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Technical Note

Support vector regression for structural identification via component-mode synthesis

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1. Introduction

There are numerous methods available such as neural networks, genetic algorithms, wavelets and fuzzy models in the structural identification field. Support vector machine (SVR) is an exclusive data based modeling method, and has a powerful potential to be applied for system identifications (Vapnik 1999, Zhang *et al*. 2006). An efficient SVR-based structural identification approach is developed through a component-mode synthesis (CMS) technique. The CMS technique transforms the structural identification equation for the whole structural system in the original coordinate to several uncoupled sub-structural formulas in the normal coordinate, which guarantees the SVR works rapidly in a greatly reduced dimension to make structural identification.

2. Structural identification algorithm

2.1 The component-mode synthesis method

Few articles have been conducted to make structural identifications by means of the component-mode synthesis (CMS) technique, even though it has been applied widely in the structural or

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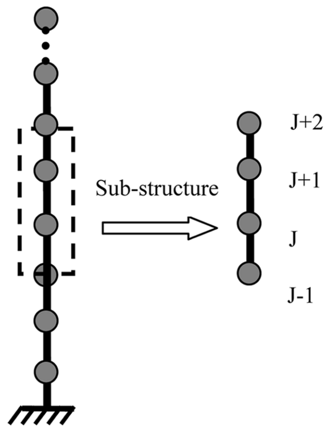

Fig. 1 The substructure model

mechanical dynamic field (Morales 2000, Takewaki and Uetani 2000). The aim of the present work is using the CMS method to reduce the SVR work dimension, thus improve the SVR-based method's efficiency for structural identification. Basically, the CMS methods are classified to free interface, fixed interface and hybrid methods, according to whether the degrees of freedom at the interfaces are free, constrained or partially constrained. In this research, the fixed interface CMS method is adopted which permits easy and precise computation.

A complex structural model is subdivided into a number of smaller substructures physically. The linear dynamic behavior of each substructure is governed by the following local equilibrium equation

$$\begin{bmatrix} \mathbf{M_{II}} & \mathbf{M_{IB}} \\ \mathbf{M_{BI}} & \mathbf{M_{BB}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_\mathbf{I}(t) \\ \ddot{\mathbf{u}}_\mathbf{B}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C_{II}} & \mathbf{C_{IB}} \\ \mathbf{C_{BI}} & \mathbf{C_{BB}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_\mathbf{I}(t) \\ \dot{\mathbf{u}}_\mathbf{B}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K_{II}} & \mathbf{K_{IB}} \\ \mathbf{K_{BI}} & \mathbf{K_{BB}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_\mathbf{I}(t) \\ \mathbf{u}_\mathbf{B}(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_\mathbf{I}(t) \\ \mathbf{f}_\mathbf{B}(t) \end{Bmatrix} \tag{1}$$

where $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the first-order and second-order time derivative of $\mathbf{u}$, the subscripts I and B refer, respectively, to the internal and boundary degrees of freedom. For instance, for the substructure in Fig. 1 being node $(J-1)$ to $(J+2)$, denoted as $S = [J-1 \ \ J+2]$ for convenience, nodes $(J-1)$ and $(J+2)$ are boundary DOFs, and the others are internal DOFs. Based on the fixed boundary CMS theory, structural responses can be rewritten

$$\begin{Bmatrix} \mathbf{u}_\mathbf{I}(t) \\ \mathbf{u}_\mathbf{B}(t) \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\varphi}_\mathbf{D} & \boldsymbol{\varphi}_\mathbf{S} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{y}_\mathbf{I}(t) \\ \mathbf{y}_\mathbf{B}(t) \end{Bmatrix} \tag{2}$$

