Structural Engineering and Mechanics, Vol. 27, No. 4 (2007) 439-456
DOI: http://dx.doi.org/10.12989/sem.2007.27.4.439

A new statistical approach for joint shear strength determination of RC beam-column connections subjected to lateral earthquake loading

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(Received December 21, 2006, Accepted June 5, 2007)

Abstract. Reinforced concrete (RC) joint shear strength models are constructed using an experimental database in conjunction with a Bayesian parameter estimation method. The experimental database consists of RC beam-column connection test subassemblies that maintained proper confinement within the joint panel. All included test subassemblies were subjected to quasi-static cyclic lateral loading and eventually experienced joint shear failure (either in conjunction with or without yielding of beam reinforcement); subassemblies with out-of-plane members and/or eccentricity between the beam(s) and the column are not included in this study. Three types of joint shear strength models are developed. The first model considers all possible influence parameters on joint shear strength. The second model contains those parameters left after a step-wise process that systematically identifies and removes the least important parameters affecting RC joint shear strength. The third model simplifies the second model for convenient application in practical design. All three models are unbiased and show similar levels of scatter. Finally, the improved performance of the simplified model for design is identified by comparison with the current ACI 352R-02 RC joint shear strength model.

Keywords: reinforced concrete; Bayesian parameter estimation; beam-column connections; experimental database; joint shear strength model.

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1. Introduction

In reinforced concrete (RC) beam-column connections subjected to earthquake loading, the main role of the joint panel is to transfer loads from and to the adjacent beam(s) and column(s). When only the flexural strength of well-detailed longitudinal beams limits overall response, RC beam-column connections typically display ductile behavior. This beam-hinging governing mode is usually considered as the most desirable for maintaining good global energy-dissipation without severe degradation of overall lateral load capacity. On the other hand, RC beam-column connections can exhibit less robust behavior when severe damage is concentrated within the joint panel. Therefore, understanding joint shear behavior is important toward controlling the overall performance of RC beam-column connections and frames.

Fig. 1 displays various types of RC beam-column connection subassemblies according to in-plane geometry. An interior connection has two longitudinal beams with a continuous column, an exterior connection has one longitudinal beam with a continuous column, and a knee connection has one longitudinal beam with a discontinuous column. Hanson and Connor (1967) first suggested a quantitative definition of RC joint shear, namely that joint shear can be determined from a free-body diagram at mid-height of a joint panel. For example, Fig. 2 shows joint shear at the mid-height of the joint panel for a typical interior connection subassembly. Paulay *et al.* (1978) described qualitative shear-resistance mechanism(s) for a joint panel, which consist of some combination of a concrete strut and/or a truss. Shear-resistance provided by the concrete strut mechanism comes from force transfer to the joint panel by bearing from concrete compression zones of adjacent beam(s) and column(s), whereas shear-resistance provided by the truss mechanism primarily comes from force transfer to the joint panel via bond between reinforcement and surrounding concrete, which is shown in Fig. 3.

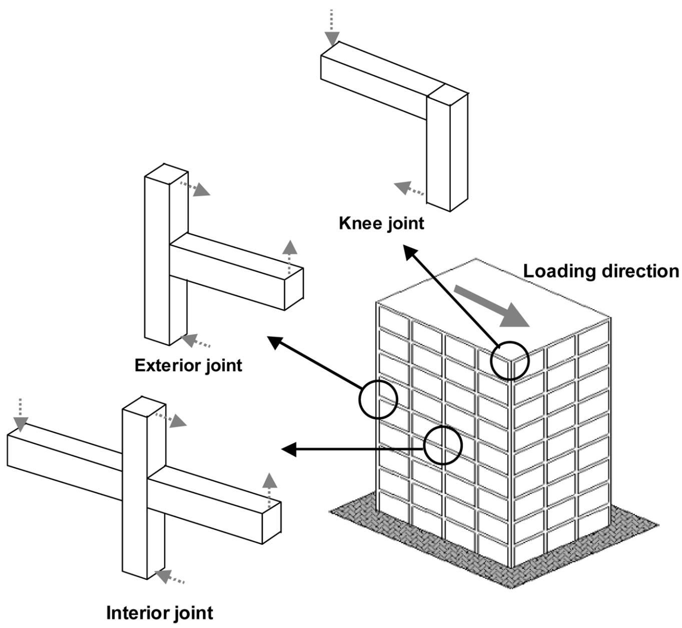
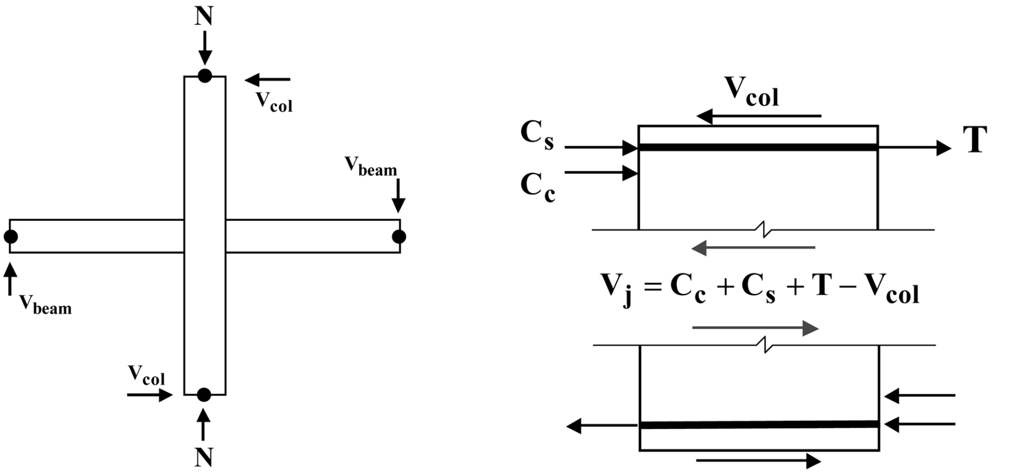
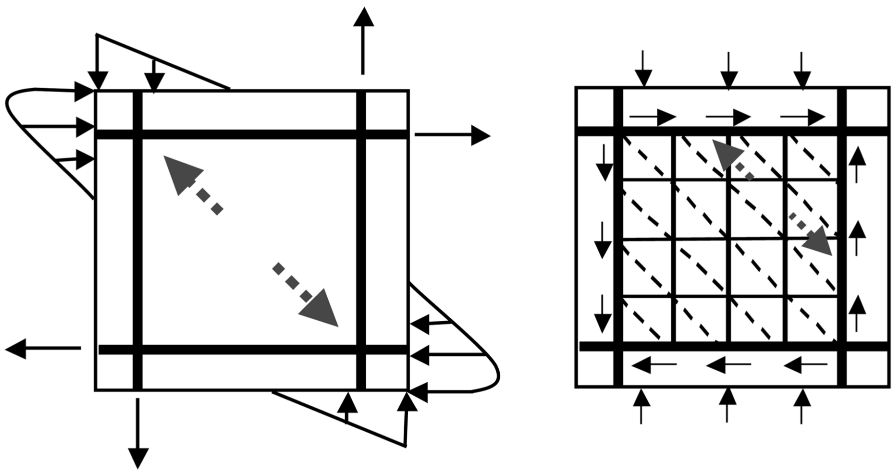


