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Ultimate flexural and shear capacity of concrete beams with corroded reinforcement

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Abstract. Assessment of structural behaviour of corrosion affected structures is an important issue, which would help in making certain decisions pertaining to the inspection, repair, strengthening, replacement and demolition of such structures. The paper presents formulations to predict the loss of weight and the loss of cross-sectional area of the reinforcing bar undergoing corrosion based on the earlier study carried out by the present authors (Bhargava *et al*. 2006). These formulations have further been used to analytically evaluate the ultimate bending moment and ultimate shear force capacity of the corroded concrete beams. Results of the present study indicate that, a considerably good agreement has been observed between the experimental and the analytically predicted values for the weight loss and reduction in radius of the corroded reinforcing bars. A considerably good agreement has also been observed between the experimental and the analytically predicted values of ultimate bending moment and ultimate shear force capacity for the corroded concrete beams.

Keywords: corrosion; concrete; ultimate bending moment; ultimate shear force.

1. Introduction

The premature structural deterioration because of the reinforcement corrosion is one of the major

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worldwide problems in reinforced concrete structures. The structural damage due to reinforcement corrosion may be caused by : (i) reduction of reinforcement cross-sectional area and its mechanical properties (Andrade *et al*. 1991), (ii) cracking and spalling of cover concrete due to the expansion of corrosion products (Andrade *et al*. 1993) and (iii) reduction of bond between the reinforcement and concrete (Rodriguez *et al*. 1994). The structural damage is caused sometimes to the extent that the structural failure becomes inevitable. Premature failure of reinforced concrete structures can therefore be prevented by proper control and monitoring of reinforcement corrosion. Decisions pertaining to the inspection, repair, strengthening, replacement and demolition of corrosion affected structures are generally governed by the assessment of performance of such structures to withstand the extreme events during their service life.

Load carrying capacity of the corroded concrete beams, columns and slabs have been studied earlier by various researchers (Okada *et al*. 1988, Uomoto and Misra 1988, Tachibana *et al*. 1990, Cabrera and Ghoddoussi 1992, Almusallam *et al*. 1996, Cabrera 1996, Almusallam *et al*. 1997, Huang and Yang 1997, Rodriguez *et al*. 1996, Rodriguez *et al*. 1997, Mangat and Elgarf 1999). These studies were mostly experimental in nature and very fewer efforts have been made towards the analytical predictions of the residual load carrying capacity of the corroded concrete structures. Therefore, it becomes necessary to develop formulations to reasonably predict the loss of weight and loss of cross-sectional area for the reinforcing bars undergoing corrosion and this in turn would be very useful in determining the residual load carrying capacity of corroded reinforced concrete structures particularly in flexure and shear. The present study differs from most of the earlier studies in a sense that it attempts to present the analytical solution for the estimation of residual load carrying capacity of the corroded concrete beams.

The paper presents formulations based on the earlier study carried out by the present authors (Bhargava *et al*. 2006) to predict loss of weight and the loss of cross-sectional area of the reinforcing bar in the corroded reinforced concrete structures. Further using these formulations, an attempt has been made for the analytical predictions of ultimate bending moment and ultimate shear force capacity of the corroded concrete beams. Although, procedures for the evaluation of ultimate bending moment and ultimate shear force capacity of RC beams are available (BIS 2000); the present paper highlights the relationship between the rebar corrosion and the remaining flexural and shear capacities of the RC beams. In the present study, the scope of the flexural failure is limited to either by the yielding of tensile reinforcement or by the crushing of concrete in compression zone. For the corroded concrete beams, both these flexural and shear failures would result basically due to the loss of reinforcement cross-section. However, the continued rebar corrosion would also affect the composite action of both concrete and rebar due to the deterioration of bond between them; the evaluation of ultimate flexural capacity due to loss of bond has been kept out of the scope of the present study. The performance of the presented formulations is then investigated through its ability to reproduce the results that are in line with the available experimental trends.

2. Previous investigations regarding load carrying capacity of corroded concrete beams

The effect of reinforcement corrosion on concrete structures has been studied extensively towards the monitoring techniques, rehabilitation and protection methods for the corroding concrete structures. Only few investigations are known to have dealt with the structural implications such as the loss of structural capacity due to corrosion of embedded steel. These investigations basically pertain to the experimental assessment of the structural capacity of corroded concrete beams. Some of the earlier experimental investigations are briefly described as follows to present the experimental assessment of the structural capacity for different loading cases and for different degrees of corrosion.

Okada *et al*. (1988) carried out tests on sound, cracked and repaired concrete beams. The reinforcement was corroded by spraying a chloride solution once a day for a period of 140-170 days. Cracks appeared on the concrete surface parallel to the main bars and the shear links with crack widths ranging between 0.05-0.15 mm. Beams were tested with different loading patterns such as static and reversed cyclic loading of constant deflection amplitude; and different shear span to effective depth ratios such as 3.4, 2.9 and 2.3. In deteriorated beams, the flexural cracks occurred in the zone of constant moment. It was also observed that the number of flexural shear cracks in the shear span was less than that in the sound beams. It was pointed out that, the bond deterioration was produced due to the presence of longitudinal cracks. Some reduction in the load carrying capacity of the corroded beams was also observed.

Uomoto and Misra (1988) carried out an experimental study on corroded beams and columns. Accelerated corrosion was induced by adding sodium chloride to the mixing water and applying a constant current density of about 280-380 $\mu A/cm^2$ for a period of 7-14 days. In one study, beams of 100 × 100 × 700 mm span, reinforced with two 10 mm diameter bars were tested and it was found that most of the beams were failed in shear. In another study, tests were carried out on beams of 100 × 200 × 2100 mm span, reinforced with two 16 mm diameter bottom bars, two 6 mm diameter top bars and shear links of 6 mm diameter at 170 mm spacing; with no links existed in the constant moment span. In this case, compression concrete failure due to flexure was observed along with the buckling of top bars. It was reported that, the reduction in load carrying capacity of the beams was not caused simply by the reduction of the effective area or the reduction in the strength of the reinforcing bars, but due to the cracks formed by the corrosion process. The weight loss of 1-2% in the bottom 16 mm diameter bars corresponded to 4-17% reduction in load carrying capacity of the corroded beams.

