

**Technical Note**

# Earthquake response of semi-rigid supported single storey frames modeled as continuous system

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## 1. Introduction

In this study, spectral analysis of semi-rigid supported single storey frames modeled as Timoshenko column with attached mass (Fig. 1) is studied for 1999 Izmit earthquake including rotatory inertia of the columns. Generalized equation of motion is derived substituting energy relations of the model into Lagrange's equation, and response is evaluated by Newmark- $\beta$  method.

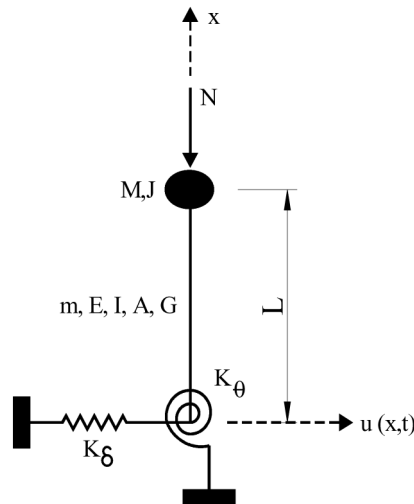


Fig. 1 Continuous mathematical model of semi-rigid supported single-storey frame

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Michaltsos (2001) studied free and forced vibration of the model in this study using Euler model. Güler (1996) searched the effects of soil flexibility on free vibration of tower-like structures using Euler model.

## 2. Seismic analysis

Lagrange's equation and kinetic, potential, load energies of the model are given in Eqs. (1), (2), (3), and (4) respectively (Demirdağ 2005).

$$\frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{Z}_n} \right) + \frac{\partial E_p}{\partial Z_n} = \frac{\partial E_g}{\partial Z_n} \quad (1)$$

$$E_k = \frac{1}{2} \left[ m \int_0^L \left( \frac{\partial u(x, t)}{\partial t} \right)^2 dx + m \frac{I}{A} \int_0^L \left( \frac{\partial \theta(x, t)}{\partial t} \right)^2 dx + M \left( \frac{\partial u(L, t)}{\partial t} \right)^2 + J \left( \frac{\partial \theta(L, t)}{\partial t} \right)^2 \right] \quad (2)$$

$$E_p = \frac{1}{2} \left[ EI \int_0^L \left( \frac{\partial \theta(x, t)}{\partial x} \right)^2 dx + \frac{AG}{k} \int_0^L \left( \frac{\partial u(x, t)}{\partial x} - \theta(x, t) \right)^2 dx - N \int_0^L \left( \frac{\partial u(x, t)}{\partial x} \right)^2 dx \right] + \frac{1}{2} \{ K_\delta [u(0, t)]^2 + K_\theta [\theta(0, t)]^2 \} \quad (3)$$

$$E_g = -m \ddot{u}_g(t) \int_0^L u(x, t) dx \quad (4)$$

where  $u(x, t)$  and  $\theta(x, t)$  are displacement and slope functions;  $Z(t)$  is modal coordinate;  $\ddot{u}_g$  is ground acceleration;  $m, I, A, E, G, k, L$  are distributed mass, moment of inertia, cross-section area, elasticity modulus, shear modulus, shear area constant, length of the column;  $M, J$  are concentrated mass and its rotational inertia;  $K_\delta, K_\theta$  are translational and rotational spring coefficients. Using the transformation

$$u(x, t) = X(x)Z(t) \quad (5)$$

$$X(x) = C_1 \sinh(m_1 x) + C_2 \cosh(m_1 x) + C_3 \sin(m_2 x) + C_4 \cos(m_2 x) \quad (6)$$

and substituting Eqs. (2), (3), (4) into Eq. (1) gives the equation of motion for the model in Fig. 1 as

$$\ddot{Z}(t) + \omega_n^2 Z(t) = -\Gamma_n \ddot{u}_g(t) \quad (7)$$

where  $\omega_n^2 = K_n^*/M_n^*$ ;  $\Gamma_n = F_n^*/M_n^*$ ;  $M_n^*, K_n^*, F_n^*$  are generalized mass, stiffness and force;  $X(x)$  is shape function obtained in free vibration according to boundary conditions (Demirdağ 2005).

## 3. Numerical analysis

Response spectrum analysis of the model subjected to Izmit earthquake is made by Newmark- $\beta$

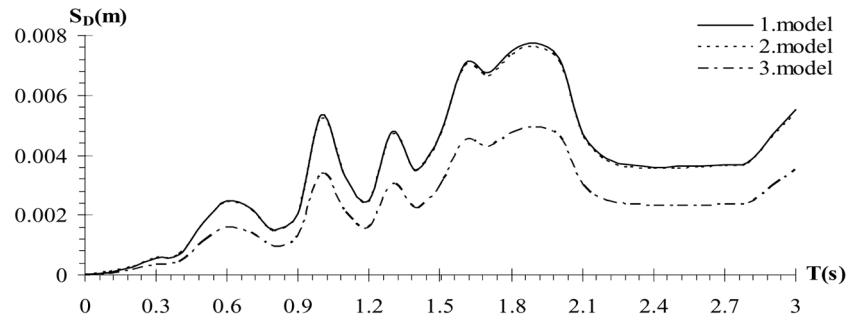


Fig. 2 Displacement response spectrum of the model for Izmit earthquake

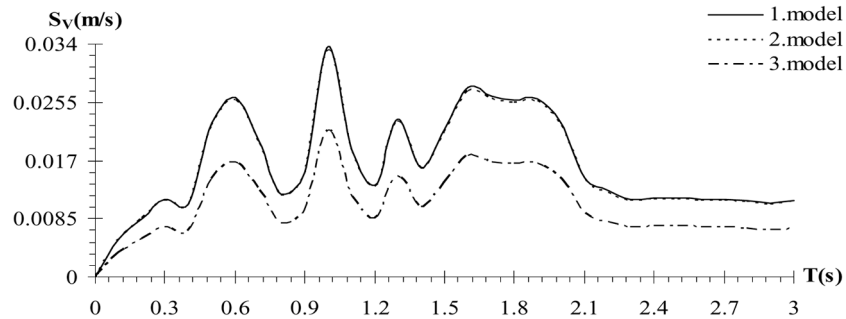


Fig. 3 Velocity response spectrum of the model for Izmit earthquake

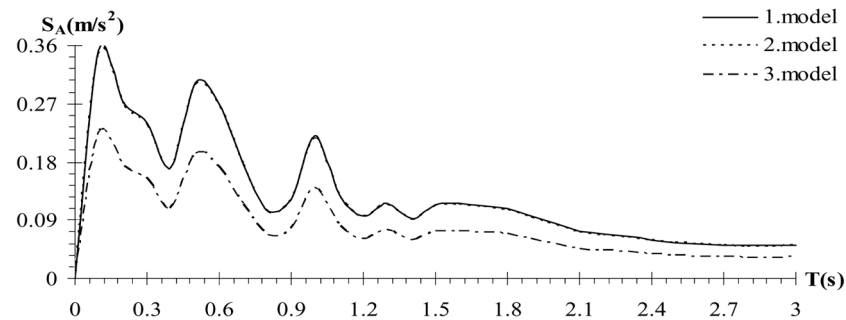


Fig. 4 Acceleration response spectrum of the model for Izmit earthquake

method for period values of 0-3 seconds using a computer program developed by the author. Displacement, velocity, acceleration response spectrum are presented in respectively Figs. 2, 3, 4 for three models having translational and rotational spring constants of respectively 10000, 100000 and 10000000 (t/m; tm/rad).

## References

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