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Reliability considerations in bridge pier scouring

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Abstract. The conventional design of bridge piers against scour uses scour equations which involve number of uncertain *flow*, *sediments* and *structural* parameters. The inherent high uncertainties in these parameters suggest that the reliability of piers must be assessed to ensure desirable safety of bridges against scour. In the present study, a procedure for the reliability assessment of bridge piers, installed in main and flood channels, against scour has been presented. To study the influence of various random variables on piers' reliability sensitivity analysis has been carried out. To incorporate the reliability in the evaluation of safety factor, a simplified relationship between safety factor and reliability index has been proposed. Effects of clear water (flood channel) and live bed scour (main channel) are highlighted on pier reliability. In addition to these, an attempt has also been made to explain the failure of Black mount bridge of New Zealand based on its pier's reliability analysis. Some parametric studies have also been included to obtain the results of practical interest.

Keywords: piers; scouring; bridges; reliability; FORM; Monte Carlo Simulation.

1. Introduction

Bridge pier scouring is an important issue in the safety evaluation of bridges. Huber (1991) reported that since 1950 over 500 bridges in USA have failed and the majority of the failures were related to the scour of foundation material. The Scholarie Creek Bridge failure in New York State of USA in 1987 killed 10 people. Following this accident, the Federal Highway Administration (FHWA), USA mandated that all state highway agencies should evaluate the existing and proposed bridges for susceptibility to scour related failure. The failure of Black mount bridge of New Zealand was also experienced due to the undermining of its piers in a riverbed (Coleman and Melville 2001). A pier is said to be failed against scour if the maximum possible life time scour depth exceeds the depth of pier foundation. As there is a high degree of uncertainties involved in the estimation of maximum possible life time scour depth the reliability study of piers against scour has paramount importance in safe and economic design of foundation of bridge piers.

In general, scour phenomenon is extremely complex in nature and consequently in the past many investigators have attempted to develop conservative, analytical, semi-empirical or empirical equations based on the understanding of mechanics of local and general scouring; dimensional analysis and data correlation of laboratory experiments and/or field observations (Breusers *et al.* 1977, Muzzammil

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1992, Melville 1997, Coleman and Melville 2001, Muzzammil and Gangadharaiah 2003, Yanmaz, 2003, Kothyari 2003). These scour equations have a considerable uncertainty due to involvement of number of uncertain *flow*, *sediments* and *structural* parameters. The inherent high uncertainties in these parameters suggest that the pier reliability, in a quantitative sense, must be studied to ensure a desirable level of safety for bridges against scour. Perhaps it is due to this reason that in the recent past some excellent papers, though very limited in numbers, appeared on reliability estimation of bridge piers against scour. The present study is also an effort in the same direction. Johnson (1992) carried out the reliability analysis of bridge piers against scouring using the Colorado State University scour equations in a modified form. Chang *et al.* (1994) also carried out a similar study but considering correlation among various random variables. They, however, observed that the effect of correlation is not significant on reliability estimation. Johnson and Ayyub (1992) presented a method to assess the risk of bridge failure due to pier scour during the life of the bridge. Ghosn and Johnson (2000) presented a reliability model for the analysis of bridges under the combined effect of scour and earthquakes both. They employed modified Ferry-Borges algorithm for this purpose.

A detailed review of limited available literature on reliability of bridge piers against scour shows that investigators have generally used HEC-18 (Richardson *et al.* 1993) equation or, equations similar to HEC-18 for the reliability analysis. However, reliability assessment based on Melville's equations (Melville 1997, Melville and Coleman 2000, Coleman and Melville 2001) is not seen in the literature, though, Melville's equations also find a high acceptability in the prediction of scour depth. It is also observed that investigators have not given a due consideration to the effect of channel nature (i.e. whether a channel is main channel or flood channel), although it is possible that a pier which is installed in main channel may have considerable difference in reliability if the same is installed in flood channel. In addition, a comprehensive sensitivity analysis, which can clearly indicate the relative influence of various random variables on pier's reliability, is also not seen widely. Effect of debris on pier's reliability is also missing in the reliability analysis of earlier investigators.

Keeping the above scope in view, in the present study, a methodology for the reliability assessment of bridge piers, installed in main and flood channels, against scour has been presented using Melville scour model. Melville's model was selected for the reliability study because in view of Melville (Melville 1997) many of the scour depth prediction equations including HEC-18 equation do not distinguish correctly between clear water and live bed scour and consequently it results in a strong dependence on the velocity/Froude number in live-bed scour relations. Most of the model study results reported in literature indicates that the scour depth is approximately proportional to flow velocity under clear water conditions but is largely independent of flow velocity under live bed conditions (Melville 1997). Moreover, Melville (1997) scour equations are based on the results from a comprehensive program of bridge scour research undertaken at the University of Auckland, New Zealand over a period of 25 years, and are also consistent with the earlier available design methods in New Zealand. Having decided the model for scour depth a limit state function is derived in terms of various governing random and deterministic parameters. This governing equation considers factors accounting the effects of non-uniformity of the pier shape, non-uniform sediment, pier alignment, floating debris, time factor etc in addition to the effect of pier width, flow depth, flow velocity and sediment size. First Order Reliability Method (FORM) and Monte Carlo simulation techniques have then been employed for reliability analyses. To study the influence of various random variables on pier reliability against scouring sensitivity analysis has been carried out. To achieve desirable safety level in the design of piers *reliability based safety factors* have also been proposed. Effects of clear water (flood channel) and live bed scour (main channel) are highlighted on pier reliability. Some parametric studies have also been carried out to obtain the results of practical interest.

2. Problem formulation

The reliability assessment of any bridge pier is concerned with the calculation and prediction of its probability of limit state violation at any stage during its entire life. In the present study, limit state violation is the exceeding the maximum local scour depth from depth of pier foundation. A limit state function is a mathematical representation of a particular limit state of failure. This function assumes a negative or zero value at failure and a positive value for safety. Thus we can define the probability of limit state violation (i.e., probability of failure) as

$$P_f = P[g(\underline{x}) \le 0] \tag{1}$$

Where, $g(\underline{x})$ is the limit state function and $\underline{x}$ is the vector of basic random variables.

