac{\partial^2}{\partial t^2}\right)C = K\theta_{,ii} \tag{4}$$

The equation of mass diffusion

$$D\beta_2 e_{,ii} + Da\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta_{,ii} + \frac{\partial C}{\partial t} - Db\left(1 + \tau^1\frac{\partial}{\partial t}\right)C_{,ii} = 0 \tag{5}$$

where

$$e_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i}); \quad i,j = 1, 2, 3$$

$$\beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu)\alpha_c$$

and λ, μ - Lame's constants, α_t - coefficient of linear thermal expansion, α_c - coefficient of linear diffusion expansion, $\theta = T - T_0$, T - absolute temperature, T_0 - temperature of the medium in its natural state assumed to be such that $|\theta/T_0| < 1$, t_{ij} - components of stress tensor, u_i -displacement vector, e_{ij} - components of strain tensor, $e = e_{kk}$, cubic dilatation, ρ - density, C - concentration , P - chemical potential per unit mass, C_E - specific heat at constant strain, K - coefficient of thermal conductivity, D - thermoelastic diffusion constant, τ_0, τ_1- thermal relaxation times, τ^0, τ^1- diffusion relaxation times, a -coefficients describing the measure of thermoelastic diffusion effects, b - coefficients describing the measure of diffusive effects, δ_{ij} - Kronecker's delta.

The thermal relaxation times τ_0 and τ_1 satisfy the inequality $\tau_1 \geq \tau_0 \geq 0$. The diffusion relaxation times τ^0 and τ^1 also satisfy the inequality $\tau^0 \geq \tau^1 \geq 0$.

3. Formulation and solution of the problem

We consider an isotropic homogeneous elastic solid with generalized thermoelastic diffusion in the undeformed state at temperature T_0. We introduce the rectangular Cartesian coordinate system (x, y, z) which has its origin on the surface $z = 0$ with the z-axis pointing normally into the medium.

For two dimensional problem, we assume

$$\vec{u} = (u_1, 0, u_3) \tag{6}$$

The initial and regularity conditions are given by

$$\begin{aligned} u_1(x,z,0) &= 0 = \frac{\partial u_1}{\partial t}(x,z,0) \\ u_3(x,z,0) &= 0 = \frac{\partial u_3}{\partial t}(x,z,0) \\ \theta(x,z,0) &= 0 = \frac{\partial \theta}{\partial t}(x,z,0) \\ C(x,z,0) &= 0 = \frac{\partial C}{\partial t}(x,z,0) \quad \text{for } z \geq 0,\ -\infty < x < \infty \end{aligned} \tag{7}$$

$$u_1(x,z,t) = u_3(x,z,t) = \theta(x,z,t) = C(x,z,t) = 0 \quad \text{for } t > 0 \text{ when } z \to \infty \tag{8}$$

Assuming time harmonic behavior as

$$(u_1, u_3, \theta, C)(x,z,t) = (u_1, u_3, \theta, C)(x,z)e^{i\omega t} \tag{9}$$

where ω is the angular frequency.

To facilitate the solution, the following dimensionless quantities are introduced

$$\begin{aligned} &x' = \frac{\omega_1^*}{c_1}x, \quad z' = \frac{\omega_1^*}{c_1}z, \quad t' = \omega_1^* t, \quad u_1' = \frac{\omega_1^*}{c_1}u_1 \\ &u_3' = \frac{\omega_1^*}{c_1}u_3, \quad t_{33}' = \frac{t_{33}}{\beta_1 T_0}, \quad t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad \omega' = \frac{\omega}{\omega_1^*} \\ &\theta' = \frac{\beta_1}{\rho c_1^2}\theta, \quad C' = \frac{\beta_2 C}{\rho c_1^2}, \quad a' = \frac{\omega_1^*}{c_1}a, \quad \tau_1' = \omega_1^* \tau_1 \\ &\tau_0' = \omega_1^* \tau_0, \quad \tau^{1'} = \omega_1^* \tau^1, \quad \tau^{0'} = \omega_1^* \tau^0, \quad P' = \frac{P}{\beta_2} \end{aligned} \tag{10}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_1^* = \frac{\rho C_E c_1^2}{K}$$

The displacement components $u_1(x, z, t)$ and $u_3(x, z, t)$ may be written in terms of potential functions $\phi(x, z, t)$ and $\psi(x, z, t)$ as

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \tag{11}$$

Using Eq. (6) and Eqs. (9)-(11), the Eqs. (3)-(5) recast into the following form (after suppressing the primes)

$$\left(\nabla^2 + \frac{\omega^2}{\delta}\right)\psi = 0 \tag{12}$$

$$(\nabla^2 + \omega^2)\phi - (1 + \tau_1 i\omega)\theta - (1 + \tau^1 i\omega)C = 0 \tag{13}$$

$$(\nabla^2 - n_1)\theta = \varepsilon_1 i\omega\nabla^2\phi + \varepsilon_1 a_1 n_2 C \tag{14}$$

$$\nabla^4\phi + a_1(1 + \tau_1 i\omega)\nabla^2\theta - \varepsilon_2[(1 + \tau^1 i\omega)\nabla^2 - a_2 i\omega]C = 0 \tag{15}$$

where

$$\delta = \frac{\mu}{\lambda + 2\mu}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho C_E(\lambda + 2\mu)}, \quad a_1 = \frac{a(\lambda + 2\mu)}{\beta_1\beta_2}, \quad \varepsilon_2 = \frac{b(\lambda + 2\mu)}{\beta_2^2}$$

$$a_2 = \frac{1}{bD\eta}, \quad \eta = \frac{\rho C_E}{K}, \quad n_1 = i\omega(1 + \tau_0 i\omega), \; n_2 = i\omega(1 + \tau^0 i\omega)$$

Applying Fourier transformation defined by

$$\hat{f}(\xi, z, \omega) = \int_{-\infty}^{\infty} f(x, z, \omega)e^{i\xi x}dx \tag{16}$$

on Eqs. (12)-(15), then eliminating $\hat{\phi}, \hat{\theta}, \hat{C}$ and $\hat{\psi}$ from the resulting expression we obtain

$$\left(\frac{d^6}{dz^6} + Q\frac{d^4}{dz^4} + N\frac{d^2}{dz^2} + I\right)(\hat{\phi}, \hat{\theta}, \hat{C}) = 0 \tag{17}$$

$$\left(\frac{d^2}{dz^2} - \lambda_4^2\right)\hat{\psi} = 0 \tag{18}$$

where

$$Q = \frac{1}{E}[F - 3\xi^2 E]$$

$$N = \frac{1}{E}[G - 2F\xi^2 + 3\xi^4 E]$$

$$I = \frac{1}{E}[F\xi^4 - G\xi^2 + H - E\xi^6]$$

$$\lambda_4^2 = \xi^2 - \frac{\omega^2}{\delta}$$

and

$$E = [-\varepsilon_2 + 1](1 + \tau^1 i\omega)$$

$$F = [\varepsilon_2 a_2 i\omega + (\varepsilon_2 - 1)n_1(1 + \tau^1 i\omega) + \varepsilon_1 a_1 n_2(1 + \tau_1 i\omega)(a_1 + 1) + i\omega(1 + \tau^1 i\omega)[i\omega\varepsilon_2 + \varepsilon_1\varepsilon_2(1 + \tau_1 i\omega) + \varepsilon_1 a_1(1 + \tau_1 i\omega)]$$

$$G = \varepsilon_2 n_1[i\omega a_2 + \omega^2(1 + \tau^1 i\omega)] + i\omega^3\varepsilon_2 a_2 + \omega^2\varepsilon_1(1 + \tau_1 i\omega)[n_2 a_1^2 + \varepsilon_2 a_2]$$