Tachibana *et al*. (1990) carried out tests on beams of 150 × 200 × 2000 mm span, reinforced with two 16 mm diameter bottom bars. The beams had no shear reinforcement. The shear and loading spans were 300 mm and 1500 mm respectively. An anodic current density of 500 $\mu A/cm^2$ was applied for a period of 3-15 days to achieve the accelerated corrosion. The maximum percentage of weight loss of reinforcement was about 5%. It was reported that, the non corroded and lowly corroded beams were failed in flexure with the yielding of reinforcement while highly corroded beams were failed in bond shear somewhat in a brittle manner.

Cabrera and Ghoddoussi (1992) carried out loading tests on beams of 125 × 165 × 1000 mm span, reinforced with two 12 mm diameter bottom bars, two 10 mm diameter top bars and shear links of 8 mm diameter at 40 mm spacing along a shear span of 384 mm. The bottom bars were corroded by applying a current density of unknown value for a period of up to 40 days, because a potentiostatic procedure was used instead of galvanostatic procedure. It was reported that, about 9% reduction in the cross section of the bottom bars resulted in about 20% reduction of ultimate bending moment and about 40% increase of mid span deflection at service loads.

Almusallam *et al.* (1996, 1997) carried out tests on beams of 150 × 150 × 1000 mm span and slabs of 310 × 60 × 710 mm span. Three degradation levels were studied namely precracking, cracking and postcracking. In both beams and slabs, the capacity increased by about 20% during the precracking stage when the percent mass loss was about 2%. The study mentioned that, this was due to the decrease of porosity at the bar surroundings during corrosion because the corrosion products fill the pores around the bars before any steel-concrete interface pressure is applied to the concrete. During the cracking and post cracking stages, the capacities of the beams and slabs were reduced to levels of 23-80% of the capacities of the non corroded control specimens.

Huang and Yang (1997) carried out tests on beams of 150 × 150 × 5000 mm span, reinforced with two 16 mm diameter bottom bars. The beams had no shear reinforcement. Corrosion was induced by an anodic current density of 5 A/mm^2. The specimens were fully immersed in artificial seawater during current application. Exposure was for 126 hours. The results showed a decrease of 48% in ultimate moment capacity after the accelerated corrosion exposure.

Rodriguez *et al*. (1997) carried out tests on beams of 150 × 200 × 2000 mm span, reinforced with two 10 mm diameter or four 12 mm diameter bottom bars, two or four 8 mm diameter top bars and shear links of 6 mm diameter at 85 or 170 mm spacing. To induce corrosion in the embedded bars, the concrete was contaminated with calcium chloride at a dosage of 3% by mass of the cement. By applying a constant anodic current density of 100 μA/cm^2 for a period of 101-190 days, further acceleration of the corrosion process was achieved. Different reinforcement configurations in the corroded beams resulted in different failure mechanisms such as flexural failure with yielding of bottom tensile reinforcement, flexural failure with crushing of concrete in compression zone, shear failure and shear combined with anchorage failure of bottom tensile bars. About 30% reduction in the bending moment capacity was observed in the corroded beams.

The present study differs from the aforementioned earlier studies in a sense that it attempts to analytically evaluate the structural capacity of corroded concrete beams. A comparison between the results obtained in this study and the earlier ones is also discussed in this paper.

3. Analytical formulations for the loss of weight and cross-section of reinforcement due to corrosion

Corrosion process is a dynamic process and as the rust layer grows thicker, the ionic diffusion distance increases and the rate of rust production decreases because the diffusion is inversely proportional to oxide thickness (Liu and Weyers 1998a). The growth of expansive corrosion products is given by following equation (Liu and Weyers 1998a, Bhargava *et al*. 2003, 2005, 2006).

$$\frac{dW_r}{dt} = \frac{k_p}{W_r} \tag{1}$$

Where, 'W_r' is the mass of expansive corrosion products per unit length of the reinforcement in mg/mm; 'k_p' is related to rate of metal loss and expressed as follows by Eq. (2) (Bhargava *et al*. 2006).

$$k_p = A_p \pi D_i i_{COR} \tag{2}$$

Where, 'A_p' is a coefficient related to the rate of metal loss; 'D_i' is the initial diameter of reinforcement in mm and 'i_{COR}' is the annual mean corrosion rate in μA/cm^2.

The mass of steel 'W_s' per unit length of the reinforcement (mg/mm) getting consumed by corrosion process is defined by the following relationship (Bazant 1979, Liu and Weyers 1998a, Bhargava *et al.* 2003, 2005, 2006).

$$W_s = \alpha W_r \tag{3}$$

Where, 'α' is the ratio of molecular weight of the iron to that of the corrosion products. For a constant corrosion rate, the integration of Eq. (1) would result in the following.

$$T = \frac{W_r^2}{2k_p} \tag{4}$$

Where, 'T' is the time to corrosion in years and at the incidence of cover cracking known as time to cover cracking (Liu and Weyers 1998a, Bhargava *et al.* 2003, 2005, 2006).

The reasonable estimates for 'A_p', and 'α' are based on the available experimental data of Liu (1996); which was chosen because it involves the outdoor corrosion testing of reinforced concrete members wherein the members are exposed to the actual environmental conditions over a longer period of time thereby considering the influence of various environmental factors such as temperature, relative humidity, rainfall and properties of concrete on the corrosion rate (Bhargava *et al.* 2006). Following reasonable estimates of the coefficients 'A_p' and 'α' are recommended (Bhargava *et al.* 2006).

$$A_p = 2.48614; \quad \alpha = 0.61309 \tag{5}$$

After knowing 'A_p', and 'α', 'W_s' at any corrosion time 'T' is evaluated as follows by making use of Eq. (2) to Eq. (4).

$$Ws = 2.42362\sqrt{D_i i_{COR} T} \tag{6}$$

By considering the mass density of steel as 7.86 mg/mm^3, the area of steel 'A_{cor}' (mm^2) getting consumed in the corrosion process and the reduction in bar radius 'X' (mm) at any time 'T' are evaluated as follows.

