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Inelastic transient analysis of piles in nonhomogeneous soil

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Abstract. In this paper, a hybrid boundary element technique is implemented to analyze nonlinear transient pile soil interaction in Gibson type nonhomeogenous soil. Inelastic modeling of soil media is presented by introducing a rational approximation to the continuum with nonlinear interface springs along the piles. Modified Özdemir's nonlinear model is implemented and systems of equations are coupled at interfaces for piles and pile groups. Linear beam column finite elements are used to model the piles and the resulting governing equations are solved using an implicit integration scheme. By enforcing displacement equilibrium conditions at each time step, a system of equations is generated which yields the solution. A numerical example is performed to investigate the effects of nonlinearity on the pile soil interaction.

Keywords: pile-soil interaction; inelastic behavior; pile dynamics; transient response of piles.

1. Introduction

Since pile soil interaction problem often has the transient loads with high degree of material nonlinearity in the solution domain, it is necessary to implement a nonlinear transient analysis algorithm of the physical problem. The various approaches that have been used for the transient problem of pile soil interaction are categorized in two main types: 1) finite and/or boundary element method formulations and 2) dynamic Winkler type formulations.

A direct boundary element method type of formulation for the transient analysis was implemented by Israil and Banerjee (1990) in 2-D and by Ahmad and Banerjee (1988) in 3-D, and indirect boundary element type of formulations were introduced for pile soil interaction problem by Mamoon and Banerjee (1992) and Guin (1997).

The frequency domain finite element method was used for pile and pile groups in Nogami and Novak (1976), Nogami and Novak (1977), Nogami (1979), Gazetas and Makris (1991), Mamoon (1990) and further developed for nonlinear interaction of a single pile by Angelides *et al.* (1980). A simple method was proposed for the analysis of vertically loaded pile groups under dynamic

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conditions (Cairo *et al.* 2005). The method makes use of the closed-form stiffness matrices to simulate the response of layered soils. These matrices are incorporated in a calculation procedure that is essentially analytical.

Nogami and Konagai (1986, 1988) presented a linear transient analysis of single piles. The soil response to the pile motion was integrated through a simplified model based on Winkler's assumption, the parameters of which were determined form the consideration of a plane strain wave propagation. Nogami and Konagai (1987, 1991) extended the time domain analysis for nonlinear effects. El Naggar and Novak (1994) studied the dynamic axial response of pile by using nonlinear models. In view of the heterogeneity of natural soil deposits and approximations made in analysis methods, a simplified dynamic analysis of a single pile may be back-calculated from the dynamic response of the pile measured in the field was presented using Winkler model and finite element methods by Anandarajah *et al.* (2005). These methods represent significant achievements in the transient analysis of pile foundations even though they have many limitations, primarily resulting from the simplified modes of wave propagation. Besides to analytical and numerical studies, dynamic experiments in lateral mode were carried out on model aluminium single piles in a simulated elastic half-space filled with clay soil to determine dynamic constants of the soil–pile system and to study the bending behaviour of piles (Boominathan 2005, 2006). A hybrid boundary element technique is implemented to get computationally most efficient results when comparing with 3D boundary element elastodynamic formulations. Linear beam column finite elements are used to model the piles and resulting governing equations are solved using an implicit integration scheme. The continuum is inelastic and modeled with modified Özdemir's nonlinear model (Ozdemir 1976). The nonlinear springs are used to describe the soil-pile interaction, whereas the pile-soil-pile interaction is analyzed by using the elasticity solution. An efficient step by step time integration scheme is implemented by using an approximate half space integral formulation (Kucukarslan 1999, 2002). By enforcing displacement equilibrium conditions at each time step, a system of equations is generated which yields the solution. A numerical example is presented for a single pile and a 3 × 3 pile group under statnamic type of loading.

2. Time domain formulations

The formulation requires the construction of a simplified integral representation for the continuum and coupling it with the finite element system of equations for the linear elastic piles.