$$H = -i\omega^3\varepsilon_2 a_2 n_1$$

The roots of the Eq. (17) are $\pm\lambda_l (l = 1, 2, 3)$ and the roots of Eq. (18) are $\pm\lambda_4$. Making use of radiation condition $\hat{\phi}, \hat{\theta}, \hat{C}$ and $\hat{\psi} \to 0$ as $z \to \infty$, the solutions of Eqs. (17) and (18) may be written as

$$\hat{\phi} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z} \tag{19}$$

$$\hat{\theta} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z} \tag{20}$$

$$\hat{C} = e_1 A_1 e^{-\lambda_1 z} + e_2 A_2 e^{-\lambda_2 z} + e_3 A_3 e^{-\lambda_3 z} \tag{21}$$

$$\hat{\psi} = A_4 e^{-\lambda_4 z} \tag{22}$$

where

$$d_l = \frac{P^*\lambda_l^2 + Q^*}{R^*\lambda_l^2 + S^*}, \qquad e_l = \frac{U^*\lambda_l^4 + V^*\lambda_l^2 + W^*}{X^*\lambda_l^2 + T^*}, \qquad (l = 1, 2, 3)$$

$$P^* = \frac{1}{1 + \tau^1 i\omega} + \frac{1}{a_1(1 + \tau^0 i\omega)}, \qquad U^* = (1 + a_1)$$

$$Q^* = -\left[\frac{\xi^2 - \omega^2}{1 + \tau^1 i\omega} + \frac{\xi^2}{a_1(1 + \tau^0 i\omega)}\right], \qquad V^* = -(2(1 + a_1)\xi^2 - a_1\omega^2)$$

$$R^* = \frac{1}{\varepsilon_1 a_1 n_2}, \qquad W^* = (1 + a_1)\xi^4 - a_1\omega^2\xi^2$$

$$S^* = -\frac{1}{\varepsilon_1 a_1 n_2}(\xi^2 + n_1) + \frac{1 + \tau_1 i\omega}{1 + \tau^1 i\omega}, \qquad X^* = a_1(1 + \tau^1 i\omega) + \varepsilon_2(1 + \tau^1 i\omega)$$

$$T^* = -\{[a_1(1 + \tau^1 i\omega) + \varepsilon_2(1 + \tau^1 i\omega)]\xi^2 + i\omega\varepsilon_2 a_2\}$$

with A_l, $(l = 1, 2, 3)$ being arbitrary constants.

4. Applications

On the half-space surface ($z = 0$) normal point force, thermal point source and chemical potential source, which are assumed to be time harmonic, are applied. We consider three types of boundary conditions, as follows:

CASE 1. The normal force on the surface of half-space
The boundary conditions in this case are

$$(i)\ t_{33}(x,z,t) = -P_1\,\psi_1(x)e^{i\omega t}, \qquad (ii)\ t_{31}(x,z,t) = 0$$
$$(iii)\ \theta = 0, \qquad (iv)\ P = 0,\ \text{at}\ z = 0 \tag{23}$$

where $\psi_1(x)$ specify source distribution function along x-axis and P_1 is the magnitude of force applied.

CASE 2. The thermal source on the surface of half-space
When the plane boundary is stress free and subjected to thermal point source the boundary conditions are

$$(i)\ t_{33}(x,z,t) = 0, \qquad (ii)\ t_{31}(x,z,t) = 0$$
$$(iii)\ \theta = P_2\psi_1(x)e^{i\omega t}, \qquad (iv)\ P = 0,\ \text{at}\ z = 0 \tag{24}$$

where $\psi_1(x)$ is the source distribution function along x-axis and P_2 is the constant temperature applied on the boundary.

CASE 3. The chemical potential source on the surface of half-space
Here the boundary is stress free and subjected to chemical potential source, therefore the boundary conditions are

$$(i)\ t_{33}(x,z,t) = 0, \qquad (ii)\ t_{31}(x,z,t) = 0$$
$$(iii)\ \theta = 0, \qquad (iv)\ P = P_3\psi_1(x)e^{i\omega t},\ \text{at}\ z = 0 \tag{25}$$

where $\psi_1(x)$ is the source distribution function along x-axis and P_3 is the constant potential applied on the boundary.

4.1 Green's functions

To synthesize the Green's functions, i.e., the solutions due to concentrated normal force/thermal source/chemical potential source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x) \tag{26}$$

in Eqs. (23), (24) and (25). Applying the Fourier transforms defined by Eq. (16) on the Eq. (26) gives

$$\hat{\psi}_1(\xi) = 1 \tag{27}$$

SUBCASE 1(a). Normal force

Making use of Eqs. (1), (2) and (9)-(11), along with $P_1' = P_1/\beta_1 T_0$ (suppressing the primes for convenience) in the boundary conditions (23) and applying the transforms defined by Eq. (16) and substitute the values of $\hat{\phi}, \hat{\theta}, \hat{C}, \hat{\psi}$ from Eqs. (19)-(22) in the resulting equations, we obtain the expressions for components of displacement, stress, temperature distribution and chemical potential distribution as

$$\hat{u}_1 = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[(-i\xi)(\Delta_1 e^{-\lambda_1 z} - \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}) + \lambda_4\Delta_4 e^{-\lambda_4 z}]\}e^{i\omega t} \quad (28)$$

$$\hat{u}_3 = \frac{-1}{\Delta}\{P_1\hat{\psi}_1(\xi)[\lambda_1\Delta_1 e^{-\lambda_1 z} - \lambda_2\Delta_2 e^{-\lambda_2 z} + \lambda_3\Delta_3 e^{-\lambda_3 z} + i\xi\Delta_4 e^{-\lambda_4 z}]\}e^{i\omega t} \quad (29)$$

$$\hat{t}_{33} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[s_1\Delta_1 e^{-\lambda_1 z} - s_2\Delta_2 e^{-\lambda_2 z} + s_3\Delta_3 e^{-\lambda_3 z} + s_4\Delta_4 e^{-\lambda_4 z}]\}e^{i\omega t} \quad (30)$$

$$t_{31} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[\lambda_1\Delta_1 e^{-\lambda_1 z} - \lambda_2\Delta_2 e^{-\lambda_2 z} + \lambda_3\Delta_3 e^{-\lambda_3 z} - m_1\Delta_4 e^{-\lambda_4 z}]\}e^{i\omega t} \quad (31)$$

$$\hat{\theta} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[d_1\Delta_1 e^{-\lambda_1 z} - d_2\Delta_2 e^{-\lambda_2 z} + d_3\Delta_3 e^{-\lambda_3 z}]\}e^{i\omega t} \quad (32)$$

$$\hat{P} = \frac{1}{\Delta}\{P_1\hat{\psi}_1(\xi)[t_1\Delta_1 e^{-\lambda_1 z} - t_2\Delta_2 e^{-\lambda_2 z} + t_3\Delta_3 e^{-\lambda_3 z}]\}e^{i\omega t} \quad (33)$$

where

$$\Delta = \{-[s_4\lambda_1 + m_1 s_1](d_2t_3 - d_3t_2) + [s_4\lambda_2 + m_1 s_2](d_1t_3 - d_3t_1) - [s_4\lambda_3 + m_1 s_3](d_1t_2 - d_2t_1)\}$$

$$\Delta_1 = m_1(d_2t_3 - d_3t_2), \quad \Delta_2 = m_1(d_1t_3 - d_3t_1), \quad \Delta_3 = m_1(d_1t_2 - d_2t_1)$$

$$\Delta_4 = [\lambda_1(d_2t_3 - d_3t_2) - \lambda_2(d_1t_3 - d_3t_1) + \lambda_3(d_1t_2 - d_2t_1)]$$

$$s_l = b_1\lambda_l^2 - b_1(1 + \tau_1 i\omega)d_l - b_1(1 + \tau^1 i\omega)e_l - b_2 i\xi, \quad (l = 1, 2, 3)$$

$$s_4 = (i\xi b_1 + b_2)\lambda_4 \quad (34)$$

$$t_l = \xi^2 - \lambda_l^2 - \varepsilon_2(1 + \tau^1 i\omega)e_l - a_1(1 + \tau_1 i\omega)d_l$$

$$b_1 = \frac{(\lambda + 2\mu)}{\beta_1 T_0}, \quad b_2 = \frac{-i\xi\lambda}{\beta_1 T_0}, \quad m_1 = \frac{\lambda_4^2 + \xi^2}{2i\xi}$$

