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Determination of collapse safety of shear wall-frame structures

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Abstract. A new finite shear wall element model and a method for calculation of 3D multi-storied only shear walled or shear walled – framed structures using finite shear wall elements assumed ideal elasto – plastic material are developed. The collapse load of the system subjected to factored constant gravity loads and proportionally increasing lateral loads is calculated with a method of load increments. The shape functions over the element are determined as a cubic variation along the story height and a linear variation in horizontal direction because of the rigid behavior of the floor slab. In case shear walls are chosen as only one element in every floor, correct solutions are obtained by using this developed element. Because of the rigid behavior of the floor slabs, the number of unknowns are reduced substantially. While in framed structures, classical plastic hinge hypothesis is used, in nodes of shear wall elements when vertical deformation parameter is exceeded ε_e, this node is accepted as a plastic node. While the system is calculated with matrix displacement method, for determination of collapse safety, plastic displacements and plastic deformations are taken as additional unknowns. Rows and columns are added to the system stiffness matrix for additional unknowns.

Keywords: finite shear wall element; elasto-plastic behaviour; lateral load parameter; plastic hinge hypothesis; matrix displacement method; a method of load increments.

1. Introduction

Although, there have been a lot of studies on finite elements in the literature, the studies on multi-storied structures considering material's nonlinear behaviour have mostly been realized for frame elements, where shear wall elements have also been idealized by transforming to frame elements. A finite element model has been developed for static and dynamic solutions of box-girder beams subjected to various loads (Saygun 1979). A general formulation of the elasto-plastic matrix for evaluating stress increments from those of stresses for any yield surface with an associated flow rule has been presented. A new "initial stress" computational process has been proposed by Zienkiewicz

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et al. (1969). By considering elasto-plastic material and geometrical changes, the plastic hinge hypothesis has been extended for combined internal forces. A general method of load increments has been developed for the determination of second-order limit load for steel plane frames. At first, a method, based on the addition of a column and a row to system equations for every plastic deformation parameter to obtain the solution, has been initiated in the study of Özer, (1987). Then, without a requirement to a sequential approach, solutions for 3D steel frame systems (İrtem 1991) and 3D RC (Reinforced Concrete) frame systems (Girgin 1996) have been obtained. The finite elements used to model concrete and steel are the eight-node isoparametric and three-node bar elements, respectively. Steel elements always coincide with the boundaries of the adjacent concrete elements (Lefas and Kotsovos 1990). A family of wall elements has been developed and a mixed finite element method of using these elements to model shear/core wall structures has been proposed. The wall elements should have in-plane rotations in order to allow direct connection and ensure compatibility with the beam elements. They should be able to represent the strain state of pure bending so as to avoid parasitic shears and also span at most only one story so that stress discontinuities at floor levels can be allowed (Kwan 1993). By using the deformation theory of linear hardening materials, a simplified finite element analysis method has been proposed. The results have been obtained by using any of initial strain method or initial stress method and by applying an iteration with the constant coefficient matrix (Xucheng and Xiaoning 1995). A method based on the application of the FEMA 273 nonlinear procedures has been described to assess the crack width of concrete shear walls for performance-based design (Chai and Guh 1999). Recently, non-linear stochastic analyses of 3D RC shear wall-frame structures under seismic excitations have been performed, where the shear wall has been separated to specific number of elements and assumed to have constant elasticity modules in the elevation. Then, stiffness matrix has been established (Taşkın 2001). For RC framed structures, a new plasticity model that combines the nonlinear material response and the geometric characteristics of a structural element has been developed (Karabinis and Kiousis 2001). An analytical model has been proposed for the nonlinear finite element analysis of RC planar structures. Load-displacement relations of shear panel beams and walls under various stress conditions have been evaluated to verify the soundness of the proposed model (Kwak and Kim 2001). An analytic method of RC structure using discrete element method has been introduced. The RC structures have been meshed with concrete discrete elements and re-bar elements (Xinzheng and Jianjing 2001). An analytical model has been developed to predict the horizontal load-displacement relationship of hybrid RC frame-corrugated steel wall systems according to the analogy of truss models (Mo and Perng 2003). A nonlinear isoparametric shell element has been developed. The load capacity and stress distribution around the openings have been analyzed by conducting 3D nonlinear pushover analysis on tunnel form building (Balkaya and Kalkan 2004). The main purpose of this study is to introduce a new finite shear wall element and to describe a simplified model for the elasto-plastic behaviour of shear walls. Additional terms for plastic nodes have been found by using the stiffness matrix of this element. A method of load increments has been used for the solutions of 3D shear wall-frame systems having material nonlinearity (Cengiz 2004).