Using above points in view, if founded depth of pier is d_p and maximum local scour depth is d_s then limit state function may be written as

$$g(x) = d_p - d_s \tag{2}$$

From above equation it is obvious that failure of bridge pier will occur if d_s is equal or greater than d_p i.e., $g(\underline{x})$ assumes a negative or zero value.

Eq. (2) shows that derivation of limit state function requires an expression for maximum local depth of scour d_s. In the present study local pier scour depth d_s, below the surrounding bed level, is estimated using the formulation of Melville (1997) and Coleman and Melville (2001). According to these investigators the local pier scour depth can be represented in a most generalized form as

$$d_s = K_{yb} K_i K_d K_s K_\theta K_g K_t \tag{3}$$

Where, K_{yb} = flow depth pier size factor; K_i = flow intensity factor; K_d = sediment size factor; K_s = foundation shape factor; K_θ = foundation alignment factor; K_g = approach channel geometry factor; and K_t = time factor.

These factors are basically adjustment factors for the governing parameters of the scour at bridge piers and are briefly presented in the Appendix in a modified form. The modified form is very much suitable for computer applications and reliability calculations.

Substituting expressions for K_{yb}, K_i, K_d, and K_t (from Appendix Equations A1, A6, A11 and A18 respectively) in Eq. (3) and then in Eq. (2), we get the following equation for limit state function $g(\underline{x})$.

$$g(\underline{x}) = dp - (2.4 b_e c_{11} + 2\sqrt{y b_e} c_{12} + 4.5 y c_{13}) \times \left\{ c_{21} * \frac{v - (v_a - v_c)}{v_c} + c_{22} \right\} \times$$

$$\left\{ c_{31} 0.57 \log\left(2.24 \frac{b_e}{d_{50a}}\right) + c_{32} \right\} \times K_s \times K_\theta \times K_g \times \left\{ c_{51} + c_{52} \times \exp\left\{ -0.03 \left| \frac{v_c}{v} \ln \frac{t}{t_e} \right|^{1.6} \right\} \right\} \tag{4}$$

where $\underline{x} = \{y, v, d_{50a}, be, K_s, K_\theta, K_g, d_p\}$ (4a) where, $\underline{x}$ = vector of random variables; d_p = depth of pier; b_e = equivalent width of the pier; y = depth of flow; v = mean velocity of flow; v_a= mean velocity of flow at armour peak; v_c = critical mean velocity; d_{50a} = median particle size of armour layer (for uniform sediments d_{50a}= d_{50}); d_{50} = median size of bed material; l = pier length; θ= foundation alignment with respect to flow direction; c_{ij} = coefficients which assume either value zero or one as described in the appendix.; t = flood peak duration; and t_e = time for equilibrium scour depth to develop (days).

3. Reliability assessment

Having known the limit state function the next step is reliability assessment of bridge pier against scouring. For this purpose, we have employed two reliability techniques, known as First Order Reliability Method (FORM) and Monte Carlo Simulation method (Nowak and Collins 2000). A brief description of these two methods is presented in the following sections.

3.1 First order reliability method (FORM)

In brief, in this approach of reliability estimation, the reliability is measured in terms of a reliability index, β, and it is related to the probability of failure or probability of limit state violation for any limit state as

$$\beta = -\Phi^{-1}(P_f) \tag{5}$$

Where P_f is the probability of failure and Φ^{-1} ( ) is the inverse of standard normal distribution function. The reliability index β is found from the solution of the constrained optimisation problem

$$\text{Minimize} \quad \beta(\mathbf{z}) = (\mathbf{z}^T\mathbf{z})^{1/2} \qquad \text{subject to } G(z) = 0 \tag{6}$$

where $\mathbf{z}$ is a vector of basic random variables in the standard normal space and $G(\mathbf{z})$ is the limit state function in the standard normal space.

3.2 Monte carlo simulation method

Monte Carlo simulation consists of drawing samples of the basic variables according to their probabilistic characteristics and then feeding them into the limit state function. It is known that failure occurs when $g(.) < 0$; therefore an estimate of the probability of failure P_f can be found by

$$P_f = \frac{N_f}{N} \tag{7}$$

Where N_f is the numbers of simulation cycles in which $g(.) < 0$, and N is the total number of simulation cycles. As N approaches infinity, the P_f approaches to the true probability of failure. The accuracy of Eq. (7) can be evaluated in terms of its variance. For a small number of simulation cycles, the variance of P_f can be quite large. Consequently, it may require a large number of simulation cycles to achieve a specified accuracy. The variance of the estimated probability of failure can be computed by assuming each simulation cycle to constitute a Bernoulli trial. Therefore, the number of failures in N trials can be considered to follow a binomial distribution. Then the variance of the estimated probability of failure can be computed approximately as

$$Var(P_f) = \frac{(1-P_f)P_f}{N} \tag{8}$$

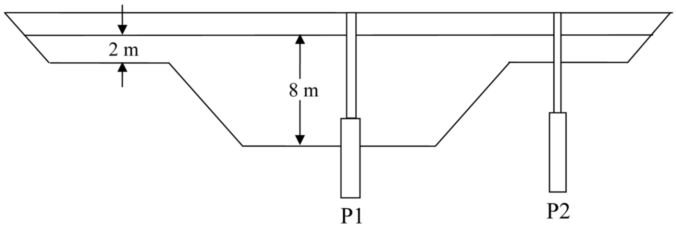
It is recommended to measure the statistical accuracy of the estimated probability of failure by computing its coefficient of variation as

$$COV(P_f) \cong \frac{\sqrt{\frac{(1-P_f)P_f}{N}}}{P_f} \tag{9}$$

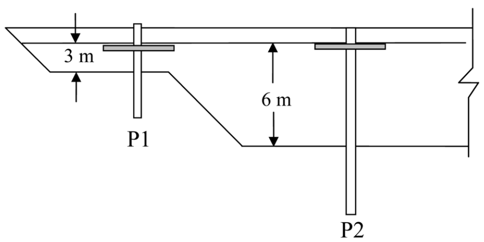
The smaller the coefficient of variation, the better is the accuracy of the estimated probability of failure. It is evident from Eqs. (8) and (9) that as N approaches infinity, *Var* (P_f) and *COV* (P_f) approaches zero. However, for all practical purposes, that number of simulation cycles for which *COV* (P_f) approaches less than 5% may be considered as appropriate number of simulation cycles.