$$A_{cor} = 0.30835\sqrt{D_i i_{COR} T} \tag{7}$$

$$X = \frac{1}{2}[D_i - \sqrt{D_i^2 - 0.39245\sqrt{D_i i_{COR} T}}] \tag{8}$$

In the present study, the analytical predictions for 'Ws' and 'X' for the available experimental data (Rasheeduzzafar *et al.* 1992, Andrade *et al.* 1993, Liu 1996, Rodriguez *et al.* 1997, Mangat and Elgarf 1999, Torres-Acosta 1999) have been carried out using Eqs. (6) and (8) respectively. To test the goodness of the Eqs. (6) and (8), the correlation between the predicted and experimental values has been estimated. Assuming that $x = Ws^P$ (the independent variable) and $y = Ws^E$ (the dependent variable), the values of both r^2_{xy} and $s^2_{y.x}$ were calculated as 0.895 and 0.126 gm^2/cm respectively; wherein 'r' is the coefficient of correlation between x and y; 's' is the root mean square error of estimate of y on x and the superscripts 'P' and 'E' correspond to the analytically predicted and the experimental observed values respectively. Similarly, by assuming $x = X^P$ and $y = X^E$, the values of both r^2_{xy} and $s^2_{y.x}$ were calculated as 0.851 and 0.028 mm^2 respectively. Fig. 1(a) presents the comparison between W_s^P and W_s^E for the available experimental data. Similarly, Fig. 1(b) presents

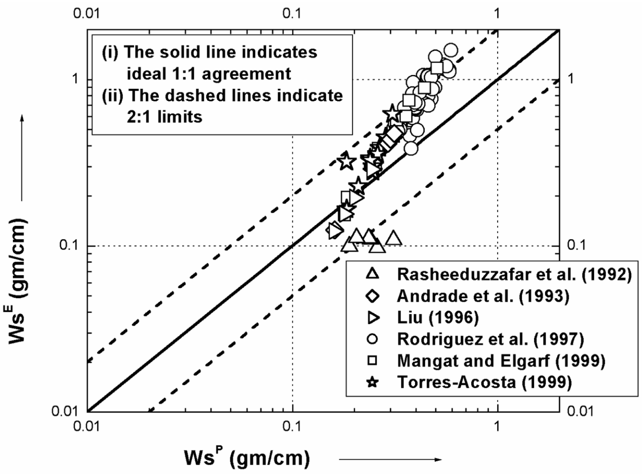

Fig. 1(a) Comparison between experimental Ws^E and Predicted Ws^P using Eq. (6)

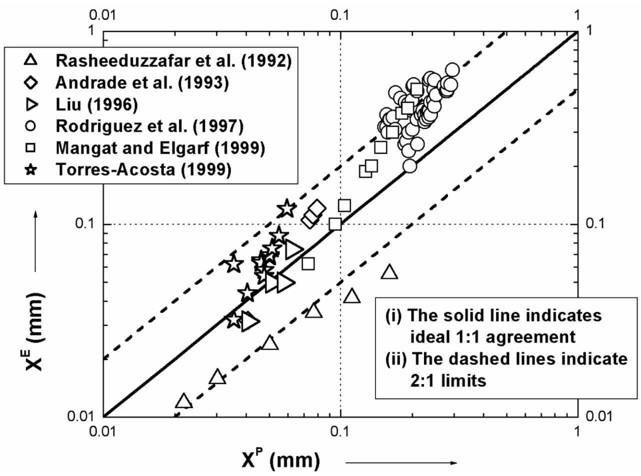

Fig. 1(b) Comparison between experimental X^E and Predicted X^P using Eq. (8)

the comparison between X^P and X^E for the same experimental data. The data in both the figures are presented by different symbols to represent the analytical predictions made for different experimental data. It is clear from the same figures that the deviations between the analytically predicted and the experimentally observed values is generally less than by a factor of two and this is a considerably good agreement in view of the large variability associated with the corrosion phenomena. Also, the quite high values associated with r^2_{xy} in both the predictions suggest that Eqs. (6) and (8) can be effectively used for estimating the values of '*Ws*' and '*X*' for the reinforced concrete members exposed to the corrosive environment.

The difference between the experimentally observed values and analytical predictions are attributed to the following reasons:

- In analytical predictions, the value of 'i_{COR}' is assumed as a constant single value. However, in concrete structures, the value of 'i_{COR}' is believed to be changing during the corrosion propagation stage (Liu and Weyers 1998b, Vu and Stewart 2000, Li 2001), and it is difficult to analytically model this variation of 'i_{COR}' with time.

- Deviations in the specified material properties for the concrete such as modulus of elasticity and tensile strength, may affect the crack propagation in the concrete (Bhargava *et al*. 2006). The corrosion rate gets affected depending on the diffusion of chloride ions during the crack propagation stage. It is noted that, deviations in the material properties of concrete may occur due to variations in the mix proportions, presence of impurities and inadequate compaction or curing.
- In concrete, the corrosion takes place mostly as pitting corrosion and therefore it is non-uniform while the corrosion-cracking model proposed by the present authors (Bhargava *et al*. 2006) assumes the uniform corrosion of the reinforcing bar. This model is further used for proposing the relationships for 'Ws' and 'X'.
- The other possible factors include the electrical properties of the minerals in concrete, composition of the reinforcing bar (it is assumed that the bar is made up of pure iron) and the presence of deposits in the salt solution. Also, the experimental data have been taken from different sources having different testing procedures and the impressed corrosion current in the experiment. All these factors are expected to affect the corrosion rate.

In view of the aforementioned reasons, it is clear that there has been a considerably good agreement between the experimentally observed and analytically predicted values of 'W_s' and 'X' considering the large variability associated with the corrosion phenomena itself. The 'A_{cor}' can also be predicted making use of Eq. (7).

4. Formulation for ultimate capacity of corroded rc beams failing in flexure

The ultimate capacity of the RC beams designed to fail in flexure has been calculated after evaluating the loss of cross-sectional area for the reinforcing bars using Eq. (7). Consider a simply