2. A new finite shear wall element model

Below, assumptions made while developing the vertical, rectangular finite shear wall element

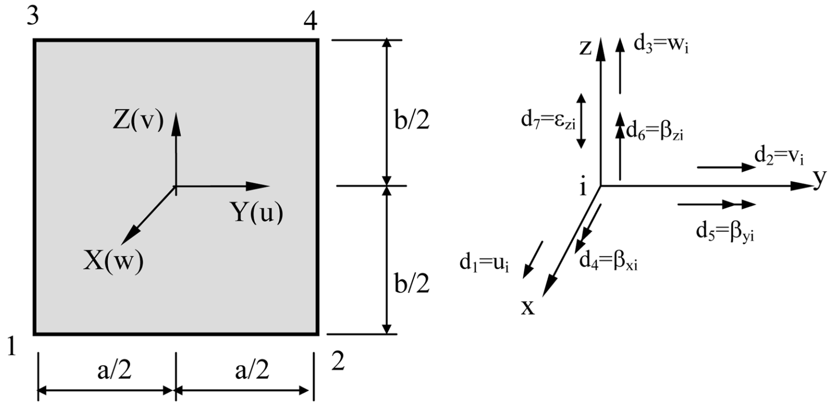

Fig. 1 The finite shear wall element, dimensions, coordinate system, displacement components and the nodal displacement parameters

(FSWE) are exhibited.

a) There are six displacement parameters in every node of the element (d_6 = 0) and totaly 24 degrees of freedom (dof).

b) The shape functions over the element are determined, so that a cubic variation along the story height and a linear variation in the plan are provided because of the rigid behaviour of the floor slabs.

The FSWE, coordinate system, nodal coordinates, numbering order of the nodes, displacement components and the nodal displacement parameters are shown Fig. 1. The nodal forces are chosen in the same directions with the nodal displacement parameters. Here, y and z are the axes within the element plane and x is the axis perpendicular to the element plane. Along these axes, displacement components are introduced as u, v and w, respectively.

β_{xi}, β_{yi} and β_{zi} are rotation displacements around these axes and also ε_z is nodal vertical deformation. $[d]_i$ is a column matrix including all nodal displacement parameters and $[d]$ is an expression consisting of all element dof.s in a matrix form. They are given in Eq. (1).

$$\left.\begin{aligned} [d]_{i\ 7x1} &= [w_i\ \ u_i\ \ v_i\ \ \beta_{xi}\ \ \beta_{yi}\ \ \beta_{zi} = 0\ \ \varepsilon_{zi}]^T \\ [d] &= [[d]_1\ \ [d]_2\ \ [d]_3\ \ [d]_4]^T_{28x1} \end{aligned}\right\} \qquad (1)$$

3. Calculation of stiffness and stress matrices of the finite element

Considering the assumptions made, following shape functions are defined as in Eq. (2).

$$\left.\begin{aligned} W(y,z) &= (a_1' + a_2' \cdot y) \cdot (a_1 + a_2 \cdot z + a_3 \cdot z^2 + a_4 \cdot z^3) \\ U(y,z) &= (b_1' + b_2' \cdot y) \cdot (b_1 + b_2 \cdot z + b_3 \cdot z^2 + b_4 \cdot z^3) \\ V(y,z) &= (c_1' + c_2' \cdot y) \cdot (c_1 + c_2 \cdot z + c_3 \cdot z^2 + c_4 \cdot z^3) \end{aligned}\right\} \qquad (2)$$

Spreading of the displacement components over the element by depending on the element's nodal displacements can be given as in Eq. (3). Here, each column of $[Ad]$ indicates the function of the displacement component over the element while the freedom related to the column is unit.

$$[U] = [w \ \ u \ \ v]^T = [Ad] \cdot [d] \tag{3}$$

$$[\varepsilon] = \begin{bmatrix} \chi_z \\ 2\tau \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial^2}{\partial z^2} & 0 & 0 \\ -\dfrac{2\partial^2}{\partial y \partial z} & 0 & 0 \\ 0 & \dfrac{\partial}{\partial y} & 0 \\ 0 & 0 & \dfrac{\partial}{\partial z} \\ 0 & \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial y} \end{bmatrix} \begin{bmatrix} w \\ u \\ v \end{bmatrix} \tag{4}$$

The relation between deformations and displacements is given in Eq. (4) by utilizing the classic elasticity theory. The variation of the displacement parameters along y direction is linear, hence, χ_y is equal to zero. If the expressions in Eq. (3) are written instead of the displacement components, then Eq. (5) is obtained.

$$[\varepsilon] = [\partial][Ad][d] = [B][d] \tag{5}$$

In case that the nodal coordinates are written instead of y and z variables in the matrix $[B]$, the deformation matrices in the nodes for the unit displacements, $[B_m]$, are found as given in Eq. (6) and the matrix $[\varepsilon_m]$, consisting of deformation matrices of the nodes written one under the other respectively, can be written in form of Eq. (7) as m = 1, 2, 3, 4 for the nodes.

$$[B_m] = [[B]_1 \ \ [B]_2 \ \ [B]_3 \ \ [B]_4]^T \tag{6}$$

$$[\varepsilon_m]_{20x1} = [B_m]_{20x28}[d]_{28x1} \tag{7}$$

$[Ad]$ and $[B]_1$, $[B]_2$, $[B]_3$, $[B]_4$ submatrices have been given according to the special element coordinate system by Cengiz (2004). By assuming linear elastic material, stresses in any of the element's nodes in matrix form can be written as follows by depending on the deformation components at that node. Here, $[\varepsilon_t]$ denotes deformations caused by heat variation. The elasticity matrix, $[D]$, is chosen assuming the material isotrophy.

