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Damage detection in truss structures using a flexibility based approach with noise influence consideration

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Abstract. The damage detection process may appear difficult to be implemented for truss structures because not all degrees of freedom in the numerical model can be experimentally measured. In this context, the damage locating vector (DLV) method, introduced by Bernal (2002), is a useful approach because it is effective when operating with an arbitrary number of sensors, a truncated modal basis and multiple damage scenarios, while keeping the calculation in a low level. In addition, the present paper also evaluates the noise influence on the accuracy of the DLV method. In order to verify the DLV behavior under different damages intensities and, mainly, in presence of measurement noise, a parametric study had been carried out. Different excitations as well as damage scenarios are numerically tested in a continuous Warren truss structure subjected to five noise levels with a set of limited measurement sensors. Besides this, it is proposed another way to determine the damage locating vectors in the DLV procedure. The idea is to contribute with an alternative option to solve the problem with a more widespread algebraic method. The original formulation via singular value decomposition (SVD) is replaced by a common solution of an eigenvector-eigenvalue problem. The final results show that the DLV method, enhanced with the alternative solution proposed in this paper, was able to correctly locate the damaged bars, using an output-only system identification procedure, even considering small intensities of damage and moderate noise levels.

Keywords: structural health monitoring; flexibility based technique; damage locating vector; noise.

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1. Introduction

Structural health monitoring (SHM) procedures, which have witnessed significant progresses in the last few decades, are based on processing vibration measurements and typically deals with structural (mass, stiffness, damping) and modal (modal frequencies, damping rates and mode shapes) parameters (e.g., Kaminski Jr. and Riera 1996, Doebling *et al.* 1996, Riera and Rios 2000, Hu *et al.* 2001, Sohn *et al.* 2003, Qu *et al.* 2003, Zhu *et al.* 2003, Cho *et al.* 2004, Riera 2004, Maity and Tripathy 2005, Amani *et al.* 2006, Kim *et al.* 2006, Zhu and Law 2006, Fadel Miguel *et al.* 2006a, Zhao and DeWolf 2007, Fadel Miguel 2007). The main objective in those studies is the evaluation of changes in the structural and modal parameters, to verify their importance and to detect correlations among individual changes, while controlling the complexity of processing collected data.

In vibration based damage detection the modal parameters extracted from dynamic tests during the lifetime of the structure, under different operating conditions, are compared with reference modal parameters corresponding to a model in a well-known *healthy* condition. Differences in the identified modal parameters may lead to the identification of structural damage. Within such context, automatic global vibration-based monitoring techniques turned out to be a useful alternative to visual inspections or local non-destructive (e.g., ultrasonic) evaluations performed manually.

Among several damage identification methods proposed in the literature, a very important class employs the dynamically measured flexibility matrix (Pandey and Biswas 1994, Topole 1997) mainly because it is insensitive to high frequency modes, which are typically difficult to determine experimentally. Hence, this matrix can be constructed by truncated modes at sensor locations without significant loss of accuracy.

The flexibility matrix can be estimated at sensor locations when the input is measured, and there is at least one co-located sensor and actuator pair. In this way, the experimental data can be used to construct the flexibility matrix and no additional information is required. However, for output-only measurements, mode shapes are not mass scaled, but recent techniques are available to normalize the experimental modes with respect to mass. The scheme has been used in a controlled mass addition experiment by Parloo *et al.* (2002), while Deweer and Dierckx (1999) employed an acoustic excitation technique on a plate structure.

Among the flexibility-based damage detection methods, the damage locating vector (DLV) proposed by Bernal (2002) is the most recent method with mathematical rigor and minor practical difficulties. Moreover, the DLV method is a very efficient approach because it is effective when operating with multiple damage scenarios, a truncated modal basis and an arbitrary number of sensors, without great computational effort.

In most previous studies, vibration characteristics of structures were estimated in laboratory tests under controlled environmental condition. However, in field studies, the variation of structural modal parameters due to environmental effects, which is always present in dynamic field tests, may hide the changes caused by structural damage. Therefore, if a method cannot properly account for the effect of noise in the damage detection process, false positive or negative damage diagnosis may occur, in which case the vibration-based health monitoring method is not reliable.

Within this context, the influence of noise on the accuracy of the damage locating vector method (DLV) is assessed in this paper. In order to verify the DLV performance for different damages intensities and, mainly, in presence of measurement noise, a parametric study was carried out. Two different excitations as well as three damage scenarios are numerically simulated in a continuous

Warren truss structure with a set of limited measurement sensors, considering five noise levels.

In addition, a new procedure to determine the damage locating vectors in the DLV method is herein presented. This procedure constitutes an alternative option to solve the problem with a better known algebraic method. The original formulation via singular value decomposition (SVD) is replaced by the solution of a simple eigenvalue problem. This is possible due to the algebraic relationship between the singular value decomposition of a matrix and the eigenproblem solution of this matrix pre-multiplied by its transpose.

