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Alternative plate finite elements for the analysis of thick plates on elastic foundations

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Abstract. A four-noded plate bending quadrilateral (PBQ4) and an eight-noded plate bending quadrilateral (PBQ8) element based on Mindlin plate theory have been adopted for modeling the thick plates on elastic foundations using Winkler model. Transverse shear deformations have been included, and the stiffness matrices of the plate elements and the Winkler foundation stiffness matrices are developed using Finite Element Method based on thick plate theory. A computer program is coded for this purpose. Various loading and boundary conditions are considered, and examples from the literature are solved for comparison. Shear locking problem in the PBQ4 element is observed for small value of subgrade reaction and plate thickness. It is noted that prevention of shear locking problem in the analysis of the thin plate is generally possible by using element PBQ8. It can be concluded that, the element PBQ8 is more effective and reliable than element PBQ4 for solving problems of thin and thick plates on elastic foundations.

Keywords: finite element method; thick plate theory; elastic foundation; Winkler model.

1. Introduction

The analysis of plates resting on elastic foundations has a wide range of applications in structural and geotechnical engineering. Most of the studies for plates resting on elastic foundation are based on classical Kirchhoff thin plate theory. However as the thickness of the plate increases, the transverse shear deformations play very important role to improve the results significantly. For this reason, many shear deformable plate theories have been proposed over the years. Reissner and Mindlin plate theory are fundamentally simpler to adopt for modeling the shear deformation behaviour of thick plates.

The primary difficulty in the analysis of the soil-foundation problems lies in determination of the contact pressure. Due to the complexity of the actual behaviour of foundations, many idealized foundation models have appeared in the literature (Selvaduari 1979). The simplest of these models was proposed in 1867 by Winkler assuming the vertical displacement of the plate at every point is

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proportional to the contact pressure at that point. This model can be considered as an idealization of the soil medium by a number of mutually independent spring elements (Hetenyi 1950). The governing differential equation of the plate when it is supported by elastic continuum is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{k}{D}w = \frac{q}{D}$$

where D is the stiffness of the plate, q is the transverse distributed load, w is the plate deflection, k is the modulus of subgrade reaction.

The analysis of plates resting on Winkler elastic foundations has been the subject of numerous investigations in the past. Voyiadjis and Kattan (1986) are used the refined theory incorporated the transverse normal strain effect in addition to the transverse shear and normal stress effects for the bending of plates on elastic foundations. Eratli and Aköz (1997), using the Gateaux differential, developed the mixed element formulation for the thick plates on elastic foundation. Yettram and Whiteman (1984) are investigated effect of thickness on the behavior of plates on elastic foundation. Al-Khaiat and West (1990) developed approximate solutions for rectangular plates on elastic foundations using initial value method. Assuming Winkler elastic foundation, they presented dimensionless deflection, moment and shear forces. Mishra and Chakrabarti (1997) investigated shear and attachment effects on the behaviour of rectangular plates resting on tensionless elastic foundation using finite element techniques. They used a nine-noded Mindlin element to account for transverse shear effects. Liu (2000), developed differential quadrature element method (DQEM) for the static analysis of homogenous isotropic rectangular plates on Winkler foundation on the basis of first-order shear deformation theory. Using nondimensional parameters, Daloglu and Vallabhan (2000) have attempted to evaluate the value of subgrade reaction modulus for use in the Winkler model for the analysis of slab. Graphs are provided from which values of an equivalent subgrade reaction modulus can be computed from the complete geometry and properties of the overall system. Sadecka (2000) analyzed thick plate on a layered foundation using Kolar-Nemec's subsoil model. Using the Reissner plate bending theory, Rashed *et al*. (1998) applied the boundary element method to analysis of thick plates on a Winkler foundation. Abdalla and Ibrahim (2006) developed a discrete Reissner-Mindlin triangular plate element for analysis of thick plates on Winkler foundation. Wang *et al.* (2003) used a semi-analytical and semi-numerical method for the analysis of plates on layered foundation. Buczkowski and Torbacki (2001), using an 18-node zero-thickness isoparametric interface element that takes into account shear deformation of the plate, analyzed thick plate resting on two-parameter elastic foundation. Çelik and Saygun (1999) developed a finite element formulation for plates on an elastic foundation in which shear deformations are included to bending behaviour, and the effect of subsoil is considered as a combination of elastic bending and shear deformation of the soil. Chucheepsakul and Chinnaboon (2002) presented an alternative domain-boundary element technique for analysis of plates on two-parameter elastic foundations. Teo and Liew (2002) used differential cubarature method for analysis of thick plates on elastic foundation. Çelik and Omurtag (2005) tried to determine of the soil parameters for plates on elastic foundation.

In this paper 4-noded element (PBQ4) and 8-noded element (PBQ8) are used to solve problems of thick plates resting on Winkler elastic foundation. A detailed analysis for the development of these elements is carried out to generate the element stiffness matrices and the load vectors taking the effect of Winkler foundation into account. The elements are tested with various values of modulus of subgrade reactions, different plate thicknesses, boundary and loading conditions.

2. Finite element modelling

The total potential energy in the soil-structure system may be written as

$$\Pi = U + V \tag{1}$$

where $U = \Pi_p + \Pi_s$, in which Π_p is the strain energy stored in the plate, Π_s is the strain energy stored in the soil and V is the potential energy of the external loads.

