Structural Engineering and Mechanics, Vol. 27, No. 1 (2007) 33-48
DOI: http://dx.doi.org/10.12989/sem.2007.27.1.033

Reliability of articulated tower joint against random base shear

Nazrul Islam†

Civil Engineering Department, Jamia Millia Islamia University, New Delhi-110025, India

Suhail Ahmad‡

Applied Mechanics Department, Indian Institute of Technology, New Delhi-110016, India

(Received May 8, 2006, Accepted March 19, 2007)

Abstract. An Articulated tower is one of the compliant offshore structures connected to the sea-bed through a universal joint which is the most vulnerable location of the tower that sustains the randomly fluctuating shear stresses. The time history response of the bottom hinge shear is obtained and presented in the spectral form. The fatigue and fracture reliability assessment of the tower joint against randomly varying shear stresses have been carried out. Non-linear limit state functions are derived in terms of important random variables using S-N curve and fracture mechanics approaches. Advanced First Order Reliability Method is used for reliability assessment. Sensitivity analysis shows the influence of various variables on the hinge safety. Fatigue life estimation has been made using probabilistic approach.

Keywords: articulated; off-shore; SPM; random sea; reliability; fatigue; compliant.

1. Introduction

Offshore compliant structures such as guyed tower, tension leg platform and articulated towers are economically attractive for deep sea conditions because of their reduced structural weight compared to the conventional platforms. An articulated tower (Fig. 1) is one of the compliant structures employed in various offshore oil exploration activities such as pre-drilling, single point mooring (SPM), flaring of waste gases, field controlling and for providing loading and unloading facility for crude oil. These towers are subjected to wind and wave induced random excitations. The present study aims at the risk assessment of a single hinged articulated tower joint under long crested random sea with and without the wind forces. The equation of motion of the tower is established by Lagrangian approach which duly takes into account the nonlinearities due to time wise variation of submergence, buoyancy, added mass, instantaneous tower orientation and resultant hydrodynamic loading. The non-linear equation of motion is solved by an iterative time integration scheme using Newmark's-β method.

† Professor, Corresponding author, E-mail: nislam2k4@yahoo.co.in
‡ Professor, E-mail: suhailahmed@hotmail.com

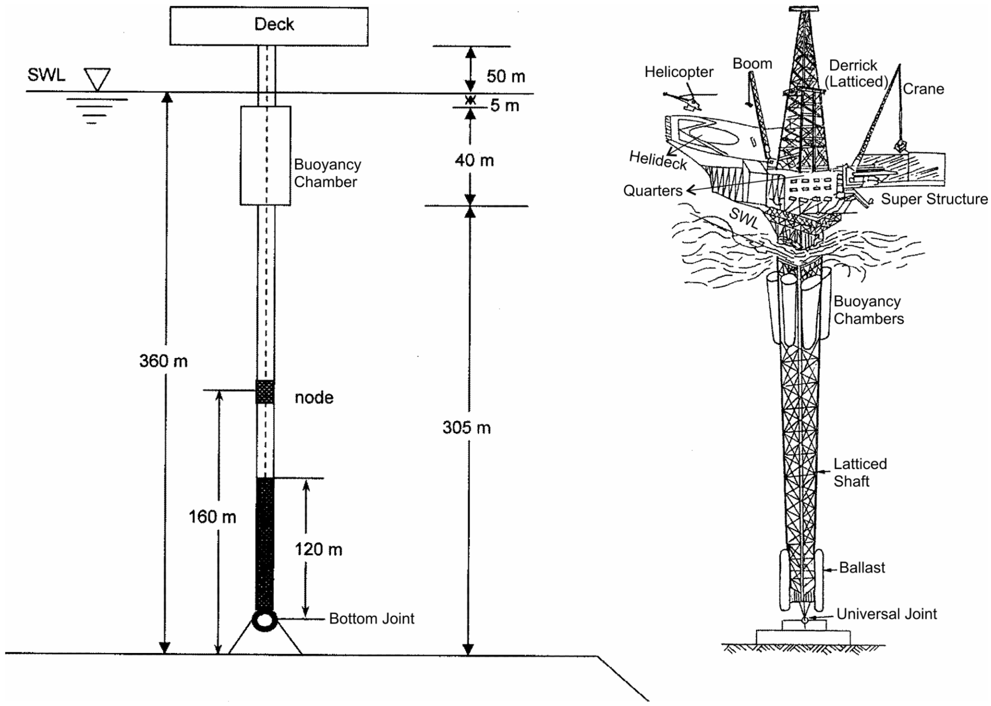

Fig. 1 Single hinged articulated tower

As shown in Fig. 1 the latticed tower is equipped with huge buoyancy chamber while it is connected to the sea floor through a universal joint which allows it to comply with the wind and wave forces instead of resisting it. This compliance of the tower avoids the concentration of high bending as well as overturning moments. The hinge provided at the base of the tower is the most critical part due to high concentration of stresses on it. Time dependent environmental loadings cause fatigue stresses at the hinge, that is highly crucial. Its fatigue and reliability assessment is one of the important design aspect. The design of the universal joint at the base of the tower is, in fact, the most challenging design part of an articulated tower.