The paper is organized as follows: First, the so-called damage locating vector is introduced, describing its methodology as well as basic aspects of the alternative formulation. Next, an approach formulated by Bernal (2004) to obtain the flexibility matrix at sensor locations in output-only system identification is described in detail, followed by the description of Gao and Spencer (2005) criterion, who showed that the modal normalization constants can be determined testing the 'healthy' structure with additional known masses and that the constants may be applied without loss of accuracy in the damaged condition. Finally, to assess the DLV performance in a structure subjected to different excitations, damage location and intensities, as well as variable noise level, a continuous Warren truss with a limited number of sensors is numerically simulated.

2. Damage locating vectors (DLV) method

The damage locating vectors (DLV) method proposed by Bernal (2002) is a general approach to extract spatial information for damage localization from changes in measured flexibility. The fundamental idea of the DLV approach is that the vectors that cover the null-space of changes in the flexibility matrix (between the pre- and post-damage states) induce no stress in the damaged elements (small in the presence of truncations and approximation) when they are treated as static loads on the structure. This unique characteristic can be employed to localize damage.

For a linear structure, the flexibility matrices can be constructed at sensor locations before and after damage occurs and are denoted as $\boldsymbol{F}_u$ and $\boldsymbol{F}_d$, respectively. It is assumed that there is a group of load vectors, defined at sensor locations, which produces the same deformations at sensors in the undamaged and damaged states. If all the linearly independent vectors that satisfy this requirement are collected in a matrix $\boldsymbol{L}$ it is evident that

$$\boldsymbol{F}_D\boldsymbol{L} = \boldsymbol{F}_U\boldsymbol{L} \quad \text{or} \quad \boldsymbol{F}_\Delta\boldsymbol{L} = (\boldsymbol{F}_D - \boldsymbol{F}_U)\boldsymbol{L} = 0 \tag{1}$$

From the definition, the DLVs are also seen to satisfy Eq. (1); that is, because the DLVs induce no stress in the damaged elements, the damage of those elements does not affect the displacements at the sensor locations. Therefore, the DLVs are indeed the vectors in $\boldsymbol{L}$.

In this paper, the solution of Eq. (1) is obtained in an alternative way. Differently from Bernal (2002), who solved this equation through singular value decomposition, it is proposed next that the vectors $\boldsymbol{L}$ can be found using a simple eigenproblem of the matrix formed by the difference flexibility matrix pre-multiplied by its transpose

$$\boldsymbol{F}_\Delta^T\boldsymbol{F}_\Delta = \boldsymbol{\lambda}\boldsymbol{V} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 & \boldsymbol{V}_0 \end{bmatrix} \tag{2}$$

in which λ_1 is diagonal and contains the non-zero eigenvalues, V_1 is a basis for the row space and $V_0 = L$ is a basis for the null space.

For ideal conditions, the DLVs are simply $V_0 = L$, that are the columns of the eigenvectors V associated with the null space. However, in practical applications the singular values corresponding to V_0 are never equal to zero due to noise and computational errors. To select the DLVs from V, an index *svn* was proposed by Bernal (2002) and it is adapted to the alternative approach proposed in the present paper

$$svn_i = \sqrt{\frac{\sqrt{\lambda_i}c_i}{\max_i \sqrt{\lambda_i}c_i}} \quad \text{for} \quad i = 1{:}m \tag{3}$$

in which m is the number of columns in V, λ_i is the ith eigenvalue of $F_\Delta^T F_\Delta$, c_i is a constant used to normalize the maximum stress in the undamaged element of the structure, induced by the static load $c_i \vec{V}_i$, to have a value of one. The vector can be taken as a DLV when $svn \leq 0.20$.

Each DLVs is then applied to the undamaged analytical model of the structure, and the stress in each structural element is calculated. The normalized cumulative stress $\bar{\sigma}_j$ for the jth element is defined as

$$\bar{\sigma}_j = \frac{\sigma_j}{\max_j (\sigma_j)} \quad \text{where} \quad \sigma_j = \sum_{i=1}^{ndlv} abs\left(\frac{\sigma_{ij}}{\max_j (\sigma_{ij})}\right) \quad \text{for} \quad j = 1{:}k \tag{4}$$

In Eq. (4) k is the number of elements, σ_j is the cumulative stress in the jth element, σ_{ij} is the stress in the jth element induced by the ith DLV and *ndlv* is the number of DLVs. If an element has zero normalized cumulative stress, then this element is a possible candidate of being damaged. However, in practice the normalized cumulative stresses induced by the DLVs in damaged elements may not be exactly zero due to noise and uncertainties. Reasonable thresholds should be chosen to select the damaged elements, such as 0.1 (Bernal 2002).

In order to introduce additional robustness into the technique, the information from multiple DLVs should be combined. The vector of weighted-average stress indices for each DLVs, WSI, needs to be calculated to select the potentially damaged elements

$$WSI = \frac{\sum_{i=1}^{ndlv} \{\sigma_{ij}/\max_j(\sigma_{ij})\}_i / \overline{svn_i}}{ndlv} \quad \text{where} \quad \overline{svn_i} = \max(svn_i, 0.015) \tag{5}$$

If an element has $WSI < 1.0$ then it is a candidate of being damaged.