$$[N] = [[M_z] \ \ [M_{yz}] \ \ [N_y] \ \ [N_z] \ \ [N_{yz}]]^T \tag{8}$$

$$[N] = [D] \cdot [[\varepsilon] - [\varepsilon_t]] \tag{9}$$

The numbers of rows and columns of the stiffness matrix are equal to 28, so the matrix is a square one. The terms for the vertical angular displacement parameter, d_6, are zero. By utilizing the theorem of virtual work, the stiffness matrix of the finite element, $[K]$

$$[K] = \iint [Ad]^T \cdot [\partial]^T \cdot [D] \cdot [\partial] \cdot [Ad] \cdot dA \tag{10}$$

$$[K] = \begin{bmatrix} [K]_{11} & [K]_{12} & [K]_{13} & [K]_{14} \\ [K]_{21} & [K]_{22} & [K]_{23} & [K]_{24} \\ [K]_{31} & [K]_{32} & [K]_{33} & [K]_{34} \\ [K]_{41} & [K]_{42} & [K]_{43} & [K]_{44} \end{bmatrix} \tag{11}$$

is obtained by using Eq. (10) and by integrating in both directions separately. The element stiffness matrix can be divided into equal square submatrices in the order of 7. $[K]_{ij}$ matrices denote the nodal forces occured in node i due to unit values of the jth node dof.s. In consequence of the Betti's reciprocal theorem and the element symmetry, it is enough to obtain only $[K]_{11}$, $[K]_{12}$, $[K]_{13}$, $[K]_{14}$ submatrices by integrating. The others on and upon the main diagonal of the element stiffness matrix in Eq. (11) can be obtained by utilizing these independent matrices and transformation matrices. Assuming the floor slabs to be infinitely rigid in their own planes, provides the computation to be faster by reducing numbers of unknowns for multi-storied structural systems. Therefore, the in-plane displacements at nodes located in the rigid plane should be expressed with an angular and two linear in-plane displacements of a selected reference node. The stiffness matrices are rearranged by transforming for the dependence to the reference node (Cengiz 2004).

4. Investigation of the elasto-plastic behaviour of shear walls

If in-plane stresses of the shear walls are scrutinized, it can be expressed that σ_y, the horizontal normal stress is equal to zero at floor levels and negligible between floor slabs along the story height, because floor slabs creates rigid diaphragm effect at floor levels. Furthermore, in the shear walls of multi-story structures which aren't too low in height and having the shear forces without great worths in nearby shear force carrying capacities of cross-sections, the effect of the shear stresses on both normal force and moment carrying capacities decrease (Çakıroğlu and Özer 1980). In this regard, by also omitting the effect of the shear stresses, it is assumed that elasto-plastic material behaviour for the shear walls can be simplified by depending on only the vertical normal stress, σ_z, as shown in Fig. 2. Accordingly, all the nodes of the shear wall element act as linear elastic until the vertical deformation parameter, ε_z, reaches the elastic deformation limit, ε_e, then σ_z steadies the state, while ε_z increases up to the plastic deformation limit, ε_p.

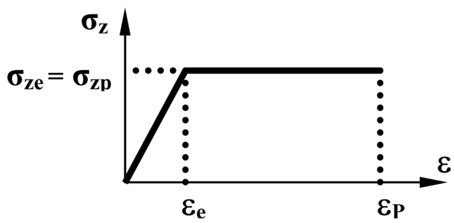

Fig. 2 Vertical normal stress and vertical deformation relationship in shear walls

In case of considering the spreading plastic deformations in the shear walls, in every load step, the plastic deformation enclaves must be defined, the element stiffness matrix must be recalculated by depending on the enclaves and the system stifness matrix must be recomposed. For every load increment, a sequential approach by considering variation of the enclave must be executed. The calculation has been explained by the approach techniques and various studies have been given as references (Zienkiewicz 1971). In this study, it is assumed that the plastic deformations are accumulated at nodes as vertical plastic displacements and also the elements between nodes keep as linear elastic. Because of this assumption which shortens the calculation in a considerable amount and is similar to the plastic hinge hypothesis in frame elements, neither the element nor the system stiffness matrices change and also in the load increments after that the vertical deformations at nodes, d_7, reach $\varepsilon_{e,}$ the relative plastic vertical displacements occur in these nodes. For example, in the load increment calculation, if a node exceeding ε_e is a node at a support, the displacement obtained by removing the vertical support restraint at this node will be assumed to be the plastic displacement of this node.

In multi-storied structures, in general, the shear walls work like a cantilever-beam. Collapse of a shear wall in such systems can be explained by two main issues:

a- In all the nodes of the support except one, a seperation in vertical direction occurs (like the element's rotation around any of the nodes in the support), in other words, reaches ε_e,

b- Reaching ε_p at any of the nodes in the support, although the deformations at all the nodes of the support don't reach ε_e.

5. The elasto-plastic behaviour of the frame elements

The plastic hinge hypothesis is accepted for the nonlinear elasto-plastic behaviour of the frame elements. As a result of the assumption that the floor slabs make the rigid diaphragm effect, only uniaxial bending moment and torsional moment occur in the beams. Plastic hinges occur after reaching the moment carrying capacity in any of the end cross-sections by omitting mutual influences of both effects. For biaxial bending combined with axial force and torsion at columns, the yield conditions have been defined (Çakıroğlu and Özer 1980).