SUBCASE 2(a). Thermal source on the surface of half-space

With the help of the Eqs. (1), (2) and (9)-(11), along with $P_2' = \dfrac{\beta_1}{\rho c_1^2}P_2$ (suppressing the primes for convenience) and the boundary conditions (24), the corresponding expressions for components of displacement, stress, temperature distribution and chemical potential distribution are as given by Eqs. (28)-(33) with Δ_l replaced by $\Delta_l^*(l = 1, 2, 3, 4)$ and P_1 replaced by P_2, respectively, in Eq. (34), where

$$
\begin{aligned}
\Delta_1^* &= m_1(s_2t_3 - s_3t_2) + s_4(\lambda_2t_3 - \lambda_3t_2) \\
\Delta_2^* &= m_1(s_1t_3 - s_3t_1) + s_4(\lambda_1t_3 - \lambda_3t_1) \\
\Delta_3^* &= m_1(s_1t_2 - s_2t_1) + s_4(\lambda_1t_2 - \lambda_2t_1) \\
\Delta_4^* &= -[s_1(\lambda_2t_3 - \lambda_3t_2) - s_2(\lambda_1t_3 - \lambda_3t_1) + s_3(\lambda_1t_2 - \lambda_2t_1)]
\end{aligned}
\tag{35}
$$

SUBCASE 3(a). Chemical potential source on the surface of half-space

Adopting the same procedure as in case of mechanical force and thermal source, along with $P_3' = P_3/\beta_2$ (suppressing the primes for convenience)and using the boundary condition (25) the expressions for components of displacement, stress, temperature distribution and chemical potential distribution are as given by Eqs. (28)-(33) by replacing Δ_l with $\Delta_l^{**}(l = 1, 2, 3, 4)$ and P_1 with P_3, respectively, in Eqs. (34), where

$$
\begin{aligned}
\Delta_1^{**} &= -[m_1(s_2d_3 - s_3d_2) + s_4(\lambda_2d_3 - \lambda_3d_2)] \\
\Delta_2^{**} &= -[m_1(s_1d_3 - s_3d_1) + s_4(\lambda_1d_3 - \lambda_3d_1)] \\
\Delta_3^{**} &= -[m_1(s_1d_2 - s_2d_1) + s_4(\lambda_1d_2 - \lambda_2d_1)] \\
\Delta_4^{**} &= s_1(\lambda_2d_3 - \lambda_3d_2) - s_2(\lambda_1d_3 - \lambda_3d_1) + s_3(\lambda_1d_2 - \lambda_2d_1)
\end{aligned}
\tag{36}
$$

The expressions for displacements, stresses, temperature distribution and chemical potential distribution can obtained for concentrated normal force/thermal source/chemical potential source, by replacing $\hat{\psi}_1(\xi)$ from Eq. (27), respectively, in Eqs. (28)-(33) along with Eqs. (35) and (36).

4.2 Influence functions

The method to obtain the half-space Influence function i.e., the solutions due to distributed load applied on the half-space is obtained by setting

$$
\psi_1(x) = \begin{cases} 1 & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases} \tag{37}
$$

in Eqs. (23), (24) and (25). The Fourier transforms with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width 2a applied at origin of the coordinate system $(x = z = 0)$ in dimensionless form after suppressing the primes becomes

$$
\hat{\psi}_1(\xi) = \left[2\sin\left(\frac{\xi c_1 a}{\omega_1^*}\right)\Big/\xi\right], \qquad \xi \neq 0 \tag{38}
$$

The expressions for displacements, stresses, temperature distribution and chemical potential distribution can be obtained for uniformly distributed normal force/thermal source/chemical potential source by replacing $\hat{\psi}_1(\xi)$ from Eq. (38), respectively, in Eqs. (28)-(33) along with Eqs. (35) and (36).

5. Particular cases

5.1: Neglecting the diffusion effects (i.e., $\beta_2 = b = a = 0$), we obtain the corresponding expressions due to normal force for displacements, stresses, and temperature distribution and given by Eqs. (28)-(33) in generalized thermoelastic half-space as

$$\hat{u}_1 = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[(-i\xi)(-\Delta_1' e^{-\lambda_1 z} + \Delta_2' e^{-\lambda_2 z}) - \lambda_4\Delta_3' e^{-\lambda_4 z}]\}e^{i\omega t} \tag{39}$$

$$\hat{u}_3 = \frac{-1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-\lambda_1\Delta_1' e^{-\lambda_1 z} + \lambda_2\Delta_2' e^{-\lambda_2 z} - i\xi\Delta_3' e^{-\lambda_4 z}]\}e^{i\omega t} \tag{40}$$

$$\hat{t}_{33} = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-s_1^*\Delta_1' e^{-\lambda_1 z} + s_2^*\Delta_2' e^{-\lambda_2 z} - s_4^*\Delta_3' e^{-\lambda_4 z}]\}e^{i\omega t} \tag{41}$$

$$t_{31} = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-\lambda_1\Delta_1' e^{-\lambda_1 z} + \lambda_2\Delta_2' e^{-\lambda_2 z} + m_1\Delta_3' e^{-\lambda_4 z}]\}e^{i\omega t} \tag{42}$$

$$\hat{\theta} = \frac{1}{\Delta^*}\{P_1\hat{\psi}_1(\xi)[-d_1\Delta_1' e^{-\lambda_1 z} + d_2\Delta_2' e^{-\lambda_2 z}]\}e^{i\omega t} \tag{43}$$

where

$$\begin{aligned} \Delta^* &= [s_4(\lambda_1 d_2 - \lambda_2 d_1) + m_1(s_1^* d_2 - s_2^* d_1)] \\ \Delta_1' &= m_1 d_2, \quad \Delta_1' = m_1 d_1, \quad \Delta_3' = [\lambda_1 d_2 - \lambda_2 d_1] \\ s_l^* &= b_1\lambda_l^2 - b_1(1+\tau_1 i\omega)d_l - b_2 i\xi, \qquad (l = 1, 2) \\ s_4 &= (i\xi b_1 + b_2)\lambda_4 \end{aligned} \tag{44}$$

The above expressions yield the corresponding expressions for concentrated and uniformly distributed normal force by replacing $\hat{\psi}_1(\xi)$ from Eqs. (27) and (38) respectively in Eqs. (39)-(43).