3. Contruction of flexibility matrices

It was pointed out that the flexibility matrices in the healthy and damage conditions need to be constructed at sensor locations to implement the DLV method. Especially in the case of output-only measurements, mode shapes are not mass scaled and its construction may appear to be difficult. Considering a linear structure, the flexibility matrix takes the form

$$\boldsymbol{F} = \Phi_m \Lambda^{-1} \Phi_m^T = (\psi\alpha)\Lambda^{-1}(\psi\alpha)^T \tag{6}$$

in which $\boldsymbol{\Phi}_m$ is the mass normalized mode shapes matrix and $\boldsymbol{\Phi}_m = \boldsymbol{\psi\alpha}$, in which $\boldsymbol{\psi}$ is the undamped arbitrarily normalized mode shapes matrix and $\boldsymbol{\alpha}$ is the matrix of modal normalization constants.

For output-only measurements, these modal normalization constants can be determined by an alternative approach, which consists of testing the structure with additional known masses at the sensor locations (Parloo *et al.* 2002, Brincker and Andersen 2003, Bernal 2004).

The mass perturbation method proposed by Bernal (2004) for a linear structure with proportional damping is presented next. The mass matrix of the modified structure can be expressed as

$$\boldsymbol{M}_1 = \boldsymbol{M}_0 + \Delta\boldsymbol{M} \tag{7}$$

in which $\boldsymbol{M}_1$ is the mass matrix of the original structure, $\boldsymbol{M}_0$ is the mass matrix of the modified structure and $\Delta\boldsymbol{M}$ is the mass perturbation matrix. The eigenvalue problem for the modified structure is

$$\boldsymbol{K}\vec{\psi}_{1,j} = \lambda_{i,j}(\boldsymbol{M}_0 + \Delta\boldsymbol{M})\vec{\psi}_{1,j} \tag{8}$$

in which $\lambda_{i,j}$ is the jth eigenvalue of the modified structure and $\vec{\psi}_{1,j}$ is the jth eigenvector of the modified structure. The corresponding eigenvalues and eigenvectors for the original structure are $\lambda_{0,j}$ and $\vec{\psi}_{0,j}$, respectively. The mode shape $\vec{\psi}_{1,j}$ can be written as

$$\vec{\psi}_{1,j} = \boldsymbol{\psi}_0\vec{q}_j + \boldsymbol{N}(\boldsymbol{\psi}_0)\vec{g}_j \tag{9}$$

in which $\boldsymbol{\psi}_0 = [\vec{\psi}_{0,1}, \vec{\psi}_{0,2}, \ldots, \vec{\psi}_{0,j}, \ldots]$, $\boldsymbol{N}(\boldsymbol{\psi}_0)$ is the column null space of $\boldsymbol{\psi}_0$, $\vec{q}_j$ and $\vec{g}_j$ are coefficient vectors. Substituting Eq. (9) into Eq. (8) and multiplying both sides by $\boldsymbol{\psi}_0^T$ yields

$$\boldsymbol{\alpha}^2(\vec{R}_j + \vec{\varepsilon}_j) = \vec{E}_j \tag{10}$$

in which

$$\begin{aligned}
\vec{R}_j &= \lambda_{1,j}\boldsymbol{\psi}_0^T\Delta\boldsymbol{M}\vec{\psi}_{1,j} \\
\vec{\varepsilon}_j &= \lambda_{1,j}\boldsymbol{\psi}_0^T\boldsymbol{M}_0\boldsymbol{N}(\boldsymbol{\psi}_0)\boldsymbol{g}_j - \boldsymbol{\psi}_0^T\boldsymbol{K}\boldsymbol{N}(\boldsymbol{\psi}_0)\vec{g}_j \\
\vec{E}_j &= \boldsymbol{\Lambda}_0\vec{q}_j - \lambda_{1,j}\vec{q}_j
\end{aligned} \tag{11}$$

in which $\boldsymbol{\Lambda}_0 = diag[\lambda_{0,1}, \lambda_{0,2}, \ldots, \lambda_{0,j}, \ldots]$ and $\vec{q}_j = (\boldsymbol{\psi}_0^T\boldsymbol{\psi}_0)^{-1}\boldsymbol{\psi}_0^T\vec{\psi}_{1,j}$. By neglecting the error term ε_j, Eq. (10) can be rewritten as

$$\boldsymbol{\alpha}^2\lambda_{1,j}\boldsymbol{\psi}_0^T\Delta\boldsymbol{M}\vec{\psi}_{1,j} = \boldsymbol{\Lambda}_0\vec{q}_j - \lambda_{1,j}\vec{q}_j \tag{12}$$

which can be de-coupled to solve for the unknown α_i as

$$\vec{\alpha}_i^2 = \frac{\lambda_{0,j} - \lambda_{1,j}}{\lambda_{1,j}}\frac{\vec{q}_{ij}}{\vec{\psi}_{0,i}^T\Delta\boldsymbol{M}\vec{\psi}_{1,j}} \tag{13}$$

Eq. (13) shows that there is one group of vectors $\vec{\alpha}_i^2$ for each vector $\vec{\psi}_{1,j}$. However, it is recommended to use $i = j$ because it gives the most accurate values of the modal normalization constants $\vec{\alpha}_i$ (Bernal 2004). The dynamic characteristics for both, the original and modified structures, must be determined by applying some stochastic system identification techniques, for instance the so-called stochastic subspace identification (SSI) method (e.g., Van Overschee and de Moor 1993, Peeters and de Roeck 1999, Fadel Miguel *et al.* 2006b).