Studies on dynamic responses of articulated towers have been carried out by Bar-Avi (1996, 1997), Adrezin (1996), Kuchuicki (2002), Datta and Jain (1990), Gunther and June (2003), Nazrul Islam and Ahmad (2006) and many others. However, studies on fatigue and reliability assessment of articulated tower joint are almost scanty in literature.

Sedillot *et al.* (1982) discussed the design aspects of universal joint for C.G. Doris gravity type articulated tower and conducted tests for fatigue and cyclic environmental loadings over a period of several years. The minimum life of the articulation system was estimated as 200 years. Likewise, Noblanc and Sehnader (1983) discussed a cardon-type articulated joint for statfjord-B articulated loading platform installed in North sea. It was composed of upper steel section welded to the column and lower cast steel section attached to the box shaped gravity base which were connected by pin. The joint was designed to accommodate upto 30 degree deflection from the vertical in any direction. However, no studies on the reliability of hinge were carried out. Khan *et al.* (2006) and Siddiqui (2001) carried out reliability analysis on TLP tether, another compliant platform under impulsive loading and random loadings respectively. Fatigue and fracture reliability and

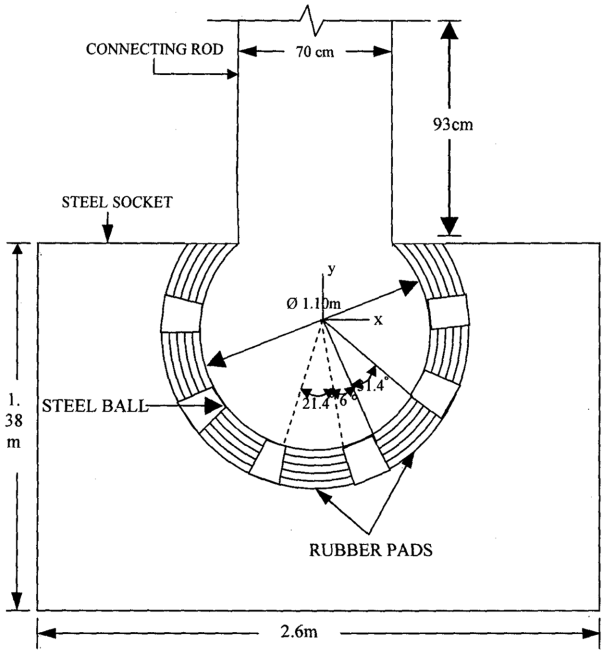

Fig. 2 Detail of articulated joint

maintainability of TLP tethers were also carried out by Kung and Wirsching (1992). For Offshore Structures fatigue reliability assessment in great detail was discussed by Wirsching (1984). But the literature is silent about the fracture and fatigue reliability analysis of articulated joint which is subjected to random shear of high magnitude.

2. Equation of motion

The nonlinear equation of motion is derived by Lagrangian approach which relates the kinetic and potential energies of the system in terms of rotational degree of freedom (θ) as follows

$$\frac{d}{dt}\left[\frac{\partial(KE)}{\partial\dot{\theta}}\right]-\frac{\partial(KE)}{\partial\theta}+\frac{\partial(PE)}{\partial\theta} = M_{\theta} \tag{1}$$

Treating expressions of Kinetic and Potential energies mathematically we get

$$\frac{\partial KE}{\partial\dot{\theta}} = \frac{1}{2}2I\dot{\theta} = I\dot{\theta}$$

$$\frac{d}{dt}\left[\frac{\partial(KE)}{\partial\theta}\right] = I\ddot{\theta} \quad \text{and} \quad \frac{\partial(KE)}{\partial\theta} = 0$$

$$\frac{\partial(PE)}{\partial\theta} = -\left(\sum_{t=1}^{np} m_{1i}r_i - \sum_{t=1}^{nsp} fb_i r_i\right)g\sin\theta - m_d g l_c \sin\theta \tag{2}$$

Putting these values in Eq. (1), Equation of motion is obtained as

$$[I]\ddot{\theta} + \left[\left(\sum_{i=1}^{np} m_{1i}r_i - \sum_{i=1}^{nsp} fb_i r_i + m_d\right)g\frac{\sin\theta}{\theta}\right] = M_\theta \tag{3}$$

or

$$[M]\{\ddot{\theta}\} + [K]\{\theta\} = \{M_\theta\} \tag{4}$$

This shows that $[M]$ consists of the mass moment of inertias of all the elements including the deck, about the hinge and M_θ is the moment due to wind and wave loadings.