2.1 Continuum equations

The elastodynamic, small displacement field in an isotropic elastic body is governed by Navier's equation

$$(\lambda+\mu)\frac{\partial^2 u_p}{\partial x_p \partial x_q} + \mu\frac{\partial^2 u_q}{\partial x_p \partial x_p} - \rho\ddot{u}_q = 0 \tag{1}$$

where λ and μ = Lame's constants

ρ = mass density of the deformed body

$\ddot{u}_q = \dfrac{\partial^2 u_q}{\partial t^2}$ are the accelerations and subscripts p and q ranges from 1 to 3.

The integral form of Eq. (1) can be written as in following

$$u_p(\xi, t) = \int_S \int_0^t G_{pq}(x, t; \xi, \tau) t_q(x, \tau) d\tau ds \tag{2}$$

where u_p is the displacement of the soil; G_{pq} is the Green's function for the half space, t_q is the tractions at the pile soil interface, ξ and x are the spatial positions of the receiver and the source point, respectively. Integration of Eq. (2) is needed to be done in both time and space. It is an implicit time domain formulation because the response of time is calculated by considering the history of surface tractions and displacements To obtain the transient response at time t^j, the time axis is discretized into j equal time intervals as

$$t^j = j\Delta t \tag{3}$$

where Δt is the time step. By substituting Eq. (3) into Eq. (2), one can write

$$u_p(\xi, t^j) = \int_{\tau = t^{j-1}}^{t^j} \int_S G_{pq} t_q ds d\tau + \int_{\tau = 0}^{t^{j-1}} \int_S G_{pq} t_q ds d\tau \tag{4}$$

where the second integral on the right hand side is the effect of the past dynamic history. The spatial integration in Eq. (4) is carried out by discretizing the pile-soil interface into cylindrical elements. This integration form yields the following matrix equation where the functional variation over the element is assumed to be linear.

$$\{u_s^j\} = \int_{\tau = t^{j-1}}^{t^j} [G]\{t_s\} d\tau + \int_{\tau = 0}^{t^{j-1}} [G]\{t_s\} d\tau \tag{5}$$

The soil displacement equation at time step j can be written as

$$\{u_s^j\} = [G_1]\{t_s^j\} + \sum_{i=1}^{j-i} [G_{j-i+1}]\{t_j^i\} = [G_1]\{t_s^j\} + \{R_s^j\} \tag{6}$$

where $[G_1]$ is the coefficient of the leading time step, $\{R_s^j\}$ represents the effects of past time histories on the current time node. The coefficients of the matrices G are calculated by using an approximate fundamental solution. That is an approximate equivalent of Mindlin's static half space point force solution. It is superposition of two Stoke's infinite space fundamental solutions and it was implemented first time by Banerjee and Mamoon (1990). In order to incorporate the effects of inhomogeneity a further approximation involving local homogeneity and a locally average speed of wave propagation is introduced in the current work.

To incorporate the nonlinear effects in this equation, an incremental form is introduced

$$\{\Delta u_s^j\} = [G_1]\{\Delta t_s^j\} + \{R_s^j\} \tag{7}$$

$$\{R_s^j\} = \sum_{i=1}^{j-i} [G_{j-i+1}]\{\Delta t_j^i\} \tag{8}$$

2.2 Inelastic model

A one dimensional simple model was developed by Özdemir (1976). Due to its simplicity and requiring only a few parameters, it is very attractive to use in this analysis. Besides its simplicity, it provides a hysteretic rule that is needed to model the cyclic behavior (Guin 1997, Kucukarslan 1999). This model is used to relate displacements and tractions. The modified Özdemir's model is written as

$$\dot{t} = K\left[\dot{w} - |\dot{w}|\left(\frac{t-\beta}{Y}\right)\right] \tag{9}$$

$$\dot{\beta} = \alpha K|\dot{w}|\left(\frac{t-\beta}{Y}\right) \tag{10}$$

where

t = traction and $\dot{t}$ = traction rates (increment)
K = elastic modulus of soil
$\dot{w}$ = displacement rates (increment)
β = back stress with $\dot{\beta}$ = incremental back stress
Y = yield stress
α = constant controlling the slope of $t - w$ curve

System nonlinearity is dominated by using the springs characterized by Eqs. (9) and (10).