4. Numerical study

To carry out the reliability analysis of bridge piers against local scour, deterministic and statistical data of bridge pier itself; flow properties of water; geometric properties of channel; and properties of sediment particles are required. Melville (1997) considered the typical numerical examples of piers situated in a compound channel, one pier in flood channel and other in the main channel, to highlight two important and practical cases of bridge pier scour. The pier in the flood channel is a case of clear water scour whereas the pier in the main channel represents a case of live bed scour. In the present study these numerical examples (Fig. 1) for scour evaluation are also considered for reliability analysis to have a wider coverage of the bridge foundation. Various statistical data that



(a) Example 1



(b) Example 2

Fig. 1 A sketch of the example compound channel

Table 1 Data for numerical study

General			Case1	Case 2
Cannel type			Main	Flood
Pier type			Non uniform	Uniform
Sediment type			Uniform	Uniform
Debris present			No	No

Statistical distribution and values

Parameter	Reference	Distribution	COV	Mean (Main Channel)	Mean (Flood Channel)
Flow depth (y)	Johnson	Normal	0.23	8.0 m	2.0 m
Mean flow velocity (v)	Assumed	Normal	0.329	1.0 m/s	0.4 m/s
Median size of sand (d_{50})	Johnson and Ayyub	Uniform	0.05	1.0 mm	1.0 mm
Pier Width (b)	Assumed	Normal	0.05	2.0 m	2.0 m
Shape factor (K_s)	Johnson and Ayyub	Normal	0.15	1.0	1.0
Alignment factor (K_θ)	Johnson and Ayyub	Normal	0.10	1.5	1.5
Channel geometry factor (K_g)	Assumed	Normal	0.10	1.0	1.0
Pier length (l)		Deterministic	-	8.0 m	9.0 m
Pile cap width (b_*)		Deterministic	-	3.0 m	3.0 m
Distance of pile cap (y_z)		Deterministic	-	1.0 m	1.0 m
Depth of pier (d_p)	Assumed	Normal	0.10	12.0 m	7.0 m
Foundation alignment (θ)		Deterministic	-	15^0	15^0

are needed for reliability analysis of the numerical example are shown in Table 1. The mean values of these data are same as nominal values given in Melville (1997). Other statistical parameters such as probability distributions and coefficient of variation (COV) are taken either from other references or assumed (if not found in approachable reference). These other references are also shown in Table 1.

5. Results and discussion

Using the numerical data shown in Table 1, the probabilities of failure and reliability indices of bridge piers installed in main and flood channels are obtained and shown in Table 2. These values are 1.26 and 1.27 for main and flood channels piers respectively (using Monte Carlo simulation method). As for most of the structural components desired or target reliability indices are generally kept close to 3 (Joint Committee on Structural Safety, JCSS), the above reliability indices are quite less for piers of main as well as flood channels. This indicates that though depth of pier is more than estimated maximum scour depth, pier reliability is not within desirable range. This suggests

Table 2 Probability of failure and reliability indices of bridge pier

Main channel (Live bed scour)		Flood channel (Clear water scour)	
P_f	β	P_f	β
10.3×10^{-2}	1.26	10.2×10^{-2}	1.27

*Results are shown for COV (P_f) less than 5%.

Table 3 Probability of failure and reliability indices of bridge pier

Main channel				Flood channel			
Monte Carlo*		FORM		Monte Carlo*		FORM	
P_f	β	P_f	β	P_f	β	P_f	β
10.3×10^{-2}	1.26	6.54×10^{-2}	1.51	10.2×10^{-2}	1.27	9.75×10^{-2}	1.29

*Results are shown for COV (P_f) less than 5%.

that pier depth should be increased to achieve a desirable range of reliability index i.e., 3-4 (if other parameters are to be kept same).

5.1 Justification for the use of FORM

Monte Carlo simulation method is considered in principle an exact method, and, First Order Reliability Method (FORM) as an approximate method. However, FORM is computationally fast and inexpensive as compared to Monte Carlo simulation method. Table 3 shows that β values and P_f obtained using FORM are having close proximity with simulation results. Therefore, it is also appropriate to use FORM for reliability assessment of bridge piers against scouring. In the present study, therefore, FORM technique in subsequent analyses is employed. As FORM is a gradient-based algorithm sometimes it shows divergence (Sindel and Rackwitz 1998). Therefore, in the present study, whenever during the β point search FORM showed divergence, gradient free Monte Carlo simulation technique is employed for reliability assessment.

5.2 Sensitivity analysis

This analysis has been carried out to study the influence of various random variables on bridge pier reliability against scouring. The influence of various random variables on bridge pier reliability is measured in terms of sensitivity factor (α_j), which for the jth random variable may be defined as (Nowak and Collins 2000).

$$\alpha_j = \frac{\left(\frac{\partial G}{\partial z_j}\right)_*}{\left[\sum_{j=1}^{n}\left(\frac{\partial G}{\partial z_j}\right)^2 *\right]^{1/2}} \tag{10}$$

Where, G and z_j indicate the limit state function and jth random variable in reduced coordinate system; and * indicate the most probable or design point on the failure surface.

The above defined sensitivity factors have following characteristics:

(i) The lower the magnitude of α_j, less is the influence of jth random variable on the reliability.

(ii) α_j is positive for load variables and negative for resistance variables.

(iii) If α_1, α_2, α_3.......α_n are the sensitivity factors for n random variables appearing in the limit state function then $\sum_{j=1}^{n} \alpha_j^2 = 1$.