5.2: Making use the values of $\hat{\psi}_1(\xi)$ from Eqs. (27) and (38) and by replacing Δ_l' with Δ_l'' ($l = 1, 2, 3$) as given below, we obtain the expressions for displacements, stresses and temperature distribution in thermoelastic medium due to concentrated and uniformly distributed thermal source, where

$$\Delta_1'' = m_1 s_2^* + s_4\lambda_2, \quad \Delta_2'' = m_1 s_1^* + s_4\lambda_1, \quad \Delta_3'' = s_2^*\lambda_1 - s_1^*\lambda_2 \tag{45}$$

6. Special case

In case of coupled thermoelasticity, the relaxation times vanish i.e., $\tau_0 = \tau^0 = \tau_1 = \tau^1 = 0$ and consequently, we obtain the corresponding expressions in thermoelastic with diffusion and thermoelasticity due normal force, thermal source and chemical potential source, respectively, with changed values in Eqs. (28)-(33) and Eqs. (39)-(43).

$$E = [-\varepsilon_2 + 1]$$

$$
\begin{aligned}
F &= [i\omega\varepsilon_2 a_2 + i\omega\varepsilon_2 + i\omega\varepsilon_1 a_1^2 - \omega^2\varepsilon_2 + i\omega\varepsilon_1\varepsilon_2 + i\omega 2\varepsilon_1 a_1 - i\omega] \\
G &= [\varepsilon_2 a_2\omega^2 + i\omega^3\varepsilon_2 a_2 + i\omega^3\varepsilon_2 + i\omega^3\varepsilon_1 a_1^2 + \varepsilon_1\varepsilon_2 a_2\omega^2] \\
H &= \omega^4\varepsilon_2 a_2 \\
P^* &= 1 + \frac{1}{a_1}, \qquad Q^* = -\left[\xi^2 - \omega^2 + \frac{\xi^2}{a_1}\right] \\
R^* &= \frac{1}{i\omega\varepsilon_1 a_1}, \qquad S^* = -\frac{\xi^2 + i\omega}{i\omega\varepsilon_1 a_1} + 1 \\
T^* &= -\{[a_1 + \varepsilon_2]\xi^2 + i\omega\varepsilon_2 a_2\}, \qquad X^* = a_1 + \varepsilon_2 \\
s_l^* &= b_1\lambda_l^2 - b_1 d_l - b_2 i\xi, \qquad (l = 1, 2) \\
s_l &= b_1\lambda_l^2 - b_1 d_l - b_1 e_l - b_2 i\xi, \qquad (l = 1, 2, 3) \\
s_4 &= (i\xi b_1 + b_2)\lambda_4 \\
t_l &= \xi^2 - \lambda_l^2 - \varepsilon_2 e_l - a_l d_l
\end{aligned}
\tag{46}
$$

7. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we invert the transforms in Eqs. (28)-(33) and (39)-(43), for the two theories, i.e., G-L and C-T theories of thermoelasticity. These expressions are functions of z and the parameter of Fourier transform ξ, and hence are of the form $\hat{f}(\xi, z)$. To obtain the function $f(x, z)$ in the physical domain, we invert the Fourier transform using,

$$f(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\hat{f}(\xi, z)d\xi = \frac{1}{\pi}\int_0^{\infty}(\cos(\xi x)f_e - i\sin(\xi x)f_0)d\xi \tag{47}$$

where f_e and f_0 are, respectively, the even and odd parts of the function $\hat{f}(\xi, z)$. The method for evaluating this integral is described by Press *et al.* (1986), which involves the use of the Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

Following Sherief and Saleh (2005) copper material is chosen for the purpose of numerical calculation.

$$T_0 = 293K,\ \rho = 8954\ kgm^{-3},\ \tau_0 = 0.02s,\ \tau^0 = 0.2s,\ C_E = 383.1\ Jkg^{-1}K^{-1}$$
$$\alpha_t = 1.78(10)^{-5}K^{-1},\ \alpha_c = 1.98(10)^{-4}m^3kg^{-1},\ K = 386\,Wm^{-1}K^{-1}$$
$$\lambda = 7.76(10)^{10}kgm^{-1}s^{-2},\ \mu = 3.86(10)^{10}kgm^{-1}s^{-2}, D = 0.85(10)^{-8}kgssm^{-3}$$

$$a = 1.2(10)^4 m^2 s^{-2} K^{-1},\ b = 0.9(10)^6 m^5 kg^{-1} s^{-2}$$

The values of the relaxation times τ_0, τ^0 have been taken from Sherief and Saleh (2005) and the values of τ_1, τ^1 are taken proportionally of comparable magnitude as $\tau_1 = 0.03$, $\tau^1 = 0.3$.

The values of normal displacement u_3, normal stress t_{33}, temperature distribution θ and chemical potential distribution P for thermoelastic diffusion (TED) and thermoelasticity (TE) are studied for normal force/thermal source/chemical potential source. The variations of the components with distance x are shown (a) solid line for TED and solid line with center symbol 'plus' for TE for G-L theory (b) small dashed line for TED and small dashed line with center symbol 'Dimond' for TE for C-T theory. The variations are shown in Figs. 1-24. The computations are carried out for non-dimensional frequency $\omega = 0.75$ and time $t = 0.5$ in the range $0 \le x \le 10$.

8.1 Normal force on the surface of half-space

8.1.1 Concentrated force

Fig. 1 shows the variations of normal displacement u_3 with distance x, which for TED increase sharply in the range $0 \le x \le 3, 5 \le x \le 7$ and decrease in the remaining range for C-T theory whereas for G-L theory the values of normal displacement u_3 have an oscillatory behavior in the range $0 \le x \le 10$. The values of normal displacement u_3 for TE have an oscillatory behavior in the whole range for both G-L and C-T theories.

Fig. 2 depicts the values of normal stress t_{33} with distance x, which for TED have similar behavior in the range $0 \le x \le 10$ for both G-L and C-T theories. The values of normal stress t_{33} for TE increase sharply in the range $0 \le x \le 3$ and then have an oscillatory behavior in the remaining range for G-L theory whereas for C-T theory the values of normal stress t_{33} show small variation near the zero value.

Fig. 3 shows the values of temperature distribution θ with distance x, which for TED have similar

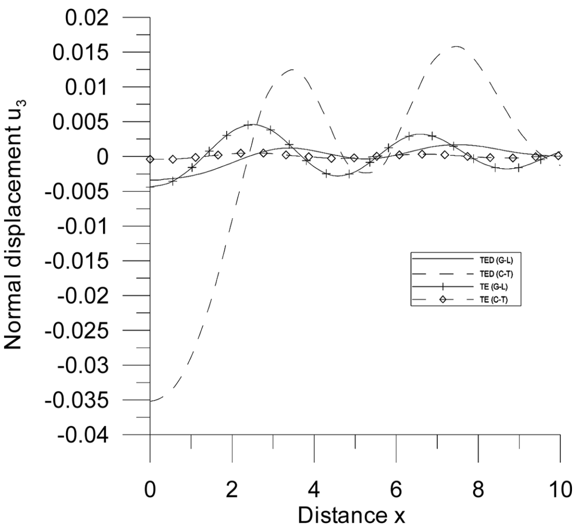

Fig. 1 Variation of normal displacement u_3 with distance x (Concentrated force)

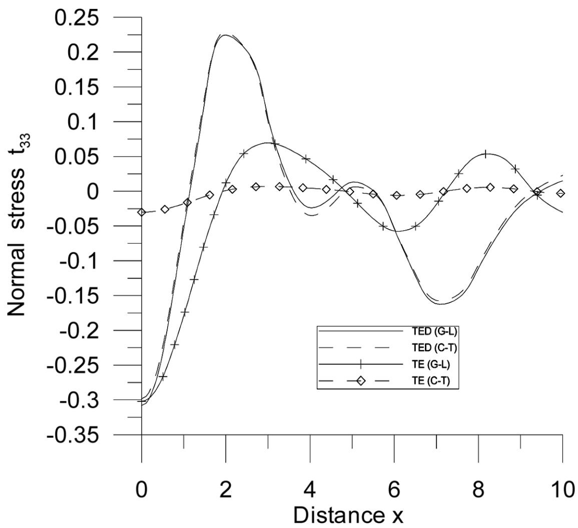

Fig. 2 Variation of normal stress t_{33} with distance x (Concentrated force)

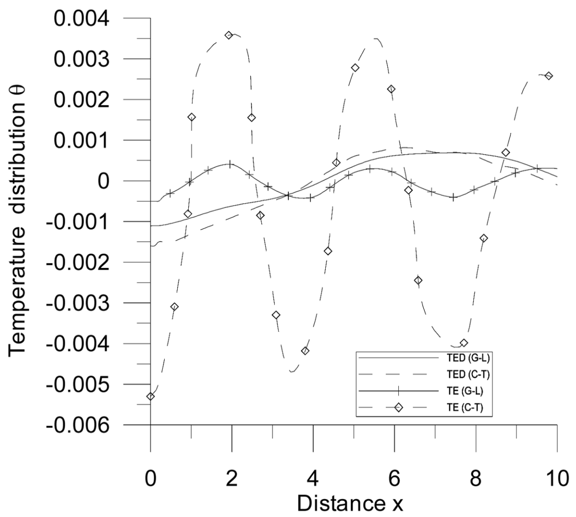

Fig. 3 Variation of temperature distribution θ with distance x (Concentrated force)