Parameters α, Y mainly control the behavior of the model. β is the function of α and Y, and is therefore not independent parameter. The verification of the inelastic model is studied in Küçükarslan and Banerjee (2003, 2004). The constant 'α' defines the post yield rate of the $t - w$ curve. For axially loaded piles, a relative slip occurs between pile and soil when tractions exceed a limiting value. In this case, the value of Y assumes the value of the limiting pile soil friction.

The purpose of this model in the lateral loading is to simulate the material nonlinearity of the soil and to take into account of local softness in the soil around the pile. The values of α and Y are depended on the soil deformation characteristics a particular depth. The elastic modulus, K in Eq. (9) is a function of soil modulus at a particular depth.

In this study following spring stiffness are used.

For lateral coefficients given by

$$\overline{K} = F_t E_s / D \tag{11a}$$

and for axial coefficients given by

$$\overline{K} = F_a G_s / \pi D \tag{11b}$$

where D is the diameter of pile, E_s is the Young's modulus of soil, G_s is the shear modulus of soil. F_a and F_t are the factors determining the level of nonlinearity existing at the pile soil interface. F_a and F_t has the main role in allowing the continuum to dominate the initial elastic response. This is adequately modeled by the elastic continuum equations. Since the gapping phenomenon is inelastic, the response by considering a gap is almost linear. The role of inelastic springs is to model the soil nonlinear behavior. Therefore, it is appropriate to use a high value of F_t so that the initial response is dictated by the elastic continuum. For higher strain levels, the springs yield and dominate the overall behavior. F_a anf F_t can be evaluated from the experimental results of the static tests.

2.3 Pile equations

The governing differential equations for time harmonic beam subjected to axial and lateral excitations are given by

$$m\ddot{u}_z^j - E_pA_p\frac{\partial^2 u_z^j}{\partial z^2} = \pi D t_z^j \tag{12}$$

$$m\ddot{u}_x^j + E_pI_p\frac{\partial^4 u_x^j}{\partial z^4} = -Dt_x^j \tag{13}$$

where, E_p is the Young's modulus of pile material; I_p is the second moment of inertia of pile; A_p is the cross-sectional area of pile; D is the diameter of the pile u_x^j and u_z^j are the lateral and axial displacements at time t^j, respectively; t_x^j and t_z^j are the lateral and axial tractions along the pile at time t^j, respectively.

The piles are modeled with linear beam column elements. The dynamic equilibrium equations of motion at time t^j are written as follow

$$\begin{bmatrix} M_{uu} & M_{u\theta} \\ M_{\theta u} & M_{\theta\theta} \end{bmatrix}\begin{Bmatrix} \ddot{u}^j \\ \ddot{\theta}^j \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\theta} \\ K_{\theta u} & K_{\theta\theta} \end{bmatrix}\begin{Bmatrix} u^j \\ \theta^j \end{Bmatrix} = \begin{Bmatrix} f_u^j \\ f_\theta^j \end{Bmatrix} \tag{14}$$

where $[M]$ and $[K]$ are the inertia and stiffness matrices. u^j, θ^j are the displacement and rotations of the beam element. f_u^j, f_θ^j are the externally applied loads and moments, respectively.