Fig. 1 Terminology of RC beam-column connections



(a) Loading condition (a) Shear free-body diagram at the mid-height of interior connections

Fig. 2 Horizontal joint shear at the mid-height of an interior connection subassembly



(a) Concrete strut mechanism (b) Concrete truss mechanism

Fig. 3 Joint shear resistance mechanisms (Paulay *et al.* 1978)

Since 1976, ACI-ASCE Committee 352 ("Joints and Connections in Monolithic Reinforced Concrete Structures") has recommended design guidelines for RC beam-column connections based on bench-marking studies such as those described above, and the committee has updated those design guidelines a number of times based on extensive additional experimental and analytical studies. Throughout this evaluation, ACI-ASCE Committee 352 has tried to keep a simple and safe joint shear strength model for use in design by considering the concrete strut mechanism. This simple ACI 352R-02 (ACI-ASCE 2002) approach is however not necessarily preferred for more detailed investigations of the performance of RC beam-column connections with respect to joint shear strength.

Researchers have tried several approaches to predict joint shear strength more accurately for various types of RC beam-column connections. For instance, Hwang and Lee (1999, 2000) developed a softened strut-and-tie model to predict joint shear strength for both interior and exterior connections. This softened strut-and-tie model satisfies equilibrium, compatibility, and constitutive relations for cracked RC. In order to satisfy these principles of mechanics, however, the distinct advantage with respect to simplicity of a strut-and-tie model is lost in the proposed softened strut-and-tie model. Their model was validated for a set of collected experimental subassemblies (56 interior and 63 exterior connections), without any restriction on the joint transverse reinforcement provided (and governing failure modes included beam flexural failure as well as joint shear failure in conjunction with or without yielding of beam reinforcement). Attaalla (2004) proposed an analytical equation to predict joint shear strength for interior and exterior RC connections. This analytical equation was developed from assuming a stress distribution around the joint panel that satisfies equilibrium, and also considering a compression-softening phenomenon associated with cracked RC. The considered parameters in the proposed equation were axial force in the beam, axial force in the column, joint reinforcement ratio in the longitudinal direction of the joint, joint reinforcement ratio in the transverse direction of the joint, and out-of-plane geometry. To validate the proposed equation, 61 interior and 69 exterior subassembly tests were used. In these collected data, the governing failure modes were joint shear failures (in conjunction with or without yielding of beam reinforcement), and there was no limitation about the amount of joint transverse reinforcement.

Shiohara (2004) also proposed a mathematical model to determine the RC joint shear strength of interior, exterior, and knee connections. In his suggestion, "quadruple flexural resistance" within the joint panel was assumed to play an important role in defining joint shear failures. Joint shear strength was determined from satisfying force equilibrium in four rigid segments within the joint panel. Finally, Russo and Somma (2004) suggested a joint shear strength model by considering three contributions-vertical stresses transmitted by the column, longitudinal beam reinforcement, and passive confinement due to transverse reinforcement in the joint. Their developed model was validated by using 50 exterior beam-column connections subjected to significant inelastic seismic actions.

Different from the above approaches, some researchers have suggested methodologies to completely predict RC joint shear stress vs. joint shear strain behavior. For example, Parra-Montesinos and Wight (2002) proposed an analytical model to predict joint shear stress vs. joint shear strain by defining plane strain conditions for the joint panel. On the other hand, Youssef and Ghobarah (2001), Lowes and Altoontash (2003), and Shin and LaFave (2004) assumed that a joint panel is a cracked RC two-dimensional membrane element and, in particular, applied the modified compression field theory (MCFT), developed by Vecchio and Collins (1986). Finally, in a somewhat different vein, Kitayama (1992) and Teraoka and Fujii (2000) simply proposed general envelope curves for joint shear behavior, which consist of line segments based only on experimental test results.

In this research, a new methodology is introduced to construct joint shear strength models for RC beam-column connections subjected to lateral loading. When RC beam-column connections are unable to keep proper confinement within the joint panel during testing, this improper confinement by itself could trigger a reduction in joint shear capacity. Therefore, an experimental database (without out-of-plane members, without eccentricity between beams and the column, and maintaining proper confinement within the joint panel) is first introduced. The probabilistic RC joint shear strength models are then constructed by a Bayesian parameter estimation method based on the introduced experimental database. From the process of construction of the joint shear strength models, key influence parameters on joint shear strength can be explicitly identified, and the governing joint shear-resistance mechanism, which is primarily dependent on certain influence parameters, may be understood. Finally, the performance of a proposed simplified joint shear strength design model is also evaluated.

2. Experimental database

Kim and LaFave (2007) collected experimental specimens from the literature into a database for RC beam-column connections by applying a consistent set of criteria. All included test specimens were subassemblies of RC moment resisting frames, at or above one-third scale. All specimens were subjected to quasi-static cyclic lateral loading, and their final governing modes were joint shear failure (either in conjunction with or without yielding of beam reinforcement). No specimens in this constructed database had out-of-plane members (such as transverse beams and/or slabs) or eccentricity between longitudinal beams and the column. The database only contained specimens with conventional types of reinforcement anchorage (*no* headed bars); longitudinal beam and column reinforcement were either anchored by hooks or passed continuously through a joint panel (according to in-plane geometry). For interior connections, longitudinal beam and column reinforcement both passed through the joint panel; in exterior connections, beam reinforcement was typically anchored by hooks and column reinforcement passed through the joint panel. Finally, in knee connections both beam and column reinforcement were typically anchored by hooks within the joint panel. Furthermore, a minimum cross-sectional area of horizontal joint transverse reinforcement for inclusion in the database (necessary for proper confinement within a joint panel) was determined, as described below.

