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Optimal stacking sequence design of laminate composite structures using tabu embedded simulated annealing

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Abstract. This paper deals with optimal stacking sequence design of laminate composite structures. The stacking sequence optimisation of laminate composites is formulated as a combinatorial problem and is solved using Simulated Annealing (SA), an algorithm devised based on inspiration of physical process of annealing of solids. The combinatorial constraints are handled using a correction strategy. The SA algorithm is strengthened by embedding *Tabu search* in order to prevent recycling of recently visited solutions and the resulting algorithm is referred to as tabu embedded simulated Annealing (TSA) algorithm. Computational performance of the proposed TSA algorithm is enhanced through *cache-fetch* implementation. Numerical experiments have been conducted by considering rectangular composite panels and composite cylindrical shell with different ply numbers and orientations. Numerical studies indicate that the TSA algorithm is quite effective in providing practical designs for lay-up sequence optimisation of laminate composites. The effect of various neighbourhood search algorithms on the convergence characteristics of TSA algorithm is investigated. The sensitiveness of the proposed optimisation algorithm for various parameter settings in simulated annealing is explored through parametric studies. Later, the TSA algorithm is employed for multi-criteria optimisation of hybrid composite cylinders for simultaneously optimising cost as well as weight with constraint on buckling load. The two objectives are initially considered individually and later collectively to solve as a multi-criteria optimisation problem. Finally, the computational efficiency of the TSA based stacking sequence optimisation algorithm has been compared with the genetic algorithm and found to be superior in performance.

Keywords: laminate composites; optimal design; simulated annealing; Tabu search; thermal buckling.

1. Introduction

Laminated composite construction of panels and other structural elements is currently being used for many applications in aerospace, automotive, civil and defence industries. Laminated composites have several advantages over more traditional materials including greater specific strength, specific stiffness, corrosion and fatigue resistance, and energy absorption among others. Multi-layer and sandwich construction also offer many opportunities for analysts and designers to tailor their properties to the specific requirements of a given application. The tailoring is mostly achieved by optimising the mechanical properties, thereby increasing the load carrying capacity of the structure. Optimisation of composite laminates with respect to ply angles to maximise the strength is

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necessary to realise the full potential of fiber reinforced materials.

In the past, ply angles are often employed as continuous design variables in the design optimisation of laminated composites and solved using gradient based methods (Jacoby *et al*. 1972, Gurdal and Haftka 1991) to improve performance of the structure. However, these methods found to have severe limitations as stacking sequence design involves discrete design variables i.e., ply angles, which must be converted to continuous variables before the problem is solved. Once the optimal continuous–valued solution is found, it must be rounded to the nearest manufacturable ply angle, which may result in a design which is either non-optimal or violates certain imposed constraints. Hence discrete optimisation techniques where discrete ply angles can be considered as design variables are more relevant for stacking sequence design problems. It is well known that most practical laminates are restricted to some discrete sets of ply orientation angles 0^o, 90^o and $\pm 45^o$ because of the availability of experimental data for structural verification of the behaviour. This practical restriction makes the stacking sequence design problem a combinatorial optimisation problem, which is not easy to solve.

For stacking sequence optimisation of laminate composite structures, genetic algorithms (GA) have been the most widely and popularly used method. A detailed survey of discrete optimisation and global design optimisation methods applied to stacking sequence optimisation can be found in the review paper of Venkataraman and Haftka (1999). Even though GA is popularly used method for stacking sequence optimisation, it is proposed to explore the effectiveness of another popular metaheuristic algorithm called simulated annealing keeping the following things in view.

i. GA is a population-based algorithm and requires considerable number of generations to converge, which involve large number of function evaluations (i.e., equal to the population size × number of generations). Practical engineering problems required detailed finite element simulations for the evaluation of the objective function, which is computationally very expensive. It is shown by Venkataraman and Haftka (2002) that rapid increases in computer processing power, memory and storage space have not eliminated computational cost and time constraints faced by engineers using structural optimisation for a design. In view of this, it is highly desirable to explore alternative optimisation algorithms, which can converge faster with the least number of function evaluations and thereby improve the computational performance of the stacking sequence optimisation problem.

ii. The No Free Lunch (NFL) theorems (Wolpert and Macready 1997) have established mathematically that the behaviour of all algorithms when analysed over all possible optimisation problems defined over some research space is same and no algorithm has performance advantage. Hence according to NFL theorem, the average behaviour of all optimisation algorithms is same. However, as shown by Droste *et al.* (1999), a particular algorithm performs better over a subset of the entire function set consisting of all optimisation problems. Hence it is worthwhile to explore alternative algorithms for stacking sequence optimisation of laminate composites, which can be more effective.

iii. SA is generally more reliable in finding global optimum and unlike GA, simulated annealing algorithm uses single solution. Several researchers while solving other combinatorial problems (Lahtinen *et al*. 1996, Mann and Smith 1996) have established that SA outperforms GA both in computational performance and also in finding the global optimum solutions.