In the present study, using above expression, sensitivity factors for each random variable have been determined. As mentioned above, the magnitude of this factor for a random variable is directly

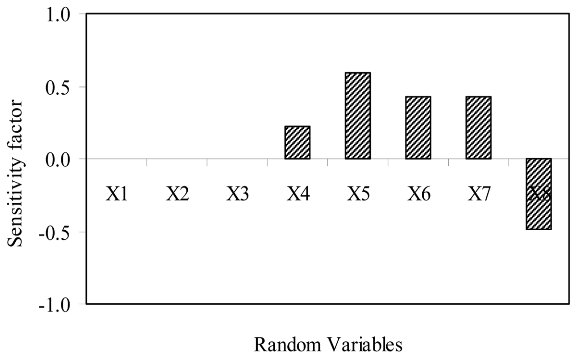


Fig. 2 Sensitivity diagram for main channel

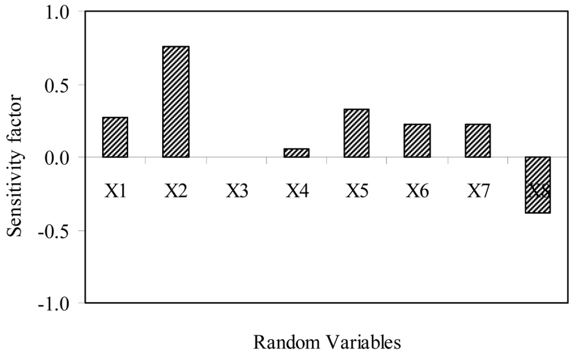


Fig. 3 Sensitivity diagram for flood channel

Note: X1 = Depth of flow (y); X2 = Velocity of flow (v); X3- = Median size of bed material (d_{50}); X4 = Width of pier (b); X5 = Foundation shape factor (Ks); X6 = Foundation alignment factor (K); X7 = Channel geometry factor (K_g) and X8- = Depth of pier (d_p)

measure of its influence on bridge pier reliability. However, its sign determines whether the random variable is a load variable or resistance variable. The negative sign of sensitivity factor indicates that the random variable is a resistance variable i.e., its increase will improve the bridge pier reliability and decrease will reduce its reliability. Similarly, positive value of sensitivity factor indicates that it is a load variable and its influence would be opposite to that of a resistance variable. The major advantage of this study is that without carrying out any separate parametric study for each variable one can directly know how a particular random variable affects the bridge pier reliability.

Figs. 2 and 3 show the results of sensitivity analysis for bridge piers installed in main and flood channels respectively. The bar chart indicates that the sensitivity factor for depth of pier is negative, hence, it is only the resistance variable and contribute to the resistance part of the limit state function. Sensitivity factor for other parameters are either positive or zero. This shows that positive parameters will affect the reliability adversely and those that are zero they will have no influence on pier reliability, at least for present set of data.

5.3 Reliability based safety factors

In a conventional approach for hydraulic design of a pier foundation; an appropriate safety factor is generally selected based on the judgement and experience. It does not account for possible

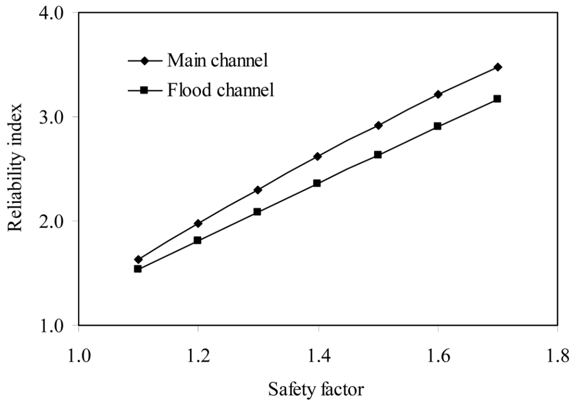


Fig. 4 Variation of reliability index with safety factor

variations associated with the variables involved in the phenomenon concerned. An application of the reliability analysis to hydraulic design practices enables the assessment of various reliability levels under different combinations of design parameters. To incorporate the reliability in the evaluation of safety factor, a relationship between safety factor and reliability index would be required. The safety factor (SF) of a foundation against scour may be expressed as d_p/d_s, wherein d_p is the depth of foundation and d_s is the depth of scour. Safety factors are widely used to incorporate uncertainties involved in the various stages of designing, construction etc. and also ensure an appropriate level of safety. These factors are generally qualitative measures of safety but do not directly tell about "quantity or magnitude" of safety. In the present study, for varied safety factors the reliability indices have been obtained after carrying out reliability analysis. The safety factor is then plotted against reliability indices and a best-fit curve is drawn (Fig. 4). After carrying out the regression analysis we then have developed following two equations for main and flood channels

For main channel, $$SF = 0.3239\,\beta_T + 0.5611 \quad (11)$$

For flood channel, $$SF = 0.3678\,\beta_T + 0.5341 \quad (12)$$

where, SF is the safety factor and β_T is the target/desired value of reliability index.

Using above two equations one can find out an appropriate value of safety factor for desired pier reliability. For example, above equations give the safety factor corresponding to target reliability index (β_T) = 3 as 1.53 and 1.63 for main and flood channels respectively. Johnson (1992) estimated the safety factor as 1.72 for a typical example problem corresponding to the allowable level of risk of 10^{-4}. A safety factor of 1.674 corresponding to risk of 10^{-3} has been obtained under clear water scour conditions in the present study.

The safety factors, however, obtained from the above equations can not be recommended for general use as these are based on only a limited set of data and assumed values of uncertainties. Other important issues such as cost issue, issue of consequences of failure etc. are not given the due consideration in the derivation of above equations. However, these equations can provide some strong basis in the decision of appropriate values of safety factor.

5.4 Parametric studies

To study the effect of various deterministic and random variables on pier reliability some parametric

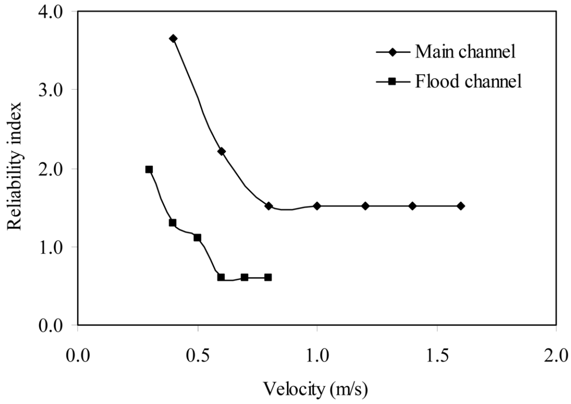


Fig. 5 Effect of approach velocity on pier reliability

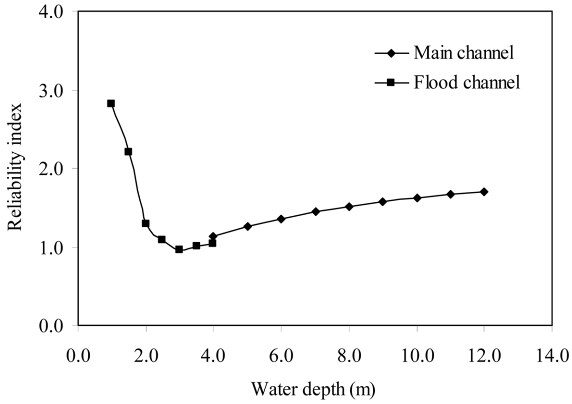


Fig. 6 Effect of water depth on pier reliability

studies have been carried out such as, effect of flow velocity; effect of water depth; effect of pier width; effect of sediment size; effect of sediment nature; effect of pier type; effect of debris; and effect of uncertainties. A brief discussion on these parametric studies may be found below.