Kim and LaFave (2007) assumed that RC joint transverse reinforcement is not effective at providing much confinement to a joint panel if and when it reaches its yield stress. In cases when joint shear capacity limits the maximum response (strength) of overall subassembly behavior, the joint transverse reinforcement almost always reaches its yield stress (often even in spite of satisfying the current area recommendations of ACI 352R-02). The presence of out-of-plane members (such as transverse beams and/or slabs) could provide additional confinement to a joint panel, so experimental specimens without out-of-plane members and that maintained *beam flexural failure* were examined with respect to their strain in joint transverse reinforcement for the purpose of assessing proper confinement within a joint panel. The strain in joint transverse reinforcement was identified from published experimental papers (or reports) that indicated beam flexural failures (without out-of-plane members and without eccentricity). When A_{sh} ratios (provided amount of joint transverse reinforcement divided by the minimum amount, in the direction of loading, following ACI 352R-02) were equal to or above 0.70, the joint transverse reinforcement did not reach yield stress throughout these tests. Therefore, when constructing the database of beam-column connections exhibiting *joint shear failure*, only specimens with A_{sh} ratios of at least 0.70 were (somewhat conservatively) included to definitely ensure a minimum adequate amount of joint confinement. Tables 1, 2, and 3 then briefly show the constructed experimental database-78 specimens for interior connections, 48 for exterior connections, and 10 for knee connections (136 in total).

Table 1 Database of interior joint subassemblies (Kim and LaFave 2007)

First author	Specimen name	f_c' (MPa)	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{ACI}}}$ [a]	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{Eq.(8)}}}$ [b]	First author	Specimen name	f_c' (MPa)	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{ACI}}}$ [a]	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{Eq.(8)}}}$ [b]
Meinheit	12	35.2	1.72	1.21	Tateishi	JCR	25.0	1.04	0.96
Briss	B1	27.9	0.97	0.70		HBS	25.0	1.24	1.10
	B2	31.5	0.97	0.70	Kamimura	No 2	28.5	1.41	0.98
Durrani	X1	34.3	1.11	1.01		No 3	28.5	1.43	0.94
	X2	33.7	1.05	0.93	Joh	PH-16	23.6	1.28	0.94
Kitayama	B2	24.5	1.81	1.13		PH-13	26.3	1.44	1.02
Watanabe	WJ-1	29.0	1.62	1.09		PH-10	25.6	1.46	1.09
	WJ-3	29.0	1.90	1.24	Shiohara	6	39.6	1.66	1.20
Leon	BCJ2	27.6	1.00	0.92		7	46.7	1.47	1.07
	BCJ3	27.6	0.98	0.87		8	46.7	1.47	1.04
Inoue	SP2	43.3	1.25	0.78	Saka	INS	55.8	1.53	0.92
Kitayama	J1	25.7	1.59	1.10	Ishida	HS-HS	70.0	1.25	0.70
	B3	24.5	1.64	0.97	Meinheit	13	41.3	1.40	1.03
Hayashi	No 34	39.4	1.13	0.71		14	33.2	1.47	1.23
	No 36	39.4	1.50	0.91	Besso	J1	31.7	2.06	1.09
Noguchi	OKJ-1	70.0	1.87	0.97	Owada	J0-1	20.1	1.21	0.86
	OKJ-4	70.0	1.97	0.99		J0-2	20.1	1.32	0.95
Goto	PH	30.5	1.30	0.84	Watanabe	WJ-6	29.0	2.26	1.34
Asou	No 1	44.1	1.63	0.90	Kitayama	A1	30.6	1.80	1.13
Kaku	J11A	57.5	1.21	0.95	Fujii	A4	40.2	1.66	0.83
	J11B	57.5	1.42	1.17	Noguchi	OKJ-3	107.0	1.96	0.96
	J12A	56.5	1.51	1.11		OKJ-5	70.0	1.99	0.97
	J12B	56.5	1.45	1.10		OKJ-6	53.5	1.88	0.97
	J12C	57.5	1.55	1.22	Tateishi	KSC	24.5	1.56	1.13
	J31A	55.2	1.17	0.88	Morita	No 1	22.1	2.24	1.20
	J31B	55.2	1.13	0.91		No 4	22.5	2.35	1.26
	J32A	55.2	1.51	1.08		No 5	21.6	2.38	1.34
	J32B	55.2	1.50	1.11	Shiohara	J2	81.2	1.45	0.83
Kamimura	A-1	19.3	1.93	1.17		J3	81.2	1.50	0.86
Teraoka	HNO1	95.4	1.41	1.05		J10	39.2	1.50	1.03
	HNO2	95.4	1.97	1.35		S3	28.0	1.72	1.03
	HNO3	95.4	1.86	1.27		1	33.6	1.52	1.13
	HNO4	95.4	1.81	1.14		2	33.6	1.52	1.10
Oda	BN-4	58.8	1.48	0.85		3	34.5	1.50	1.12
Tochio	J1	27.0	2.27	1.38		4	36.6	1.66	1.19
	No 3	28.5	2.23	1.46		5	36.6	1.66	1.16
Tateishi	AIJ	24.5	0.89	0.81		9	30.5	1.38	1.10
	HRP	25.0	1.04	0.88		10	30.5	1.39	1.05
	CSP	33.3	0.93	0.87		12	32.2	1.54	1.11

[a]: Experimental joint shear stress to the ACI 352R-02 model, [b]: Experimental joint shear stress to the simplified model

Table 2 Database of exterior joint subassemblies (Kim and LaFave 2007)

First author	Specimen name	f_c' (MPa)	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{ACI}}}$ [a]	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{Eq.(8)}}}$ [b]	First author	Specimen name	f_c' (MPa)	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{ACI}}}$ [a]	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{Eq.(8)}}}$ [b]
Hanson	1-A	36.4	0.97	0.98	Tsonos	MS4	33.7	1.02	0.93
Megget	Unit A	22.1	1.13	1.03	Hamada	J-10	34.3	1.24	0.92
Uzumeri	6	36.2	0.81	0.85	Chutarat	Specimen1	27.6	1.08	0.92
	7	30.8	0.85	0.90	Ehsani	1B	33.6	1.05	0.95
	8	26.3	1.06	0.99		2B	35.0	1.12	0.98
Paulay	Unit 2	22.5	0.93	0.91		5B	24.3	1.09	0.98
Kaneda	U42L	30.1	0.74	0.74	Joh	HO-NO	29.6	1.20	0.87
Ehsani	3B	40.9	1.35	1.19		MM-NO	27.7	1.18	0.87
	4B	44.6	1.42	1.22		HH-NO	29.3	1.37	0.98
Ehsani	3	64.7	1.05	0.96		H'O-NO	31.5	1.09	0.79
	4	67.3	1.12	0.97		HH-N96	30.5	1.27	0.92
Nishiyama	RC2	29.8	0.92	1.24	Joh	NRC-J1	51.5	1.66	1.08
Ehsani	LL8	55.9	0.97	0.88		NRC-J2	81.7	1.47	0.99
	LH8	55.9	0.94	0.83		NRC-J4	88.9	1.56	1.05
	LH11	73.8	1.06	0.96	Fujii	B4	30.0	1.28	0.89
	HH8	55.9	1.17	0.97	Ehsani	HL8	55.1	1.17	1.00
	HH11	73.8	1.01	0.86	Tsonos	S4	21.0	0.82	0.79
Tsonos	S3	19.0	0.83	0.85		S5	25.0	0.86	0.81
	S6'	29.0	0.98	0.93		S6	33.0	0.84	0.79
Mitsuwa	No 18	20.8	0.85	0.88	Ishida	A-O	27.0	0.97	1.04
	No 20	25.6	0.71	0.71		A-O-F	27.0	0.91	1.08
	No 21	19.2	0.92	0.90	Yamada	BUC	19.0	0.78	0.80
	No 22	21.0	0.87	0.83		BUH	19.0	0.82	0.84
Joh	NRC-J13	79.4	1.36	1.00	Lee	NJ2+0.0	23.5	0.85	0.79