In the equilibrium equation, degrees of freedom corresponding to rotations at the pile nodes need to be eliminated since boundary element formulations contain translation degrees of freedom only. The rotational degree of freedom of the pile top is left in the system of equation, since it will form a part of unknowns to be solved. The Guyan reduction technique was used to eliminate the degrees of freedom corresponding to rotations at the pie node since BEM formulation includes only displacements and tractions. By using Guyan reduction technique (Bathe 1993), the Eq. (14) yield to following equation in incremental form

$$[K^{eff}]\{\Delta u^j\} = \{\Delta\bar{f}^j\} + \{\bar{B}^j\} \tag{15a}$$

where

$$[K^{eff}] = [\bar{K}] + a_o[\bar{M}] \tag{15b}$$

$$\{\bar{B}^j\} = [\bar{M}]\{a_0 u^{j-1} + a_2\dot{u}^{j-1} + a_3\ddot{u}^{j-1}\} \tag{15c}$$

$$a_0 = \frac{1}{\gamma\Delta t^2};\quad a_2 = \frac{1}{\gamma\Delta t};\quad a_3 = \frac{1}{2\gamma} - 1 \tag{15d}$$

For given values of $\delta = 0.5$ and $\gamma = 0.25$, the suggested method is unconditionally stable (Bathe 1993). Eq. (15a) needs to be cast in a form so that it can be coupled with the boundary element equations by enforcing displacement compatibility. By keeping the pile soil degrees of freedom as unknowns, one can write the following

$$\{\Delta u_p^j\} = [D]\{\Delta t_p^j\} + [b_p]\{\Delta u_c^j\} + \{B_p^j\} \tag{16}$$

where $[D]$ is inversion of $[K^{eff}]$ with the equivalent areas of the pile nodes absorbed in it, $[b_p]$ is a boundary condition matrix for a unit pile head displacements and rotations, $\{\Delta u_c^j\}$ is the vector of pile head displacements and rotations and the subscript 'p' represents the displacements and tractions obtained by pile domain only.

2.4 Global equilibrium equations

Eqs. (7) and (16) can be coupled only by imposing a constraint on the incremental traction vector, $\{\Delta t_p^j\}$, i.e., $\{\Delta t_p^j\} = -\{\Delta t_s^j\}$. This constraint arises from a consideration of global equilibrium of the entire system, which can be written in incremental form as

$$[B_1]\{\Delta t_p^j\} - [B_2]\{\ddot{u}_p^j - \ddot{u}_p^{j-1}\} - [B_3]\{\ddot{u}_c^j\} = \{\Delta F^j\} \tag{17}$$

where $[B_1]$ and $[B_2]$ are coefficient matrices obtained from area and mass properties of pile segment respectively. $[B_3]$ is a diagonal matrix constituting of the inertial properties of the pile cap. $\{\Delta F^j\}$ is the externally applied incremental transient load on the pile head. By substituting current acceleration and Eq. (16) into Eq. (17), one can write

$$[B]\{\Delta t_p^j\} - [E]\{\Delta u_c^j\} = \{\Delta F^j - H^j\} \tag{18}$$

where

$$[B] = [B_1] + a_o[B_2][D] \tag{19a}$$

$$[E] = a_o[B_2][b_p] + a_o[B_3] \tag{19b}$$

$$\{H^j\} = -a_o[B_2][B_p^j] - \{\tilde{B}^j\} + [B_3]\{a_2\dot{u}_c^{j-1} + \bar{a}_3\ddot{u}_c^{j-1}\} \tag{19c}$$

$$\{\tilde{B}^j\} = -B_2\{a_2\dot{u}_p^{j-1} + \bar{a}_3\ddot{u}_p^{j-1}\} \tag{19d}$$