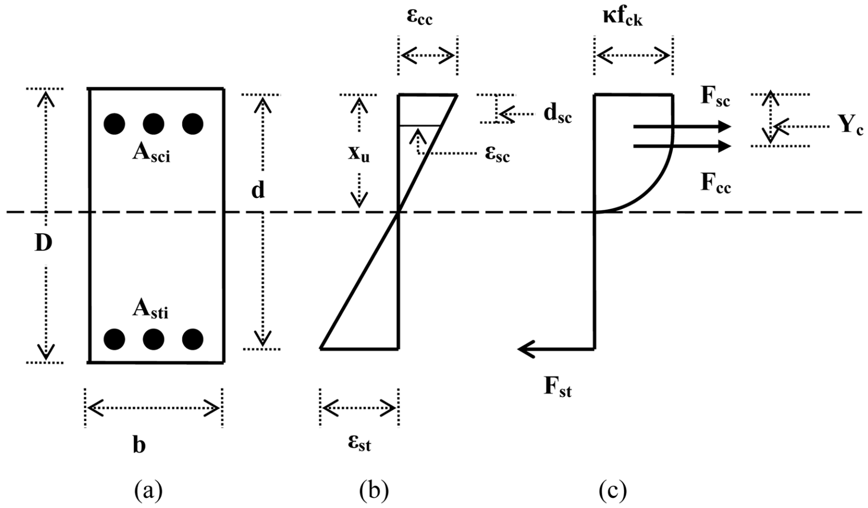

Fig. 2 Formulation of flexural strength of reinforced concrete beams (BIS 2000): (a) typical reinforced concrete beam section, (b) strain distribution, (c) stress distribution

supported reinforced concrete (RC) beam subjected to flexure under loads. Fig. 2(a) shows the beam section in which the beam is reinforced with bottom tensile reinforcing steel bars having initial area 'A_{sti}' and top compressive reinforcing steel bars having initial area 'A_{sci}'. 'b' and 'D' are the width and depth of the beam; 'D_{it}' and 'D_{ic}' are the initial diameters of the tensile and compressive reinforcing bars. The distance between the centroid of tensile steel and the edge of the compression zone is 'd' (also known as effective depth). Figs. 2(b) and 2(c) present the strain and stress distribution across the cross-section of the beam; wherein 'ε_{cc}', 'ε_{st}' and 'ε_{sc}' are the strains in concrete, tensile steel and compressive steel and 'f_{ck}' is the 28 days characteristic compressive strength of concrete (BIS 2000).

To evaluate the ultimate bending moment capacity of the reinforced concrete beam, 'ε_{cc}' has been considered as 0.0035 (BIS 2000). Considering the simple bending theory, 'ε_{st}' and 'ε_{sc}' are given as follows.

$$\varepsilon_{st} = \left(\frac{d-x_u}{x_u}\right)\varepsilon_{cc}; \quad \varepsilon_{sc} = \left(\frac{x_u-d_{sc}}{x_u}\right)\varepsilon_{cc} \tag{9}$$

Where, 'x_u' is the height of compression zone and 'd_{sc}' is the distance between the centroid of the compression steel and the edge of the compression zone. The total force of compression is given as follows.

$$F_c = F_{cc} + F_{sc} \tag{10}$$

Where, 'F_{cc}' is the force of compression in concrete and 'F_{sc}' is the force of compression in the compressive steel. Considering a parabolic stress-strain relationship for the concrete and the stress distribution across the cross-section of the beam as shown in Fig. 2(c) (BIS 2000), 'F_{cc}' and its point of application 'Y_c' from the edge of compression zone are given as follows.

$$F_{cc} = \kappa f_{ck} b x_u\left[\frac{3\varepsilon_{cc}-0.002}{3\varepsilon_{cc}}\right] \tag{11}$$

Where, 'κ' is a factor which is decided based on the design compressive strength of the concrete in the structures and the partial safety factor appropriate to the material strength of concrete and in the present study it is taken as 0.67 (BIS 2000).

$$Y_c = x_u - x_u\left[\frac{6\varepsilon_{cc}-\left(\frac{0.000004}{\varepsilon_{cc}}\right)}{12\varepsilon_{cc}-0.008}\right] \tag{12}$$

The force of compression in compressive steel 'F_{sc}' is given as follows.

$$F_{sc} = f_{sc}A_{sc} \tag{13}$$

The force of tension 'F_{st}' in tensile steel is given as follows.

$$F_{st} = f_{st}A_{st} \tag{14}$$

Where, 'f_{sc}' and 'f_{st}' are the stresses in compressive and tensile steels respectively; 'A_{sc}' and 'A_{st}'

are the areas of compressive and tensile steels respectively. 'f_{sc}' and 'f_{st}' are determined corresponding to 'ε_{sc}' and 'ε_{st}' from the stress-strain curves for steel as described in BIS (2000). Both 'f_{sc}' and 'f_{st}' are limited to 'ηf_y'; where, 'f_y' is the yield strength of the steel; 'η' is a factor which is decided based on the partial safety factor appropriate to the material strength of steel and in the present study it is taken as 1.0 (BIS 2000).

By equating the force of compression given by Eq. (10) and the force of tension given by Eq. (14), the height of compression zone 'x_u' can be evaluated. The ultimate bending moment capacity is then determined as follows.

$$Xu = F_{cc}(d - Y_c) + F_{sc}(d - d_{sc}) \tag{15}$$

A scheme of the procedure to evaluate the ultimate bending moment capacity of the corroded RC beams corresponding to any corrosion time 'T' is shown in the Fig. 3. In the same figure, 'A_{cort}' and 'A_{corc}' are the areas of steel getting consumed in the corrosion process for tension and compression reinforcements at any time 'T' and are evaluated using Eq. (7) by putting 'D_{it}' and 'D_{ic}' in place of 'D_i' respectively; 'A_{stR}' and 'A_{scR}' are the reduced areas of steel for tension and compression reinforcements at any time 'T'.

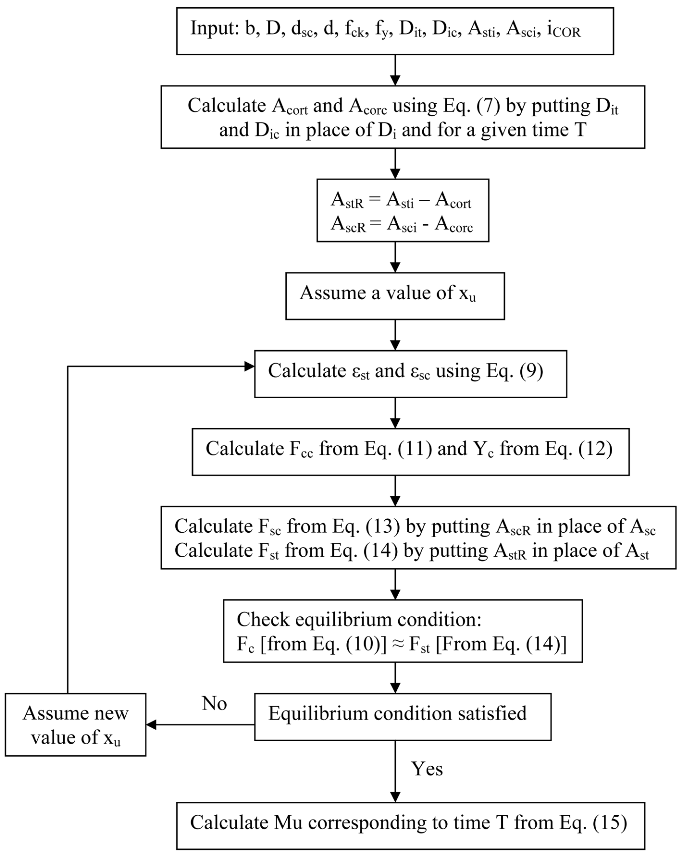

Fig. 3 Evaluation of ultimate bending moment capacity corresponding to any corrosion time 'T'