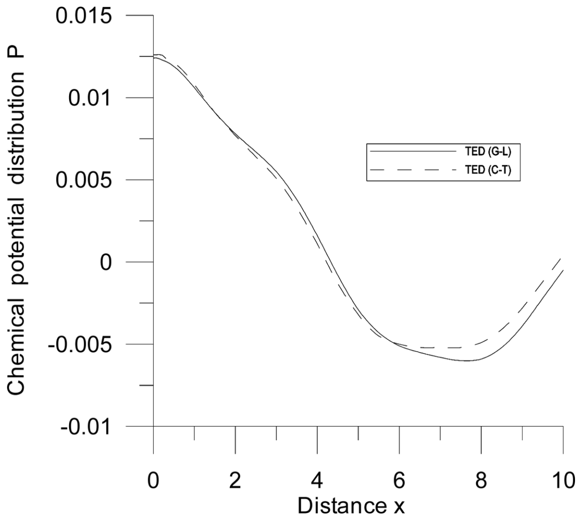

Fig. 4 Variation of chemical potential distribution P with distance x (Concentrated force)

behavior in the range $0 \le x \le 10$ for both G-L and C-T theories. The variation of temperature distribution θ for TE has similar behavior in the range $0 \le x \le 10$ for both G-L and C-T theories with difference in their magnitude. The values of temperature distribution θ for TE are magnified by multiplying by 10 for C-T theory.

Fig. 4 shows the values of chemical potential distribution P with distance x, which for TED decrease sharply in the range $0 \le x \le 6$ and then increase with increase in horizontal distance x for both G-L and C-T theories.

8.1.2 Uniformly distributed force

Fig. 5 shows the variations of normal displacement u_3 with distance x, which for TED show small variation near zero value in the whole range for G-L theory whereas for C-T theory the values of normal displacement u_3 increase sharply in the range $0 \le x \le 3$ and have an oscillatory behavior in the remaining range. The values of normal displacement u_3 for TE have an oscillatory behavior in the range $0 \le x \le 10$ for G-L theory whereas for C-T theory the values of normal displacement u_3 show small variation near the zero value.

Fig. 6 depicts the values of normal stress t_{33} with distance x, which for TED have similar behavior i.e. increase sharply in the range $0 \le x \le 1.5$, $7 \le x \le 10$ and decrease in the remaining range for both G-L and C-T theories. The values of normal stress t_{33} for TE increase sharply in the range $0 \le x \le 3$ and then have an oscillatory behavior in the remaining range for G-L theory whereas for C-T theory the values of normal stress t_{33} show small variation near the zero value.

Fig. 7 shows the variations of temperature distribution θ with distance x, which for TED in case of G-L theory are greater than in case of C-T theory in the range $0 \le x \le 10, 7 \le x \le 10$ and are smaller in the remaining range. The values of temperature distribution θ for TE have an oscillatory behavior in the range $0 \le x \le 10$ for both G-L and C-T theories.

Fig. 8 shows the values of chemical potential distribution P with distance x, which for TED decrease sharply in the range $0 \le x \le 6$ and increase in the remaining range for G-L and C-T theories.

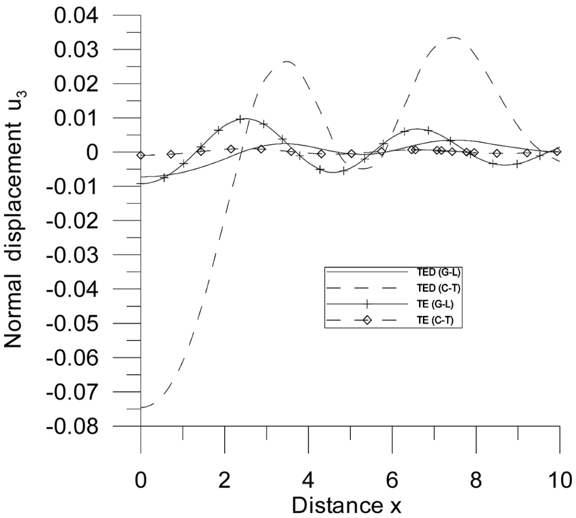

Fig. 5 Variation of normal displacement u_3 with distance x (Uniformly distributed force)

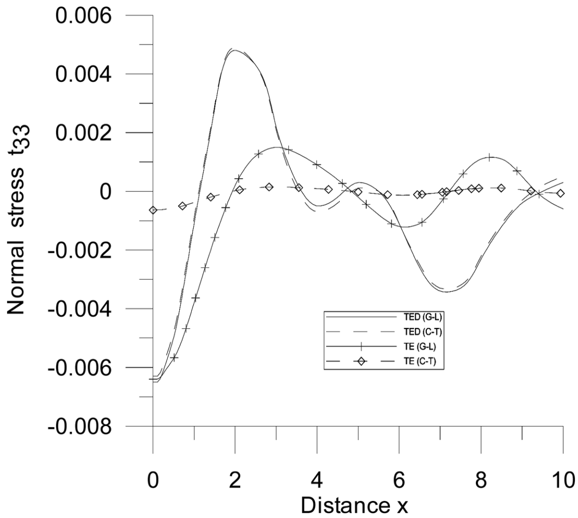

Fig. 6 Variation of normal stress t_{33} with distance x (Uniformly distributed force)

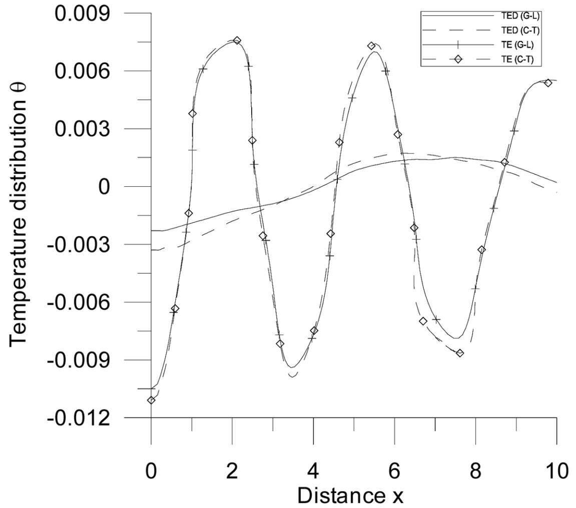

Fig. 7 Variation of temperature distribution θ with distance x (Uniformly distributed force)

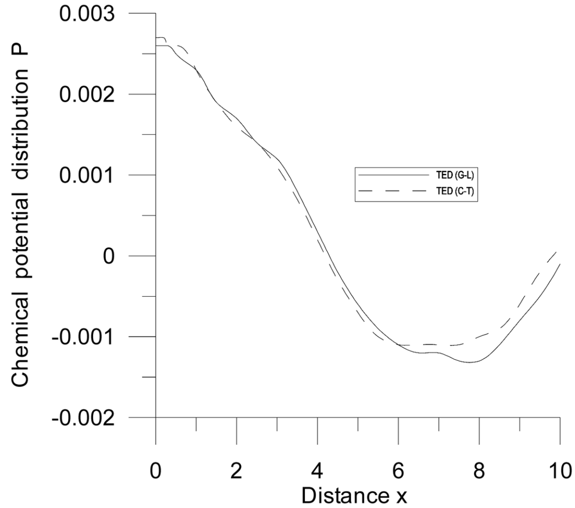

Fig. 8 Variation of chemical potential distribution P with distance x (Uniformly distributed force)

8.2 Thermal source on the surface of half-space

8.2.1 Concentrated thermal source

Fig. 9 shows the variations of normal displacement u_3 with distance x, which for TED decrease sharply in the range $0 \le x \le 2, 4 \le x \le 6, 8 \le x \le 10$ and increase sharply in the remaining range for both G-L and C-T theories. The values of normal displacement u_3 for TE show small variation near zero value in the whole range for G-L theory whereas for C-T theory it has an oscillatory behavior in the range $0 \le x \le 10$.