2.5 Assembly of pile and soil equations

The developed equations for the pile and soil domain are assembled by imposing a displacement compatibility conditions at the pile-soil interface. Nonlinear behavior is incorporated by relating pile and soil displacements through a spring. The incremental displacements in the spring is expressed as

$$\Delta w = \Delta u_s - \Delta u_p \tag{20}$$

The traction in the spring is same as that at the interface but signs are opposite. Eqs. (7) and (8) are coupled using compatibility relations with Eqs. (17) and (18). The resulting form is as follow

$$\begin{bmatrix} G + D + K_{ozd}^{-1} & b_p \\ B & -E \end{bmatrix} \begin{Bmatrix} \Delta t_p^j \\ \Delta u_c^j \end{Bmatrix} = \begin{Bmatrix} R_s^j \\ \Delta F_c^j - H^j \end{Bmatrix} \tag{21}$$

where K_{ozd} is the equivalent nonlinear stiffness given by Özdemir's model and can be represented as

$$K_{ozd} = K\left[1 - \text{sgn}(\dot{w})\left(\frac{t-\beta}{Y}\right)\right] \tag{22}$$

2.6 Formulation for a pile group

For a two pile group, the equilibrium equation is given by using same procedure applied in single pile formulation, one can write the following

$$\begin{bmatrix} G_{11} + D_{11} + K^{-1}_{ozd,\,11} & G_{12} & b_{p_1} \\ G_{21} & G_{22} + D_{22} + K^{-1}_{ozd,\,22} & b_{p_2} \\ B_1 & B_2 & -E \end{bmatrix} \begin{Bmatrix} \Delta t^j_{p1} \\ \Delta t^j_{p2} \\ \Delta u^j_c \end{Bmatrix} = \begin{Bmatrix} R^j_{s1} - B^j_{p1} \\ R^j_{s2} - B^j_{p2} \\ \Delta F^j_c - H^j \end{Bmatrix} \tag{23}$$

In Eq. (23), the indices '1' and '2' resembles to piles in the group. The pile cap is rigid and its contact with the ground is assumed to be ineffective.

2.7 Extension to multiple groups

Extension to multiple groups is useful for coupling a superstructure with multiple supports to the entire foundation system. An assembled equation for two independent groups of piles is presented as

$$\begin{bmatrix} G + D + K^{-1}_{ozd,\,11} & G_{12} & b_{p_1} & 0 \\ G_{21} & G + D + K^{-1}_{ozd,\,22} & 0 & b_{p_2} \\ B_1 & 0 & -E_1 & 0 \\ 0 & B_2 & 0 & -E_2 \end{bmatrix} \begin{Bmatrix} \Delta t^j_{p1} \\ \Delta t^j_{p2} \\ \Delta u^j_{c1} \\ \Delta u^j_{c1} \end{Bmatrix} = \begin{Bmatrix} R^j_{s1} - B^j_{p1} \\ R^j_{s2} - B^j_{p2} \\ \Delta F^j_{c1} - H^j_1 \\ \Delta F^j_{c2} - H^j_2 \end{Bmatrix} \tag{24}$$

In this equation, the indices '1' and '2' refer to the two independent groups. It shows that between the two groups only a weak coupling exits through the terms G_{12} and G_{21}. For groups that are very far to each other, these coupling terms are close to zero and the behavior of the groups are uncoupled.

3. Numerical example

A statnamic type of loading typically shown in Fig. 1 is applied to a single pile and 3 × 3 pile group (Fig. 2) under axial and lateral directions for Gibson soils (Fig. 3). Following parameters are used for analysis.

Pile length, L = 20. meter
Pile diameter, D = 1. meter
Number of elements, N = 20
Pile to pile spacing for groups, S = 3D
Pile modulus, E_p = 25.5 GPa
Density of pile, ρ_p = 2400 kg/m^3