In this paper, rather than using SA in it's traditional form, we preferred to synthesize the simulated annealing with another metaheuristic algorithm called tabu Search (TS) in order to avoid recycling and also improve the diversification mechanism, thereby the convergence characteristics.

```
Simulated Annealing (S0, T0, α, β, M, Maxtime)
Begin
        T= T0;
        Cur_S = S0;
        Best_S = Cur_S ;  / * Best_S is the best solution so far */
        Cur_Fitness = Fitness( Cur_S);
        Best_Fitness = Fitness( Best_S);
        Repeat
                Call Metropolis ( Cur_S Cur_Fitness, Best_S, Best_Fitness, T, M);
                Time = Time + M;
                T = α x T;
                M = βx M;
                Until (T < 0.001)
                Return ( Best_S);
 End / * End of Simulated annealing */
```

Fig. 1 Simulated annealing algorithm

The resulting algorithm is referred to as tabu embedded Simulated Annealing (TSA) algorithm. The proposed TSA algorithm has been employed to solve the combinatorial optimisation problem of composite laminate stacking sequence for buckling load maximisation of composite panels and cylindrical shells. Eventhough there are some earlier works on application of simulated annealing for optimal design of composite laminate (Sadagopan and Pitchumani 1998, Di Sciuva and Gherlone 2003), the authors are not aware of any efforts in the earlier works towards detailed investigation on sensitivities of various parameter settings in SA algorithm while applied to laminate composite design. It is well known that SA is highly sensitive to parameter settings and these parameter settings are highly specific applications. Keeping this in view, numerical investigations have been conducted to study the influence of various parameter settings in TSA and also the influence of several neighbourhood search algorithms. A new adaptive neighbourhood search algorithm is proposed in the TSA algorithm for combinatorial optimisation. The proposed TSA algorithm has later been employed for the multi-criteria optimal design of composite cylinder made of two materials for simultaneously optimising the weight and also cost with constraint on buckling.

2. Simulated annealing

Simulated annealing (SA) is an iterative search method inspired by the annealing of metals (Kirkpatrick Jr. *et al.* 1983, Cerny 1985). Starting with an initial solution and armed with adequate perturbation and evaluation functions, the algorithm performs a stochastic partial search of the state space. Uphill moves are occasionally accepted with a probability controlled by a parameter called temperature (T). The probability of acceptance of uphill moves decreases as T decreases. At high temperature, the search is almost random, while at low temperature the search becomes almost greedy. At zero temperature, the search becomes totally greedy, i.e., only good moves are accepted (Kirkpatrick Jr. *et al.* 1983, Cerny 1985).

```
Algorithm Metropolis ( Cur_S, Cur_Fitness, Best_S, Best_Fitness, T, M)
Begin
        Repeat
          New_S = Neighbourhood_ search (Cur_S);
          New_Fitness = Fitness(New_S);
          ΔFitness = (New_Fitness_ Cur_Fitness);
          If (ΔFitness>0) then
                Cur_S = New_S;
                If New_Fitness > Best_Fitness, then
                        Best_S = New_S;
                Endif
          Else
                If(RANDOM < e^(ΔFitness/T)) then
                Cur_S = New_S;
                Endif
        Endif
        M = M - 1;
        Until (M=0)
End / * End of Metropolis algorithm */
```

Fig. 2 Metropolis algorithm

The basic SA algorithm is shown in Fig. 1. The core of the algorithm is the Metropolis procedure (Metropolis *et al.* 1953), which simulates the annealing process at a given temperature T (Fig. 1). The Metropolis procedure is shown in Fig. 2. The Metropolis procedure receives as input, the current temperature T, and the current solution *Cur_S* which it improves through neighbourhood search. *Metropolis* must also be provided with the value M, which is the amount of time for which annealing must be applied at temperature T. The procedure *Simulated_annealing* simply invokes *Metropolis* at decreasing temperatures. Temperature is initialized to a value T_0 at the beginning of the procedure, and is slowly reduced (in a geometric or arithmetic progression). The annealing procedure gets terminated when temperature, T is reduced to a very small value say, 0.001. Eventhough SA has been used extensively for solving combinatorial problems, there are certain problems associated with setting of the cooling schedule of the algorithm, which consists of the following four components.