Gao and Spencer (2005) presented a criterion which uses modal normalization constants α_i determined for the healthy condition, in output-only damage detection procedures, on damaged state to construct the flexibility matrix in this situation. Moreover, they contend that this can be done, since in large civil structures damage has a local effect and is not expected to significantly change the global structural characteristics, including these modal normalization constants.

4. Numerical example and discussion

The damage detection approach proposed in this paper is numerically tested in the continuous Warren truss shown in Fig. 1. This planar truss consists of 37 nodes and 71 steel bars, which have a cross section of 2×10^{-3} m^2. Young's modulus of the material is 2×10^{11} N/m^2 and its mass density 7.86×10^{3}kg/m^3. The height of the truss is 9 m while the total length is 168 m. The supports of the structure are modeled as two hinged supports at nodes 1 and 37 and as a roller support at node 19. The pinned end allows nodes to rotate freely with all three translations restricted. It must be pointed out that the model considered in the example does not correspond to a built structure, able to withstand design loads for a bridge of those dimensions. However, the amplitude of the excitation was chosen to result in response amplitudes, in terms of displacements, expected in real conditions, which is an important factor in health monitoring. As a consequence, simulated deformations and displacements are inside a band that may be considered typical in field monitoring.

The structure is numerically modeled using a MatLab finite element code. The dynamic problem is solved by numerical integration of the equations of motion using Newmark method, with an integration time step equal to 0.0005s. Damping of the structure is assumed proportional to the mass and stiffness matrices. The proportionality constants were determined to yield damping ratios in both the 1st and 5th modes equal to $\xi = 1\%$.

As indicated before, two standard excitations were simulated: an impulsive and an ambient excitation. The latter was modeled by 69 uncorrelated Gaussian white noise signals (generated with MatLab), with zero mean and standard deviation equals to one, applied at all generalized coordinates of the structure. This representation seems adequate to represent a broad band, ambient excitation of the structure, as suggested in several experimental studies (Brownjohn *et al.* 1999, Peeters and de Roeck 2001, Maeck and de Roeck 2000, Cunha *et al.* 2001, Ren *et al.* 2004, He *et al.* 2005). The impulsive loading is represented by the application of a sequence of four impact

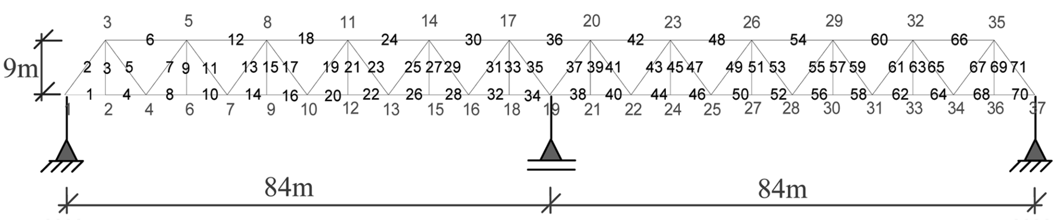

Fig. 1 Continuous Warren truss

excitations at node 7 in the y-direction. As mentioned before, the excitation magnitude was chosen to result in the response amplitudes, in terms of displacements, similar expected in real conditions. The excitations are shown in Figs. 2 and 3. In order to have a clearer visualization, the ambient excitation signal is partly shown.

For the identification procedure, the response is calculated for a time interval of 80s for the transient condition and for a time interval of 200s for ambient vibrations. To reduce the number of data points and to make the identification more accurate in the range of frequency of interest, the output data are filtered with an eight-order Chebyshev type I lowpass filter and the data is re-sampled at a rate of 100 Hz.

To better represent the experimental conditions imposed to the structures during real tests in output-only identification procedures, just the 22 nodal responses (y-direction) of the lower chord are considered for the modal parameter estimation. In this context, just mode shapes at these 22 DOFs can be established, and besides this, the responses are collected in terms of accelerations, since accelerometers are usually used as the measurement transducers. Figs. 4 and 5 shows the 10 node y-displacement response, both for the impact loading and for the ambient excitation. In order to have a clearer visualization, the ambient excitation signal is partly shown.