5. Formulation for ultimate capacity of corroded RC beams failing in shear

The ultimate capacity of the RC beams designed to fail in shear has been calculated after evaluating the loss of cross-sectional area for the reinforcing bars using Eq. (7). Consider a simply supported reinforced concrete (RC) beam subjected to shear under loads. The shear strength of the concrete is given as follows (BIS 2000).

$$\tau_c = \frac{0.85\sqrt{0.8 f_{ck}}(\sqrt{1+5\beta}-1)}{6\beta} \tag{16a}$$

$$\beta = \frac{0.8 f_{ck}}{6.89 P_t} \tag{16b}$$

$$P_t = \frac{100 A_{st}}{bd} \tag{16c}$$

The shear capacity of concrete is given as follows (BIS 2000).

$$V_{uc} = \tau_c bd \tag{17}$$

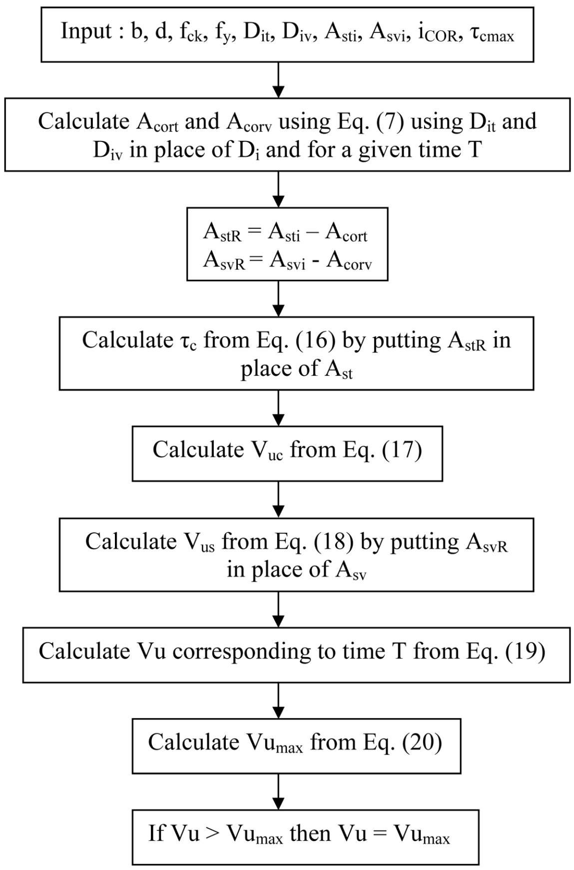

Fig. 4 Evaluation of ultimate shear force capacity corresponding to any corrosion time '*T*'

The shear capacity of the shear stirrups is given as follows (BIS 2000).

$$V_{us} = \frac{\eta f_y A_{sv} d}{S_v} \tag{18}$$

Where, 'A_{sv}' is the area of shear stirrups and 'S_v' is the spacing of shear stirrups.
The total shear capacity of the RC beam section is given as follows.

$$Vu = V_{uc} + V_{us} \tag{19}$$

The maximum value of the shear capacity of the RC beam section is given as follows.

$$Vu_{\max} = \tau_{c\,\max} bd \tag{20}$$

Where, '$\tau_{c\max}$' is the maximum shear stress as described in BIS (2000) for different 'f_{ck}'.

A scheme of the procedure to evaluate the ultimate shear force capacity of the corroded RC beams corresponding to any corrosion time 'T' is shown in the Fig. 4. In the same figure, 'A_{corv}' is the area of steel getting consumed in the corrosion process for shear reinforcement at any time 'T' and is evaluated using Eq. (7) by putting 'D_{iv}' in place of 'D_i'; 'A_{svR}' is the reduced area of steel for shear reinforcement at any time 'T'.

6. Comparison with test results

Analytical predictions have been presented for the loss of weight and reduction in radius of the reinforcing bars for the available experimental data (Rasheeduzzafar *et al.* 1992, Andrade *et al.* 1993, Liu 1996, Rodriguez *et al.* 1997, Mangat and Elgarf 1999, Torres-Acosta 1999) using the formulations proposed in Eqs. (6) and (8). These results are presented in Figs. 1(a) and 1(b). As discussed in section 3.0, Eqs. (6) and (8) can be effectively used for estimating the values of 'Ws' and 'X' for the reinforced concrete members exposed to the corrosive environment. Meanwhile good agreement between the calculated and experimental corrosion penetration depths corresponding to through thickness cracking of the cover concrete has also been reported by the present authors in one of their earlier study (Bhargava *et al.* 2006) using the corrosion cracking model which forms the basis of the present study for the evaluation of 'Ws' and 'X'.

Analytical formulations and the proposed methodology for the evaluation of ultimate capacity of corroded RC beams in flexure and shear as described under sections 4.0 and 5.0 and shown in Figs. 3 and 4 have been used to predict the residual load carrying capacity of some of the degraded RC beams for which the experimentally observed trends are available (Rodriguez *et al.* 1997). Rodriguez *et al.* (1997) tested five different types of corroded RC beams of sections 150 mm × 200 mm with spans ranging from 2050 mm to 2300 mm. The beams were provided with different ratios of tensile and compressive reinforcements, different spacing of shear stirrups and different locations of curtailment of tensile reinforcing bars. The various reinforcing bars were submitted to a constant current density of about 100 $\mu A/cm^2$ applied on the concrete surface through the stainless steel counter electrodes for a period of 101-190 days. After corroding the bars till the planned deterioration level, a detailed map of the concrete cracking was obtained for each beam. The attack penetration i.e. reduction in bar radius produced at the main bars and the shear stirrups was

Table 1 Characteristics of the tested corroded concrete beams (Rodriguez *et al*. 1997)

Beam type	Concrete compressive strength (*)	Reinforcement details		
		Tensile bars	Compressive bars	Shear stirrups
11	34	2-Φ10	2-Φ8	Φ6/17 cm
12	35	4-Φ12	2-Φ8	Φ6/17 cm
13	37	2-Φ12 + 2-Φ12 (–)	2-Φ8	Φ6/17 cm
21	35	4-Φ12	4-Φ8	Φ6/17 cm
31	37	4-Φ12	4-Φ8	Φ6/8.5 cm

(*) : Compressive strength at the date of the loading test in MPa.
(–) : 2-Φ12 mm cut-off bars at appropriate locations.