The subsoil has a finite depth on a rigid base at the bottom (Fig. 1). The total potential energy in Eq. (1) can be expanded as

$$\Pi = \frac{1}{2}\int_{\Omega}[B]^T[D][B]dA + \frac{1}{2}\int_{\Omega}[w(x,y)]^T k[w(x,y)]dA + \int_{\Omega}N^T q dA \tag{2}$$

In this study 4-noded (PBQ4) and 8-noded (PBQ8) quadrilateral rectangular finite elements based on Mindlin plate theory are used to develop the element stiffness matrix (Fig. 2) (Weaver and Johnston 1984).

Nodal displacements at each node are

$$w,\ \varphi_x,\ \varphi_y \tag{3}$$

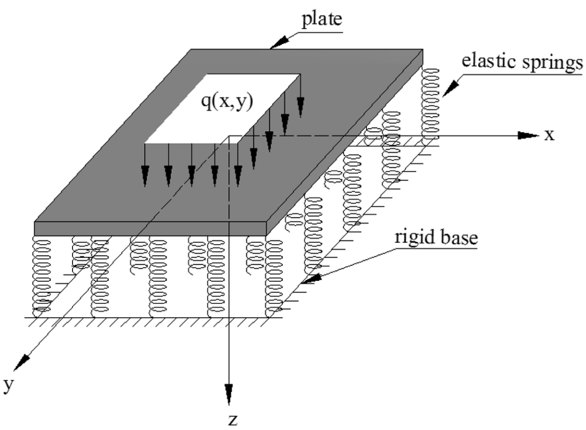

Fig. 1 A loaded plate resting on elastic foundation

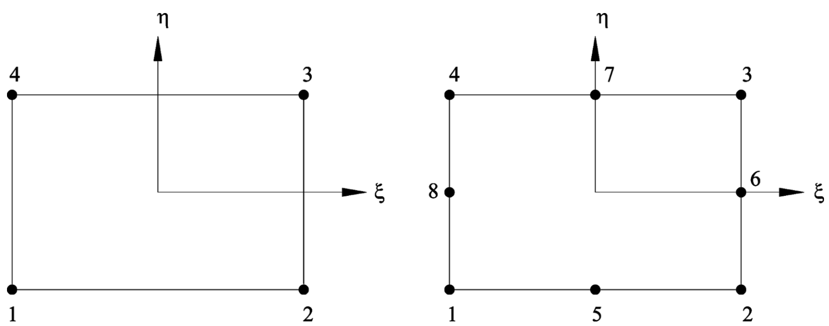

Fig. 2 The finite elements (a) PBQ4, (b) PBQ8

where w is the transverse displacement, φ_x, φ_y are the rotations of the normal to the undeformed middle surface. It is assumed that w, φ_x and φ_y vary quadratically over the element so that

$$\begin{aligned} u &= z\varphi_y = z\sum_{i=1}^{n} N_i \varphi_{yi} \\ v &= -z\varphi_x = -z\sum_{i=1}^{n} N_i \varphi_{xi} \\ w &= \sum_{i=1}^{n} N_i w_i \end{aligned} \tag{4}$$

in which n is equal to 4 for PBQ4 and 8 for PBQ8. So the displacement shape functions for PBQ4 are given as

$$[N_i] = [N_1 \;\; 0 \;\; 0 \;\; N_2 \;\; 0 \;\; 0 \;\; N_3 \;\; 0 \;\; 0 \;\; N_4 \;\; 0 \;\; 0] \tag{5}$$

where N_1, N_2, N_3, and N_4 are given in Weaver and Johnston (1984). Shape functions for PBQ8 are

$$[N_i] = [N_1 \;\; 0 \;\; 0 \;\; N_2 \;\; 0 \;\; 0 \;\; \ldots \;\; N_8 \;\; 0 \;\; 0] \tag{6}$$

and N_1, N_2,…, N_8 for PBQ8 are given in Weaver and Johnston (1984). The rotations φ_x and φ_y are independent, and are not related to w by differentiation.

The strain-displacement matrix is

$$[B]_i = \begin{bmatrix} 0 & 0 & \dfrac{\partial N_i}{\partial x} \\ 0 & -\dfrac{\partial N_i}{\partial y} & 0 \\ 0 & -\dfrac{\partial N_i}{\partial x} & \dfrac{\partial N_i}{\partial y} \\ \dfrac{\partial N_i}{\partial x} & 0 & N_i \\ \dfrac{\partial N_i}{\partial y} & -N_i & 0 \end{bmatrix} \quad [B] = [[B]_1 [B]_2 \ldots [B]_n] \tag{7}$$

where n is 4 for PBQ4 and 8 for PBQ8 element.

The stresses and corresponding strains for the case of plane stress are

$$\begin{aligned} \{\sigma\} &= \{\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz}\} \\ \{\varepsilon\} &= \{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}\} \\ \{\sigma\} &= [D]\{\varepsilon\} \end{aligned} \tag{8}$$

where $[D]$ is the material matrix relating the strains and stresses (Bathe 1996).

Substituting Eq. (4) into Eq. (2) the stiffness matrices of the plate-soil system can be evaluated as

$$U = \frac{1}{2}\{w_e\}^t([k_p] + [k_w])\{w_e\} \tag{9}$$

where $[k_p]$ and $[k_w]$ are stiffness matrix of the plate and stiffness matrix of the Winkler foundation. $\{w_e\}$ is the nodal displacement vector for an element containing 12 components for PBQ4 and 24 components for PBQ8.