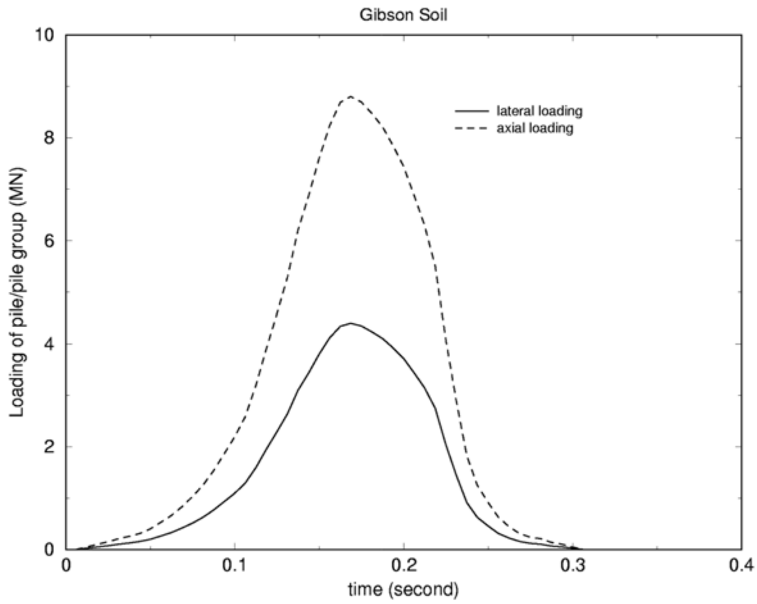

Fig. 1 Statnamic loads applied to pile head

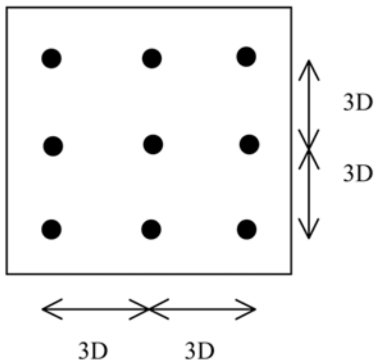

Fig. 2 A 3 × 3 pile goup used in analysis

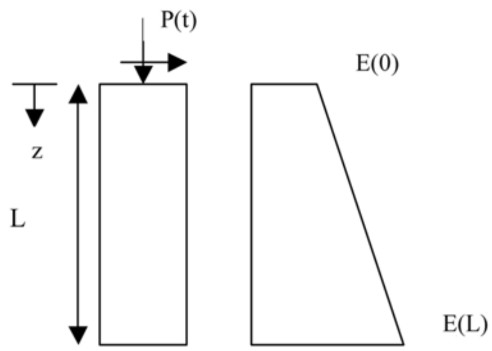

Fig. 3 Gibson soil and pile domain

Density of soil, ρ_s = 1800 kg/m^3

Poisson's ratio of soil, $\upsilon_s = 0.4$

For Gibson soil ($E_s = E_o + mz$), E_o = 12.75 MPa and m = 1.9125 MPa/meter and z is the depth. The reason to choose these parameters is to attempt to simulate a realistic problem. In other words, $L/D = 20$, and $E_p/E_s = 400$ resembles a concrete pile embedded in Gibson soil. Nonlinear analyses in time domain, the post yield parameter was used as 0.02 in axial and lateral modes of deformation and F factors as 2 in all modes.

In all plots, the x-axis is used for nondimensional time τ, and given by

for Gibson soil, $\tau = \dfrac{t}{R}\sqrt{\dfrac{G_{s,o}}{\rho_s}}$

where, t is total duration of impact loading

R is radius of pile

$G_{s,o}$ is shear modulus of Gibson soil at the top (z = 0)

ρ_s is mass density of soil.

The linear and nonlinear behavior of single pile for lateral and axial modes are presented for