[a]: Experimental joint shear stress to the ACI 352R-02 model, [b]: Experimental joint shear stress to the simplified model

After concrete crushing occurred within a joint panel, the joint shear-resistance was usually then reduced, which limited the overall connection capacity and triggered a story shear decrease. Thus, maximum story shear can be considered as corresponding to the joint shear capacity of a tested specimen when the final governing failure mode is joint shear failure. The larger of the peak values at positive and negative drifts is considered as the maximum story shear, and this maximum story shear is used to compute maximum joint shear demand. For interior and exterior connections, the other direction typically has about 95% of the overall maximum at its peak. For knee connections, the peak value under closing action is higher than the peak value under opening action (and the peak value under closing action is used in computing maximum joint shear demand). Similar to as shown in Fig. 2 (joint shear demand for interior connections), experimental joint shear demand was calculated from force and moment equilibria and a free-body diagram at mid-height of the joint panel for exterior and knee connections. Experimental joint shear stress was then calculated as this

Table 3 Database of knee joint subassemblies (Kim and LaFave 2007)

First author	Specimen name	f'_c (MPa)	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{ACI}}}$ [a]	$\frac{v_{j,\,\mathrm{exp}}}{v_{j,\,\mathrm{Eq.(8)}}}$ [b]
Kramer	Joint 4	34.6	0.74	1.03
McConnell	KJ 5	31.5	1.11	1.20
	KJ 7	32.9	1.19	1.29
	KJ 8	36.4	0.95	1.12
	KJ 10	38.0	0.92	1.09
	KJ 11	35.0	0.97	1.06
	KJ 13	31.7	1.01	1.09
Mazzoni	2-Hoop	42.1	1.07	1.16
	4-Hoop	42.1	1.08	1.11
Shimonoka	L-U	32.0	1.03	0.99

[a]: Experimental joint shear stress to the ACI 352R-02 model.
[b]: Experimental joint shear stress to the simplified model.

maximum joint shear demand divided by the effective joint shear area, which was taken as the product of effective joint width (average of beam and column widths) and column depth.

3. Bayesian methodologies for model construction

Gardoni *et al.* (2002) introduced a Bayesian methodology for developing probabilistic shear capacity models of RC columns based on experimental observations. The probabilistic models were constructed by correcting the biases in existing deterministic models and by quantifying the remaining errors. The model predicts the shear capacity (C) in the form

$$C(\mathbf{x}, \Theta) = c_d(\mathbf{x}) + \gamma(\mathbf{x}, \boldsymbol{\theta}) + \sigma\varepsilon \tag{1}$$

where $\mathbf{x}$ is the vector of input parameters that were measured during tests, $\Theta = (\boldsymbol{\theta}, \sigma)$ denotes the set of unknown model parameters that are introduced to fit the model to the test results, $c_d(\mathbf{x})$ is the existing deterministic model, $\gamma(\mathbf{x}, \boldsymbol{\theta})$ is the bias-correction term, ε is the normal random variable with zero mean and unit variance, and finally σ is the unknown model parameter representing the magnitude of the model error that remains after the bias-correction. Since the true form of the bias-correction term is unknown, the bias-correction term $\gamma(\mathbf{x}, \boldsymbol{\theta})$ could be expressed by using a suitable set of p "explanatory" functions, $h_i(\mathbf{x})$ where $i = 1, \ldots, p$, in the form

$$\gamma(\mathbf{x}, \boldsymbol{\theta}) = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) \tag{2}$$

Eq. (1) also assumes that the variance of the model error is independent of the input parameters $\mathbf{x}$ (the so-called "homoskedasticity" assumption). Gardoni *et al.* (2002) and Song *et al.* (2007) applied the natural logarithms to satisfy this condition in their applications; that is

$$\ln[C(\mathbf{x}, \boldsymbol{\Theta})] = \ln[c_d(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon \tag{3}$$

Bayesian parameter estimation is used in order to find the unknown parameters, $\boldsymbol{\Theta} = (\boldsymbol{\theta}, \sigma)$, that make the models in Eqs. (1) or (3) best fit the test results. In this Bayesian approach, the "prior" distributions represent the uncertain parameters based on subjective information, and they are updated to the "posterior" distribution based on objective information such as test results. This well-known updating procedure is described in the form (Box and Tiao 1992)

$$f(\boldsymbol{\Theta}) = \kappa L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) \tag{4}$$

where $p(\boldsymbol{\Theta})$ is the prior distribution, $f(\boldsymbol{\Theta})$ is the posterior distribution, $L(\boldsymbol{\Theta})$ is the "likelihood" function that represents the likelihood of the test results, and $\kappa = [\int L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) d(\boldsymbol{\Theta})]^{-1}$ is the normalizing factor. Details about the selection of prior distributions, the formulation of likelihood functions, and the computational method for obtaining posterior statistics can be found in Gardoni *et al.* (2002).

For the Bayesian methodology explained above, Gardoni *et al.* (2002) proposed a systematic procedure to construct a probabilistic capacity model. First, a deterministic capacity model is selected. Then, key influence parameters are selected for explanatory terms in the bias-correction function based on an understanding of the physical mechanisms. The distributions of uncertain parameters $\boldsymbol{\Theta}$ are updated by the aforementioned Bayesian parameter estimation method. The explanatory term with θ_i having the largest coefficient of variation (c.o.v.) is considered the least informative term of the various influence parameters and is hence removed. This process of updating and subsequent removal is repeated until the posterior mean of σ starts increasing significantly (because σ represents the magnitude of the error after bias-corrections). Finally, the posterior means of θ_i's and σ are substituted into the Eqs. (1) or (3) to complete the model construction.