The stiffness matrix $[K]$ consists of moments due to buoyancy and weight forces, described as under

$$[K] = \left(\sum_{i=1}^{np} m_{1i}r_i - \sum_{i=1}^{nsp} fb_i r_i + m_d\right)g\frac{\sin\theta}{\theta} \tag{5}$$

3. Wind driven waves

A wave pattern is built-up in the sea due to its interaction with wind. When the wind blows for a certain time duration its pressure executes work on the sea leading to an increase in wave amplitudes. An increased energy enhances the wave lengths maintaining their possible steepness. Slowly the wave crests travel with the wind speed resulting in rising of waves over the entire area.

The energy equation in terms of spectral moment may be expressed as

$$0.5\frac{\rho_a}{g}V^2\left(1 - \frac{g}{V\Omega}\right)^2\Omega^4 = -0.5\rho_w g^2\frac{1}{\Omega}\frac{\partial\Omega}{\partial x} - \rho_w g\frac{\partial\Omega}{\partial t} \tag{6}$$

where, ρ_a, ρ_w are the air and water densities respectively; g is the acceleration due to gravity; Ω is the cut-off frequency or weighted mean frequency, V is the nominal wind velocity observed at the standard height of 10 m above the sea surface.

In order to represent the energy in terms of a dimensionless sea state variable y, putting $y = g/V\Omega$

or

$$\Omega = g/Vy \tag{7}$$

The value of y lies between zero and one. For fully developed seas at infinite space fetch, this may be taken as unity. The energy equation in terms of the sea state variable y may now be represented by the following differential equation

$$(1-y)^2 = y^3\frac{\partial y}{\partial\xi} + y^2\frac{\partial y}{\partial\tau} \qquad 0 < y < 1 \tag{8}$$

where, ξ is the fetch length; and τ is the wind duration.

The fetch length ξ and wind duration τ may be obtained by the solution of the above equation as follows

$$\xi = \frac{y}{1-y} + 3\ln(1-y) + 2y + \frac{1}{2}y^2 \tag{9}$$

and $$\tau = \frac{1}{1-y} + 2\ln(1-y) - (1-y) \tag{10}$$

This shows that the growth of the sea-state, y is a function of dimensionless fetch length ξ and storm duration τ. The corresponding relation between actual time t and dimensionless duration τ is

$$\tau = 0.60 \times 10^{-4}\frac{g}{V}t \tag{11}$$

For known ξ and τ the sea state y may now be obtained from Eqs. (9) and (10). The sea state parameters are then estimated in terms of significant wave height (H_s) and average zero up crossing period (T_z) by the following equations

$$H_s = 0.18\left(\frac{V^2}{g}\right)y^2 \tag{12}$$

$$T_z = \sqrt{2}\pi\left(\frac{V}{g}\right)y \tag{13}$$

In this way the entire ocean wave loading is simulated in terms of wind velocity (V).

4. Dynamic analysis

The solution of the above equation of motion has been carried out in time domain iteratively using Newmark's β integration scheme. The description of load modeling due to wind and wave has been given in detail by the same authors (Ahmad and Islam 1997, Ahmad Suhail 1996).

The present analysis includes the effects of non-linearities due to hydrodynamic load, large deformation, mean and fluctuating wind loads represented by Email Simiu's wind spectrum. Random sea is idealized by Pierson Moskowitz (PM) spectrum. The response is assumed to follow a zero mean Gaussian process. The base shear response time history as shown in Fig. 3, thus obtained are transformed into frequency domain using Fast-Fourier transformation and represented in terms of power spectral density function (PSDF) as shown in Fig. 4.

Once the spectrum of the stress variation $S(\omega)$ is obtained, its variance σ_s^2 and zero-crossing