5.4.1 Effect of velocity

Fig. 5 shows the influence of the approach velocity on the reliability index of the pier against scour. It has been reported that the scour depth is approximately proportional to flow velocity under clear water conditions but it is largely independent of the flow velocity under live bed conditions (Melville 1997). The mean velocities at the threshold condition for the main and the flood channels are 0.6 and 0.56 m/s respectively. Consequently the effect of velocity may be observed only for the velocity less than the threshold velocity. It is evident from Fig. 5 that the reliability index decreases with the increase in the velocity only up to a critical velocity (for both the cases).

5.4.2 Effect of water depth

The influence of the flow water depth on the reliability index has been indicated in Fig. 6. Water depth (y) may affect maximum local scour depth and consequently reliability index. As we can see from limit state function (Eq. (9)), the influence of y is present on third and fourth terms only and these terms will influence the reliability if b/y is lying either between 0.7 and 5 or beyond 5 (Eq. (4)). For b/y less than 0.7, these two terms disappears and therefore, influence of y also disappears.

5.4.3 Effect of pier width

Fig. 7 shows the influence of the pier width on the pier reliability for live bed and clear water scours. It may be observed that as pier width (b) is increasing the pier's reliability is decreasing for both the conditions of scour. This is so because width (b) of pier directly affects the scour depth up to five times water depth ($5y$), and in the present parametric study it is varied up to 3 m that is less than $5y$.

5.4.4 Effect of sediment size

The data that has been considered (Table 1 and Table 2) for reliability analysis of bridge piers give the value of b/d_{50} much above the critical value (i.e., 25) for piers in main as well as flood

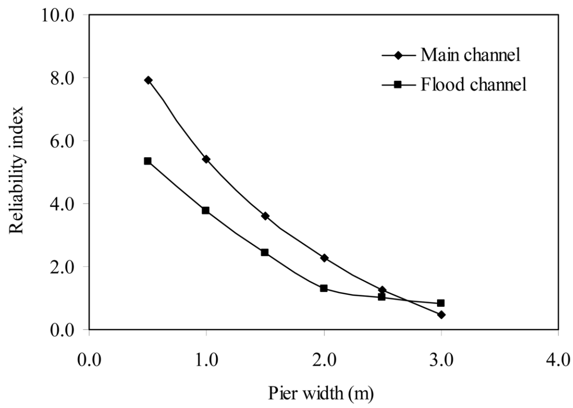


Fig. 7 Effect of pier width on pier reliability

Table 4 Effect of sediment nature for main channel

Uniform sediment		Non uniform sediment	
P_f	β	P_f	β
9.22×10^{-3}	2.36	3.33×10^{-6}	4.51

channels. It is due to this reason we do not observe any variation in reliability index with sediment size (Eq. (6)). Sensitivity diagram (Figs. 2 and 3) also indicate the same.

5.4.5 Effect of sediment nature

Due to armouring effect of non-uniform sediments, pier scour depth would be lesser compared to a pier surrounded by uniform sediments. Using the fact "less is the scour depth more is the reliability" we observe a higher reliability under non-uniform sediments (Table 4).

5.4.6 Effect of pier type

Since the effective size of the non-uniform pier (i.e., b_e) is more than the size of the uniform diameter pier, the scour depth would be more for non-uniform pier. It is due to this reason in Table 5 a lesser reliability for non-uniform pier than uniform pier has been observed.

5.4.7 Effect of debris

The effect of debris on pier reliability has been studied and results are shown in Table 6. The table shows that there is a dramatic reduction in reliability index due to the presence of debris around the piers. This is due to the fact that, debris in general enhances the scouring and consequently reliability decreases and probability of pier's failure increases.

Table 5 Effect of pier type for main channel

Non uniform pier		Uniform pier	
P_f	β	P_f	β
9.22×10^{-3}	2.36	8.86×10^{-4}	3.13

Table 6 Effect of debris

	Debris excluded		Debris included	
	P_f	β	P_f	β
Main channel	2.25×10^{-1}	2.270	4.96×10^{-2}	1.649
Flood channel	9.75×10^{-2}	1.295	1.16×10^{-2}	0.754

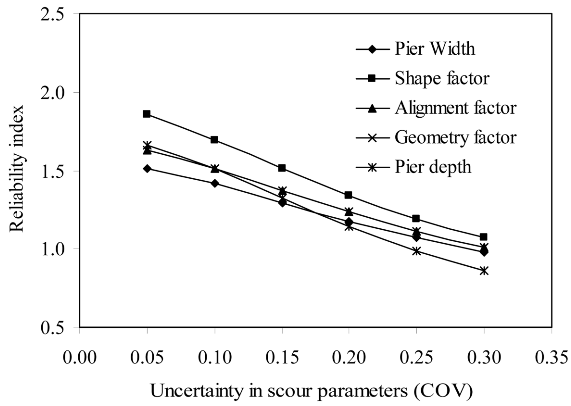


Fig. 8 Effect of uncertainty in scour parameters

5.4.8 Effect of uncertainties

Fig. 8 shows that as the uncertainty measured in terms of the coefficient of variation (COV), in pier width, depth of pier, K_s, K_θ and K_g increases; there is corresponding continuous decrease in the reliability index magnitude. This shows that it is not only the mean value that controls the reliability or safety of the pier against the scouring but the COV also place a very significant role in determining the reliability or safety of piers against scouring. This study shows that if through proper care, inspections and quality control COV is reduced reliability can be improved.