The stiffness matrix for the plate element can be evaluated as

$$[k_p] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\det(J)d\xi d\eta \tag{10}$$

the stiffness matrix for the Winkler foundation can be derived by

$$[k_w] = k\int_{-1}^{1}\int_{-1}^{1}[N]^T[N]\det(J)d\xi d\eta \tag{11}$$

and, the load vector for a plate element with a uniformly distributed load can be evaluated as

$$\{f\} = \int_{-1}^{1}\int_{-1}^{1}[N]^T q\det(J)d\xi d\eta \tag{12}$$

The element stiffness matrices for the plate and for the Winkler foundation were integrated from above equations. The Winkler foundation element stiffness matrices for PBQ4 and PBQ8 are given in explicit forms in Appendix. The plate element stiffness matrices are not presented in the paper since the matrices takes a lot of space.

Assembling each element stiffness matrix obtained from above equations, global stiffness matrix is evaluated as

$$[K] = \sum_{i=1}^{n_e}([k_p]+[k_w]) \tag{13}$$

where n_e is the nodal number of plate finite elements. Finally the equation to be solved is

$$[K]\{W\} = \{F\} \tag{14}$$

Here $[K]$ is the global stiffness matrix, $\{W\}$ represents the global nodal displacement, and $\{F\}$ is the applied equivalent load vector of the system (Turhan 1992).

3. Numerical examples

3.1 Example-1

In the first example, a square plate is analyzed to demonstrate the accuracy and the efficiency of the present formulation. The examples are considered for two different types of boundary conditions and for two load cases. Clamped supported plate with uniformly distributed load, clamped supported plate with central point load, simply supported plate with uniformly distributed load and simply supported plate with central point load are considered respectively. Poisson's ratio of the plate equals 0.30. The ratio of the length to the plate thickness is 15. A convergence test is performed for two boundary conditions without foundations. Mesh size of 20 × 20 for PBQ4, and

10×10 for PBQ8 are decided for a reasonable result. The effects of foundation stiffness and boundary conditions on solutions have been studied. The results obtained in the study are compared with the results of past studies (Al-Khaiat and West 1990, Mishra and Chakrabarti 1997, Timoshenko and Krieger 1970).

The effects of both l/h and K on the plate behavior have been studied for all four edges free plate subjected to concentrated load. The ratio of the span length to the plate thickness is taken as 40, 20, 15, 10, 5, 4, and the Poisson's ratio is 0.30. The nondimensional modulus of subgrade reaction is taken as 3, 6, 9, 12, 15. In this case 16×16 mesh sizes are used for element PBQ4 and element PBQ8 as in the study by Mishra and Chakrabarti (1997).

In these examples, the nondimensional modulus of subgrade reaction and factors are used and they are defined as follows

$$K = l(k/D)^{0.25}$$
$$a_m = wD/(Pl^2)$$
$$a_m = wD/(Ql^4)$$

where l is the span length, D is flexural rigidity of the plate, P is concentrated load and Q is distributed load. Other terms have been defined before.

In Table 1 and Fig. 3 nondimensional central deflections for the clamped supported plate with uniformly distributed load are presented and compared with the result obtained in (Al-Khaiat and West 1990, Mishra and Chakrabarti 1997, Timoshenko and Krieger 1970). The similar comparisons

Table 1 Nondimensional central deflections for the clamped supported plate with uniformly distributed load

$wD100/(Ql^4)$					
	Mishra and Chakrabarti (1997)		Al-Khaiat and West (1990)	Present study	
K	Thick	Thin		PBQ4	PBQ8
0	0.1360	0.1249	0.1260	0.1228	0.1369
1	0.1350	0.1249	0.1260	0.1227	0.1367
2	0.1340	0.1235	0.1240	0.1212	0.1350
3	0.1270	0.1174	0.1180	0.1154	0.1277
4	0.1110	0.1037	0.1040	0.1021	0.1114
5	0.0870	0.0829	0.0830	0.0818	0.0874
6	0.0620	0.0603	-	0.0596	0.0622
7	0.0410	0.0410	-	0.0405	0.0414
8	0.0270	0.0269	-	0.0266	0.0267
9	0.0170	0.0175	-	0.0173	0.0172
10	0.0110	0.0115	-	0.0113	0.0112
11	0.0070	0.0076	-	0.0076	0.0074
12	0.0050	0.0053	-	0.0052	0.0051
13	0.0040	0.0038	-	0.0037	0.0036
14	0.0030	0.0028	-	0.0027	0.0026
15	0.0020	0.0021	-	0.0020	0.0020

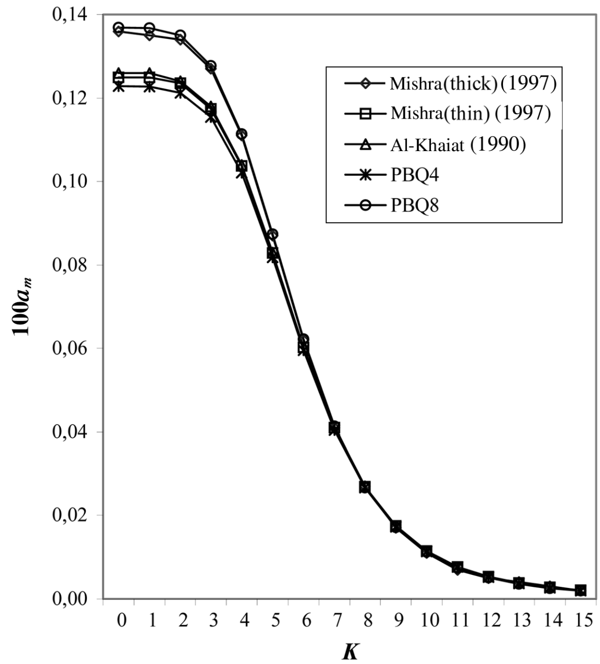

Fig. 3 Variation of the nondimensional central deflection with K for clamped supported plate subjected to uniformly distributed load