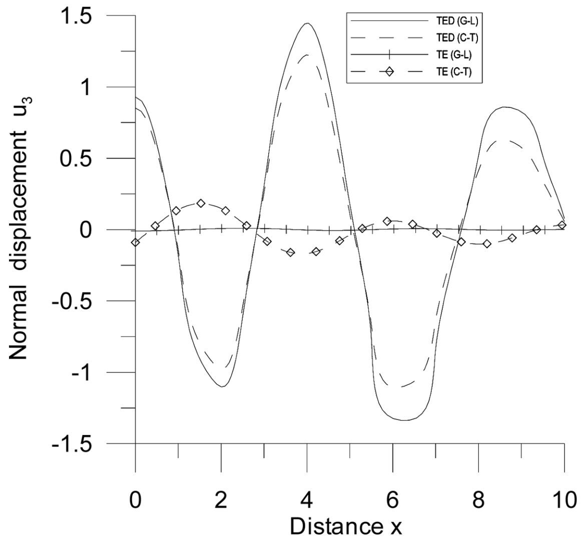

Fig. 9 Variation of normal displacement u_3 with distance x (Concentrated thermal source)

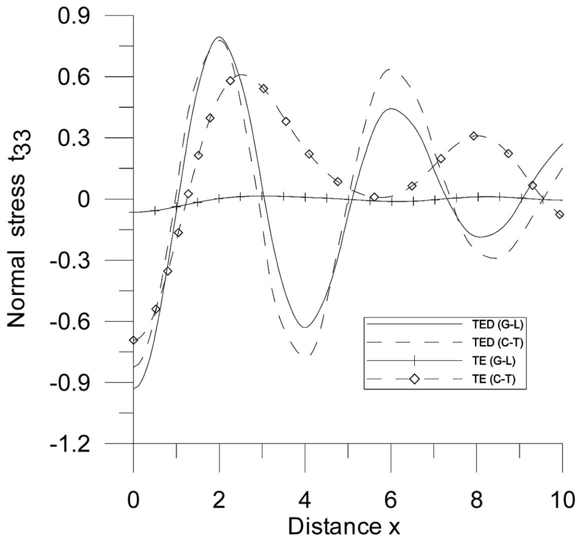

Fig. 10 Variation of normal stress t_{33} with distance x (Concentrated thermal source)

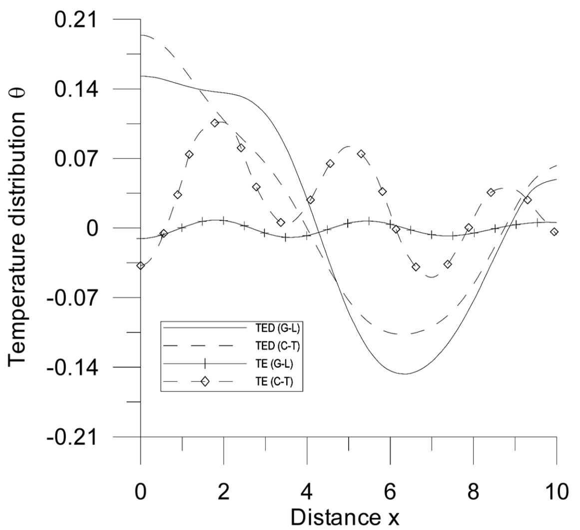

Fig. 11 Variation of temperature distribution θ with distance x (Concentrated thermal source)

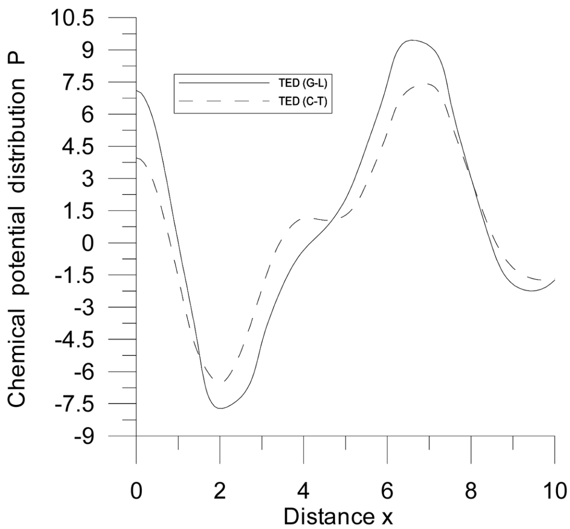

Fig. 12 Variation of chemical potential distribution P with distance x (Concentrated thermal source)

Fig. 10 shows the variations of normal stress t_{33} with distance x, which for TED increase in the range $0 \le x \le 2, 4 \le x \le 6, 8 \le x \le 10$ and decrease sharply in the remaining range (i.e., it has an opposite behavior to normal displacement of thermal source). The values of normal stress t_{33} for TE shows small variation about zero value in the range $0 \le x \le 10$ for G-L theory whereas for C-T theory it increase sharply in the range $0 \le x \le 2$ and then has an oscillatory behavior in the remaining range.

Fig. 11 shows the variations of temperature distribution θ with distance x, which for TED in case of G-L theory is smaller than C-T theory in the range $0 \le x \le 1.8, 4.5 \le x \le 10$ and is greater in the remaining range. The values of temperature distribution θ for TE have an oscillatory behavior in the range $0 \le x \le 10$ for both G-L and C-T theories.

Fig. 12 shows the variations of chemical potential distribution P with distance x, which for TED decrease sharply in the range similar $0 \le x \le 2, 7 \le x \le 10$ and increase in the remaining range for both G-L and C-T theories.

8.2.2 Uniformly distributed thermal source

Fig. 13 shows the variations of normal displacement u_3 with distance x, which for both TED and TE has an oscillatory behavior in the whole range for both G-L and C-T theories with difference in their magnitude.

Fig. 14 shows the variations of normal stress t_{33} with distance x, which for TED increase sharply in the range $0 \le x \le 2, 4 \le x \le 6, 8 \le x \le 10$ and decrease sharply in the remaining range for G-L theory whereas for C-T theory the values of normal stress t_{33} has an oscillatory behavior in the range $0 \le x \le 10$. The variations of normal stress t_{33} for TE increase sharply in the range $0 \le x \le 2$ and then has an oscillatory behavior for G-L theory whereas for C-T theory the values of normal stress t_{33} shows small variation near zero value.

Fig. 15 shows the variations of temperature distribution θ with distance x, which for TED in case of G-L theory is greater than C-T theory in the range $0 \le x \le 4$ and is smaller in the remaining range whereas for TE the values of temperature distribution θ have similar behavior (i.e., oscillatory behavior) for both G-L and C-T theories in the range $0 \le x \le 10$.

Fig. 16 shows the variations of chemical potential distribution P with distance x, which for TED increase sharply in the range $2 \le x \le 7$ and decrease in the remaining range for both G-L and C-T theories.