i. Initial temperature
ii. Terminating temperature
iii. Temperature decrement
iv. Number of iterations at each temperature

At present there are no known methods to calculate the cooling schedule for a large range of problems and the values are often set using empirical evidence from experimental runs of the algorithm. The same approach has been employed here. The details of the cooling schedule adopted in this paper are presented here.

Initial temperature

The initial temperature value should be such that it allows virtually all proposed uphill or down hill moves to be accepted. There are several empirical suggestions to fix the initial temperature. For example, Rayward-Smith *et al.* (1996) suggested starting initially with a high temperature and then cooling it rapidly until about 60% of worst states are being accepted. This temperature is regarded as the best initial temperature and can be cooled more slowly. Similarly, Dowsland (1995) suggested to heat the system rapidly until a certain proportion of worse solutions are accepted and then start cooling slowly. However, in the present work it is preferred to initialise the temperature using a different procedure as outlined here.

The idea is basically to use the Metropolis function $e^{\Delta Fitness/T}$ to determine the initial value of the temperature parameter. Before the start of actual SA procedure, a constant number of moves, say M, in the neighborhood of the current solution are made, and the respective fitness values of these moves are determined. The fitness difference for each move i: $\Delta Fitness_i$ is given by $\Delta Fitness_i = Fitness_i - Fitness_{i-1}$. Let M_u and M_d be the number of uphill and downhill moves, respectively (downhill refers to inferior moves) ($M = M_u + M_d$). The average $\Delta Fitness_d$ is then given by

$$\Delta Fitness_d = \frac{1}{M_d}\sum_{i=1}^{M_d} \Delta Fitness_i \tag{1}$$

Since we wish to keep the probability, say P_0, of accepting downhill moves high in the initial stage of SA, we estimate the value of the temperature parameter by substituting the value of P_0 in the following expression derived from the Metropolis function:

$$T_0 = \frac{\Delta Fitness_d}{\ln(P_0)} \quad \text{where} \quad P_0 \approx 1 \ (P_0 = 0.999) \tag{2}$$

Terminating temperature

It is usual to set the terminating temperature as zero. However, it is not necessary to let the temperature reach zero because as it approaches zero the chances of accepting a worse move are almost the same as the temperature being equal to zero. Keeping this in view, in the present work, the simulated annealing algorithm is terminated when at least one of the following criteria is satisfied.

i. Maximum number of iterations
ii. When the temperature is equal or nearly equal to zero i.e., $T < 0.001$
iii. When there is no improvement in the solution in the last N consecutive moves
iv. When there is no change in the solution for the last NC moves, where NC is an user specified parameter

Temperature decrement

Choosing appropriate temperature decrement is highly critical to the success of the simulated annealing algorithm. The temperature can be decremented by using either geometric decrement or arithmetic decrement.

$$\left.\begin{array}{ll}\text{Geometric:} & t = \alpha \cdot T \quad \text{Where} \quad \alpha < 1 \\ \text{Arithmetic:} & t = T - \alpha\end{array}\right\} \tag{3}$$

In this paper, studies have been reported using both arithmetic and geometric decrement procedures to compare the computational performance of the two procedures while solving the stacking sequence optimisation problem.

Number of iterations at each temperature

The number of iterations at each temperature can be changed by varying the parameter β in the simulated annealing algorithm as given in Fig. 1. A constant number of iterations at each temperature by setting the parameter β to one is an obvious scheme. Alternatively, one can adaptively change the iterations as the algorithm progresses. At lower temperatures it is important to have a large number of iterations so that the local optimum can be fully explored. At higher temperatures, the number of iterations can be less. The third alternative is as suggested by Lundy and Mees (1986), to perform only one iteration at each temperature, but to decrease the temperature very slowly. The formula suggested for temperature decrement is

$$t = t/(1 + \gamma t) \tag{4}$$

where γ is a suitably small value. In the present work, all the three schemes are implemented and their influence on the computational performance of SA in the optimal stacking sequence design of the laminate composites is studied.