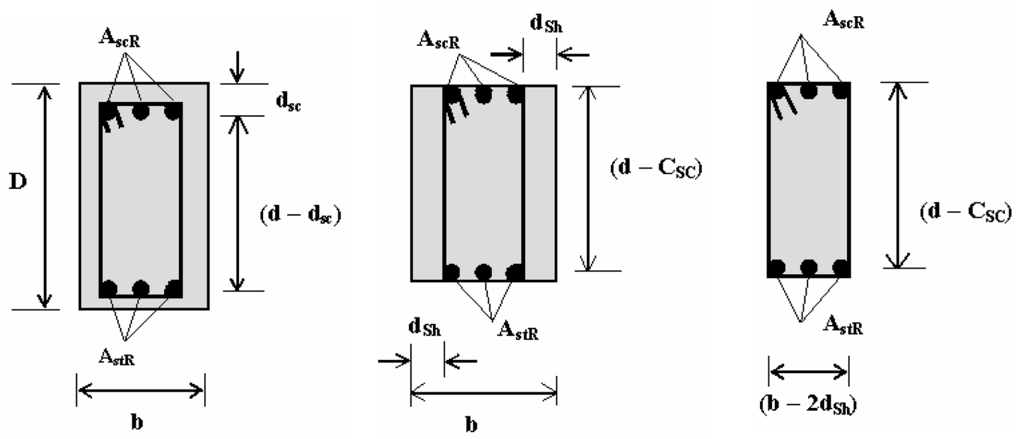

Fig. 5 Schemes of deteriorated reinforced concrete sections (Rodriguez *et al*. 1997)

recorded at the end of the corrosion period. After having corroded the reinforcement, the beams were tested up to the failure. Table 1 summarizes the characteristics of the tested concrete beams.

In the present study, the areas of steel 'A_{cort}', 'A_{corc}' and 'A_{corv}' getting consumed in the corrosion process for tension, compression and shear reinforcements at any time 'T' have been evaluated from Eq. (7); wherein 'T' has been considered as 101-190 days for different tested beams as given in the reference literature (Rodriguez *et al.* 1997). In the reference literature it is also mentioned that some of the type 11 and type 31 beams were kept under natural corrosion conditions for other 180 days. The residual ultimate bending moment capacity and ultimate shear force capacity have then been evaluated using the formulations and the methodology described in sections 4.0 and 5.0 and shown in Figs. 3 and 4.

The analytical predictions for the residual ultimate bending moment and shear force capacity have been made by considering the different schemes of deteriorated reinforced concrete sections as shown in Fig. 5 (Rodriguez *et al.* 1997). The section 1 indicates the reduced section of the main bars at pitting corrosion and the intact concrete section with effective depth being 'd'. The section 2 indicates the reduced section of the main bars at pitting corrosion and the reduced concrete section with effective depth being '$(d - C_{SC})$'; where 'C_{SC}' is clear cover to the compression steel. The section 3 indicates the reduced section of the main bars at pitting corrosion and the reduced

Table 2(a) Comparison of experimental and analytically predicted ultimate bending moments for corroded concrete beams (Rodriguez *et al*. 1997)

Beam details			Ultimate bending moment (kN-m)			
Type	No.	Corrosion time (Days)	Mu^E	Mu^P Section 1	Mu^P Section 2	Mu^P Section 3
11	115	101	11.6	13.551 (1.168)	11.886 (1.025)	11.813 (1.018)
	114	117	10.5	13.486 (1.284)	11.803 (1.124)	11.750 (1.119)
	113	160	10.1	13.329 (1.320)	11.653 (1.154)	11.600 (1.149)
12	126	104	29.0	35.970 (1.240)	31.522 (1.087)	28.770 (0.992)
	123	115	27.2	35.883 (1.319)	31.442 (1.156)	28.721 (1.056)
13	134	108	25.3	33.833 (1.337)	29.513 (1.167)	27.168 (1.074)
	133	116	25.2	33.688 (1.337)	29.389 (1.166)	27.045 (1.073)
	135	175	24.7	32.742 (1.327)	28.580 (1.157)	26.251 (1.063)
21	215	108	28.2	36.419 (1.292)	32.741 (1.161)	29.501 (1.046)
	216	127	26.4	36.266 (1.374)	32.449 (1.229)	29.215 (1.107)
31	313	111	28.2	36.588 (1.297)	33.553 (1.190)	32.484 (1.152)
	314	128	28.5	36.452 (1.279)	33.415 (1.173)	32.348 (1.135)
	316	164	27.5	36.190 (1.316)	33.149 (1.205)	32.086 (1.167)

Note : R_m is presented within brackets where $R_m = (Mu^P/Mu^E)$
Mu^P is the predicted ultimate bending moment capacity
Mu^E is the experimental ultimate bending moment capacity

Table 2(b) Comparison of experimental and analytically predicted ultimate shear force for corroded concrete beams (Rodriguez *et al*. 1997)