6. Solution of the nonlinear shear wall-frame structures

The unknowns of the system with plastic nodes and plastic hinges at a specific time of the load increment are:

a- The unknowns of the linear elastic solution, shown with $[d]$, are two linear (d_1, d_2) and a vertical angular (d_6) in-plane displacement parameters of the reference nodes (master joint) selected for each one of the floor slabs and also the other independent displacement parameters of the system nodes,

b- The unknowns called as the plastic deformation parameter and shown with $[\Delta]$ being a column matrix are Δ_k, the plastic vertical displacement parameter at nodes of the shear walls exceeding ε_e and ϕ_k, the plastic rotation displacement parameters at end cross-sections occuring the plastic hinge of the frame elements.

To obtain these unknowns, following equations can be used:

a- The moment and the projection equilibrium equations written in the directions of the displacement parameters at nodes and reference nodes,
b- The vertical equilibrium equation at the node that plastic deformation occurs, because of the vertical seperation of the finite shear wall elements joined above this node,
c- The equilibrium equation written in the normal direction to the yield surface at the end cross-section of the frame element occuring the plastic hinge.

The solution of the nonlinear shear wall-frame structures will be done by using the matrix displacement method. The expanded equation system is given as Eq. (12).

$$\begin{bmatrix} [S_{dd}] & [S_{d\Delta}] \\ [S_{\Delta d}] & [S_{\Delta\Delta}] \end{bmatrix} \begin{bmatrix} [d] \\ [\Delta] \end{bmatrix} = \begin{bmatrix} [q] \\ [0] \end{bmatrix} \tag{12}$$

$[S_{dd}]$ is the stiffness matrix of the system omitting plastic nodes and plastic end cross-sections. It is symmetric according to the main diagonal. It is obtained by transforming the element stiffness matrices according to the global coordinate system and dependence to the displacements of the reference nodes and then by being placed to the own location.

$[q]$ is a column matrix consisted of nodal external loads in the global coordinate system.

$[S_{d\Delta}]$ is a rectangular matrix which the number of columns equal to the total of the plastic nodes and the plastic end cross-sections. The column identified by k of this matrix gives the forces occuring in the direction of $[d]$, the nodal displacement parameters at the global coordinate system, for a unit value of Δ numbered with k. If it is a plastic node on the shear wall, by using the columns belonging to the nodal vertical displacement in the stiffness matrices of the finite shear wall elements joined above this node, or if it is a plastic end cross-section on the frame element, by using the column belonging to the rotation displacement parameter on the above yield condition in the stiffness matrix of the frame element and then by doing the transformation processes which are explained for $[S_{dd}]$, the column numbered with k is obtained.

$[S_{\Delta d}]$ is equal to the transpose of $[S_{d\Delta}]$ according to the Betti's reciprocal theorem.

$[S_{\Delta\Delta}]$ is a square matrix. If the total of the plastic nodes and the plastic end cross-sections is m, the order of this matrix is m. The column numbered with k of this matrix shows the variations of the internal forces in the directions of the plastic deformation parameters both at the plastic nodes and the plastic end cross-sections for a unit value of Δ numbered with k. It is symmetric to the main diagonal according to the Betti's reciprocal theorem. In general, the terms except those on the main diagonal are equal to zero. If there are plastic displacements at each of the bottom nodes of a finite shear wall element or if there are plastic hinges at both end cross-sections of a frame element, then some of them will be different from zero.

7. A method of load increments for determination of collapse safety of shear wall-frame structures

A method, based on load increments for the determination of collapse safety of the nonlinear shear wall-frame structures subjected to constant gravity loads and proportionally increasing lateral loads is given as follows:

1- At first, $[S_{dd}]$ is constructed. The nodal displacements, stresses in the shear walls and the end forces of the frame elements are obtained for the system subjected to factored constant gravity loads. It is stipulated that there should not be any plastic deformations under the effects of vertical loads.

2- The linear elastic system is solved for a unit value of H_p, the lateral load parameter. The nodal displacements and internal forces are obtained.

3- The smallest lateral load parameter initiating a plastic deformation is determined. The solution belonging to the second step multiplies by this load parameter and then adds to the solution belonging to the first step. Thus, both the nodal displacements and internal forces for the current load parameter causing the first plastic deformation are obtained.

4- The values of bending moment carrying capacity in every end cross-section of frame elements under biaxial bending combined with axial force are determined in the special coordinate system of the frame elements.

5- The step number is increased by one. Because of the plastic nodes or the plastic end cross-sections, supplement rows and columns which must be added to $[S_{dd}]$ are determined. Only the supplement columns and rows are eliminated. The elimination procces is continued for the supplement row or rows which have to be added to the column of constants for a unit H_p. The supplement nodal displacement parameters and Δ are obtained with the replacing process for a unit H_p. The supplement deformation occuring in the plastic node of the shear wall because of Δ_k is obtained for the current step by using the releated terms of the element stiffness matrix. Either new plastic nodes or new plastic end cross-sections, their numbers, supplement H_p are determined. The process is repeated starting from 3.

6- The smallest of H_p is determined. It is chosen among the values belonging ε_p exceeded at any of plastic nodes and the plastic rotation capacity value exceeded at any of plastic end cross-sections. Either the determinant value of the expanded equation system is less than or equal to zero or the supplement horizontal displacement parameter's sign changes in any of steps at reference node where the maximum horizontal displacement is occured, it can be told that the system reaches the lack of stability situation.

7- The computation continues until collapse of the system. The collapse occurs in case of instability or in case top lateral displacement exceeds an assumed extreme value or in case the plastic rotational capacities at the plastic end cross-sections of frames or ε_p at the plastic nodes of shear walls are exceeded.