6. Case Study: reliability assessment of black mount bridge pier

To illustrate the application of present reliability procedure to real problems reliability analysis of Black Mount road bridge pier, existed in New Zealand and failed due to scour in August 1980, was carried out. Fig. 9 shows the details and salient features of Blackmount Road Bridge. Data available in the literature (Coleman and Melville 2001) shows that the pile of this bridge was founded to a depth of 9.1 m and it was thought to be sufficiently deep with respect to maximum estimated possible scour depth of 4.5 m. The factor of safety thus used was 1.46. Since the founded depth of pier was deep enough than maximum possible depth of scour the investigators of the past could not provide a sound reason for the failure. In the present study, we have carried out the reliability analysis of the same pier considering uncertainties involved in various design parameters. The statistical data used for the analysis are shown in Table 7 and the results of the analysis for various depths of pier (or for various safety factors) are presented in Table 8. The results show that for 9.1 m pile depth the reliability index value is only 1.77. This value is quite small for any structural component of importance. The value of reliability index for desirable safety should be greater or

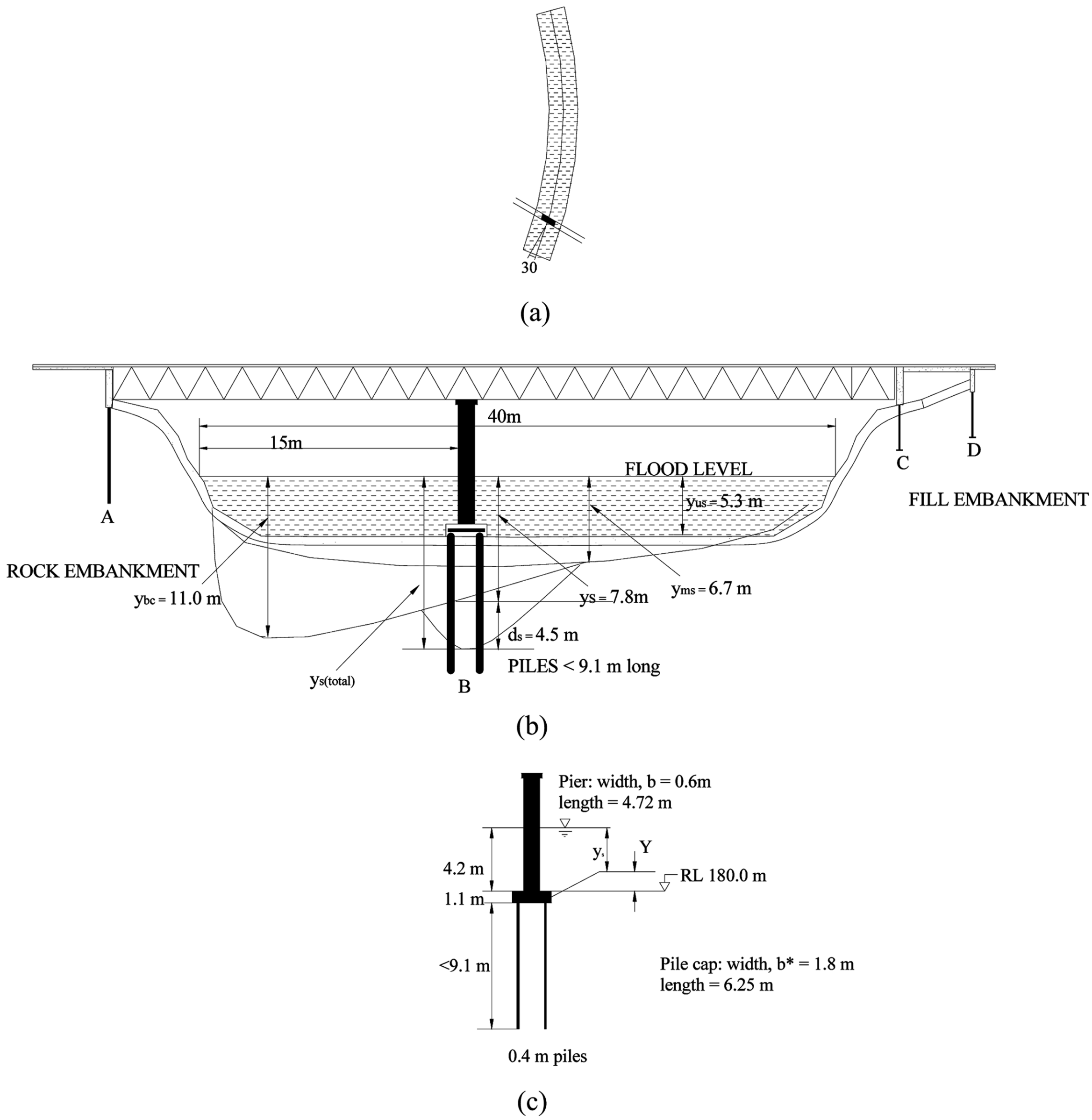


Fig. 9 Details of Blackmount Road Bridge; (i) schematic plan view of bridge, (ii) Elevation of bridge looking downstream, and (iii) Pier elevation

equal to 3 (Joint Committee on Structural Safety, Siddiqui and Ahmad 2000, 2001, 2003). Table 8 also shows that if the pile depth was kept close to 11.5 m or factor of safety was taken 2.0 the reliability had been achieved greater than 3.0. The result of this analysis indicates that the pier was vulnerable to failure since its installation due to its pier's smaller value of reliability index (this may be attributed to high uncertainties involved in the various design parameters, as listed in Table 7). However, a desirable safety would have been achieved if it were founded to a depth close to 11.5 m.

Table 7 Statistical data required for reliability analysis of Black mount Road bridge pier

Parameter	Reference	Distribution	COV	Mean
Flow depth (y_s)	Johnson	Normal	0.23	7.8 m
Mean flow velocity (v)	Assumed	Normal	0.329	2.88 m/s
Median size of sand (d_{50})	Johnson and Ayyub	Uniform	0.05	30.0 mm
Pier Width (b)	Assumed	Normal	0.05	0.6 m
Shape factor (K_s)	Johnson and Ayyub	Normal	0.15	1.0
Alignment factor (K_θ)	Johnson and Ayyub	Normal	0.10	1.91
Channel geometry factor (K_g)	Assumed	Normal	0.10	1.0
Pier length (l)		Deterministic	-	4.72 m
Pile cap width (b_*)		Deterministic	-	1.8 m
Distance of pile cap (y_z)		Deterministic	-	3.6 m
Depth of pier (d_p)	Assumed	Normal	0.10	6.6 m
Foundation alignment (θ)		Deterministic	-	30^0