Table 2 Nondimensional central deflections for the clamped supported plate with concentrated load

	$wD100/(Pl^2)$			
	Mishra and Chakrabarti (1997)		Present study	
K	Thick	Thin	PBQ4	PBQ8
0	0.6540	0.5505	0.5914	0.6507
1	0.6540	0.5501	0.5911	0.6503
2	0.6480	0.5448	0.5861	0.6441
3	0.6230	0.5233	0.5656	0.6186
4	0.5660	0.4740	0.5187	0.5614
5	0.4820	0.3990	0.4468	0.4764
6	0.3920	0.3163	0.3667	0.3854
7	0.3150	0.2440	0.2959	0.3077
8	0.2570	0.1887	0.2410	0.2489
9	0.2140	0.1486	0.2006	0.2062
10	0.1830	0.1197	0.1708	0.1749
11	0.1590	0.0983	0.1485	0.1514
12	0.1410	0.0821	0.1309	0.1332
13	0.1270	0.0696	0.1170	0.1188
14	0.1150	0.0596	0.1057	0.1071
15	0.1050	0.0516	0.0963	0.0974

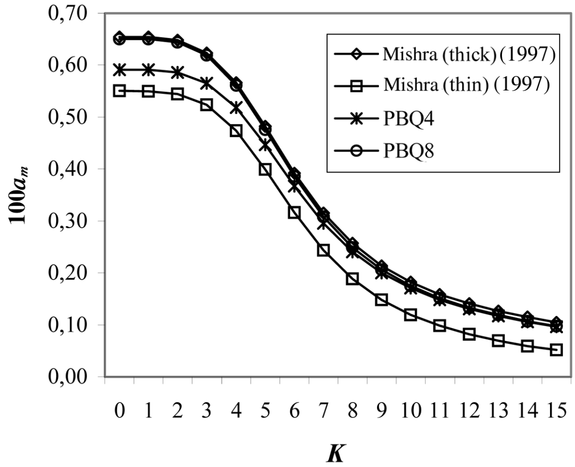

Fig. 4 Variation of the nondimensional central deflection with K for clamped supported plate subject to concentrated load

Table 3 Nondimensional central deflections for the simply supported plate with uniformly distributed load

	$wD1000/(Ql^4)$					
	Mishra and Chakrabarti (1997)		Al-Khaiat (1990)	Timoshenko (1970)	Present study	
K	Thick	Thin			PBQ4	PBQ8
0	4.1300	4.0060	4.0600	4.0525	3.8487	4.1539
1	4.1200	3.9950	4.0500	4.0420	3.8391	4.1427
2	3.9600	3.8450	3.9000	3.8893	3.7017	3.9820
3	3.3900	3.3040	3.3400	3.3400	3.2025	3.4066
4	2.4400	2.3870	2.4000	2.4111	2.3416	2.4426
5	1.5100	1.4910	1.4900	1.5039	1.4797	1.5121
6	0.8700	0.8690	-	0.8751	0.8695	0.8741
7	0.5000	0.5010	-	0.5034	0.5035	0.5002
8	0.2900	0.2940	-	0.2946	0.2960	0.2916
9	0.1800	0.1780	-	0.1777	0.1791	0.1758
10	0.1100	0.1120	-	0.1114	0.1125	0.1104
11	0.0800	0.0760	-	0.0728	0.0736	0.0724
12	0.0500	0.0530	-	0.0497	0.0502	0.0496
13	0.0400	0.0380	-	0.0352	0.0356	0.0353
14	0.0300	0.0290	-	0.0259	0.0261	0.0260
15	0.0200	0.0220	-	0.0195	0.0196	0.0197

are presented for the clamped supported plate with concentrated load in Table 2 and Fig. 4, for the simply supported plate with uniformly distributed load in Table 3 and Fig. 5, for the simply supported plate with concentrated load in Table 4 and Fig. 6 and for the free square plate subjected to concentrated load with various values of l/h and K in Table 5 and Figs. 8-10.

As seen form Figs. 3-6, the effect of the shear deformation on the displacement is larger for concentrated load than for distributed load case. For very stiff foundation ($K > 6$), the effect of the

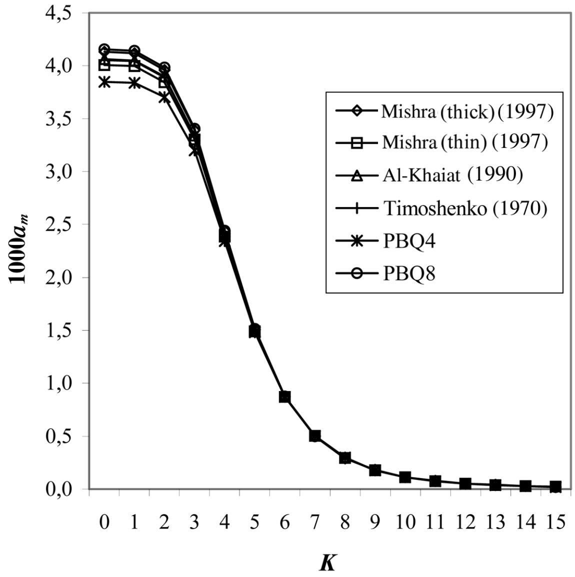

Fig. 5 Variation of the nondimensional central deflection with K for simply supported plate subjected to uniformly distributed load

Table 4 Nondimensional central deflections for the simply supported plate with concentrated load