where $y(t)$ denotes the displacement weight in each mode, $\boldsymbol{\varphi}_\mathbf{D}$ is the main mode of a substructure obtained from $(\mathbf{K_{II}} - \varpi^2\mathbf{M_{II}})\boldsymbol{\varphi}_\mathbf{D} = \mathbf{0}$, and $\boldsymbol{\varphi}_\mathbf{S}$ is the additive mode produced from $\boldsymbol{\varphi}_\mathbf{S} = -\mathbf{K_{II}}^{-1}\mathbf{K_{IB}}$. The sub-structural modal shape matrix $\boldsymbol{\varphi} = \begin{bmatrix} \boldsymbol{\varphi}_\mathbf{D} & \boldsymbol{\varphi}_\mathbf{S} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$ can be assembled from the main and additive mode matrices. The first line of Eq. (2) is $\mathbf{u}_\mathbf{I}(t) = \boldsymbol{\varphi}_\mathbf{D}\mathbf{y}_\mathbf{I}(t) + \boldsymbol{\varphi}_\mathbf{S}\mathbf{u}_\mathbf{B}(t)$, which means the sub-structural dynamic response consists of the main vibration of the fixed boundary substructure and the additive vibration caused by the boundary node movements. Substituting Eq. (2) to Eq. (1) and multiplying φ from left to make the coordinate transform, the sub-structural motion equations in the normal coordinate are derived

$$\begin{bmatrix} \mathbf{I} & \mathbf{M}^*_{\mathbf{IB}} \\ \mathbf{M}^*_{\mathbf{BI}} & \mathbf{M}^*_{\mathbf{BB}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{y}}_{\mathbf{I}}(t) \\ \ddot{\mathbf{y}}_{\mathbf{B}}(t) \end{Bmatrix} + \begin{bmatrix} A\mathbf{I} + B\Lambda_{\mathbf{II}} & A\mathbf{M}^*_{\mathbf{IB}} \\ A\mathbf{M}^*_{\mathbf{BI}} & A\mathbf{M}^*_{\mathbf{BB}} + B\mathbf{K}^*_{\mathbf{BB}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{y}}_{\mathbf{I}}(t) \\ \dot{\mathbf{y}}_{\mathbf{B}}(t) \end{Bmatrix} + \begin{bmatrix} \Lambda_{\mathbf{II}} & \\ & \mathbf{K}^*_{\mathbf{BB}} \end{bmatrix} \begin{Bmatrix} \mathbf{y}_{\mathbf{I}}(t) \\ \mathbf{y}_{\mathbf{B}}(t) \end{Bmatrix} = \begin{bmatrix} \varphi^*_{\mathbf{D}} & \varphi_{\mathbf{S}} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^T \begin{Bmatrix} \mathbf{f}_{\mathbf{I}}(t) \\ \mathbf{f}_{\mathbf{B}}(t) \end{Bmatrix} \quad (3)$$

where $\mathbf{M}^*_{\mathbf{IB}} = \varphi_{\mathbf{D}}^T \mathbf{M}_{\mathbf{II}} \varphi_{\mathbf{S}} + \varphi_{\mathbf{D}}^T \mathbf{M}_{\mathbf{IB}}$, $\Lambda_{\mathbf{II}} = \varphi_{\mathbf{D}}^T \mathbf{K}_{\mathbf{II}} \varphi_{\mathbf{D}}$ is a diagonal matrix, and $\mathbf{I} = \varphi_{\mathbf{D}}^T \mathbf{M}_{\mathbf{II}} \varphi_{\mathbf{D}}$ is a unit matrix as the orthogonal properties of the sub-structural main mode. Note that $\mathbf{M}_{\mathbf{IB}}$ will vanish for lumped mass structures, and the Rayligh damping $(\mathbf{C} = A\mathbf{M} + B\mathbf{K})$ is adopted here. An approximation is constructed by assuming the velocity-dependent part in the interface is negligible, as the damping force is usually very smaller than the inertia force. For the 4-node substructure in Fig. 1, the interval node motion equations are