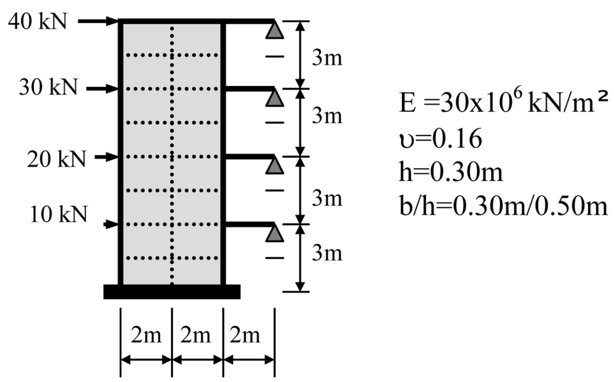

Fig. 3 Planar shear wall-tie beam system

Table 1 Comparisons of the results

Values		1	2	3	4
Overturning moment at base		77.6	78.52	78.81	78.18
Normal force at base		−3.099	−2.87	−2.8	−2.956
Top-story displacement		0.6777×10^{-3}	0.6923×10^{-3}	-	0.7292×10^{-3}
Moment of tie beams in each floor	Floor4.	1.774	1.562	1.438	1.504
	Floor3.	1.793	1.697	1.689	1.788
	Floor2.	1.568	1.495	1.487	1.576
	Floor1.	1.043	0.985	0.981	1.042

8. Examples

8.1 The elastic solution of a planar shear wall-tie beam system

Different solutions for the elastic solution of the system shown in Fig. 3 are compared and detailed as follows:

Solution 1: Using the developed finite shear wall element as a single element in each floor,

Solution 2: Using the developed element by performing a 2 × 2 division in each floor,

Solution 3: Using SAP2000 with an element system, where shear walls in each floor are divided to finite elements of 12 by 8 vertically and horizontally and tie beams are divided to finite elements of two by four in vertical and horizontal directions, respectively,

Solution 4: Considering Pala and Saygun's work (1992), similar shear walls have been used as plate finite elements and divided as in solution 3. Tie beams have been considered to be prismatic elements.

The results of these solutions are tabulated in Table 1. As is seen in Table 1, adequate results are obtained by using the developed finite element in few number.

8.2 Convenience of the finite shear wall element

A shear wall given in Fig. 4 is divided in axes y as 1, 2, 3, 6 and finally 6 × 2 making a total of

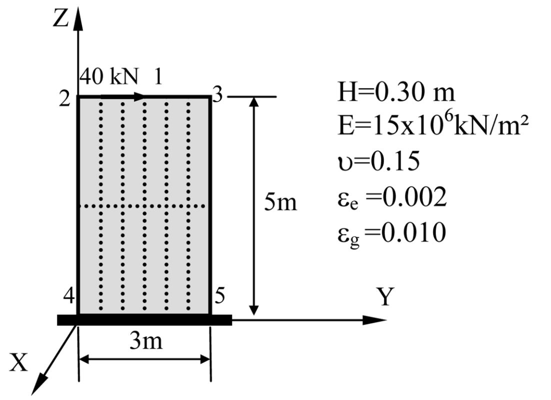

Fig. 4 6 × 2 modeling shear wall

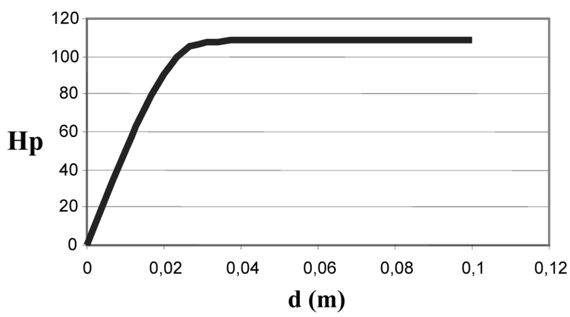

Fig. 5 H_p-d diagram of 6 × 2 modeling shear wall

Table 2 Values of d according to element numbers

Element numbers	d_2 ($\cdot 10^{-6}$ meter)
1	194.980
2	197.074
3	200.316
6	202.768
12	203.092
Exact result for the cantilever beam	(164.6 + 40.9) = 205.500

Table 3 Variation of H_p for the first plastic node according to element numbers

Element numbers	H_p
1	69.054
2	68.649
3	65.758
6	63.591
12	60.828
Exact result for the cantilever beam	69.050

12 finite elements. d, top lateral displacement values, obtained at node 1 are given in Table 2.

As clearly seen, a convenient result for d is obtained with only a single element. H_p, lateral load parameter values, pertaining to lower right node being the first plastic node are given Table 3. For the cantilever beam, H_p is exactly calculated as 69.05 at lower right node having ε_e. As seen, this value is very close to the value obtained for one element. Stress values at the support cross-section do not change linearly if the shear wall is divided into more number of elements. Hence, H_p decreases a bit. H_g, the collapse load parameter, for the shear wall with six finite elements is calculated as 107.974 (Fig. 5). For ideal elasto-plastic material, M_p, moment carrying capacity of columns with rectangular cross-section is equal to 1.5 times of M_e, elastic moment carrying capacity. Hence, H_g is calculated as 69.05 × 1.5 = 103.6.