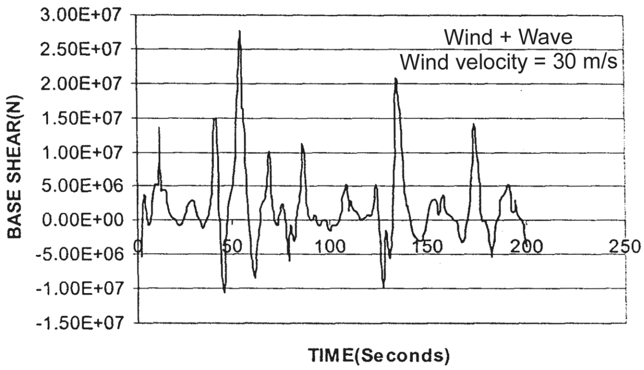

Fig. 3 Base shear time history

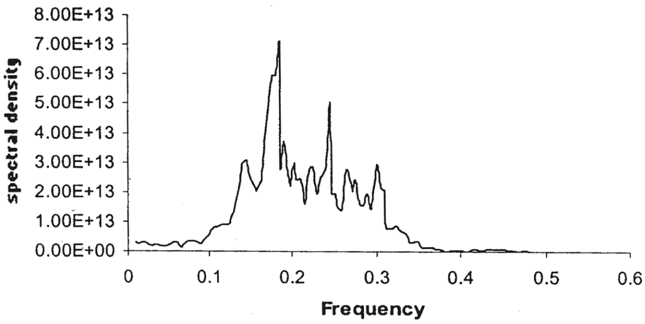

Fig. 4 Base shear PSDF

period T_0 is estimated as

$$\sigma_s^2 = \int_0^{\alpha} S_{ss}(\omega)d\omega \tag{14}$$

and

$$T_0 = \frac{2\pi\sigma_s}{\sqrt{\int_0^{\infty} \omega^2 S_{\omega}(\omega)d\omega}} \tag{15}$$

Taking the stress variation as a narrow band zero mean process, the statistical measures of the stress variations are obtained. The reliability index is then estimated following the procedure described by Siddiqui and Ahmad (2001), outlined here as under.

5. Fatigue reliability formulation

5.1 Limit state function

Fatigue failure has been defined through the limit state function $g(\underline{z})$ which is negative or zero at failure. $\underline{z}$ is the vector of basic variables describing loads, material properties, geometrical variables statistical estimates and model uncertainties. The probability of failure due to limit state violation is given as

$$p_f = p[g(\underline{z}) \leq 0] = \int_{g(\underline{z})<0} f_z(\underline{z})d\underline{z} \tag{16}$$

where, $f_z(\underline{z})$ is the joint probability density function of vector $\underline{z}$ which is equal to the product of individual probability density functions for uncorrelated random variables. For fatigue of offshore platforms the major uncertainties involved are due to estimation of environmental parameters, estimation of hydrodynamic and wind loadings, calculation of structural response, calculation of local stresses (stress concentration factors), stress intensity factors and analysis of crack growth.

Two models followed for the fatigue reliability are S-N curves/Miner-Palmgren damage models and crack growth rate curves/fracture mechanics models.

5.2 Minor palmgren damage model

The fatigue strength is expressed through the S-N relation which gives the number of stress cycles N with stress range S required to cause failure. The S-N model generally used for high-cycle fatigue is given as

$$NS^m = A \tag{17}$$

where, S is the stress range; m, A are empirical constants; and N is the number of cycles to cause failure.

The estimation of fatigue damage under stochastic loading is commonly done by the Miner-Palmgren model. Assuming that the damage on the structure per load cycle D_j is constant at a given range S_j and is equal to

$$D_j = \frac{1}{N(S_j)} \tag{18}$$

where, $N(S_j)$ is the number of cycles to failure at stress range (S_j). The Total damage accumulated in life time Ts of the structure is given by

$$D_j = \sum_{j=1}^{N(Ts)} \frac{1}{N(S_j)} \tag{19}$$

where, $N(T_s)$ is the total number of stress cycles in time T_s. In this formulation it is assumed that the accumulated damage D is independent of the sequences in which the stress cycles have occured.

Using the S-N curve, accumulated damage D is given as

$$D = \sum_{j=1}^{N(Ts)} \frac{S_j^m}{A} \tag{20}$$

Since each stress range is a random variable, therefore, $\sum_{j=1}^{N(Ts)} S_j^m$ is also a random variable. If $N(T_s)$ is sufficiently large, then the uncertainty associated with the sum is to be a very small and the sum can be replaced by its expected value. Therefore,

$$E\left|\sum_{j=1}^{N(T_s)} S_j^m\right| = E[N(T_s)]E[S_j^m] \tag{21}$$

For a narrow-band Gaussian process, stress range are Rayleigh-distributed. The mean value of the stress range follows directly as

$$E[S_i^m] = \int_0^x (2x)^m \frac{x}{\sigma_x} \exp\left(-\frac{1}{2}\left(\frac{x}{\sigma_x}\right)^2\right) dx$$

$$= (2\sqrt{2})^m \sigma^m{}_x \Gamma\left(1 + \frac{m}{2}\right) \tag{22}$$

Hence, the accumulated damage is given as

$$D = \frac{1}{A}E[N(T_s)E(S^m)] \tag{23}$$

If the environmental conditions are described by a set of stationary short-term sea states, then the total damage can be obtained by summing up the accumulated damage over all the sea sates. Thus, the total damage D yields

$$D = \frac{T_s}{A}Q \tag{24}$$

where, Q is a stress parameter given as

$$Q = (2\sqrt{2})^m \Gamma\left(1 + \frac{m}{2}\right) \sum_{q=1}^{n} f_q \nu_{0_q} \sigma_q^m \tag{25}$$

where, ν_{0_q} is the zero mean crossing frequency of stress process in qth sea state, f_q is the fraction of time in qth sea state, σ_q is the standard deviation of stress process in qth sea state.