Beam details			Ultimate shear force (kN)			
Type	No.	Corrosion time (Days)	Vu^E	Vu^P Section 1	Vu^P Section 2	Vu^P Section 3
11	115	101	15.9	18.563 (1.167)	16.255 (1.022)	16.182 (1.018)
	114	117	14.4	18.474 (1.283)	16.168 (1.123)	16.096 (1.118)
	113	160	13.9	18.259 (1.314)	15.963 (1.148)	15.891 (1.143)
12	126	104	39.8	49.274 (1.238)	43.181 (1.085)	39.411 (0.990)
	123	115	37.3	49.155 (1.318)	43.071 (1.155)	39.344 (1.055)
13	134	108	34.6	46.347 (1.340)	40.429 (1.168)	37.216 (1.076)
	133	116	34.5	46.148 (1.338)	40.259 (1.167)	37.048 (1.074)
	135	175	33.9	44.852 (1.323)	39.151 (1.155)	35.960 (1.061)
21	215	108	38.6	49.889 (1.292)	44.851 (1.162)	40.412 (1.047)
	216	127	36.2	49.679 (1.372)	44.451 (1.228)	40.021 (1.106)
31	313	111	38.7	50.121 (1.295)	45.963 (1.188)	44.499 (1.150)
	314	128	39.0	49.934 (1.280)	45.774 (1.174)	44.312 (1.136)
	316	164	37.7	49.575 (1.315)	45.410 (1.205)	43.953 (1.166)

Note : R_v is presented within brackets where $R_v = (Vu^P/Vu^E)$
Vu^P is the predicted ultimate shear force capacity
Vu^E is the experimental ultimate shear force capacity

concrete section with effective depth being '$(d - C_{SC})$' and effective width being '$(b - 2d_{sh})$'; where 'd_{sh}' is the clear cover to the shear stirrups. In corroded concrete beams the sections 2 and 3 may represent the realistic deteriorated scenario because of the peeling of concrete cover due to the action of expansive corrosion products for the cases where $C > (S - D_i)/2$ (Bazant 1979); wherein 'C' is the clear cover to reinforcement, 'S' is the spacing of reinforcement and 'D_i' is the initial diameter of the reinforcement. However, the results with section 1 deteriorated scenario are also presented in the present study to reflect the upper bound values for the ultimate bending moment and ultimate shear force and for the comparison purposes.

The results of the analytical predictions have been presented in Tables 2(a) and 2(b). Table 2(a) presents the comparison of experimental and analytically predicted ultimate bending moment capacity in type 11, 12, 13, 21 and 31 beams with corroded reinforcements. Table 2(b) presents the comparison of experimental and analytically predicted ultimate shear force capacity for the same beams. The same tables also present 'R_m' and 'R_v'. 'R_m' is the ratio of predicted ultimate bending moment capacity (Mu^P) to the experimental ultimate bending moment capacity (Mu^E). 'R_v' is the ratio of predicted ultimate shear force capacity (Vu^P) to the experimental ultimate shear force capacity (Vu^E). The same tables depict the following:

(a) Increase in corrosion time would result in the reduction of ultimate bending moment and shear force capacity for the beams for all the schemes of the deteriorated reinforced concrete sections as shown in Fig. 5.
(b) For the section 1 deteriorated scheme 'R_m' is found to vary between 1.168 and 1.374; 'R_v' is found to vary between 1.167 and 1.372. For most of the beams 'R_m' and 'R_v' is less than 1.32.
(c) For the section 2 deteriorated scheme 'R_m' is found to vary between 1.025 and 1.229; 'R_v' is found to vary between 1.022 and 1.228. For most of the beams 'R_m' and 'R_v' is less than 1.17. This shows good agreement between the experimental and predicted values.
(d) For the section 3 deteriorated scheme 'R_m' is found to vary between 0.992 and 1.167; 'R_v' is found to vary between 0.990 and 1.166. For most of the beams 'R_m' and 'R_v' is less than 1.08. This also shows good agreement between the experimental and predicted values.

In the present study the reduction of ultimate bending moment and shear force capacities with increase in corrosion time is mainly attributed to the increase in loss of cross-section of the reinforcing bars with the increase in the corrosion time. It is also clear from the tables that, for most of the beams Mu^P and Vu^P are in good agreement with Mu^E and Vu^E for the section 2 and 3 schemes as compared to the section 1 scheme for deteriorated reinforced concrete sections on account of the lesser 'R_m' and 'R_v' associated with them. This is because of the assumption of larger sectional dimensions of the corroded concrete beams for the section 1 scheme. These results highlight the fact that Eq. (7) predicts reasonably good estimates of the loss of cross-section for the reinforcing bars as a function of corrosion time. This further emphasizes that; it is possible to make reasonable analytical estimates of residual ultimate bending moment and shear force capacities for corrosion damaged concrete beams by considering the reduced sections for both steel and concrete; thereby also confirming the realistic scenario of cover peeling in actual structures due to reinforcement corrosion as assumed in section 2 and 3 schemes for deteriorated reinforced concrete sections. The similar analytical trends have also been reported by Rodriguez *et al.* (1997) in their study. However, in the reference literature, steady state linear function was adopted to evaluate the loss of steel area. The present study considers a non-linear function to express the growth of corrosion products and this is an improvement over the steady state linear function for the growth of corrosion products (Bhargava *et al.* 2006).

The difference between the actual experimental and analytically predicted values in Tables 2(a) and 2(b) may be mainly attributed to various factors such as the variability in materials and their mechanical properties, impressed current variability, deposits in salt solution, local loss of concrete cover in experiments etc. One more factor that would also possibly be responsible for the difference in experimentally observed and analytically predicted values can be the nature of corrosion taking place in the concrete. In concrete the corrosion takes place mostly as a pitting corrosion around the surface and along the length of the reinforcement and therefore it is non-uniform while in the model proposed by the present authors (Bhargava *et al.* 2006) the uniform corrosion has been assumed around the surface and along the length of the reinforcement. Therefore, considering the large variability associated with the corrosion phenomena itself, the proposed formulations predict the analytical trends which are in considerably good agreement with those of the observed experimental trends.

7. Conclusions

The paper presents the analytical formulations to predict the loss of weight and loss of cross-section of reinforcement due to corrosion upto and beyond the period of cover cracking. These formulations are proposed considering the growth of corrosion products as a non-linear function which is an improvement over the steady state linear function for the growth of corrosion products (Bhargava *et al.* 2006). Results indicate that the Eqs. (6)-(8) can be considered as good estimates for 'W_s', 'A_{cor}' and 'X' respectively.

It has been found that both experimentally observed and analytically predicted values of ultimate bending moment and shear force are in good agreement for the sections 2 and 3 schemes of the deteriorated RC sections (Fig. 5). Thus it is possible to make reasonable analytical estimates of residual ultimate bending moment and shear force capacity for corrosion damaged concrete beams, by considering the reduced sections for both steel and concrete. Similar results were also reported by Rodriguez *et al.* (1996) for corrosion damaged RC beams; however, they adopted steady state linear function to evaluate the loss of steel area.