	$wD100/(Pl^2)$				
	Mishra and Chakrabarti (1997)		Timoshenko (1970)	Present study	
K	Thick	Thin		PBQ4	PBQ8
0	1.2500	1.1498	1.1581	1.1578	1.2460
1	1.2470	1.1472	1.1555	1.1554	1.2433
2	1.2070	1.1096	1.1176	1.1212	1.2034
3	1.0650	0.9737	0.9812	0.9968	1.0605
4	0.8260	0.7430	0.7499	0.7815	0.8201
5	0.5920	0.5155	0.5218	0.5640	0.5854
6	0.4270	0.3540	0.3600	0.4062	0.4197
7	0.3230	0.2531	0.2588	0.3061	0.3159
8	0.2580	0.1902	0.1956	0.2429	0.2503
9	0.2140	0.1487	0.1539	0.2008	0.2063
10	0.1830	0.1197	0.1247	0.1708	0.1749
11	0.1590	0.0985	0.1031	0.1484	0.1514
12	0.1410	0.0824	0.0867	0.1310	0.1332
13	0.1270	0.0698	0.0739	0.1171	0.1188
14	0.1150	0.0559	0.0637	0.1057	0.1071
15	0.1050	0.0519	0.0555	0.0964	0.0974

shear deformation on the displacements becomes insignificant. As can also be seen from all Figures and Tables, the displacement of the plate always decrease with increasing K for any boundary conditions or any loading cases. This behaviour is understandable because a plate resting on stiffer

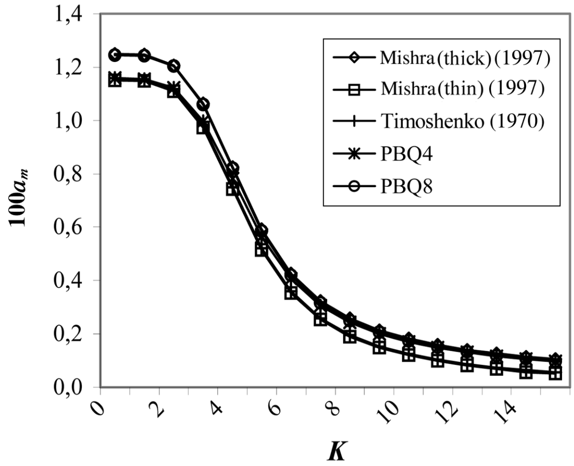

Fig. 6 Variation of the nondimensional central deflection with K for simply supported plate subjected to concentrated load

Table 5 Nondimensional central deflections for the free square plate subjected to concentrated load with various l/h and K

		$w1000D/(Pl^2)$				
		K				
	l/h	3	6	9	12	15
Mishra (1997)		17.97	3.84	1.70	0.97	0.64
PBQ4	40	15.05	2.60	1.13	0.63	0.42
PBQ8		18.26	3.76	1.62	0.93	0.61
Mishra (1997)		18.31	4.14	1.97	1.22	0.86
PBQ4	20	17.19	3.57	1.64	1.00	0.70
PBQ8		18.71	4.09	1.91	1.19	0.85
Mishra (1997)		18.66	4.14	2.24	1.46	1.07
PBQ4	15	18.03	4.00	1.94	1.24	0.90
PBQ8		19.09	4.41	2.19	1.45	1.08
Mishra (1997)		19.62	5.27	2.97	2.11	1.64
PBQ4	10	19.39	4.89	2.63	1.81	1.38
PBQ8		20.14	5.31	2.99	2.16	1.72
Mishra (1997)		24.77	9.66	6.70	5.21	4.11
PBQ4	5	24.46	8.84	5.80	4.32	3.29
PBQ8		25.76	10.06	7.04	5.61	4.65
Mishra (1997)		28.62	12.86	9.29	7.19	5.11
PBQ4	4	28.03	11.63	7.93	5.86	4.33
PBQ8		29.94	13.50	9.87	7.92	6.51

elastic foundation become less flexible and has a smaller displacement. But it should be noted that the decrease in the displacement of the plate with increasing K for any boundary conditions or any

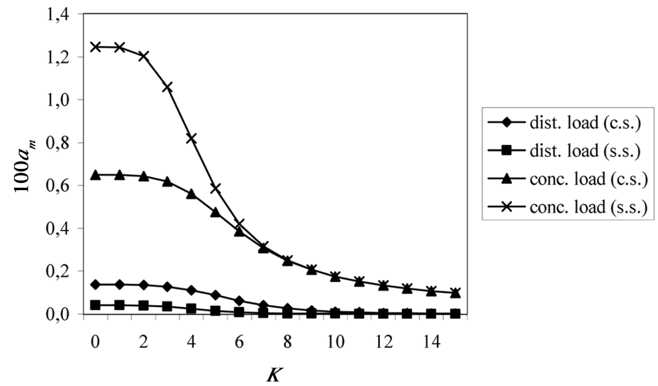

Fig. 7 The effected of boundary conditions for two loading cases for PBQ8

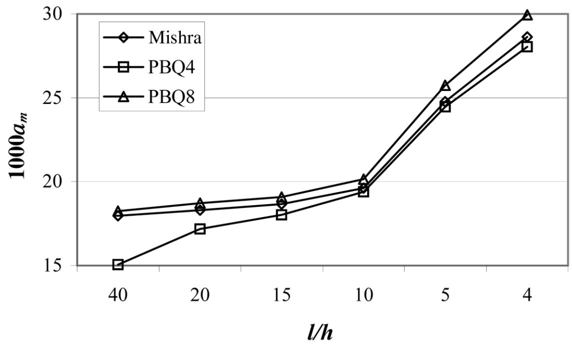

Fig. 8 Variation of the nondimensional central deflection of free square plate subjected to concentrated load with l/h for $K = 3$