In the above expression T_s is years in service

$\nu_{\sigma_q} = \dfrac{1}{2\pi}\sqrt{\dfrac{m_2}{m_0}}$, is the zero crossing frequency of the stress in qth sea state,

$\sigma_q = \sqrt{m_0}$, is R.M.S. value of the stress process in qth sea state,

$m_q = \int_0^\infty \omega^n S(\omega) d\omega$ is nth moment of the stress spectrum

and f_q is the fraction of the time spent in the qth sea state (to account for long-term sea effect).

Failure occurs if $D > \Delta_F$ where Δ_F is the value of the Miner-Palmgren damage index at failure. Often Δ_F is taken as 1.

Letting $D = \Delta_F$, the time for fatigue failure T of the tower is obtained as

$$T = \frac{\Delta_F A}{Q} \tag{26}$$

In order to take into account the uncertainties associated with the above expression, the factors involved in the expression shall be modeled as random variables. The time of failure T_s for the joint may be given as

$$T_i = \frac{\Delta_F A_i}{B_i^m Q_i} \tag{27}$$

where, Δ_F, A_i, B_i are random variables.

In the above equation B_i describes the inaccuracies in estimating the fatigue stresses. The actual stress range is assumed to be equal to the product of B_i and the estimated stress range S_i. The uncertainties in fatigue strength, as evidence by the scatter in S-N data, are accounted by considering A_i, to be a random variable. The random variable quantifies modeling error associated with the Miner-Palmgren rule.

The fatigue failure when the variable T_t is smaller than T_s where T_s is the lifetime of the structure. Thus, the limit state function is

$$g(\underline{z}) = \frac{\Delta_F A_i}{B_i^m Q_i} - T_s \tag{28}$$

where,

$$\underline{z} = (\Delta_F, A_i, B_i)$$

The surface $g(\underline{z})$ is the limit state surface, and $\underline{z}$ is the vector of basic random variables in the problem. The failure probability is computed using the First Order Reliability Method (FORM) and Monte Carlo simulation technique.

If $z_1 = \Delta_F$; $z_2 = A_i$ and $z_3 = B_i$ then limit function is

$$g(\underline{z}) = g(z_1, z_2, z_3) = \frac{z_1 z_2}{z_3^m Q} - T_s \tag{29}$$

and the probability of failure p_f is given as

$$P_f = P(T_i \leq T_s) = P[g(\underline{z}) \leq 0] \tag{30}$$

The reliability or safety index is thus obtained by

$$\beta = \Phi^{-1}(P_f) \tag{31}$$

5.3 Fracture mechanics model

Miner Palmgren damage model defines the time for fatigue failure as the time required for crack initiation in a material. However, for many design applications; time for initiation is a very small percentage of the total life of the structure. Much of the time is spent in sub critical crack growth. In the linear elastic fracture mechanics approach, relationship between average increment in crack growth (*da/dN*) during a load cycle and a global parameter are developed. The most popular global parameter used is the stress-intensity factor, *k*, which gives the magnitude of the stresses in the crack-tip region as a function of type and magnitude of loading and geometry of the cracked body. *k* is usually expressed as

$$k = Y(a)S\sqrt{\pi a} \tag{32}$$

where, α is the crack size, S is the far-field stress due to applied load and $Y(a)$ is the geometry function which takes into account the crack geometry and specimen shape as defined as follows

$$Y(a) = 1.0\alpha^{-0.125} \tag{33}$$

Crack growth relationship developed by Paris and Erdogan, described below is used for predicting the crack growth rate in the present study.

$$\frac{da}{dN} = C_2(\Delta k)^m$$

$$\Delta k = Y(a)\Delta S\sqrt{\pi a} \tag{34}$$

where, Δk is the stress-intensity factor range in a stress and C_2 and m are the material constants.

Substituting for Δk into above and integrating over da and dN, the following relation between crack size "α" and the number of cycles N in time T_s is obtained as

$$\int_{a_0}^{a} \frac{dz}{Y(z)^m(\sqrt{\pi z})^m} = N(T_s)E(S^m) \tag{35}$$

The sum of the values of stress range in each cycle $\left(\sum_{j=1}^{N(Ts)} S_j\right)$ has been approximated by $N(T_s)E(S^m)$; neglecting the effects of load cycle sequence and assuming the sea as a long-term sea states; also the stress range is assumed to follow a Rayleigh distribution in each sea state, it is obtained that

$$\frac{1}{C_2}\int_{a_0}^{a} \frac{dz}{Y(z)^m(\sqrt{\pi z})^m} = T_sQ \tag{36}$$

where, Q is given by Eq. (25). The failure criteria can then be formulated as a function of crack size. Failure occurs when the crack size exceeds a critical value a_0 which is based on a serviceability conditions. The probabilistic model for the time to failure T_i of the hinge with due consideration of the uncertainties involved in the fracture mechanics is given as follows

$$T_i = \frac{1}{C_2B_i^mQ_i^m}\int_{a_0}^{a} \frac{dz}{\gamma_iY(z)^m(\sqrt{\pi z})^m} \tag{37}$$

where, C_2, B_i, a_0, γ_i are the random variables.