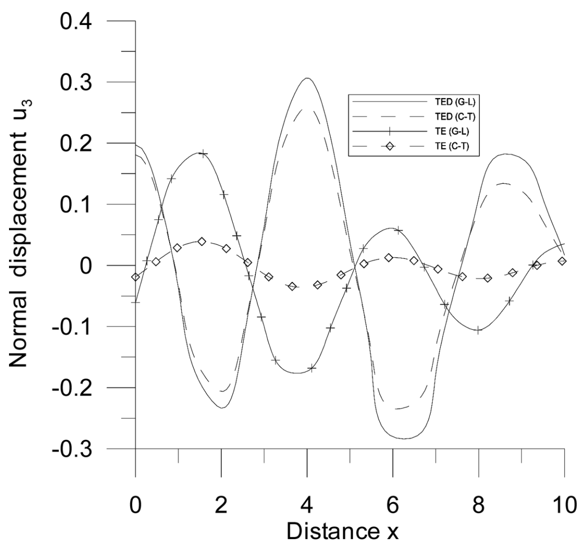

Fig. 13 Variation of normal displacement u_3 with distance x (Uniformly distributed thermal source)

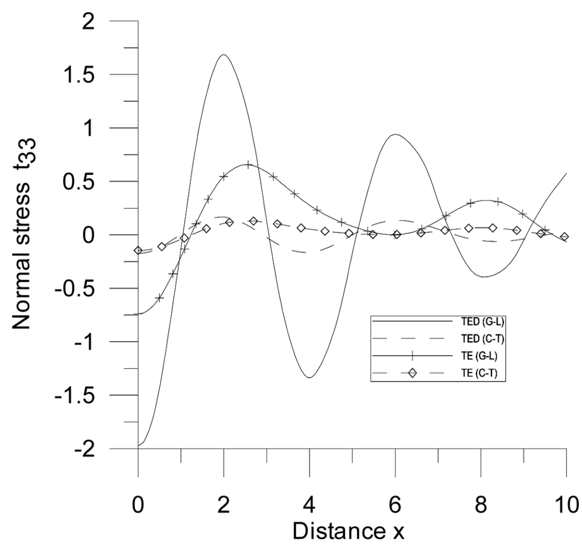

Fig. 14 Variation of normal stress t_{33} with distance x (Uniformly distributed thermal source)

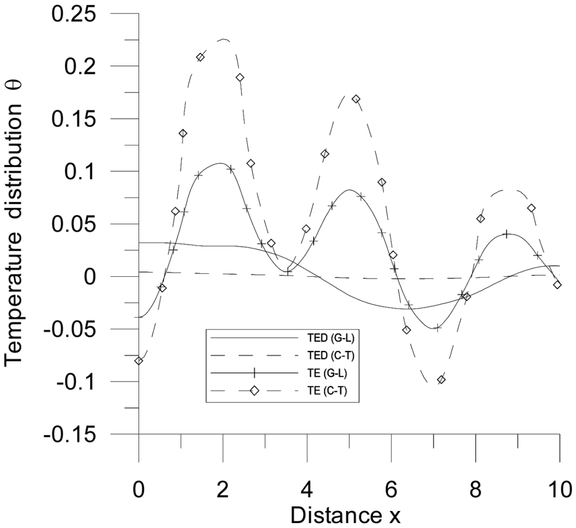

Fig. 15 Variation of temperature distribution θ with distance x (Uniformly distributed thermal source)

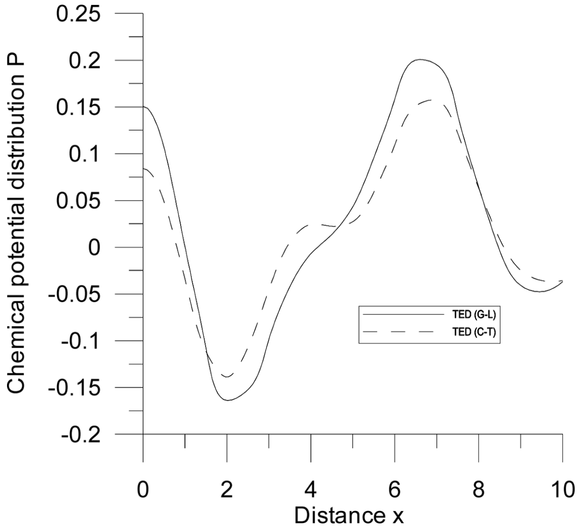

Fig. 16 Variation of chemical potential distribution P with distance x (Uniformly distributed thermal source)

8.3 Chemical potential source on the surface of half-space

8.3.1 Concentrated chemical potential source

Fig. 17 shows the variations of normal displacement u_3 with distance x, which for both TED has similar behavior (i.e., oscillatory behavior) for both G-L and C-T theories in the range $0 \le x \le 10$.

Fig. 18 shows the variations of normal stress t_{33} with distance x, which for TED increase in the

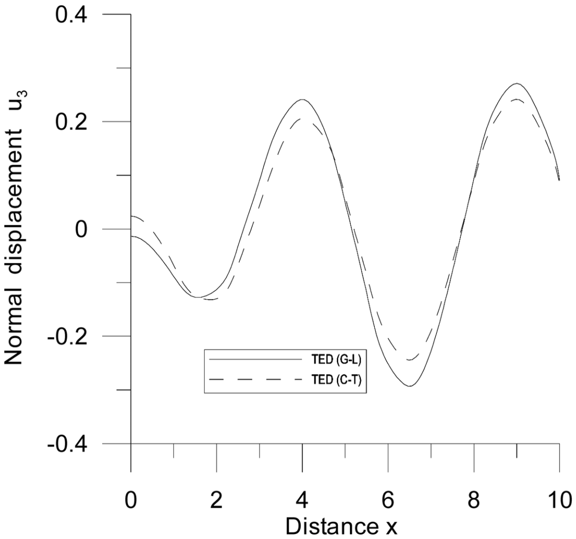

Fig. 17 Variation of normal displacement u_3 with distance x (Concentrated chemical potential source)

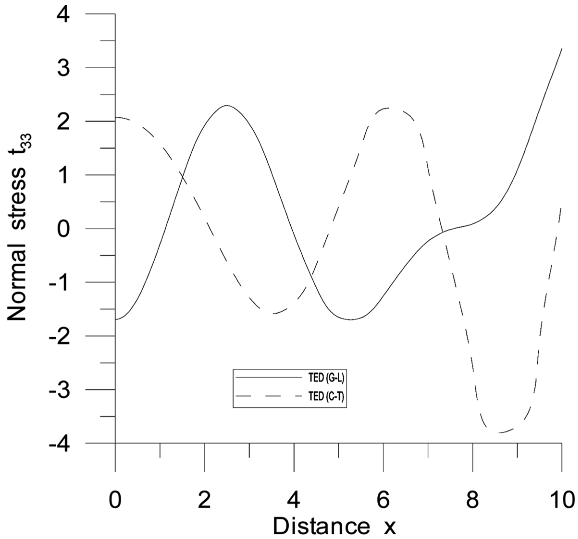

Fig. 18 Variation of normal stress t_{33} with distance x (Concentrated chemical potential source)

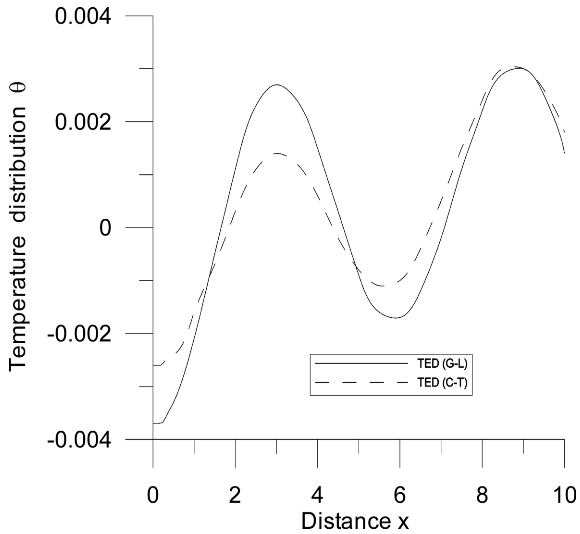

Fig. 19 Variation of temperature distribution θ with distance x (Concentrated chemical potential source)