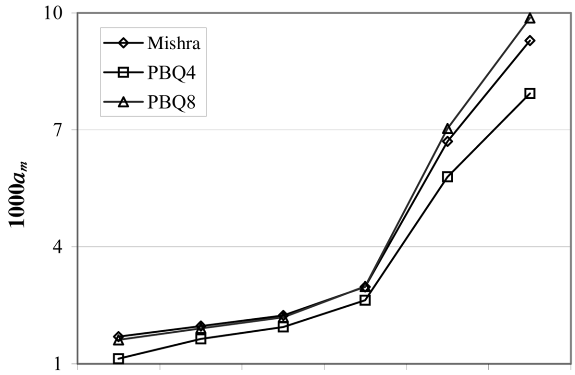

Fig. 9 Variation of the nondimensional central deflection of free square plate subjected to concentrated load with l/h for $K = 9$

loading cases gets less for larger values of K. Shear locking problem in the PBQ4 element is observed for small value of K. As can also be seen from Fig. 7, the curves for simply and clamped

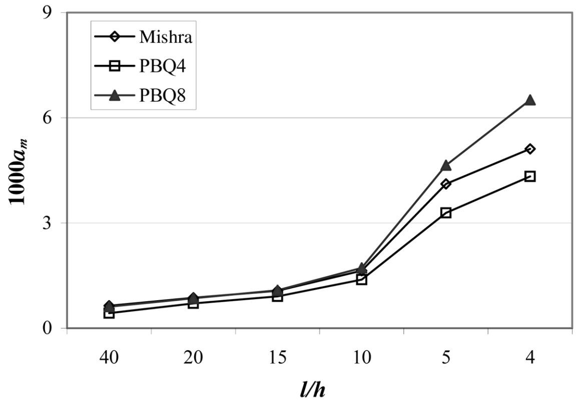

Fig. 10 Variation of the nondimensional central deflection of free square plate subjected to concentrated load with l/h for K = 15

supported plate get fairly closer to each other as the value of K increases for the same loading case. Especially for $K > 7$, effects of the boundary conditions on the results can be neglected.

As seen from Table 5 and Figs. 8-10, the displacement factors of the plate always increase with decreasing l/h, (or in other words, with increasing plate thickness) for all values of K, while the displacements of the plate always decrease. The shear locking problem were observed very clearly in the PBQ4 for small value of K and h. This problem disappears as the plate gets thicker and the foundation gets stiffer.

Table 6 Central deflections for the free square centrally loaded plate

			w(m) (h = 0.05 m)				w(m) (h = 0.5 m)			
			Yettram and Whiteman (1984)		Present study		Yettram and Whiteman (1984)		Present study	
E (MN/m^2)	ν	k (MN/m^3)	Thin	Thick	PBQ4	PBQ8	Thin	Thick	PBQ4	PBQ8
300	0.35	20	0.0476	0.0480	0.0469	0.0478	0.0128	0.0134	0.0132	0.0132
		50	0.0213	0.0215	0.0212	0.0213	0.0053	0.0059	0.0057	0.0057
		80	0.0138	0.0139	0.0137	0.0137	0.0034	0.0040	0.0038	0.0038
3000	0.30	20	0.0278	0.0280	0.0263	0.0279	0.0125	0.0126	0.0126	0.0125
		50	0.0144	0.0145	0.0139	0.0145	0.0050	0.0051	0.0051	0.0051
		80	0.0101	0.0102	0.0097	0.0101	0.0032	0.0032	0.0032	0.0032
6850	0.25	20	0.0218	0.0219	0.0204	0.0219	0.0125	0.0125	0.0125	0.0124
		50	0.0116	0.0117	0.0109	0.0116	0.0050	0.0050	0.0050	0.0050
		80	0.0083	0.0084	0.0078	0.0083	0.0031	0.0031	0.0031	0.0031
14000	0.20	20	0.0182	0.0183	0.0171	0.0182	0.0125	0.0125	0.0125	0.0122
		50	0.0095	0.1000	0.0088	0.0095	0.0050	0.0050	0.0050	0.0050
		80	0.0069	0.0069	0.0064	0.0069	0.0031	0.0031	0.0031	0.0031

Table 7 Bending moments for the free square centrally loaded plate

			M (MN-m/m) (h = 0.05 m)				M (MN-m/m) (h = 0.5 m)			
			Yettram and Whiteman (1984)		Present study		Yettram and Whiteman (1984)		Present study	
E (MN/m^2)	ν	k (MN/m^3)	Thin	Thick	PBQ4	PBQ8	Thin	Thick	PBQ4	PBQ8
300	0.35	20	0.00335	0.00331	0.00322	0.00328	0.0213	0.0242	0.0209	0.0212
		50	0.00147	0.00144	0.00149	0.00141	0.0211	0.0239	0.0204	0.0206
		80	0.00085	0.00084	0.00093	0.00082	0.0208	0.0236	0.0199	0.0201
3000	0.30	20	0.01200	0.01200	0.01056	0.01200	0.0210	0.0236	0.0208	0.0211
		50	0.00781	0.00778	0.00713	0.00776	0.0210	0.0235	0.0207	0.0210
		80	0.00594	0.00590	0.00552	0.00588	0.0209	0.0235	0.0207	0.0210
6850	0.25	20	0.01500	0.01500	0.01263	0.01524	0.0205	0.0227	0.0203	0.0206
		50	0.01110	0.01050	0.00969	0.01104	0.0205	0.0227	0.0203	0.0206
		80	0.00890	0.00890	0.00799	0.00888	0.0205	0.0226	0.0203	0.0206
14000	0.20	20	0.01630	0.01630	0.01297	0.01628	0.0195	0.0209	0.0194	0.0197
		50	0.01310	0.01310	0.01094	0.01315	0.0195	0.0209	0.0194	0.0197
		80	0.01110	0.01110	0.00953	0.01115	0.0195	0.0209	0.0194	0.0197