In the above equation B_i and γ_i are introduced to model the errors in the estimation of the stress range S and in the geometry function $Y(a)$, respectively.

The fatigue failure occurs when random variable T_i is smaller than T_s.

where, T_s is the lifetime of the structure.

The limit state function, therefore, is described as

$$g(\underline{z}) = \frac{1}{C_iB_i^mQ_i^m}\int_{a_0}^{a} \frac{dz}{\gamma_iY(z)^m(\sqrt{\pi z})^m} - T_s \tag{38}$$

$$\underline{z} = (C_2, B_i, a_0, \gamma_i) \tag{39}$$

The surface $g(\underline{z})$ is the limit state surface, and $\underline{z}$ is the vector of basic random variables in the problem:

If $z_1 = C_2$; $z_2 = B_i$; $z_3 = a_0$ and $z_4 = \gamma_1$ we get the following equation

$$g = g(\underline{z}) = g(z_1, z_2, z_3, z_4)$$

$$= \frac{1}{z_1z_2^mz_4^m\pi^{m/2}\Omega}\int_{z_3}^{a} \frac{dz}{[Y(z)]^mz^{m/2}} - T_s \tag{40}$$

The probability of failure, P_F is given as

$$P_F = P(T_i \le T_s) = P[g(\underline{z}) \le 0] \tag{41}$$

The failure probability is again computed using advanced first order reliability method (FORM) and Monte Carlo simulation technique.

The reliability or safety index is thus obtained by

$$\beta = \Phi^{-1}(P_t) \tag{42}$$

where, Φ^{-1} is the inverse of the standardized normal distribution function.

5.4 Wide band correction

Fatigue stresses are assumed to be narrow band random process. However, if they are wide band random process then the stress parameter Q expressed by Eq. (25) is to be modified accordingly through a correction factor as described by Wirsching (1984). In the present study a wide band correction factor (λ_q) has been applied to modify the expression of Q. Therefore, the corrected expression for stress parameter (Q) is described as

$$Q = (2\sqrt{2})^m \Gamma\left(1 + \frac{m}{2}\right) \sum_{q=1}^{n} f_q \nu_{0_q} \sigma_q^m \lambda_q \tag{43}$$

where, λ_q is a wide band correction factor for qth sea state. Estimation of λ_q is obtained in the following form of empirical expression derived by Wirsching (1984).

$$\lambda_q(\varepsilon_q, am) = a(m) + [1 - a(m)](1 - \varepsilon_q)^{b(m)} \tag{44}$$

where,

$a(m) = 0.926 - 0.033m$

$b(m) = 1.587m - 2.323$

and ε_q is the spectral width parameter for qth sea state.

For a typical ocean structure problem; if $\varepsilon_q > 0.5$, then $\lambda_q \cong 0.79$ for $m = 4.38$ and

$$\lambda_q \cong 0.86 \text{ for } m = 3.$$

6. Numerical study

The details of articulated joint are shown in Fig. 2, the cylindrical rod connecting to the ball and socket joint and tower base has 70 cm. diameter, which is vulnerable for fatigue failure due to shear reversals.

Numerical study yields the response time histories of base shear under random wind plus wave loadings as shown in Fig. 3 and the corresponding PSDF are shown in Fig. 4. Table 2 shows the

Table 1 Tower's properties

Property	Value
Tower's length	400 m
Ballast Height	120 m
Height of buoyancy chamber	40 m
position of the buoyancy	325 m
Tower Mass	2.0 E4 Kg/m
Deck Mass	2.5 E6 Kgs
water depth	350 m
Pivot's internal diameter	1.1 m
Time period	38 sec
Drag coefficient	0.6
Inertia coefficient	2.0
Wind drag co-efficient	0.002
equivalent area of superstructure for wind load	1288.5 m^2
Aerodynamic center above SWL	27.216 m
Email Simiu's constants β, f_m and f_s	6.0, 0.07 and 0.2

Table 2 Base shear (Newton) for SHAT under wind, wind induced waves and current

Mean Wind Vel.	Wind Driven Waves	Statistics	Wave Alone (1)	Wave + Wind (2)	Wind + wave + Curent (v_c = 1.0 m/s)
10.0 m/s	H_s = 2.015 m T_z = 4.94 sec.	Max.	2.11E5	2.5E4	−2.61E6
		Min.	−2.59E6	−5.96E5	−3.71E6
		RMS	6.2456E4	2.109E5	3.0395E6
		Mean	1.3376E3	−1.94E5	−3.0377E6
		S.D (σ)	6.2441E4	8.159E4	1.4396E5
30.0 m/s	H_s = 17.76 m T_z = 14.64 sec.	Max.	6.89E6	2.53E7	4.6E6
		Min.	−2.27E7	−1.00E7	−5.16E6
		RMS	1.736E6	1.5008E7	2.2574E6
		Mean	7.752E5	−9.31E5	−1.9776E6
		S.D (σ)	1.565E6	1.17E6	1.0901E6