Song *et al.* (2007) applied this Bayesian methodology to construct probabilistic shear strength models for RC beams with no shear reinforcement (stirrups). Using Eq. (3), they developed shear strength models by correcting the biases inherent in six existing deterministic models and then quantifying the remaining scatters. Gardoni *et al.* (2002) used normalized influencing parameters as explanatory functions; whereas, Song *et al.* (2007) noted that the natural logarithms of the normalized parameters captured the biases more efficiently for the given problem. They also suggested a method to construct a model without relying on starting with an existing deterministic model. One of the proposed model forms is then:

$$\ln[C(\mathbf{x}, \boldsymbol{\Theta})] = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon \tag{5}$$

Due to the lack of a deterministic model in this form, the Bayesian methodology should be used with caution such that the dimension of Eq. (5) is the same as that of the quantity of interest, even after removing less informative terms. One possible way to assure this is to select a set of explanatory terms that constitutes the same dimension as the quantity of interest and to exclude them from the removal process. One may then add as many possible dimensionless terms as needed, which are subject to the removals. Using this method, concrete beam shear strength models that are as accurate as those built upon deterministic models were obtained.

In resisting RC beam-column connection joint shear input demand, the concrete strut is generally considered as the main shear-resistance mechanism. ACI-ASCE Committee 352 accounts for the concrete strut mechanism by using a joint shear strength that is a function of the square root of concrete compressive strength. In this research, joint shear strength models are developed by the Bayesian methodology employing the form in Eq. (5) without relying on a deterministic model such as from ACI 352R-02. This approach can still model the contribution of concrete compressive strength accurately, without being limited by the specific descriptions of existing models. Moreover, this approach can also identify the contributions of various other key parameters on joint shear strength in a more explicit manner.

4. Development of joint shear strength models

4.1 Selection of explanatory terms

Probabilistic joint shear strength models for RC beam-column connections are developed based on the constructed database from Kim and LaFave (2007). They also investigated various influence parameters on joint shear strength for interior, exterior, and knee connections by introducing several parameters for examination. These parameters were classified into the following 5 groups: material property (concrete compressive strength), joint panel geometry, column axial compression, normalized reinforcement ratios, and bond. The relation between RC beam-column connection joint shear strength and the examined parameters was further quantified by the correlation coefficient. The correlation coefficient of two quantities X and Y is computed as $(\sum x_i y_i - N\bar{x}\,\bar{y})/(N s_x s_y)$ where x_i, y_i, $i = 1, \ldots, N$ are available data for X and Y, respectively, $\bar{x}$ and $\bar{y}$ are the sample means, and s_x and s_y are the sample standard deviations (Ang and Tang 2006). A correlation coefficient near 1.0 indicates a strong positive relationship while a near-zero coefficient implies little

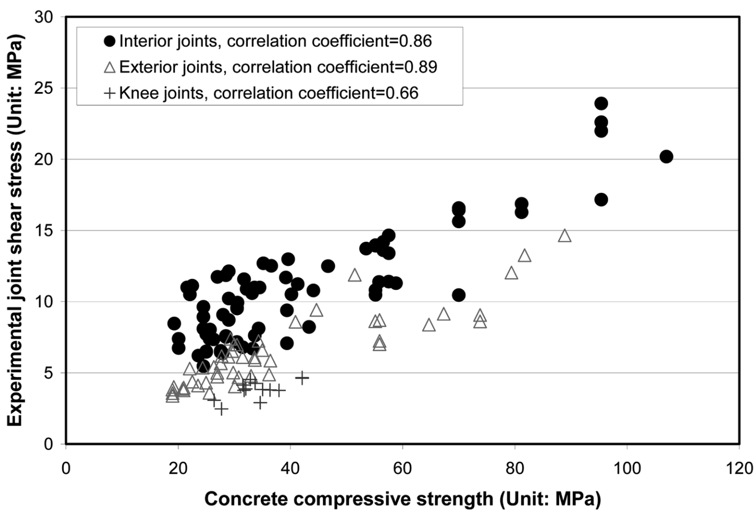


Fig. 4 Joint shear stress vs. concrete compressive strength

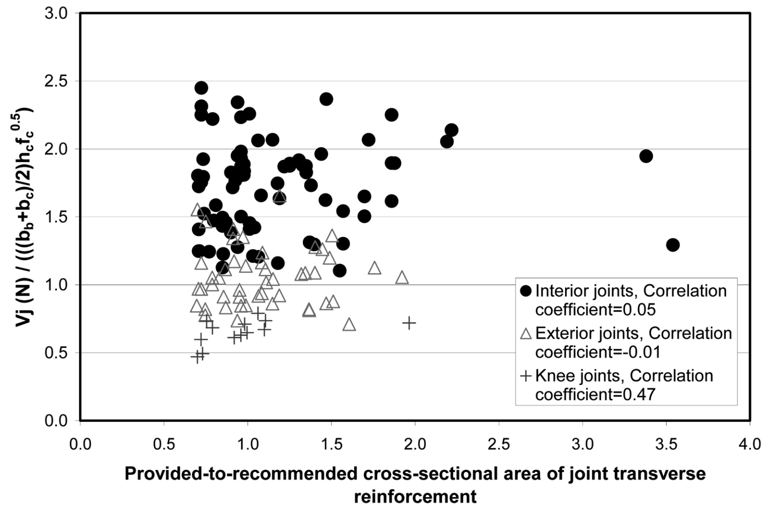


Fig. 5 Normalized joint shear stress vs. A_{sh} ratio

correlation. For example, Figs. 4 and 5 plot joint shear stress vs. concrete compressive strength, and normalized joint shear stress vs. A_{sh} ratio, respectively. The scatter plots indicate that joint shear strength is more dependent on concrete compressive strength than on A_{sh} ratio, as confirmed quantitatively by the correlation coefficients. Figs. 4 and 5 also show that in-plane geometry is important in determining joint shear strength.

The investigation results from Kim and LaFave (2007) can be summarized as follows: the strongest influence parameters on joint shear strength were a bit different according to in-plane geometry and governing failure mode sequence (joint shear failure either in conjunction with or without yielding of beam reinforcement); however, concrete compressive strength was the main governing parameter on RC joint shear strength. Furthermore, a reduction in design joint shear strength according to in-plane geometry should be considered. The research reported herein is to investigate some questions triggered by the prior work of Kim and LaFave (2007), such as what is the exact (and relative) contribution of various governing parameters on joint shear strength and how could a joint shear strength model be efficiently improved. So, most of the examined parameters introduced by Kim and LaFave (2007) are also applied here as possible explanatory terms in Eq. (5) to develop probabilistic joint shear strength models.