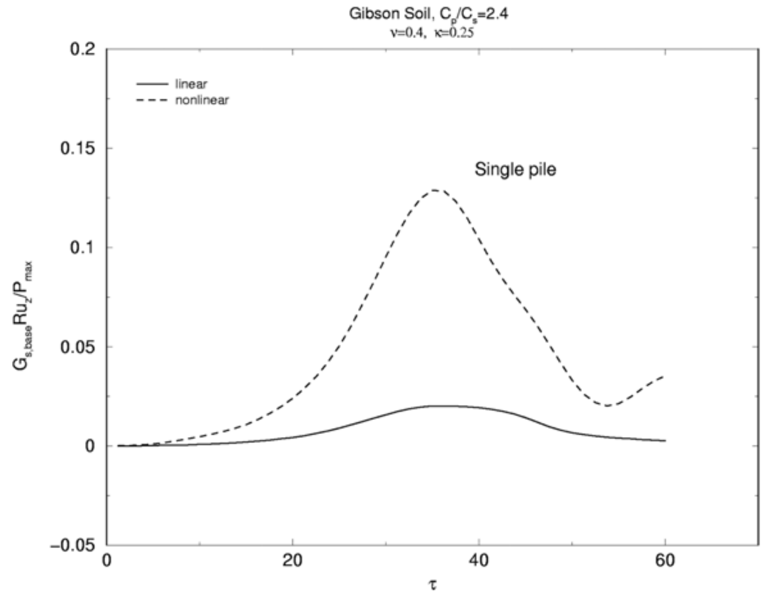

Fig. 4 Axial response of single pile

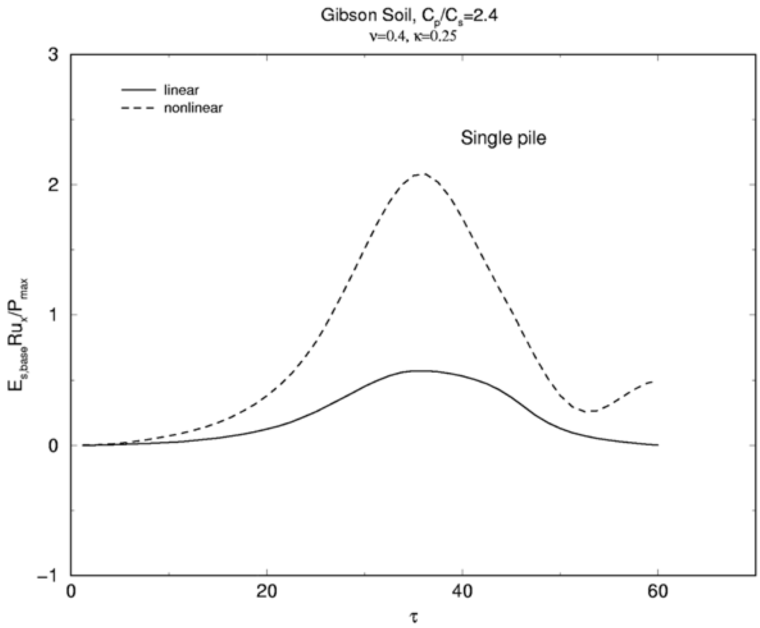

Fig. 5 Lateral response of single pile

Gibson soil in Figs. 4 and 5 for axial and lateral cases respectively. For nonlinear case, it can be seen that there is a non-recoverable plastic displacements.

Peak pile axial response of single pile is about 1% of the pile diameter at which level considerable pile soil slippage occurs. It can be seen from the figures that the behavior of the nonlinear case is different from that of the linear case due to this slipping effect. And another observation is that the nonlinear effect does not seen to produce additional phase lag in the response. This is probably not surprising because the wave speeds are not significantly altered by nonlinearities.

The behavior of a single pile under lateral load is similar to that of the axial case, but this time the nonlinear effects increase due to material near the ground surface yielding significantly. Beside a single pile, a 3 × 3 pile group is also analyzed under the same type of loading for Gibson soils. Pile group is rigidly capped with a concrete block of two pile diameter thickness. Analysis is done with cap and without cap inertia effects and the results are presented in Figs. 6 and 7, respectively for Gibson soil.

For lateral loading, the group shows a higher stiffness in linear case because of the push pull action on a per pile basis. Cap inertia affect the behavior of group by increasing in peak displacement magnitude and delaying in the response. Increasing levels of nonlinearity can be observed at higher load levels.