statistical characteristic of random base shear for wind driven waves corresponding to wind velocities of 10 m/sec and 30 m/sec. Tables 3, 4 show the random data considered in the reliability studies using S-N curve and fracture mechanics models. Fatigue strength coefficient (A), Stress modeling error (B) and Miner Palmgren damage index at failure (ΔF) in S-N curve model while Paris coefficient (C), Stress modeling error (B), initial crack length (a_0) and uncertainty factor (γ) for fracture mechanics models are considered respectively. The distribution of all the random variables are lognormal except "a_0" which is an exponential in nature with specified coefficient of variance (COV). Table 5 shows the probabilities of hinge failures in S-N and Fracture mechanics approaches in terms of sensitivity factors and service life.

Table 3 Data for reliability study (S.N. Model)

Variable	Distribution	Mean/Median	COV
Fatigue strength coefficient, A	lognormal	$\tilde{A} = 5.27 \times 10^{12}$ MPa	0.63
Stress modeling error, B	lognormal	$\tilde{B} = 1.00$	0.20
Miner-Palmgren damage index at failure Δ_F	lognormal	$\tilde{\Delta}_F = 100$	0.30
Fatigue exponent, m	Constant	3.0	

Table 4 Data reliability study (Fracture Mechanics Model)

Variable	Distribution	Mean/Median	COV
Paris Coefficient, (c)	Lognormal	$\tilde{C}_2 = 1.8 \times 10^{-12}$ MPa	0.63
Stress modeling, error B	Lognormal	$\tilde{B} = 1.00$	0.20
Initial crack length a_0 (mm)	exponential	$\mu_{a_0} = 0.005$	
Modeling error in $Y(a)$ γ_i	Lognormal	$\mu_\gamma = 1.00$	0.10
Critical crack length a_0 (mm)	Constant	33.4	
Paris exponent, m	Constant	3.0	

Data in Tables 3 and 4 (Kung and Wiresching (1992))

Table 5 Sensitivity factors (S-N Model)

Sensitivity factors (→)	α_1	α_2	α_3
Sea state (↓)			
Long crested sea	−0.3338	−0.65740	0.67557
Long crested sea + wind	−0.3222	−0.6430	0.67667

7. Discussion of results

The paper presents base shear responses for forces due to random sea waves, wave plus wind together with and without current. Reliability indices are obtained against fatigue damages. The statistical characteristics of random base shear are not always higher in case of wind plus waves than waves alone. It shows the attenuation effect due to wind forces on super structure in presence of waves on substructure.

Sensitivity factor α_i for the jth random variable, the failure surface "g_i" in the normalized co-ordinate "Y_j" and the value of the variable at the design point is given as

$$\alpha_i = -\left(\frac{\partial g_i}{\partial y_j}\right) \Big/ \left(\sum_{j=1}^{n}\left(\frac{\partial g_i}{\partial y_j}\right)^2\right)^{1/2} \tag{46}$$

Lesser value of α_i implies the lesser influence of the jth random variable on reliability. Positive α_i is for load variables where-as negative for resistance variables. Table 5 shows the values of sensitivity factors for fatigue reliability under random base shear due to environmental loads given by long crested random sea together with and without random wind. In S-N curve approach (Table 5) sensitivity factor for shear stress modeling error or response uncertainty factor (α_3) is negative hence it will contribute to resistance part of the limit state function. Therefore, for the given uncertainty an increase in the Miner Palmgren damage index (Δ_F) and fatigue strength co-efficient (A) will improve the reliability of the joint whereas, an increase in response uncertainty factor (B) will reduce the hinge reliability. Out of the two resistance variables; reliability is more sensitive to fatigue strength co-efficient (A) than Miner Palmgren damage index (Δ_F) for both the sea idealizations.

Likewise, Table 4 shows the sensitivity values of various random variables appearing in the facture mechanics based limit state function. Sensitivity factor for all the random variables viz. Paris Co-efficient "C" stress modeling error B, initial crack length "α_0" and modeling error in geometry function (γ_i) are positive which shows that these variables will contribute to the load part only. As all the random variables are load variables (+*ve*), therefore, increase in their magnitude for a given uncertainty will decrease the reliability of the joint. Furthermore, reliability is most sensitive and least sensitive to the stress modeling error (B) and the modeling error in geometry function (γ_i) respectively for both the sea idealizations; i.e., long crested random sea plus wind and random sea alone. This is due to the highest and lowest magnitude of sensitivity factors for stress modeling error "α_2" and modeling error in geometry function (γ_i) respectively. However, the sensitivity magnitude for Paris Co-efficient and initial crack length are also significantly sensitive to reliability because of their sensitive magnitudes, though smaller, yet closer to α_2.