Table 8 Corner deflections for the free square corner loaded plate

				w (m)						
				Yettramm and Whiteman (1984)					Present study	
E (MN/m^2)	ν	k (MN/m^3)	h (m)	Thin	Thick	F.E. Thin	F.E. Thick	F.E. 3-D	PBQ4 element	PBQ8 element
300	0.35	50	0.05	0.005370	0.005830	0.004780	0.005140	-	0.005444	0.005791
			0.10	0.002210	0.002520	0.002000	0.002280	-	0.002451	0.002507
			0.20	0.000865	0.001060	0.000814	0.000992	-	0.001049	0.001053
			0.30	0.000541	0.000686	0.000517	0.000646	-	0.000676	0.000676
			0.40	0.000431	0.000544	0.000413	0.000512	-	0.000534	0.000533
			0.50	0.000386	0.000481	0.000376	0.000449	0.000503	0.000468	0.000466
14000	0.15	50	0.05	0.000980	0.001010	-	-	-	0.000952	0.001011
			0.10	0.000456	0.000472	-	-	-	0.000467	0.000471
			0.20	0.000352	0.000357	-	-	-	0.000357	0.000356
			0.30	0.000340	0.000343	-	-	-	0.000343	0.000341
			0.40	0.000337	0.000340	-	-	-	0.000339	0.000337
			0.50	0.000336	0.000380	-	-	-	0.000338	0.000335

3.2 Examle-2

A centrally loaded square plate on the Winkler foundation solved by Yettram and Whiteman (1984) is considered as a second example. The authors solved the plate problem using both thin plate and thick plate theory by Fourier series. The properties of the plate-soil system are as follows.

Table 9 Soil parameter, maximum displacement and bending moment of the plate on elastic foundation subjected to concentrated load at the center

h (m)	Reference	k (kN/m^3)	w (cm)	M_x (kNm/m)
3.048	Çelik and Saygun (1999)	31898	0.0818	15.047
	Buczkowski and Torbacki (2001)	31898	0.0894	-
	PBQ4 element	31898	0.0401	1.416
	PBQ8 element	31898	0.0983	11.356
6.096	Çelik and Saygun (1999)	24256	0.0845	14.563
	Buczkowski and Torbacki (2001)	24256	0.0912	-
	PBQ4 element	24256	0.0462	1.505
	PBQ8 element	24256	0.1145	12.046
9.144	Çelik and Saygun (1999)	23737	0.0846	14.510
	Buczkowski and Torbacki (2001)	23738	0.0912	-
	PBQ4 element	23738	0.0467	1.512
	PBQ8 element	23738	0.1159	12.101
15.240	Çelik and Saygun (1999)	23710	0.0846	14.510
	Buczkowski and Torbacki (2001)	23717	0.0912	-
	PBQ4 element	23717	0.0467	1.513
	PBQ8 element	23717	0.1159	12.103

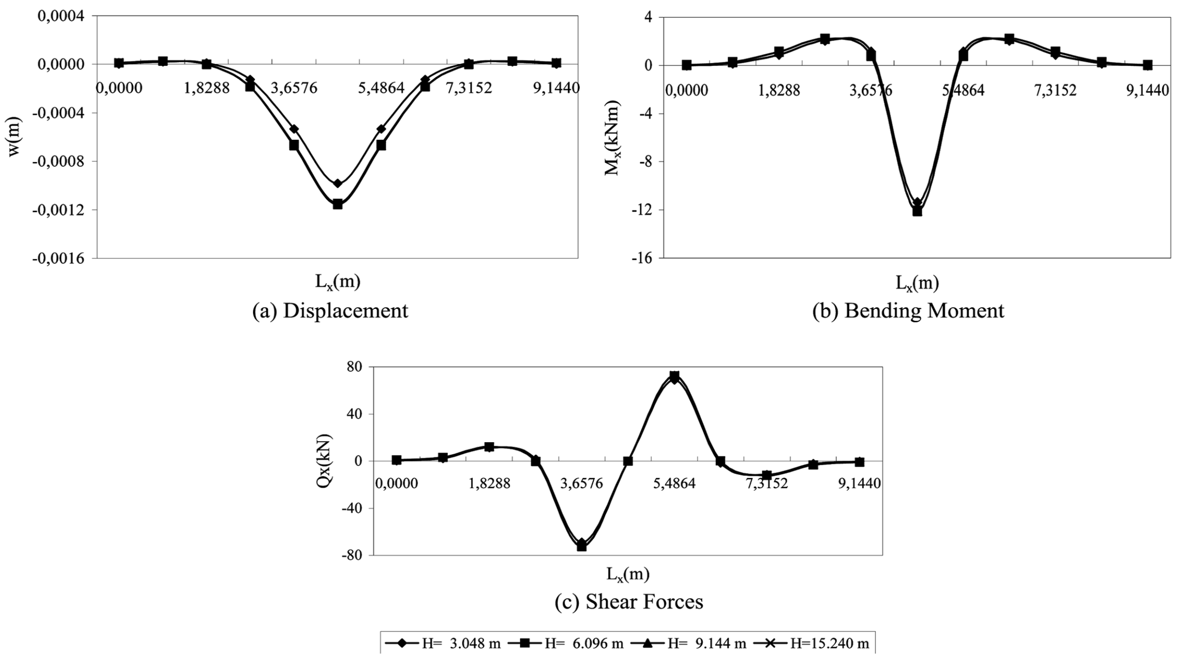

Fig. 11 Variation of displacement, bending moment and shear force of the plate for various value of subsoil depth for concentrated load at the center by PBQ8