8.3 A two-storied, planar shear wall-frame system

For the system shown in Fig. 6, uniaxial bending moment and torsion moment carrying capacities for beams are given as: M_{pe} = 150.0 kNm, M_{pb} = 60.0 kNm.

Normal force, uniaxial bending moments in both of axes and torsion moment carrying capacities for columns are given as: N_p = 3000.0 kN, M_{pe} = 360.0 kNm, M_{pb} = 90.0 kNm.

The results of the analysis are given in Table 4 and the diagram of H_p-d is given in Fig. 7. The rotational capacity of the node 14 is chosen as 3.2 × 10^{-3}. At this node, the rotational capacity is

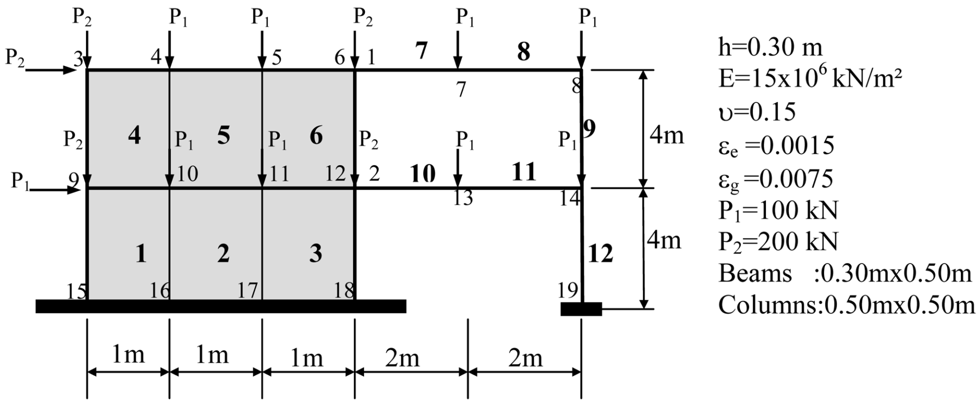

Fig. 6 A two-storied, planar shear wal-frame system

Table 4 H_p-d values at 1, the reference node

d (meter)	H_p, Lateral load parameter	Plastic node
0.01008	2.696	14 (in beam)
0.01160	3.098	8 (in beam)
0.01909	4.981	18 (in shear wall)
0.01954	5.066	6 (in beam)
0.02106	5.340	15 (in shear wall)
0.02373	5.599	
0.03750	6.874	17 (in shear wall)
0.05213	6.933	19 (in column)

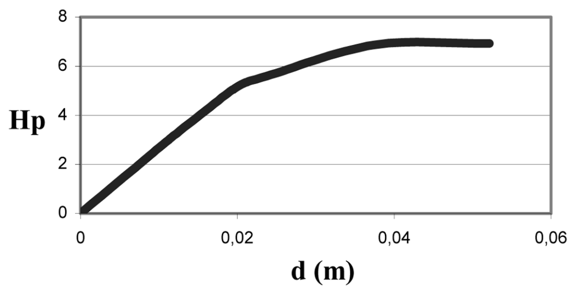

Fig. 7 H_p-d diagram

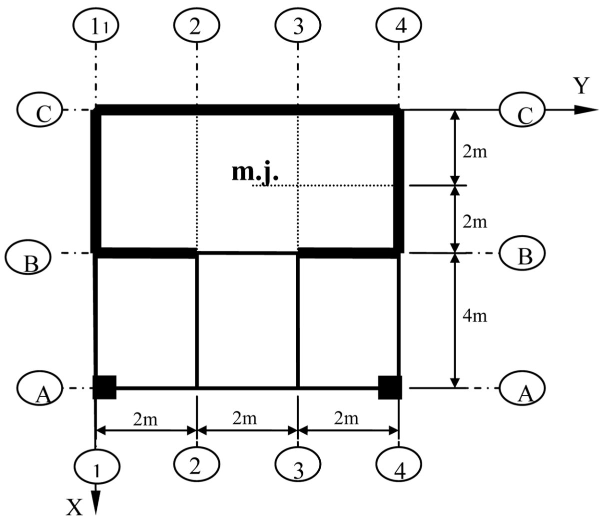

Fig. 8 Three-storied 3D core shear wall-frame system

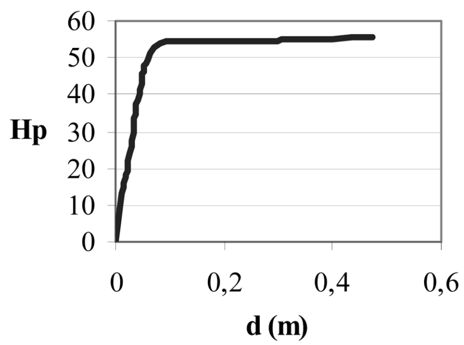

Fig. 9 H_p-d diagram

exceeded at a lateral load parameter of 5.38. This rotational capacity changes depending on assumptions, ratio of longitudinal and transverse reinforcement.

8.4 Three-storied 3D core shear wall-frame system

The H_p-d diagram of the three storied 3D core shear wall-frame system, subjected to both lateral loads effecting the reference nodes (m.j.) in the negative X direction and vertical loads which are concentrated and applied at the nodes, in Fig. 8 is exhibited in Fig. 9.