8. Conclusions

It is observed that S-N curve approach significantly yields a conservative estimate of probability of failure as compared to the fracture mechanics approach. The inclusion of mean and fluctuating wind with random sea in the dynamic analysis causes a reduction of probability of failure in some cases differing in terms of mean wind, wave height and wave period. The reliability is found to be more sensitivity to fatigue strength co-efficient (A) than Miner Palmgren damage index (Δ_F). In fracture mechanics approach limit state function consists of random load variables viz. Paris Co-efficient (C) stress modeling error "B", initial crack length "a_0" and modeling error in geometry function γ_i. Out of these the reliability is most sensitive to stress modeling error (B) and least sensitive to modeling error in geometry function γ_i. Fatigue life of base hinge of articulated tower is inversely proportional to the increase in service life.

References

Adrezin, R., Bar-Avi, P. and Benaroya, H. (1996), "Dynamic response of compliant offshore structures-Review", *J. Aerospace Eng.,* ASCE, **9**(4), 114-131.

Ahmad, S. (1996), "Stochastic TLP response under long crested random sea", *Comput. Struct.,* **61**(6), 975-993.

Ahmad, S., Islam, N. and Ali, A. (1997), "Wind induced response of tension: Leg platform", *J. Wind Eng. Ind. Aerod.,* **72**, 225-240.

Bar-Avi, P. and Benaroya, H. (1996), "Non-linear dynamics of an articulated tower in the ocean", *J. Sound Vib.,* **190**(1), 77-103.

Bar-Avi, P. and Benaroya, H. (1997), "Stochastic response of two DOF articulated tower", *Int. J. Nonlinear Mech.,* **32**(4), 639-655.

Dutta, T.K. and Jain, A.K. (1990), "Response of articulated tower platforms to random wind and wave forces",

Comput. Struct., **34**(1), 137-144.
Gunther, F. Clauss and June Young Lee (2003), "Dynamic behaviour of compliant towers in deep sea", *22nd Int. Conf. on Offshore Mechanics and Artic Engineering* (*OMAE2003-37173)*, 1-10.
Islam, N. and Ahmed, S. (2006), "Earthquake response of articulated offshore tower", *Int. J., Euro. Earthq. Eng.* **1**, 48-58.
Khan, R.A., Siddiqui, N.A., Naqvi, S.Q. and Ahmad, S. (2006), "Reliability analysis of TLP tethers under impulsive loading", *Reliab. Eng. Syst. Safe*, **91**(1), 73-83.
Kuchnicki, S.N. and Benaroya, H. (2002), "A practical study of flexible ocean tower", *Int. J. Chaos Soliton. Fract.*, **14**, 183-201.
Kung and Wrisching (1992), "Fatigue and fracture reliability and maintainability of TLP tensions", *ASME*, **II**, 15-21.
Noblanc, A. and Sehnader, H.E. (1983), "Precise seabed emplacement of an articulated laoding platform in the North Sea", *Proc. of the 15th Annual offshore Technology Conference, (OTC 4571), Huston Texas*, 481-488.
Sedillot, F., Doris, C.G. and Stevension, A. (1982), "Laminated rubber articulated joint for the deep water gravity tower", *Proc. of the 14th Annual Offshore Technlogy Conference (OTC 4195)*, Huston, Texas, 1, 341-350.
Siddiqui, N.A. and Ahmad, S. (2001), "Fatigue and fracture of TLP tethers under random loading", *Marine Struct.,* **14**(3), 331-352.
Wirsching, P.H. (1984), "Fatigue reliability for offshore structures", *J. Struct. Div.,* ASCE, **110**(10), 2340-2356.

Notation

A_i	: random variable
(a_0)	: initial crack length
(A)	: Fatigue strength coefficient
α_i	: Sensitivity factor
α_1	: Sensitivity factor for Miner Palmgren damage index at failure
α_2	: Sensitivity factor for fatigue strength coefficient (A)
α_3	: Sensitivity factor for stress modeling error (B)
α	: rack size
β	: reliability index
B_i	: random variable
B	: Stress modeling error
C	: Paris coefficient
C_2	: material constants
COV	: Coefficient of variation
ΔF	: Miner-Palmgren damage index at failure
Δk	: stress-intensity factor range in a stress
m_d	: deck mass
m	: material constants
(γ)	: uncertainty factor
(λ_q)	: wide band correction factor
ε_q	: spectral width parameter
τ	: wind duration
ξ	: fetch length
ρ_a, ρ_w	: air and water densities respectively
Ω	: cut-off frequency
Q	: stress parameter
S	: far-field stress
V	: nominal wind velocity
$Y(a)$	: geometry function

$\sim$	: median value
μ	: mean value
(σ)	: standard deviation
Φ^{-1}	: inverse of the standardized normal distribution function