2.1 Neighbourhood search techniques

Several neighborhood search techniques have been implemented in the SA algorithm to effectively explore the search space. Before discussing on neighbourhood search techniques, it is essential to mention the representation of a solution coding, which encodes alternative candidate solutions for manipulation. The choice of the coding that provides an efficient way of implementing the moves and evaluating the solutions is essential for the success of the neighbourhood search heuristic. For laminate lay-up sequence optimization, the candidate solution represents a design, i.e., lay-up sequence of a composite laminate. The solution is encoded by arranging all ply angles of the given composite laminate in an array v (v_i : i = 1, 2, ..., n), where v_i is an encoded value corresponding to a ply angle and n stands for the number of plies in the laminate composite. The ply angles are encoded as 1, 2, 3, which stand for the three possible stacks, $0^o{}_2$, $\pm45^o$, $90^o{}_2$ respectively. For example, the laminate $[90^o{}_2, \pm45^o, 90^o{}_2, 0^o{}_2]_s$ is encoded as 3 2 3 1. The rightmost 1 corresponds to the layer closest to the laminate plane of symmetry. The leftmost 3 describe the outermost layer. Details of various neighborhood search algorithms implemented in the present work are as follows:

The neighborhood search technique called *Invert* assigns each variable a small probability to switch to any other permissible integer value (except the value before alteration) with higher fitness. Similarly, the neighbourhood search technique called *permutation* chooses two locations randomly in the string of design variables and reverses the order of the variables between the two chosen locations. *Swap* is another neighbourhood search technique, which is less disruptive when compared to permutation. The *swap* technique is implemented by randomly selecting two unique design variables in the string and switching their positions. The *swap* and *permutation* search techniques have been used in combination with *invert* to improve the exploratory characteristics of neighbourhood search.

Apart from these neighbourhood search algorithms, a new adaptive search algorithm has been proposed in this paper for the combinatorial optimisation using TSA. The idea behind the proposed adaptive search technique is to disrupt (or alter) the design variables more intensely at the beginning of the annealing process and it should be least disruptive as annealing process is close to convergence. For this purpose a parameter related to temperature is used to dictate the number of design variables to be altered at a particular state. At the beginning of the annealing process, all the design variables will be permitted to alter with some probability and during the process of annealing, the number of randomly chosen design variables in the string permitted to alter is adaptively reduced. This reduction is closely associated with the reduction in the temperature during the annealing process.

2.2 Performance enhancement techniques

In order to improve the search characteristics and also computational performance, the Tabu search features are built into the algorithm. Further, the computational performance is enhanced through the implementation of cache-fetch techniques.

```
Algorithm Metropolis ( Cur_S, Cur_Fitness, Best_S, Best_Fitness, T, M)
Begin
        Repeat
          New_S = Neighbourhood_ search (Cur_S);
          New_Fitness = Fitness(New_S);
          ΔFitness = (New_Fitness -  Cur_Fitness);
          If (ΔFitness >0) then
            Check_for_tabu_active(New_S)
            If(Tabu_active) check_for_Aspiration(New_S)
             If ((not. Tabu_active ). or. (Aspiration is true)), then
                 Cur_S = New_S;
          Endif
            If (New_Fitness > Best_Fitness)  then
                         Best_S = New_S;
                Endif
          Else
                 If(RANDOM < e^(ΔFitness/T))  then
                 Check_for_tabu_active(New_S)
               If(Tabu_active) check_for_Aspiration(New_S)
                If ((not. Tabu_active ). or. (Aspiration is true))
                 Cur_S = New_S;
                 Endif
          Endif
        M = M - 1;
        Until (M=0)
End / * End of Metropolis algorithm */
```

Fig. 3 Tabu embedded Metropolis algorithm

Tabu embedding

Tabu search is a heuristic search technique introduced by Glover (1989) and is being used in wide variety of applications. The tabu search technique uses short-term memory to avoid recycling. Recycling refers to the process of obtaining the same solution repeatedly several times. This recycling is quite common in neighbourhood search algorithms, which generates solutions in a greedy fashion. Recycling ultimately leads the optimisation algorithm to converge to a local optimum. To circumvent this situation and also to cover a wider solution space, tabu search technique uses short-term memory to mark the recently visited best solutions through neighbourhood search techniques that cannot be accepted as the best solution for a certain number of iterations. These marked solution options are known as *tabu active* and the number of iterations for which the move remains *tabu active* is known as *tabu tenure*. In the present work, the recently visited best solutions are recorded in a tabu list and tabu tenure is set as 4. The *aspiration criterion*, commonly used in tabu search, is also built into the algorithm. The *aspiration criterion* overrules the *tabu active* status, if the set criterion is satisfied. In the stacking sequence optimisation algorithm, the *aspiration criterion* requires that the solution being considered is superior to the best-ever solution rather than the best among the recently visited solutions. Fig. 3 shows the Metropolis algorithm embedded with tabu search.