9. Conclusions

The following results are obtained from this study and discussed below:

1- A new finite shear wall element model has been developed. It has been shown that solutions of this model, even introducing a single element in each floor, gives acceptable results. Thus, numbers of unknowns are reduced substantially.

2- The solutions of the shear walls with sectional shapes of *V*, *U*, *L* or *T* are possible.

3- It is assumed that the plastic deformations are accumulated at nodes as vertical plastic displacements after that the vertical deformations reach the elastic deformation limit, ε_e, for shear walls while the plastic hinge hypothesis is applied for frame elements.

4- The limit load of multi-story shear wall-frame structural systems is calculated by using a method of load increments and collapse safety is determined.

References

Balkaya, C. and Kalkan, E. (2004), "Three-dimensional effects on openings of laterally loaded pierced shear walls", *J. Struct. Eng.*, **130**(10), 1506-1514.

Cengiz, E.Y. (2004), "A new finite wall element model and a method of load increments for determination of collapse safety of multi story shear walled and framed structures", *Doctorate thesis,* İTÜ Institute of Science and Technology, İstanbul.

Chai, W. and Guh, T.J. (1999), "Performance-based design of concrete shear wall building", *SEAOC Convention*, Santa Barbara, CA, September.

Çakıroğlu, A., Özden, E. ve Özmen, G. (1992), *Yapı Sistemlerinin Hesabı İçin Matris Metodları ve Elektronik Hesap Makinası Programları,* Cilt **1-2**, İTÜ İnşaat Fakültesi Matbaası, İstanbul.

Çakıroğlu, A. ve Özer, E. (1980), *Malzeme ve Geometri Değişimi Bakımından Lineer Olmayan Sistemler*, Matbaa Teknisyenleri Basımevi, İstanbul.

Girgin, K. (1996), "Method of load increments for the determination of second-order limit load and collapse safety of reinforced concrete framed structures", *Doctorate thesis,* İTÜ Institute of Science and Technology, İstanbul.

İrtem, E. (1991), "Determination of second-order limit load of framed space structures by a method of load increments", *Doctorate thesis,* İTÜ Institute of Science and Technology, İstanbul.

Karabinis, A.I. and Kiousis, P.D. (2001), "Plasticity model for reinforced concrete elements subjected to overloads", *J. Struct. Eng.*, **127**, 1251-1256.

Kwak, H.G. and Kim, D.Y. (2001), "Nonlinear analysis of RC shear walls considering tension-stiffening effect", *Comput. Struct.,* **79**, 499-517.

Kwan, A.K.H. (1993), "Mixed finite element method for analysis of coupled shear/core walls", *J. Struct. Eng.,* **119**(5), 1388-1401.

Lefas, I.D. and Kotsovos, M.D. (1990), "Nlfe analysis of RC structures and design implications", *J. Struct. Eng.*, **116**, 146-164.

Mo, Y.L. and Perng, S.F. (2003), "Analytical model for hybrid RC frame-steel wall systems", *Struct. Eng. Mech.*, **16**(2), 127-139.

Özer, E. (1987), "Determination of second-order limit load by a method of load increments"*, Bulletin of Tecnical University of İstanbul,* **40**(4), 815-836.

Pala, S. ve Saygun, A. (1992), "A rectangular plane stress element with 12 degrees of freedom", *Bulletin of the Tecnical University of İstanbul,* **45**, 1-3.

Przemieniecki, J.S. (1968), *Theory of Matrix Structural Analysis*, Mcgraw-Hill, Book Company.

Saygun, A.I. (1979), "Eğri Eksenli, Kutu Kesitli Kirişlerin Hesabı İçin Bir Sonlu Elemanlar Yöntemi", *Doçentlik Tezi,* İTÜ Institute of Science and Technology, İstanbul.

Taşkın, B. (2001), "Non-linear stochastic of 3D reinforced concrete shear wall-frame structures under seismic excitation", *Doctorate thesis*, İTÜ Institute of Science and Technology, İstanbul.

Xinzheng, L. and Jianjing, J. (2001), "Elastic-plastic analysis of RC shear wall using discrete element method", *Proc. Int. Conf. on Enhancement and Promotion of Computational Methods in Engineering and Science*, Shanghai, Jul.

Xucheng, W. and Xiaoning, W. (1995), “Simplified method for elasto-plastic finite element analysis of hardening materials”, *Comput. Struct.*, **55**, 703-708.
Zienkiewicz, O.C. (1971), *The Finite Element Method In Engineering Science*, Mcgraw-Hill, London.
Zienkiewicz, O.C., Valliapan, S. and King, I.P. (1969), “Elasto-plastic solutions of engineering problems ‘Initial Stress’, finite element approach”, *Int. J. Numer. Meth. Eng.,* **1**, 75-100.