The thickness of the plate are taken as 0.05 m and 0.5 m, the dimensions of the plate are 1×1 m and patch load on the plate is 1 MN/m^2 for 0.5×0.5 m square area. For the sake of the accuracy in the results, 20 elements in each direction are considered. The same example is solved by both PBQ4 and PBQ8, and results are presented in tabular forms.

The displacements and bending moments occurred in the middle of the plate are given in Table 6 and Table 7. The approximate results for both the elements PBQ4 and PBQ8 considered here are in excellent agreement with corresponding results in the study by Yettram and Whiteman (1984). But it is obvious that the results are more accurate by element PBQ8 compared to the ones obtained by PBQ4.

3.3 Example-3

As a third example, the plate on elastic foundation solved by Buczkowski-Torbacki (2001) and Çelik-Saygun (1999) using modified Vlasov model for various soil depth is considered. The same problem is analyzed using the Winkler foundation model for the same subsoil depth. The modulus of subgrade reactions to be used in the Winkler model are taken from Buczkowski-Torbacki (2001). The modulus of elasticity of the subsoil is 68950 kN/m^2, the poisson ratio of the subsoil is 0.25, the modulus of elasticity of the plate is 20685000 kN/m^2, the poisson ratio of the plate is 0.20, the thickness of the plate is 0.1524 m, the dimensions are 9.144×12.192 m and concentrated load at the center of the plate is 133.34 kN. Four different values of subsoil depth are considered such as 3.048, 6.096, 9.144 and 15,240.

From the comparison of the results in Table 9, it can be seen that the results obtained by using element PBQ8 are quite close to the results obtained in other studies while the shear locking problem are clearly observed in the result by PBQ4. The plate deflections and bending moments increase with the depth of the soil but the changes of these values become small as the depth of the soil increases.

It is seen in Fig. 11 that the curves are getting fairly close to each other as the value of soil depth increases.

4. Conclusions

In this study 4-noded (PBQ4) and 8-noded (PBQ8) quadrilateral rectangular finite elements based on Mindlin plate theory are adopted for the analysis of thick plates resting on Winkler elastic foundation. The accuracy and the efficiency of the elements are tested for different boundary condition, the load cases and the modulus of subgrade reaction. It is seen that the model adopted in the present study can be effectively and easily used for analyzing rectangular thick plates on elastic foundations with any type of boundary conditions and loading case. It is noted that the shear locking problem occurs under thin plate limit for PBQ4, and PBQ8 is more reliable than PBQ4 for thin plates as expected. But it is observed that as the plate becomes thicker or the foundation becomes stiffer, this problem disappears. The results for simply supported and clamped plate get fairly close to each other as the value of K increases for the same loading case so that the effect of the boundary conditions can be neglected. The effect of shear deformation is more considerable for larger values of plate thickness and smaller values of subgrade reaction modulus.

One of the most important shortcomings of using the Winkler model is that it assumes no effect of shear interaction among adjacent points in the foundation. The authors have been studying on more realistic foundation models to improve the results.

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Appendix

The stiffness matrix of the Winkler elastic foundation for PBQ4 finite element can be partitioned into submatrices of 6 × 6

$$k_w = kab\begin{bmatrix} k_{w1} & k_{w2} \\ k_{w2} & k_{w1} \end{bmatrix}$$

These submatrices are

$$k_{w1} = \begin{bmatrix} \frac{4}{9} & 0 & 0 & \frac{2}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{9} & 0 & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad k_{w2} = \begin{bmatrix} \frac{1}{9} & 0 & 0 & \frac{2}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The stiffness matrix of the Winkler elastic foundation for PBQ8 finite element can be partitioned into submatrices of 6 × 6

$$k_w = kab\begin{bmatrix} k_{w1} & k_{w2} & k_{w3} & k_{w4} \\ k_{w2} & k_{w1} & k_{w4} & k_{w3} \\ k_{w3} & k_{w4} & k_{w5} & k_{w6} \\ k_{w4} & k_{w3} & k_{w6} & k_{w5} \end{bmatrix}$$

These submatrices are

$$k_{w1} = \begin{bmatrix} \frac{2}{15} & 0 & 0 & \frac{2}{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{45} & 0 & 0 & \frac{2}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad k_{w2} = \begin{bmatrix} \frac{1}{15} & 0 & 0 & \frac{2}{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{45} & 0 & 0 & \frac{1}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_{w3} = \begin{bmatrix} -\frac{2}{15} & 0 & 0 & -\frac{8}{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2}{15} & 0 & 0 & -\frac{2}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad k_{w4} = \begin{bmatrix} -\frac{8}{45} & 0 & 0 & -\frac{2}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{45} & 0 & 0 & -\frac{8}{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_{w5} = \begin{bmatrix} \frac{32}{45} & 0 & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{9} & 0 & 0 & \frac{32}{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad k_{w6} = \begin{bmatrix} \frac{16}{45} & 0 & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{9} & 0 & 0 & \frac{16}{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$