To consider the variation of structural modal parameters due to noise effects and to evaluate the DLV robustness in field conditions, five levels of noise are simulated through the addition of five different white noise signals with RMS amplitude varying from 0% to 2.5%, 5%, 10% and 15% of

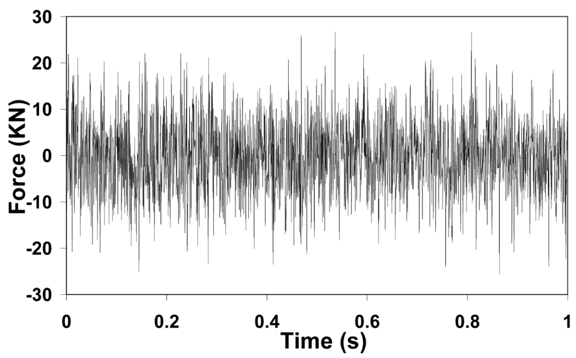

Fig. 2 Ambient excitation: Sample of white noise process

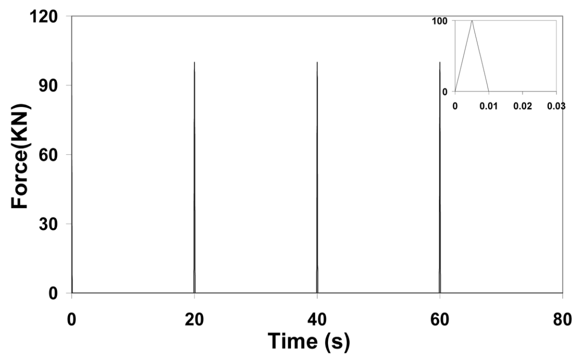

Fig. 3 Impulsive excitation

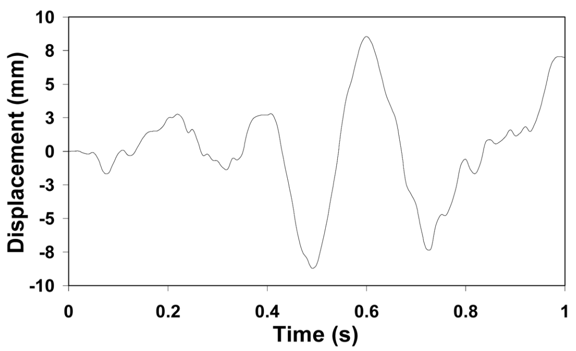

Fig. 4 Part of the ambient response

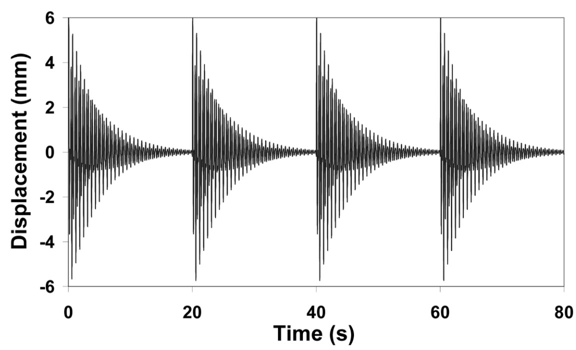

Fig. 5 Impulsive response

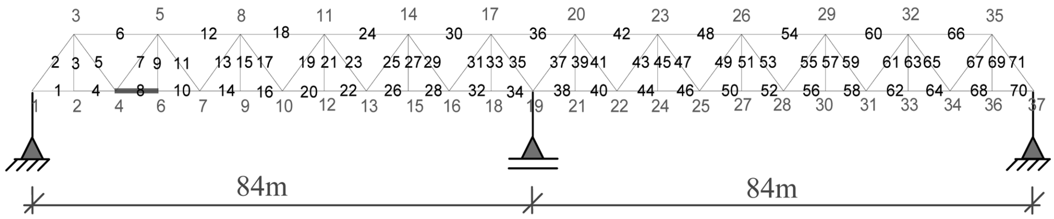

Fig. 6 Case 1 - Bar 8 damaged

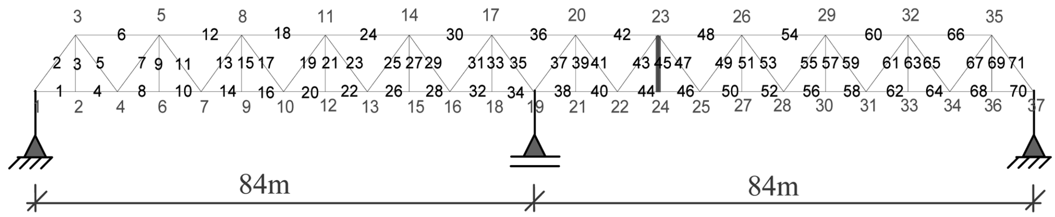

Fig. 7 Case 2 - Bar 45 damaged

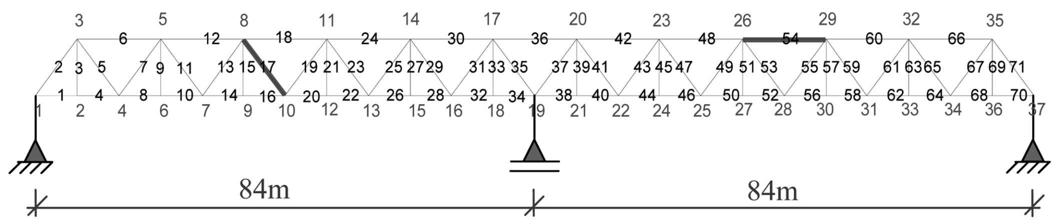

Fig. 8 Case 3 - Bar 17 and 54 damaged

the measured response. This noise is applied at all loaded nodal coordinates.