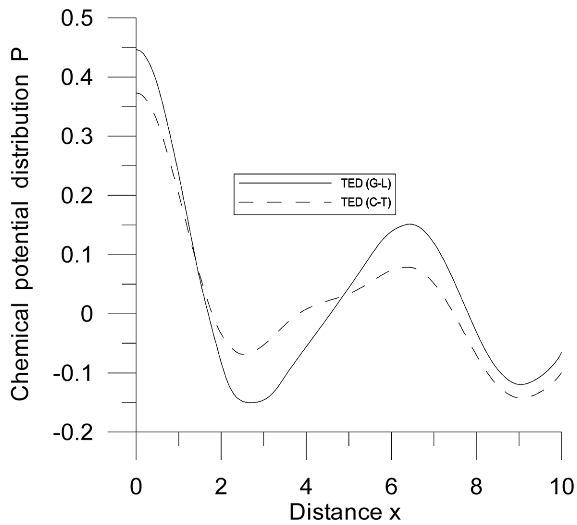

Fig. 20 Variation of chemical potential distribution P with distance x (Concentrated chemical potential source)

range $0 \le x \le 2, 5 \le x \le 10$ and decrease in the remaining for G-L theory whereas for C-T theory the values of normal stress t_{33} decrease in the range $0 \le x \le 3, 6 \le x \le 8$ and increase in the remaining range.

Fig. 19 shows the variations of temperature distribution θ with distance x, which for TED in case of G-L theory is smaller than C-T theory in the range $0 \le x \le 1.2, 5 \le x \le 10$ and is greater in the remaining range.

Fig. 20 shows the variations of chemical potential distribution P with distance x, which for TED decrease sharply in the range $0 \le x \le 2, 7 \le x \le 10$ and increase in the remaining range for both G-L and C-T theories.

8.3.2 Uniformly distributed chemical potential source

Fig. 21 shows the variations of normal displacement u_3 with distance x, which for both TED in case of G-L theory is greater than C-T theory in the range $2 \le x \le 4, 8 \le x \le 10$ and is smaller in the remaining range.

Fig. 22 shows the variations of normal stress t_{33} with distance x, which for TED has an oscillatory behavior in the range $0 \le x \le 10$ for C-T theory .The values of normal stress t_{33} for G-L theory increase in the range $0 \le x \le 2, 5 \le x \le 10$ and decrease in the remaining range.

Fig. 23 shows the variations of temperature distribution θ with distance x, which for TED increase sharply in the range $0 \le x \le 3, 6 \le x \le 9$ decrease sharply in the remaining range for both G-L and C-T theories.

Fig. 24 shows the variations of chemical potential distribution P with distance x, which for TED decrease sharply in the range $0 \le x \le 2.3$ and then has an oscillatory behavior for both G-L and C-T theories.

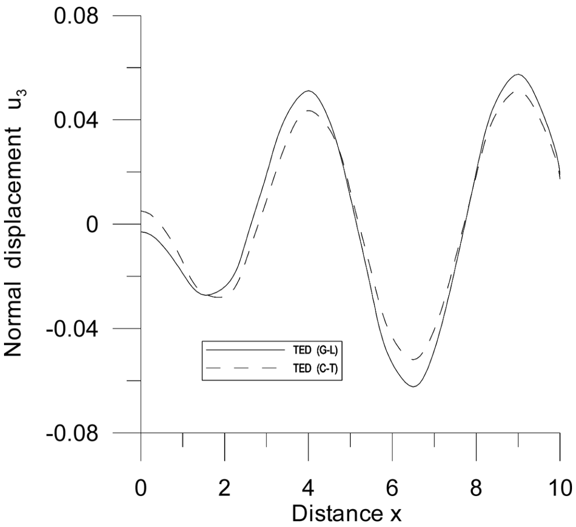

Fig. 21 Variation of normal displacement u_3 with distance x (Uniformly distributed chemical potential source)

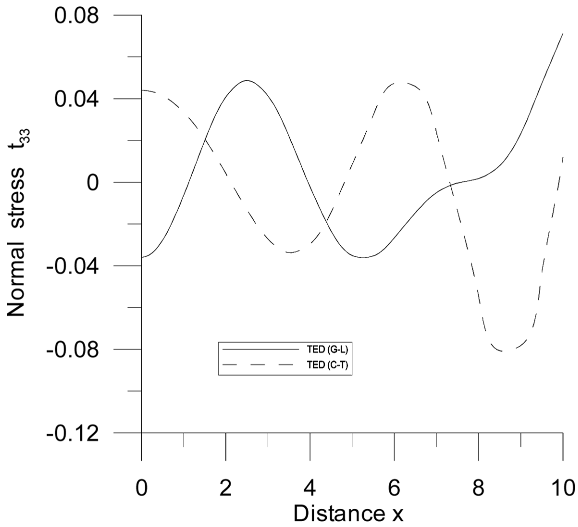

Fig. 22 Variation of normal stress t_{33} with distance x (Uniformly distributed chemical potential source)

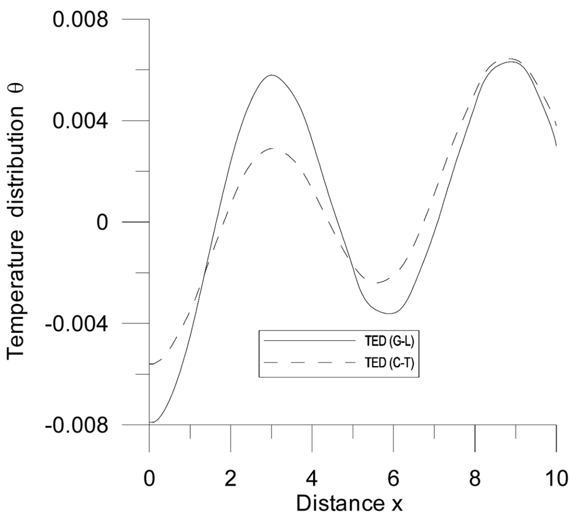

Fig. 23 Variation of temperature distribution θ with distance x (Uniformly distributed chemical potential source)

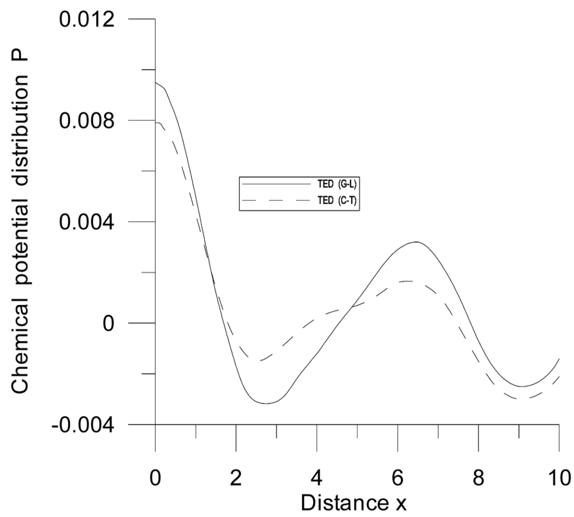

Fig. 24 Variation of chemical potential distribution P with distance x (Uniformly distributed chemical potential source)

9. Conclusions

Effect of diffusion plays important role in the deformation of the body. As disturbances travel through different constituents of the medium, it suffers sudden changes, resulting in an inconsistent/non-uniform pattern of curves. It is observed from the figures that the diffusivity effects the disturbances produced due to concentrated and distributed loads. The trend of curves exhibits the properties of thermo-diffusivity of the medium and satisfies the requisite condition of the problem. It is observed that the values of normal displacement u_3, normal stress t_{33}, temperature distribution θ and chemical potential distribution P in case of concentrated and uniformly distributed normal force and chemical potential source have almost similar behavior with difference in their magnitude. The results of this problem are very useful in the two dimensional problem of dynamic response due to various sources of the thermoelastic diffusion which has the various geophysical and industrial application. Study of phenomenon of thermoelastic diffusion is used to improve the conditions of oil extraction.

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