Table 4 shows the ranges and brief definitions of the included parameters. Concrete compressive strength is included to accurately investigate the contribution of concrete strength on overall joint shear strength. To consider a possible reduction in joint shear strength according to the in-plane geometry (interior, exterior, and knee connections), JP represents a ratio of the number of not-free in-plane surfaces around a joint panel to the total number of in-plane surfaces of the joint panel (4). Thus, JP is 1.0 (4/4) for interior connections, 0.75 (3/4) for exterior connections, and 0.5 (2/4) for knee connections. The ratios of beam height to column depth (h_b/h_c) and beam width to column width (b_b/b_c) are used to examine whether the shape of the joint panel in the in-plane direction, and the out-of-plane dimensions of in-plane members, respectively, might affect joint shear strength. Within a joint panel, the amount and yield stress of longitudinal beam and joint transverse reinforcement vary according to different experimental objectives. Indices that represent the amount and yield stress of reinforcement are introduced to examine the effects of longitudinal beam (BI)

Table 4 Explanatory terms

Parameters	Symbol	Min.	Max.
Concrete compressive strength	f_c' (Unit: MPa)	19	107
In-plane geometry	JP (interior = 1.0, exterior = 0.75, knee = 0.5)	0.50	1.00
Beam height to column depth	h_b/h_c	0.80	1.60
Beam width to column width	b_b/b_c	0.56	1.00
Joint transverse reinforcement index	JI: $(\rho_{joint\ reinf} f_{y,joint\ reinf})/f_c'$	0.03	0.26
Beam reinforcement index	BI: $(\rho_{beam\ reinf} f_{y,beam\ reinf})/f_c'$	0.07	1.08
Provided-to-recommended cross-sectional area of joint transverse reinforcement	A_{sh} ratio: $A_{sh,pro}/A_{sh,req}$	0.70	3.54
Provided-to-recommended spacing of joint transverse reinforcement	Spacing ratio: s_{pro}/s_{req}	0.37	2.14

and joint transverse reinforcement (JI) on joint shear strength. The joint transverse reinforcement ratio ($\rho_{joint\ reinf}$) is calculated as the volume of joint transverse reinforcement (located between the top and bottom beam reinforcement) divided by the joint volume (product of column width, column depth, and the distance between the top and bottom beam reinforcement). The beam reinforcement ratio ($\rho_{beam\ reinf}$) is calculated as the total volume of beam reinforcement (within the joint panel) divided by the product of beam width, beam height, and column depth. In the recommendations of ACI 352R-02, the cross-sectional area and spacing of joint transverse reinforcement tend to represent the degree of confinement within the joint panel. The A_{sh} ratio ($A_{sh,pro}/A_{sh,req}$) and the spacing ratio (s_{pro}/s_{req}) of joint transverse reinforcement are therefore also included to examine whether joint shear strength is at all influenced by the A_{sh} ratio and/or the spacing ratio.

4.2 Construction of joint shear strength models

The first joint shear strength model for RC beam-column connections is developed by a Bayesian method employing Eq. (5). The natural logarithms of the eight aforementioned parameters (f_c', JP, h_b/h_c, b_b/b_c, JI, BI, $A_{sh,pro}/A_{sh,req}$, and s_{pro}/s_{req}) are used as explanatory terms $h_i(x)$ $i = 1, \ldots, 8$. The Bayesian updating performed based on all 136 experimental subassemblies leads to the following joint shear model (with the specific exponents noted)

$$v_j(\text{MPa}) = 1.04\left(\frac{s_{pro}}{s_{req}}\right)^{-0.00513}\left(\frac{b_b}{b_c}\right)^{0.0151}\left(\frac{A_{sh,pro}}{A_{sh,req}}\right)^{0.0241}\left(\frac{h_b}{h_c}\right)^{-0.301}\times$$

$$(\text{JI})^{0.0886}(\text{BI})^{0.236}(\text{JP})^{1.28}(f_c')^{0.765} \tag{6}$$

where the uncertain error of the model, (i.e., $\sigma\varepsilon$) is not shown for simplicity. The posterior mean of σ is 0.148, which is approximately the c.o.v. of the joint shear strength predicted by the developed model (Song *et al.* 2007). Table 5 summarizes the step-wise process that systematically identifies and removes the less significant parameters. In Eq. (6), the spacing ratio (s_{pro}/s_{req}) is the least informative of the initial 8 parameters (i.e. the c.o.v. of θ for the spacing ratio is the largest). The remaining 7 parameters are then updated after removing the spacing ratio, and the mean of σ is 0.147. Because the removal of the spacing ratio does not significantly increase the mean of σ, the

Table 5 Step-wise removal process

	1[a]	2	3	4	5	6	7	8
f_c'	O	O	O	O	O	O	O	O
JP	O	O	O	O	O	O	O	X
BI	O	O	O	O	O	O	X	X
JI	O	O	O	O	O	X	X	X
h_b/h_c	O	O	O	O	X	X	X	X
A_{sh} ratio	O	O	O	X	X	X	X	X
b_b/b_c	O	O	X	X	X	X	X	X
Spacing ratio	O	X	X	X	X	X	X	X
Mean of σ	0.148	0.147	0.147	0.149	0.151	0.156	0.190	0.366

[a] : step

spacing ratio could be considered as a fairly insignificant parameter on joint shear strength. This process is repeatedly performed until only one parameter remains. As shown in Table 5, the mean of σ distinctively increases upon removing JI. Thus, the joint shear strength per Eq. (7) is developed from the parameters (JI, BI, JP, and f_c') surviving a step-wise removal process; that is

$$v_j(\text{MPa}) = 0.950(\text{JI})^{0.0728}(\text{BI})^{0.259}(\text{JP})^{1.31}(f_c')^{0.777} \tag{7}$$

for which the mean of σ is 0.151, which implies that its uncertainty is similar to that of the first joint shear strength model in Eq. (6) despite its simplicity.

Table 5 shows that concrete compressive strength is the strongest parameter affecting joint shear strength; thus, this updating process re-confirms the concrete strut as the main shear-resistance mechanism against RC joint shear input demand, which was apparently the basis for the design approach in ACI 352R-02. However, the details of the contributions of concrete compressive strength to joint shear strength are different between the developed model in Eq. (7) and the ACI 352R-02 joint shear strength models; the power terms for concrete compressive strength are 0.5 for ACI 352R-02 and 0.777 for Eq. (7). (There has been little consensus in design specifications about the exact contribution of concrete compressive strength to joint shear strength; for instance, the power terms for concrete compressive strength are 0.7 in Japan and 1.0 in New Zealand (Teraoka and Fujii 2000, Russo and Somma 2004)). Similar to the general findings by Kim and LaFave (2007), in-plane geometry (JP) is also important toward determining joint shear strength. When concrete compressive strength, JI, and BI are fixed as constant values, the developed joint shear strength model indicates that the joint shear strengths of exterior and knee connections are on average about 69% and 40% of those for interior connections, respectively.