After getting the outputs for each noise level, the output-only system identification is carried out using stochastic subspace identification (SSI), which presents the main advantage of avoiding any preprocessing to obtain spectra or covariances, identifying models directly from time signals.

In addition, three different damage cases are studied to provide details of the performance of the DLV method concerning the effect of noise. In the first case of damage, i.e., Case 1, shown in Fig. 6, the damaged element is bar number 8. In Case 2 (Fig. 7) the stiffness of element 45 is reduced, which was chosen because it presents small sensitivity to damage. Finally, Case 3 consists of a multiple damage condition, in which diagonal element 17 and longitudinal element 54 are damaged simultaneously. This last case is illustrated in Fig. 8. The structural damage is simulated by uniformly reducing the member area along its length. In addition to this, the percentage of reduction in the cross section of the elements considered in the studies was 5%, 10%, 15% and 20%.

Due to the presence of noise, virtual tests of the structure were conducted 50 times for every noise, excitation and damage case and the modal parameters determined as the average of the fifty simulated responses. A sample size of 50 was chosen to be statistically representative.

In order to apply the DLV method, it is required to assemble the flexibility matrix for the 'healthy' state, with mass addition and in the damaged state. Since in real situations, the healthy and mass added modal parameters may be obtained in more controlled experiments, the noise is fixed as 5% for these conditions. Additional noise levels were considered for damaged structures. In order to apply the mass perturbation method, 50 kg masses were added in each of the nodes of the lower chord of the truss, which represent less than 10% of its total mass. In most cases such an excess mass would be admissible in real monitoring procedures. It is important to point out that according to the modal normalization constants precision determined through this criterion, there is a clear indication that these values could still be lower. In addition, following Gao and Spencer (2005), the modal normalization constants determined in the healthy state are used on damaged condition to construct the flexibility matrix for this situation. As it was pointed out, this is a reasonable assumption since the damage has a local effect and is not expected to significantly change the global structural characteristics for large civil structures.

After determining flexibility matrices, damage locating vectors are evaluated using the alternative solution proposed in this paper, and applied as static forces on the undamaged structure. It is important to point out that the index *svn* application presented some difficulty to determine the correct vectors. In this example, the last eight DLVs provided good results with the current sensor configuration and therefore they were used to localize damage in the different cases. The DLV determination is one of the most important points in the methodology, since if the selection is not effective to taken account the correct vectors, false positive or negative damage diagnosis may occur, so that DLV method becomes unreliable. Considering that *svn* index is not always efficient, it is not recommended its employment in a blind fashion, but as a reference to the true DLV determination. It may be numerically interesting to pre-select a set of damage cases results and then to compare which results are obtained by adding or removing some vectors from the group. Following this procedure, generally false DLVs can be identified and then discarded.

Since the WSI index and the normalized cumulative stress presented quite similar results, just those from the later are showed. The normalized cumulative stress for three damage cases are shown in Figs. 9, 10 and 11. They correspond to the ambient vibration, 5% of reduction in the cross section and 5% of noise level.

The results indicate that the normalized cumulative stress for damaged elements, in the three cases, is considerably smaller than other elements and smaller than the 0.1 threshold (Bernal 2002). Apparently, the reason why bar 10 also has a small cumulative stress is due to the equilibrium of forces in the node 6 of the truss. Under the application of the DLVs just in the lower chord, if either of these two elements has small stress, so does the other.

In damage Case 2, the neighbor bars 44 and 46, which are in the original condition, also have normalized cumulative stress small than the 0.1 threshold. This may be not a problem because the elements are inside of the same bay. Besides this, bar 36 also has a small normalized cumulative stress. Finally, for Case 3, just the real damaged elements presented normalized cumulative stress small than the threshold. Actually, the DLV method identifies a small group of potentially