For axial loading case, the pile head displacement response is double that of a single pile on a per pile basis comparison. This is caused by interaction among the piles in the group. Cap inertia has little effect on the behavior unlike the lateral case. The reason for this is that a larger soil is exited in axial mode and in lateral mode only near ground surface is playing the dominant role. The cap inertia therefore can have bigger effect under the lateral loading.

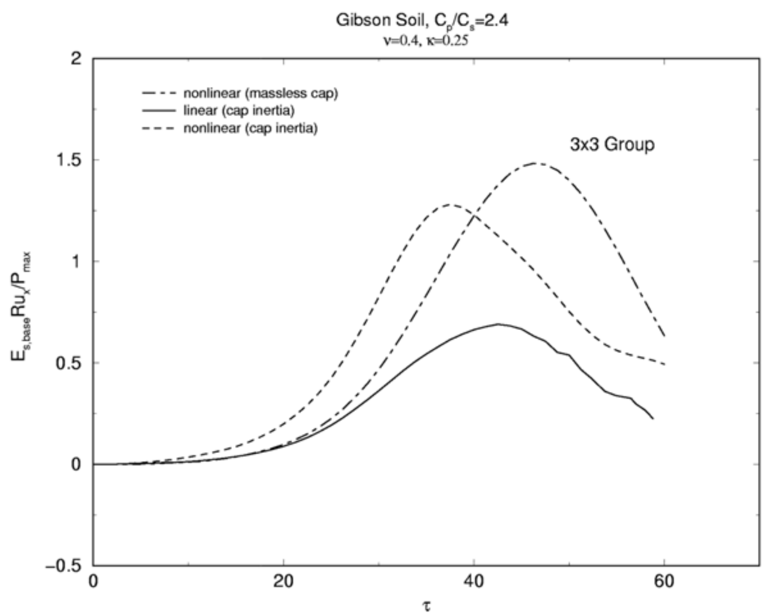

Fig. 6 Lateral response of 3 × 3 pile group

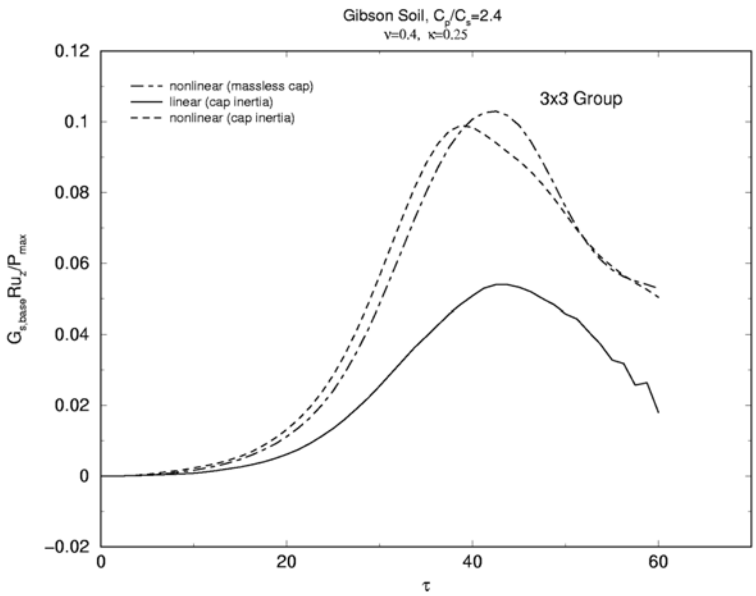

Fig. 7 Axial response of 3 × 3 pile group

4. Conclusions

Nonlinear transient dynamic analysis of pile soil interaction in Gibson soil was presented. The hybrid method employed a time stepping BEM algorithm together with an implicit time integration FEM scheme used for modeling the piles. Nonlinear effects were accommodated by modified Özdemirs's model. A single pile and 3 × 3 pile group was analyzed to investigate the effects nonlinear media on the pile soil interaction. The proposed model is the computationally most efficient and most accurate algorithm since it uses a simple nonlinear model and couples finite and boundary element methods.

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