ACI 352R-02 does not explicitly include the effects of longitudinal beam and joint transverse reinforcement in the joint shear strength definition because their roles have not been conclusively determined. In other specification examples, New Zealand considers the effects of longitudinal beam and joint transverse reinforcement on joint shear strength, but Japan does not (Russo and Somma 2004, Teraoka and Fujii 2000). From this current research, helpful information about the absolute and relative roles of longitudinal beam and joint transverse reinforcement in joint shear strength are provided. In general, joint shear failure without yielding of beam reinforcement is induced by using a high amount and/or a high yield stress of beam reinforcement, and the joint shear strength of this failure type is typically higher than that of joint shear failure in conjunction with yielding of beam reinforcement; the formation of plastic hinge(s) between a joint panel and the longitudinal beam(s) can reduce confinement within the joint panel originally provided by the longitudinal beam(s). The beam reinforcement index (BI) roughly represents this phenomenon (in that the beam reinforcement indices without yielding of beam reinforcement were higher than BI in conjunction with yielding of beam reinforcement). Finally, JI appears to be more appropriate than the A_{sh} ratio and/or the spacing ratio to express the beneficial joint confinement provided by joint transverse reinforcement (i.e., the removal of JI triggers a greater increase in the mean of σ than does the removal of the A_{sh} ratio or the spacing ratio). However, JI is not a particularly strong influence parameter on joint shear strength within this database, in part because the joint panels of the experimental database were selected considering minimum proper confinement within the joint panel.

Finally, Eq. (7) was converted into a simple model that could be conveniently used in practical design

$$v_j(\text{MPa}) = \alpha\gamma(\text{JI})^{0.07}(\text{BI})^{0.25}(f_c')^{0.75} \tag{8}$$

where α is a parameter for describing in-plane geometry: 1.0 for interior connections, 0.7 for exterior connections, and 0.4 for knee connections; γ is 1.02, which adjusts the difference between Eqs. (7) and (8) in an average sense (and could probably even just be taken as unity, with little loss in accuracy).

4.3 Performance of simplified joint shear strength model

The performance of the simplified joint shear strength model described above is evaluated in comparison with the current ACI 352R-02 joint shear strength model. In ACI 352R-02, RC beam-column connection design joint shear strength is determined by a shear stress limit and the joint shear area. The joint shear stress limit is the product of a joint shear stress factor and the square root of concrete compressive strength, as shown

$$v_j(\text{MPa}) = \gamma\sqrt{f_c'} \tag{9}$$

where the joint shear strength factor (γ) is simply determined from the number of effectively confined faces around a joint panel; for planar connections (with no out-of-plane members), ACI 352R-02 considers that a joint panel is effectively confined (in-plane) when the beam-to-column width ratio is equal to or above 0.75. For interior connections, γ is 1.00 when the joint panel is not effectively confined, and γ is 1.25 when the planer joint panel is effectively confined; γ is 1.00 for exterior connections, and γ is 0.67 for knee connections. The joint shear area is taken as the product of effective joint width (average of beam and column widths) and column depth. The overall bias and scatter of a deterministic model can be evaluated by introducing a constant bias-correction term θ (Song *et al.* 2007), that is

$$\ln[C(\mathbf{x},\Theta)] = \ln[c_d(\mathbf{x})] + \theta + \sigma\varepsilon \tag{10}$$

When the simplified joint shear strength model (Eq. (8)) is used as a deterministic model ($c_d(\mathbf{x})$ in Eq. (10)), the means of θ and σ are –0.0117 and 0.154, respectively. When the ACI 352R-02

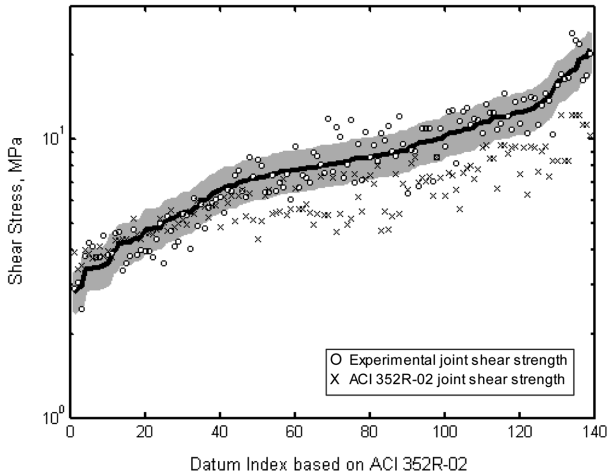


Fig. 6 Performance of the simplified joint shear strength model

joint shear strength model is used as a deterministic model in Eq. (10), the means of θ and σ are 0.366 and 0.284, respectively. This evaluation provides two interesting results. First, the simplified joint shear strength model of Eq. (8) is also unbiased and maintains a similar level of scatter (similar value of the mean of σ) compared to the first and second developed joint shear strength models (Eqs. (6) and (7)). Second, the simplified model clearly reduces scatter compared to the current ACI 352R-02 joint shear strength model.

The overall performance of the simplified joint shear strength model can be visually confirmed from Fig. 6 plotting joint shear stress vs. datum index. The solid line is constructed by rearranging the collected experimental data based on increasing order of the simplified joint shear strength model (mean strength). After that, the shaded region is determined by the mean ± one standard deviation interval. The experimental joint shear strength and the ACI 352R-02 joint shear strength are also shown by the void circles and x-marks, respectively. The mean strength of the simplified model passes through the center of the experimental joint shear strength results, which reconfirms that the simplified joint shear strength model is unbiased. The ACI 352R-02 model predicts the joint shear strength with larger scatter and bias compared to the simplified joint shear strength model.

The performance of the simplified joint shear strength model is also investigated by plotting the joint shear stress ratio vs. various examined parameters. The joint shear stress ratio (Tables 1-3) is calculated as the experimental joint shear stress divided by either the ACI 352R-02 or the simplified joint shear strength model, and the examined parameters are concrete compressive strength (f_c'), in-plane geometry (JP), beam-to-column width ratio (b_b/b_c), beam height to column depth ratio (h_b/h_c), joint transverse reinforcement index (JI), beam reinforcement index (BI), A_{sh} ratio, and spacing ratio. Because the simplified joint shear strength model is unbiased, the average of the joint shear ratios of experimental joint shear stress to the simplified joint shear strength model were always 1.0, and these ratios do not show any bias toward the examined parameters. On the other hand, the joint shear ratios of experimental joint shear stress to the ACI 352R-02 joint shear strength model were biased with respect to f_c', JP, JI, b_b/b_c, h_b/h_c, and spacing ratio.