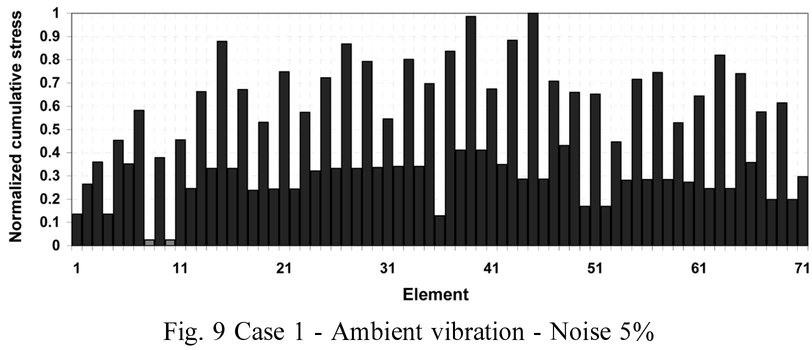

Fig. 9 Case 1 - Ambient vibration - Noise 5%

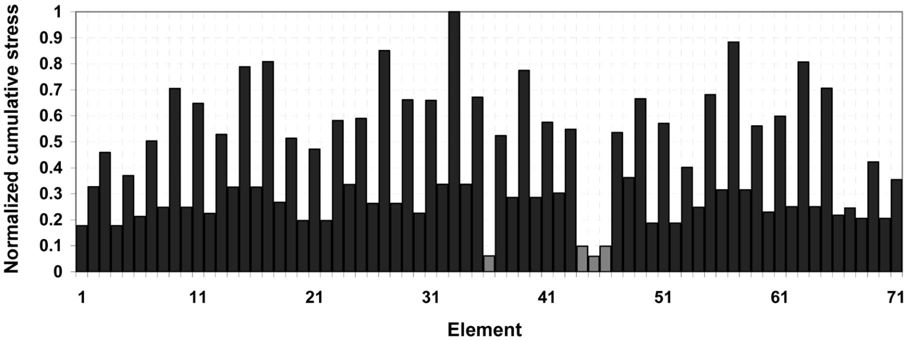

Fig. 10 Case 2 - Ambient vibration - Noise 5%

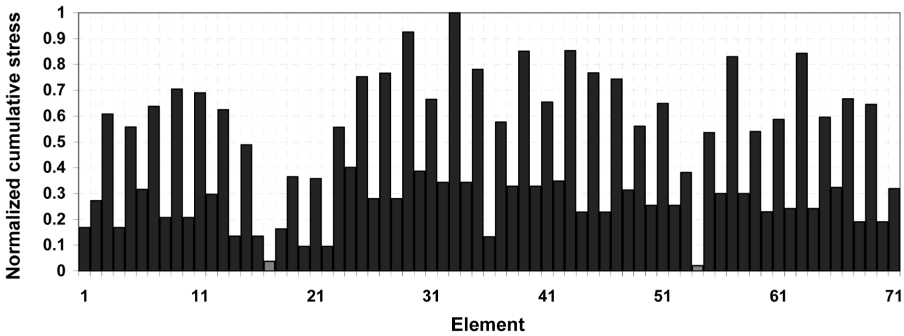

Fig. 11 Case 3 - Ambient vibration - Noise 5%

Table 1 Localization comparison: ambient vibration

Noise		Damage			
		5%	10%	15%	20%
0%	Case 1	X	X	X	X
	Case 2	X	X	X	X
	Case 3	X	X	X	X
2.5%	Case 1	X	X	X	X
	Case 2	X	X	X	X
	Case 3	X	X	X	X
5%	Case 1	X	X	X	X
	Case 2	+	X	X	+
	Case 3	X	X	X	X
10%	Case 1	+	X	X	X
	Case 2	◊	◊	+	+
	Case 3	+	X	X	X
15%	Case 1	+	X	X	X
	Case 2	◊	◊	◊	+
	Case 3	+	X	X	X

candidates to damage that contain the real damaged elements.

Table 1 shows a comparison of the results of the DLV method for ambient vibration, considering the different damage cases and noise levels. The 'X' symbol indicates that only the damaged bar was identified as damaged. As already pointed out, element 10 is always identified jointly with 8. The symbol '+' indicates that a small group of elements was identified, with normalized cumulative stress small than the threshold, including the actually damaged elements. Finally, the '◊' symbol indicates that a small group of elements has a normalized cumulative stress slightly higher than the threshold, including actually damaged elements.

In general, the DLV method presented very good results. The results in Table 1 indicate that damage localization is easier for low noise levels. Even in cases of small intensity of damage, the method proved able to identify the damaged elements in almost all conditions. However, in Case 2, which presents a very small variation of modal parameters, for high noise levels (10% and 15%), the normalized cumulative stress is lower than in some undamaged bars, but slightly higher than the threshold.

For the impulsive vibration again the case with 5% of reduction in the cross section and 5% noise level, the normalized cumulative stress for the three damage cases are shown in Figs. 12, 13 and 14.

As in ambient vibration, it was observed that the damaged elements have a smaller normalized cumulative stress than undamaged elements and is smaller than the 0.1 threshold. In Case 2, just bar 36 and bar 45 presented normalized cumulative stress smaller than the threshold. For the third case, the bars 17, 36 and 54 presented normalized cumulative stress smaller than the threshold.

Table 2 shows a comparison of results of the DLV method for impulsive loading, considering the different damage cases and noise levels. The criterion adopted in the classification is the same used