Table 8 Reliability analysis results of Black mount Road bridge pier

Pile depth below general scoured Bed level, (m)	Pile depth below the pile cap, (m)	Factor of safety applied to d_s= 4.5 m	Probability of failure, P_f	Reliability index, β
4.50	7.00	1.00	4.37×10^{-1}	0.16
6.60	9.10	1.46*	3.82×10^{-2}	1.77
9.00	11.50	2.00	6.27×10^{-4}	3.23
11.25	13.75	2.50	9.14×10^{-6}	4.28
13.50	16.00	3.00	1.42×10^{-7}	5.13

*Pile length provided against local scouring

7. Conclusions

In the present study, a methodology for the reliability assessment of bridge piers against scouring has been presented using Melville scour model. To illustrate the methodology two numerical examples were taken, one for the main channel and other for the flood channel. It has been observed that the piers of present examples are not reliable as desired against scouring. To make the present study effective for field applications, reliability based safety factors have also been derived. However, these equations need thorough calibration for general use. An overall influence of various random variables on pier's reliability was assessed through sensitivity analysis. A detailed parametric study established the fact: "Less is the scour depth more is the reliability". The study, *effect of uncertainty* on pier's reliability showed if through better quality control; regular maintenance and proper care uncertainties can be minimized, reliability of bridge piers could be improved. The present approach of reliability analysis has also been applied to explain the failure of Black mount bridge of New Zealand through its pier's reliability analysis. It is observed that this bridge was not having a desirable level of reliability index at the time of construction itself.

References

Breusers, H.N.C., Nicollet, G. and Shen, H.W. (1977), "Local scour around cylindrical piers", *J. Hydraul. Res.*, IAHR, **15**(3), 211-252.
Chang, C., Tung, Y. and Yang, J. (1994), "Monte Carlo simulation for correlated variables with marginal distribution", *J. Hydraul. Eng.*, **120**(3), 313-331.
Coleman, S.E. and Melville, B.W. (2001), "Case study: New Zealand bridge scour experiences", *J. Hydraul. Eng.*, ASCE, **127**(7), 535-546.
Ghosn, M. and Johnson, P. (2000), "Reliability analysis of bridges under the combined effect of scour and Earthquakes", *8th ASCE Specialty Conference on Probability Mechanics and Structural Reliability*, PMC 2000-164, 1-6.
Huber, F. (1991), *Updated Bridge Scour.* American Society of Civil Engineers (ASCE), N.Y.
Johnson, P.A. (1992), "Reliability-based pier scour engineering", *J. Hydraul. Eng.*, ASCE, **118**(10), 1344-1358.
Johnson, P.A. and Ayyub, B.M. (1992), "Assessing time-variant bridge reliability due to pier scour", *J. Hydraul. Eng.*, ASCE, **118**(6), 887-136.
Joint Committee on Structural Safety (JCSS). http://www.jcss.ethz.ch/.
Kothyari, U.C. (2003), "Scour around bridge piers", *Proceedings, Workshop on Bridge Scour, River Training and Protection Works*, India International Centre, New Delhi, India.
Melville, B.W. and Sutherland, A.J. (1988), "Design method for local scour at bridge piers", *J. Hydraul. Eng.*, ASCE, **114**(10), 1210-1226.
Melville, B.W. (1997), "Pier and abutment scour: integrated approach", *J. Hydraul. Eng.*, ASCE, **123**(2), 125-136.
Melville, B.W. and Coleman, S.E. (2000), *Bridge Scour.* Water Resources Publications, Littleton, Colorado.
Muzzammil, M. (1992), "Characteristics of horseshoe vortex at cylindrical bridge piers", *Ph.D. thesis*, Indian Institute of Technology, Kanpur, India.
Muzzammil, M. and Gangadhariah, T. (2003), "The mean characteristics of horseshoe vortex at a cylindrical pier", *J. Hydraul. Res.*, IAHR, **41**(3), 285-297.
Muzzammil, M., Gangadhariah, T. and Gupta, A.K. (2004), "An experimental investigations of bridge pier induced horse shoe vertex", *Water Management*, Institution of Civil Engineers, London, U.K.
Nowak, A.S. and Collins, K.R. (2000), *Reliability of Structures*. Mc Graw Hill, Singapore.
Richaradson, E.V., Harison, J.R. and Davis, R. (1993), "Evaluating scour at bridges", *Hydraulic Engineering Circular*, No. 18 (HEC-18), FHWA, US Department of Transportation, Washington, DC.
Siddiqui, N.A. and Ahmad, S. (2000), "Reliability analysis against progressive failure of TLP tethers in extreme tension", *Reliab. Eng. Syst. Safe.*, **68**, 195-205.
Siddiqui, N.A. and Ahmad, S. (2001), "Fatigue and fracture reliability of TLP tethers under random loading", *Marine Struct.,* **14**, 331-352.
Siddiqui, N.A. and Ahmad, S. (2003), "Reliability of TLP tethers under extreme tensions", *Struct. Eng. Mech.,* **16**(3), 317-326.
Sindel, R. and Rackwitz, R. (1998), "Problems and solution strategies in reliability updating", *J. Offshore Mech. Arctic Eng.,* **120**, 109-114.
Yanmaz, A.M. (2003), "Reliability simulation of Scouring downstream of outlet facilities", *Turkish J. Eng. Environ. Sci.*, **27**, 65-71.

Notation

The following symbols are used in this paper:

b : width of pier
b^* : width of caisson, slab footing or pile cap
b_e : equivalent width of the pier

d_{50} : median size of bed material
d_{50a} : median particle size of armour layer (for uniform sediments $d_{50a} = d_{50}$)
d_{84} : particle size which is finer than 84% of sample particles
d_{max} : maximum particle size
d_p : depth of pier
d_s : maximum scour depth
d_c : diameter of circular pile
$g(\underline{x})$: limit state function
K_d : sediment size factor
K_g : approach channel geometry factor
K_i : flow intensity factor
K_s : foundation shape factor
K_t : time factor
K_{yb} : flow depth pier size factor
K_θ : foundation alignment factor
l : pier length
P_f : probability of failure
SF : safety Factor against Local scour
t : flood peak duration
t_e : time for equilibrium scour depth to develop (days)
t_d : thickness of a debris raft
d_d : diameter of a debris raft
v : mean velocity of flow
v_a : mean velocity of flow at armour peak
v_c : critical mean velocity
$\underline{x}$: vector of random variables
y : average depth of flow; flow depth appropriate to equation
y_p : Pile depth below Highest Flood Level (HFL)
y_s : flow depth for the combination of general scour and contraction scour
y_{ts} : total scour depth below Highest Flood Level (HFL)
$\underline{z}$: vector of random variables in reduced coordinate system
y_z : distance from the top of pile cap or caisson below the surrounding bed level
c_{ij} : coefficients which assume either value zero or one
θ : foundation alignment with respect to flow direction
σ_g : geometric standard deviation of particle size distribution