$$\begin{Bmatrix} \ddot{y}_j(t) \\ \ddot{y}_{j+1}(t) \end{Bmatrix} + \begin{bmatrix} AI + B\Lambda_j & 0 \\ 0 & AI + B\Lambda_{j+1} \end{bmatrix} \begin{Bmatrix} \dot{y}_j(t) \\ \dot{y}_{j+1}(t) \end{Bmatrix} + \begin{bmatrix} \Lambda_j & \\ & \Lambda_{j+1} \end{bmatrix} \begin{Bmatrix} y_j(t) \\ y_{j+1}(t) \end{Bmatrix} = \begin{Bmatrix} \bar{f}_j(t) \\ \bar{f}_{j+1}(t) \end{Bmatrix} \quad (4)$$

where $\begin{Bmatrix} \bar{f}_j(t) \\ \bar{f}_{j+1}(t) \end{Bmatrix} = \varphi_{\mathbf{D}}^T \begin{Bmatrix} f_j(t) \\ f_{j+1}(t) \end{Bmatrix} \varphi_{\mathbf{D}}^T \begin{bmatrix} m_j & \\ & m_{j+1} \end{bmatrix} \varphi_{\mathbf{S}} \begin{Bmatrix} y_{j+2}(t) \\ y_{j-1}(t) \end{Bmatrix}$

The derived uncoupled sub-structural identification formulas (Eq. (4)) can be seen as SDOF structural motion equations, and can be rewritten in a linear form. Taking the node j as an example

$$y_j(k+3) = \theta_{j1} y_j(k+1) + \theta_{j2} y_j(k+2) + \tilde{\theta}_{j1} \bar{f}_j(k+1) + \tilde{\theta}_{j2} \bar{f}_j(k+2) + e_t \quad (5)$$

where coefficient vector $\{\theta_{j1} \ \ \theta_{j2} \ \ \tilde{\theta}_{j1} \ \ \tilde{\theta}_{j2}\}$ are functions of the sub-structural generalized stiffness Λ_j. Eq. (5) will be solved by the novel SVR technique, which will be described later. In brief, by using the CMS method, structural equations first are transformed to equations in the sub-structural domain, next are uncoupled by the normal transform. The derived structural identification of the sub-structure in the normal coordinate (Eq. (5)) only consists of several unknown parameters to identify, thus the SVR can work in a greatly reduced low dimension efficiently.

2.2 Iterative algorithm

The modal responses of substructures are required when the SVR technique is used to solve the unknown coefficients $\{\theta_{j1} \ \ \theta_{j2} \ \ \tilde{\theta}_{j1} \ \ \tilde{\theta}_{j2}\}$ in Eq. (9). It can be derived from Eq. (2)

$$\mathbf{y}_{\mathbf{I}}(t) = \varphi_{\mathbf{D}}^{-1}(\mathbf{u}_{\mathbf{I}}(t) - \varphi_{\mathbf{S}} \mathbf{u}_{\mathbf{B}}(t)) \quad (6)$$

However, it is difficult to measure the complete mode shapes of substructures in the real world. A way to overcome that difficulty is substituting an approximated mode shape matrix for the true one, and using an iterative strategy to guarantee the convergence. In the iterative algorithm, an initial sub-structural element stiffness vector first is assumed, and the corresponding $\varphi_{\mathbf{D}}$ is produced through Eq. (2). Next the SVR is utilized to identify $\{\theta_{j1} \ \ \theta_{j2} \ \ \tilde{\theta}_{j1} \ \ \tilde{\theta}_{j2}\}$. Thirdly, the sub-structural generalized stiffness, Λ_j and Λ_{j+1} are extracted from the identified coefficient matrix. Consequently, through $\begin{bmatrix} \Lambda_j & 0 \\ 0 & \Lambda_{j+1} \end{bmatrix} = eig(\mathbf{K}_{\mathrm{II}}, \mathbf{M}_{\mathrm{II}})$, the substructure element stiffness vector, $\{k_j \;\; k_{j+1} \;\; k_{j+2}\}$, is recovered. Once a new substructure element stiffness vector is produced, next iteration begins until the identified results converge to accurate enough values.