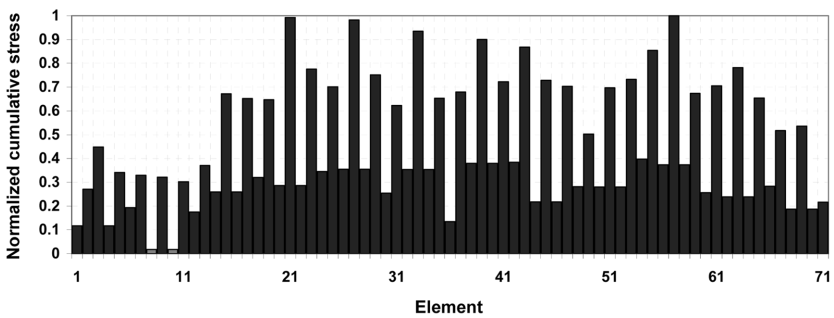

Fig. 12 Case 1 - Impulsive vibration - Noise 5%

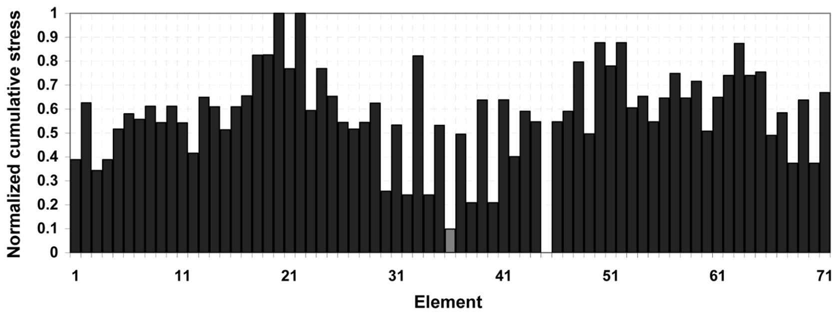

Fig. 13 Case 2 - Impulsive vibration - Noise 5%

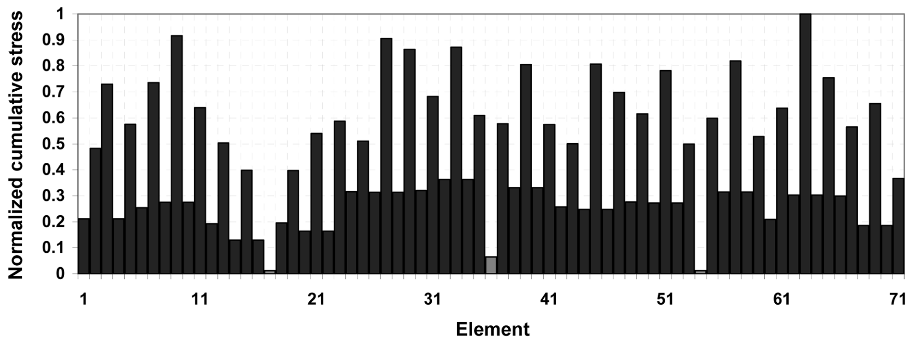

Fig. 14 Case 3 - Impulsive vibration - Noise 5%

Table 2 Localization comparison: impulsive vibration

Noise		Damage			
		5%	10%	15%	20%
0%	Case 1	X	X	X	X
	Case 2	X	X	X	X
	Case 3	X	X	X	X
2.5%	Case 1	+	X	X	X
	Case 2	+	X	X	X
	Case 3	X	X	X	X
5%	Case 1	X	X	X	+
	Case 2	+	X	X	X
	Case 3	+	X	X	+
10%	Case 1	+	+	X	X
	Case 2	◊	+	X	X
	Case 3	+	X	X	X
15%	Case 1	+	X	X	+
	Case 2	◊	◊	+	◊
	Case 3	X	X	X	X

for ambient vibrations.

In a similar way, the DLV method provided good results for impulsive loading. By observing Table 2, it can be seen that even for 5% of damage it was possible to identify the correct element in all analyzed cases. However, as in the former situation, at some noise levels, in Case 2 damaged bars presented lower normalized cumulative stress than other bars, but slightly higher than the threshold. It must be pointed out that for 20% damage, although all damage elements were successfully identified, some localizations are not clearly identified for high noise intensity (10% and 15%). This may be explained by the use of modal normalization constants determined in the healthy state to construct the flexibility matrix for the damaged condition. As damage levels increase, these values tends to move away from each other, leading to a loss in the approach capacity, until a point in which this consideration is no longer valid. However, this criterion is satisfactory for most values of interest.

5. Conclusions

In this paper, a vibration based damage detection approach that employs the so-called damage locating vector (DLV) method was described. This is a very efficient flexibility-based methodology which has the main advantage that it can be constructed by truncated modes at sensors locations. However, for the important output-only system identification procedures, the construction of the flexibility matrix is not an easy task. To circumvent the difficulty, vibration tests of the structure with additional known masses at sensor locations have been recently proposed, approach that is also discussed in the paper.

It is shown that damage locating vectors may be obtained in an alternative manner, which consists of replacing the original singular value decomposition by a simple eigenproblem. Thus, DLVs can be found as the eigenvectors that cover the null-space of the matrix formed by the difference flexibility matrix pre-multiplied by its transpose.

A simulation study of a continuous Warren truss structure with a set of limited measurement sensors was conducted. In order to assess the DLV performance under different damage location and intensity and, mainly, in presence of measurement noise, a parametric study was carried out. Although it is recognized that the final validation of the method must necessarily be given by successful damage identification using field data from real structures, the numerical simulation presented provides an indication of the expected performance of the approach in field applications. This is an important step to evaluate the global feasibility of the approach, such as its accuracy and minimum monitoring instrumentation requirements.

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