For example, Fig. 7 plots joint shear stress ratios vs. concrete compressive strength and also shows the linear regression lines for the joint shear stress ratios. The range of experimental joint

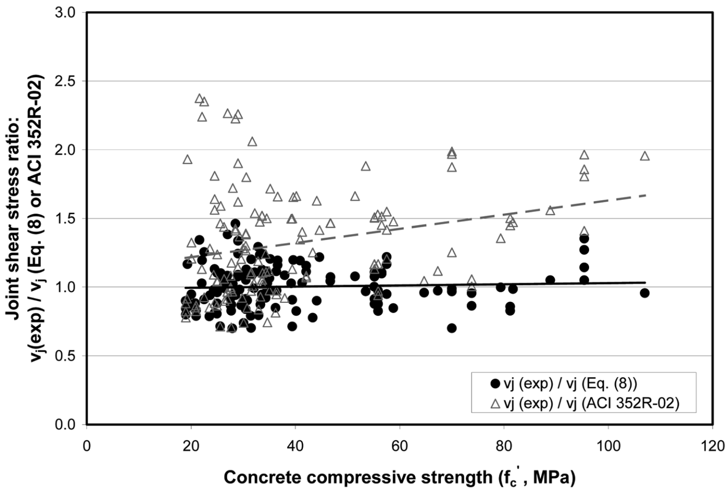


Fig. 7 Joint shear stress ratio vs. concrete compressive strength (f_c')

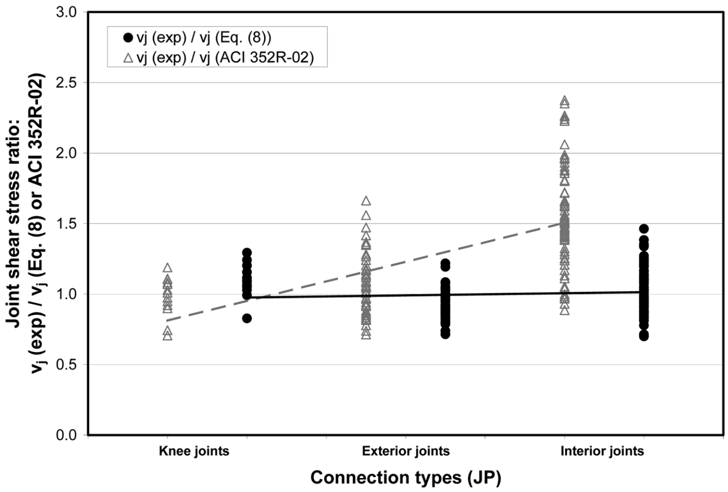


Fig. 8 Joint shear stress ratio vs. connection type (JP)

shear stress to the simplified model is from 0.70 to 1.46; whereas, the range of experimental joint shear stress to the ACI 352R-02 model is from 0.71 to 2.38 (i.e., the simplified joint shear strength model can significantly reduce the range of joint shear stress ratios compared to the ACI 352R-02 joint shear strength model). Different from the ACI 352R-02 model, the simplified model is unbiased toward concrete compressive strength. Similar to Fig. 7, Fig. 8 (plotting joint shear stress ratio vs. connection type) shows that the current ACI 352R-02 joint shear strength model is also biased in determining the joint shear strength according to various connection types, which can be adjusted by the proposed simplified joint shear strength model.

In summary, the performance of the simplified RC beam-column connection joint shear strength model (in comparison to the ACI 352R-02 joint shear strength model) is successfully demonstrated in three ways: evaluation of overall bias and scatter by Eq. (10), plotting joint shear stress vs. datum index, and plotting joint shear stress ratio vs. selected parameters. The simplified joint shear strength model distinctively improves accuracy and reduces scatter in determining joint shear strength compared to the current ACI 352R-02 joint shear strength model.

5. Conclusions

For RC beam-column connections (without out-of-plane members, without eccentricity between the beams and the column, and with proper confinement maintained within the joint panel) subjected to lateral earthquake loading, joint shear strength models have been constructed by using an introduced experimental database in conjunction with a Bayesian parameter estimation method. Important conclusions may be summarized as follows:

- An experimental database with joint shear failure (either in conjunction with or without yielding of beam reinforcement) is introduced. To exclude joint shear failure triggered by improper confinement within the joint panel, all included experimental specimens had equal to or above 70% of the typically recommended cross-sectional area of joint transverse reinforcement. From this database, it was observed that joint shear strength is primarily dependent on concrete compressive strength, and that in-plane geometry is also important in determining joint shear strength.
- Bayesian parameter estimation is introduced as a methodology to develop probabilistic joint shear strength models. The applied Bayesian parameter estimation method did not rely on starting with a deterministic capacity model and could identify the contribution of various influence parameters on joint shear capacity in a more explicit manner.
- Three types of joint shear strength models are developed. The first model is constructed by using all influence parameters on joint shear strength that are considered to be important. The second model uses the parameters surviving a step-wise removal process, which identifies insignificant parameters. The third model is a simplified form of the second model for convenient application to practical design. All three models are unbiased with similar levels of scatter. From examining the considered parameters in these models, concrete compressive strength is confirmed as the most important parameter affecting joint shear strength. In-plane geometry, beam reinforcement index, and joint transverse reinforcement index are also somewhat important, whereas beam-to-column width ratio, beam depth to column width ratio, A_{sh} ratio, and spacing ratio are fairly insignificant toward determining joint shear strength.
- The performance of the simplified joint shear strength model is evaluated by comparison with the ACI 352R-02 joint shear strength model. Evaluation of overall bias and scatter, as well as plotting joint shear stress vs. datum index and joint shear stress ratio vs. certain examined parameters, clearly indicates that the simplified joint shear strength model has distinctively reduced bias and scatter compared to the ACI 352R-02 joint shear strength model.
- Joint shear strength models are so far constructed only for subassemblies without out-of-plane members and without eccentricity between the beams and the column. An expanded experimental database for all types of RC beam-column connections (with/without out-of-plane members and/or eccentricity between the beams and the column) will eventually be constructed, and the procedures established herein can then also be applied to the construction of joint shear strength models applicable for all types of RC beam-column connections.

Acknowledgements

The authors wish to thank Graduate Research Assistant, Won Hee Kang in the Department of Civil and Environmental Engineering at the University of Illinois at Urbana-Champaign for his valuable assistance with the calculations for Bayesian parameter estimation.

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