Appendix

The independent submatrices for the new finite shear wall element are given with the shortenings as follows:

$$k_1=(1-\upsilon)\cdot a/b \qquad k_2=(1-\upsilon)\cdot b/a \qquad k_3=(\upsilon+(1-\upsilon)/2) \qquad k_4=h^2\cdot(1-\upsilon)/a\cdot b$$

$$k_5=(\upsilon-(1-\upsilon)/2) \qquad k_6=(1-\upsilon)\cdot a\cdot b \qquad k_7=h^2\cdot(1-\upsilon)/24$$

$$[K_{11}]=\begin{bmatrix}
\frac{h^2a}{3b^3}+\frac{6k_7}{5ab} & 0 & 0 & 0 & -\frac{h^2a}{6b^2}-\frac{k_7}{10a} & 0 & 0\\
 & \frac{13b}{35a}+\frac{k_1}{5} & \frac{k_3}{4} & -\frac{11b^2}{210a}-\frac{k_6}{60b} & 0 & 0 & -\frac{bk_5}{20}\\
 & & \frac{2a}{5b}+\frac{13k_2}{70} & -\frac{bk_5}{20} & 0 & 0 & \frac{a}{30}+\frac{11k_2b}{420}\\
 & & & \frac{b^3}{105a}+\frac{k_6}{45} & 0 & 0 & 0\\
 & & SMT & & \frac{h^2a}{9b}+\frac{2k_7b}{15a} & 0 & 0\\
 & & & & & 0 & 0\\
 & & & & & & \frac{2ba}{45}+\frac{k_2b^2}{210}
\end{bmatrix}_{7\times 7}\frac{Eh}{(1-\upsilon^2)}$$

$$[K_{12}]=\begin{bmatrix}
\frac{h^2a}{6b^3}-\frac{6k_7}{5ab} & 0 & 0 & 0 & -\frac{h^2a}{12b^2}+\frac{k_7}{10a} & 0 & 0\\
0 & -\frac{13b}{35a}+\frac{k_1}{10} & \frac{k_5}{4} & \frac{11b^2}{210a}-\frac{k_6}{120b} & 0 & 0 & -\frac{bk_3}{20}\\
0 & -\frac{k_5}{4} & \frac{a}{5b}-\frac{13k_2}{70} & \frac{bk_3}{20} & 0 & 0 & \frac{a}{60}-\frac{11k_2b}{420}\\
0 & \frac{11b^2}{210a}-\frac{k_6}{120b} & -\frac{bk_3}{20} & -\frac{b^3}{105a}-\frac{k_6}{90} & 0 & 0 & 0\\
-\frac{h^2a}{12b^2}+\frac{k_7}{10a} & 0 & 0 & 0 & \frac{h^2a}{18b}-\frac{2k_7b}{15a} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \frac{bk_3}{20} & \frac{a}{60}-\frac{11k_2b}{420} & 0 & 0 & 0 & \frac{ba}{45}-\frac{k_2b^2}{210}
\end{bmatrix}_{7\times 7}\frac{Eh}{(1-\upsilon^2)}$$

$$[K_{13}] = \begin{bmatrix} -\frac{h^2a}{3b^3}-\frac{6k_7}{5ab} & 0 & 0 & 0 & -\frac{h^2a}{6b^2}-\frac{k_7}{10a} & 0 & 0 \\ 0 & \frac{9b}{70a}-\frac{k_1}{5} & -\frac{k_5}{4} & \frac{13b^2}{420a}-\frac{k_6}{60a} & 0 & 0 & \frac{bk_5}{20} \\ 0 & \frac{k_5}{4} & -\frac{2a}{5b}+\frac{9k_2}{140} & \frac{bk_5}{20} & 0 & 0 & \frac{a}{30}-\frac{13k_2b}{840} \\ 0 & -\frac{13b^2}{420a}+\frac{k_6}{60b} & \frac{bk_5}{20} & -\frac{b^3}{140a}-\frac{k_6}{180} & 0 & 0 & -\frac{b^2k_5}{120} \\ \frac{h^2a}{6a^2}+\frac{k_7}{10a} & 0 & 0 & 0 & \frac{h^2a}{18b}-\frac{k_7ba}{30} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{bk_5}{20} & -\frac{a}{3}+\frac{13k_2b}{840} & \frac{b^2k_5}{120} & 0 & 0 & -\frac{ba}{90}-\frac{k_2b^2}{280} \end{bmatrix}_{7\times 7} \frac{Eh}{(1-\upsilon^2)}$$

$$[K_{14}] = \begin{bmatrix} -\frac{h^2a}{6b^3}+\frac{6k_7}{5ab} & 0 & 0 & 0 & -\frac{h^2a}{12b^2}+\frac{k_7}{10a} & 0 & 0 \\ 0 & -\frac{9b}{70a}-\frac{k_1}{10} & -\frac{k_3}{4} & -\frac{13b^2}{420a}-\frac{k_6}{120b} & 0 & 0 & \frac{bk_3}{20} \\ 0 & -\frac{k_3}{4} & -\frac{a}{5b}-\frac{9k_2}{140} & -\frac{bk_3}{20} & 0 & 0 & \frac{a}{60}+\frac{13k_2b}{840} \\ 0 & \frac{13b^2}{420a}+\frac{k_6}{120b} & \frac{bk_3}{20} & \frac{b^3}{140a}-\frac{k_6}{360} & 0 & 0 & -\frac{b^2k_3}{120} \\ \frac{h^2a}{12b^2}-\frac{k_7}{10a} & 0 & 0 & 0 & \frac{h^2a}{36b}+\frac{k_2ba}{30} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{bk_3}{20} & -\frac{a}{60}-\frac{13k_2b}{840} & -\frac{b^2k_3}{120} & 0 & 0 & -\frac{ba}{180}+\frac{k_2b^2}{280} \end{bmatrix}_{7\times 7} \frac{Eh}{(1-\upsilon^2)}$$