The present study could be very useful for predicting the remaining service-life of RC structures serving in aggressive environment by taking into account the deterioration of cross-section for concrete and steel due to reinforcement corrosion. In this regard, the earlier study by the present authors (Bhargava *et al.* 2006) could be very useful in evaluating the time of concrete cover peeling due to the action of expansive corrosion products based on the failure mode condition of cover peeling given by Bazant (1979).

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References

Almusallam, A.A., Al-Gahtani, A.S., Aziz, A.R. and Rasheeduzzafar (1996), "Effect of reinforcement corrosion on bond strength", *Constr. Build. Mater.*, **10**(2), 123-129.

Almusallam, A.A., Al-Gahtani, A.S., Maslehuddin, M., Khan, M.M. and Aziz, A.R. (1997), "Evaluation of repair materials for functional improvement of slabs and beams with corroded reinforcement", *Proc. Ins. Civil Eng., Struct. Build.*, **122**, 27-34.

Andrade, C., Alonso, C., Garcia, D. and Rodriguez, J. (1991), "Remaining lifetime of reinforced concrete structures : effect of corrosion on the mechanical properties of steel", *Proc. Int. Conf. on 'Life Prediction of Corrodible Structures'*, Cambridge, UK, 12/1-12/11.

Andrade, C., Alonso, C. and Molina, F.J. (1993), "Cover cracking as a function of rebar corrosion : Part I – experimental test", *J. Mat. and Struct.,* Paris, **26**, 453-464.

Bazant, Z.P. (1979), "Physical model for steel corrosion in concrete sea structures – application", *J. Struct. Div.,* ASCE, **105**(6), 1155-1166.

Bhargava Kapilesh, Ghosh, A.K., Mori Yasuhiro and Ramanujam, S. (2003), "Analytical model of corrosion-induced cracking of concrete considering the stiffness of reinforcement", *Struct. Eng. Mech.,* **16**(6), 749-769.

Bhargava, Kapilesh, Ghosh, A.K., Mori, Y. and Ramanujam, S. (2005), "Modeling of time to corrosion-induced cover cracking in reinforced concrete structures", *Cement Concrete Res.*, **35**(11), 2203-2218.

Bhargava, Kapilesh, Ghosh, A.K., Mori, Y. and Ramanujam, S. (2006), "Model for cover cracking due to rebar corrosion in RC structures", *Eng. Struct.,* **28**(8), 1093-1109.

BIS (2000), "IS 456 : 2000 – Plain and reinforced concrete : Code of practice", Bureau of Indian Standards, New Delhi, India.

Cabrera, J.G. and Ghoddoussi, P. (1992), "The effect of reinforcement corrosion on the strength of the steel/concrete bond", *Proc. Int. Conf. on Bond in Concrete*, CEB, Riga, Latvia, 10/11-10/24.

Cabrera (1996), "Deterioration of concrete due to reinforcement steel corrosion", *Cement Concrete Comp.*, **18**, 47-59.

Huang, R. and Yang, C.C. (1997), "Condition assessment of reinforced concrete beams relative to reinforcement corrosion", *Cement Concrete Comp.*, **19**, 131-137.

Li, C.Q. (2001), "Initiation of chloride-induced reinforcement corrosion in concrete structural members – Experimentations", *ACI Struct. J.*, **98**(4), 502-510.

Liu, Y. (1996), *"Modeling the Time to Corrosion Cracking of the Cover Concrete in Chloride Contaminated Reinforced Concrete Structures"*, Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Liu, Y. and Weyers, R.E. (1998a), "Modelling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures", *ACI Mat. J.,* **95**(6), 675-681.

Liu, T. and Weyers, R.E. (1998b), "Modelling the dynamic corrosion process in chloride contaminated concrete structures", *Cement Concrete Res.,* **28**(3), 365-379.

Mangat, Pritpal S. and Elgarf, Mahmoud S. (1999), "Flexural strength of concrete beams with corroding reinforcement", *ACI Struct. J.*, **96**(1), 149-158.

Okada, K., Kobayashi, K. and Miyagawa, T. (1988), "Influence of longitudinal cracking due to reinforcement corrosion on characteristics of reinforced concrete members", *ACI Struct. J.*, **85**(2), 134-140.

Rasheeduzzafar, Al-Saadoun, S.S. and Al-Gahtani, A.S. (1992), "Corrosion cracking in relation to bar diameter, cover and concrete quality", *J. Mater. Civil. Eng.*, ASCE, **4**(4), 327-343.

Rodriguez, J., Ortega, L. and Garcia, A. (1994), "Corrosion of Reinforcing Bars and Service Life of reinforced concrete Structures : Corrosion and Bond Deterioration", *Proc. Int. Conf. on 'Concrete Across Borders'*, Odense, Denmark, Vol. II, 315-326.

Rodriguez, J., Ortega, L.M. and Casal, J. (1996), "Load bearing capacity of concrete columns with corroded reinforcement", *Proc., 4th SCI, International Symposium on 'Corrosion of Reinforcement in Concrete Construction'*, C.L. Page, P.B. Bamforth and J.W. Figg, eds., E&FN Spon, Cambridge, UK, 220-230.

Rodriguez, J., Ortega, L.M. and Casal, J. (1997), "Load carrying capacity of concrete structures with corroded reinforcement", *Constr. Build. Mater.*, **11**(4), 239-248.

Tachibana, Y., Maeda, K.I., Kajikawa, Y. and Kawamura, M. (1990), "Mechanical behaviour of RC beams damaged by corrosion of reinforcement", *Proc. 3rd International Symposium on 'Corrosion of Reinforcement in Concrete Construction'*, Wishaw, UK, 178-187.

Torres-Acosta, A.A. (1999), "Cracking induced by localized corrosion of reinforcement in chloride contaminated concrete", Ph.D. Dissertation, Dept. of Civil and Environmental Engineering, University of South Florida, USA.

Uomoto, T. and Misra, S. (1988), "Behaviour of concrete beams and columns in marine environment when corrosion of reinforcing bars takes place", ACI Special Publication SP-109, 127-145.

Vu, K.A.T. and Stewart, M.G. (2000), "Structural reliability of concrete bridges including improved chloride-induced corrosion models", *Struct. Safety*, **22**, 313-333.