2.3 Support vector regression

Eq. (5) will be solved by the novel SVR technique, due to its excellent robustness. The novel SVR make data processing by using the 'Max-Margin' idea and the ε-insensitive loss function (Vapnik 1999). The function to be estimated is defined by

$$f(\mathbf{x}) = \langle \mathbf{w}, \phi(\mathbf{x}) \rangle + b \tag{7}$$

where $< \cdot , \cdot >$ denotes the dot product, $\phi(\mathbf{x})$ is constructed by kernel functions. When the objective function is linear, $\phi(\mathbf{x})$ equals to $\mathbf{x}$. The SVR produce the regression results (Eq. (7)) by solving an optimization problem with inequality constraints

$$\text{Minimize} \quad R_{reg} = \frac{1}{2}\|\mathbf{w}\|^2 + \bar{C}\sum_{i=1}^{n}(\xi_i + \xi_i^*) \tag{8}$$

$$\text{Subject to} \quad \begin{cases} y_i - \langle \mathbf{w}, \phi(x_i) \rangle > -b \le \varepsilon + \xi_i \\ -y_i + \langle \mathbf{w}, \phi(x_i) \rangle > +b \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases} \tag{9}$$

where $\bar{C}$ and ε are pre-determined coefficients, ξ_i and ξ_i^* are training errors in two subsets defined by

$$\xi(\xi_i^*) = \begin{cases} |f(\mathbf{x}) - y| - \varepsilon & \text{for } |f(\mathbf{x}) - y| \ge \varepsilon \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The SVR technique is closely related to the classic regularization theory. The first term of Eq. (2) can be seen as a regularization term to produce stable solutions, and parameter $\bar{C}$ to be the regularization parameter. This feature equips the SVR with a greater potential to make better predictions than the classic least squares method. It is seen that Eq. (5) has the same form as Eq. (7), thus can be transformed to the optimization problem (Eq. (8)) with the constraints (Eq. (9)), and be solved by the SVR procedure robustly.

3. Structural identification examples

3.1 10-DOF structural identification

To test the performance and efficiency of the SVR-CMS structural identification approach, a 10-

Table 1 10-DOF structural stiffness identified results

Number	Exact	5% noise	10% noise
		Estimated (Error)	Estimated (Error)
1	7.2E+05	6.95E+05 (3.47%)	7.66E+05 (6.34%)
2	7.2E+05	7.11E+05 (1.26%)	7.21E+05 (0.08%)
3	7.2E+05	6.88E+05 (4.45%)	6.22E+05 (13.57%)
4	7.2E+05	7.19E+05 (0.10%)	6.95E+05 (3.53%)
5	7.2E+05	7.75E+05 (7.57%)	6.14E+05 (14.68%)
6	7.2E+05	6.83E+05 (5.20%)	6.55E+05 (8.99%)
7	7.2E+05	6.79E+05 (5.65%)	6.80E+05 (5.59%)
8	7.2E+05	6.79E+05 (5.71%)	6.30E+05 (12.46%)
9	7.2E+05	6.98E+05 (3.10%)	6.67E+05 (7.31%)
10	7.2E+05	6.84E+05 (4.97%)	6.24E+05 (13.41%)
Mean absolute error		4.15%	8.60%

DOF lumped mass structure first is investigated. The mass lumped to each floor is denoted by m_i = 1253.3 kg (i = 1, …, 10). The stiffness of each floor is k_i = 720000 N/m (i = 1, …, 10). The Rayleigh damping with $A = 0.005$ and $B = 0.0015$ is adopted. The Kobe (NS 1995) earthquake is added in the tenth floor of the structure model. The structural vibration data are simulated by the Newmark method. To test the applicability of the proposed method in presence of noise, both 5% and 10% standard Gaussian white noises without frequency band limitation are added as observation noise, where 5% (10%) means the standard deviation of noise is 5% (10%) of that of the simulated observed data.