Appendix

(i) Flow depth pier size factor (K_{yb})
This factor may be expressed in a generalized form as

$$K_{yb} = 2.4 b_e c_{11} + 2\sqrt{y_s c_e} c_{12} + 4.5 y_s c_{13} \quad \text{(A1)}$$

Where, y = flow depth; y_s = flow depth for the combination of general and contraction scour; b_e = equivalent width of a pier. The expirations for the equivalent width of pier is given as

$$b_e = b; \text{ uniform pier of width } b; \quad \text{(A2)}$$

$$= b\left(\frac{y_s + y_z}{y_s + b^*}\right) + b^*\left(\frac{b^* - y_z}{b^* + y_s}\right); \text{ non-uniform pier} \quad \text{(A3)}$$

$$= \frac{0.52t_d t_d + (y_s - 0.52t_d)}{y_s}\text{; floating debris raft} \quad \text{(A4)}$$

Wherein b^* = width of caisson, slab footing or pile cap; y_z = distance from the top of pile cap or caisson below the surrounding bed level; c_{ij} = coefficients which depend on b_e/y and assume either value zero or one as given as

$$c_{11} = 1;\ c_{12} = 0;\ c_{13} = 0; \qquad \frac{b_e}{y} < 0.7 \quad \text{(A5)}$$

$$c_{11} = 0;\ c_{12} = 1;\ c_{13} = 0; \qquad 0.7 < \frac{b_e}{y} < 5 \quad \text{(A6)}$$

$$c_{11} = 0;\ c_{12} = 0;\ c_{13} = 1; \qquad \frac{b_e}{y} > 5 \quad \text{(A7)}$$

(ii) Flow intensity factor (K_i)

The flow intensity factor may be expressed as

$$K_i = c_{21} * \frac{v - (v_a - v_c)}{v_c} + c_{22} \quad \text{(A8)}$$

Where, v = mean flow velocity; v_a = mean velocity of flow at the armour peak; v_c = critical mean velocity of flow at the threshold condition for sediment movement; $v_a \equiv v_c$ for uniform sediments (i.e., $d_{50a} \equiv d_{50}$); $v_a = 0.8v_{ca}$ for nonuniform sediments (i.e., $d_{50a} = d_{max}/1.8 \approx d_{84}/1.8 = \sigma_g d_{50}/1.8$); d_{max} = maximum particle size; d_{84} = particle size which is finer than 84% of sample particles; σ_g = geometric standard deviation of particle size distribution; and c_{ij} = coefficients which depend on $v - (v_a - v_c)/v_c$ and assume either value zero or one as given below

$$c_{21} = 1;\ c_{22} = 0 \qquad \frac{v - (v_a - v_c)}{v_c} < 1 \quad \text{(A9)}$$

$$c_{21} = 0;\ c_{22} = 1 \qquad \frac{v - (v_a - v_c)}{v_c} \geq 1 \quad \text{(A10)}$$

(iii) Sediment size factor (K_d)

The sediment size factor may be expressed as

$$K_d = c_{31} 0.57 \log\left(2.24 \frac{b_e}{d_{50a}}\right) + c_{32} \quad \text{(A11)}$$

Where, d_{50a} = median particle size of armour layer; and c_{ij} = coefficients which depend on b_e/d_{50a} and assume either value zero or one as

$$c_{31} = 1;\ c_{32} = 0; \qquad \frac{b_e}{d_{50a}} \leq 25 \quad \text{(A12)}$$

$$c_{31} = 0;\ c_{32} = 1; \qquad \frac{b_e}{d_{50a}} > 25 \quad \text{(A13)}$$

(iv) Foundation shape factor (K_s)

The foundation shape factor assumes a different value for different shapes of piers e.g., a value of 1.0 is assigned to K_s for a circular shape pier. For other shapes we can refer Melville and Coleman (2001).

(v) Foundation alignment factor (K_θ)

The foundation alignment factor may be expressed as

$$K_\theta = c_{41}\left(\frac{l}{b_e}\sin\theta + \cos\theta\right)^{0.65} + c_{42} \quad \text{(A14)}$$

Where, l = pier length; θ = foundation alignment with respect to flow direction; and c_{ij} = coefficients which depend on pier's shape and assume either value zero or one as

$$c_{41} = 0;\ c_{42} = 1;\ \text{circular pier} \tag{A15}$$

$$c_{41} = 1;\ c_{42} = 0;\ \text{noncircular pier} \tag{A16}$$

(vi) Approach channel geometry factor (K_g)

The approach channel geometry factor assumes a value equals to 1.0 if values of y and v are selected to be representative of the flow approaching the particular pier.

(vii) Time factor (K_t)

The time factor may be expressed as

$$K_t = c_{51} + c_{52} \times \exp\left\{-0.03\left|\frac{v_c}{v}\ln\frac{t}{t_e}\right|^{1.6}\right\} \tag{A18}$$

Where, t = flood peak duration; t_e = time for equilibrium scour depth to develop (days) which is given as

$$t_e = 48.26\ \frac{b_e}{v}\left(\frac{v}{v_c} - 0.4\right);\quad y/b_e > 6,\ v/v_c > 0.4 \tag{A19}$$

$$= 30.89\ \frac{b_e}{v}\left(\frac{v}{v_c} - 0.4\right)\left(\frac{y}{b_e}\right)^{0.25};\quad y/b_e < 6,\ v/v_c > 0.4 \tag{A20}$$

c_{ij} = coefficients which depend on v/v_c and assume either value zero or one as given

$$c_{51} = 1;\ c_{52} = 0;\qquad \frac{v}{v_c} \geq 1 \tag{A21}$$

$$c_{52} = 1;\qquad \frac{v}{v_c} < 1 \tag{A22}$$