The 10-DOF structure is divided into three substructures. The first substructure being node from 7 to 10, denoted as $S_1 = [7 \quad 10]$, which is utilized to identify k_8 to k_{10}. The second is $S_2 = [4 \quad 7]$ and the third $S_3 = [1 \quad 4]$. $k = \{300000\ 200000\ 500000\}$N/m is assumed as the initial element stiffness vector for each substructure. The choice of SVR parameters can be found in Cherkassky and Ma (2004). Identified results in the presence of 5% and 10% noises are shown in Table 1. Because the velocity-dependent part is assumed to be negligible in Eq. (5), the damping coefficients are not identified in this example.

3.2 Thirty-floor structural identification

Structural identification becomes more difficult when the DOF of the structure studied increases. A 30-DOF lumped mass structure is investigated to verify the SVR-CMS method for large-scale structural identifications. All the mass, the stiffness and the damping values are the same as those in the first example. The 30-DOF structure is divided to 10 substructures, $S_1 = [1 \quad 4]$, $S_2 = [4 \quad 7]$, …, $S_9 = [25 \quad 28]$ and $S_{10} = [27 \quad 30]$. The input motion is the Kobe earthquake data added in all the sub-structural interface nodes. Both 5% and 10% standard Gaussian white noises are added as observation noise. As the same sub-structures are divided, the SVR-CMS procedure for the sub-structural identification is same as that in the first example. For conciseness, the identified structural stiffness values are plotted in Fig. 2. The absolute mean identified errors under 5% and 10% noises respectively are 5.07% and 10.32%. Note that in this example, the displacements in all the nodes of

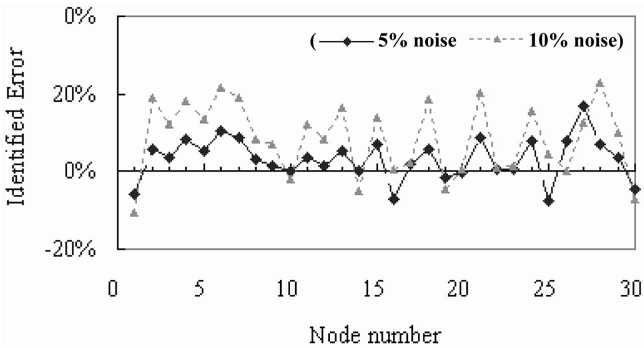

Fig. 2 Thirty-floor structural stiffness identification results (—◆—5% noise, ---▲--- 10% noise)

the substructure and the acceleration in boundary nodes are needed for structural identification. In the proposed SVR-CMS identification approach, the input force need not be measured when the substructure interval nodes are not suffered by the input excitation, because the effect of the input force can be expressed in terms of the interface node responses.

4. Conclusions

This research investigated support vector regression for structural identification through the component-mode synthesis technique. The novel data processing method, SVR, work robustly for structural parameter identification, and the CMS method reduced the number of unknown parameters comprised in the identification equation, thus guarantee the SVR work efficiently in a low dimension.

References

Cherkassky, V. and Ma, Y. (2004), “Practical selection of SVM parameters and noise estimation for SVM regression”, *Neural Networks,* **17**, 113-126.

Morales, C.A. (2000), “Dynamic analysis of frames by a Rayleigh-Ritz based component mode method”, *Eng. Struct.,* **22**, 1632-1640.

Takewaki, I. and Uetani, K. (2000), “Inverse component-mode synthesis method for damped large structural systems”, *Comput. Struct.*, **78**, 415-423.

Vapnik, V.N. (1999), *The Nature of Stastical Learning Theory*, 2nd edn., Springer-Verlag.

Zhang, J., Sato, T. and Iai, S. (2006), “Large-scale structural health monitoring by increment support vector regression with local strategy”, *J. Struct. Safety*, **28**, 392-406.