Cache fetch

The performance of the optimisation algorithm is generally measured by the number of function evaluations, as the function evaluation is the most time consuming part of the algorithm. In the present work, an attempt has been made to minimise the number of function evaluations by setting appropriate parameters for simulated annealing and also by *cache-fetch* implementation. Already evaluated string patterns (stacking sequences) during the search and their corresponding objective values, constraints etc., are stored in a separate location in the memory referred to as cache. These values are stored in a modular fashion for minimising search during fetching the solution from cache. Whenever a new string (stacking sequence) is generated in the Metropolis algorithm through neighbourhood search, and it is not tabu active, then the *cache* is searched for the matching pattern. The matching pattern if found in the cache, the objective value and other constraints corresponding to the newly generated string are directly fetched from the cache instead going through the detailed function evaluation which is rather expensive than directly fetching from cache. Modularity in the storage of string patterns in the cache is maintained by employing binary tree data structure, which helps in minimising the search efforts during *cache-fetch.*

3. Laminate composite models for numerical studies

In this paper two laminate composite models are considered as case studies to demonstrate the effectiveness of the proposed lay-up sequence optimisation technique employing the TSA algorithm. The first case study is thermal buckling of laminate composite panel and the second one is buckling load optimisation of composite cylindrical shells made of single and multiple materials.

3.1 Formulation details of laminate composites

Composite laminates are formed by stacking different composite materials and/or fiber

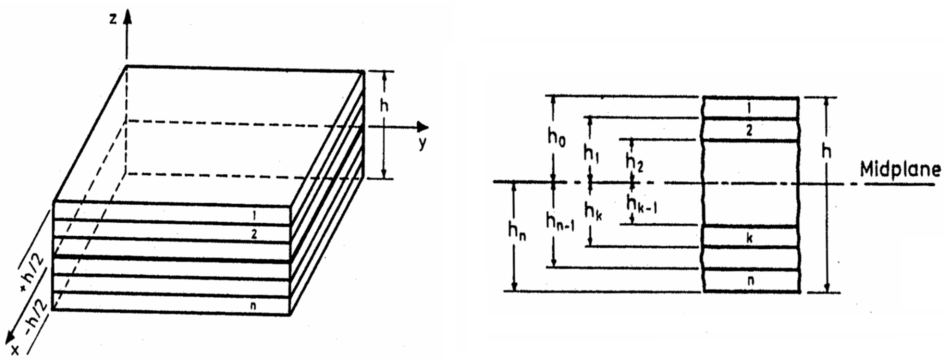

Fig. 4 A laminate composite made up of n stacked plies

orientations. These composite laminates are used in applications that require axial and bending strengths. Therefore, composite laminates are treated as plate elements. Several theories exist to model laminate composites. They are equivalent to single layer theory, classical laminate theory, shear deformation laminate theory, three-dimensional elasticity theory and finally multiple modal methods. The details of these formulations can be found in Reddy (2001). In the present formulation, we use classical laminate theory. Consider a laminate shown in Fig. 4 of total thickness, h composed of n orthotropic layers with the principal natural coordinates X, Y and Z directions with Z axis is taken as positive upward at middle plane.

The assumptions made in the formulations based on classical laminate theory include (i) lamina thickness is uniform and small compared to its lateral dimensions. Therefore, stresses acting on the interlaminar planes in the interior of the laminate, that is, away from the free edges, are negligibly small. (ii) There is a perfect bond between any two laminae and therefore, the laminae are not capable of sliding over each other and displacements are continuous across the bond. (iii) A line originally straight and perpendicular to the laminate mid plane remains so after deformation. (iv) Finally, the Kirchhoff assumption which states that in-plane displacements are linear functions of the thickness, and therefore the interlaminar shear strains, ε_{xz} and ε_{yz} are negligible. With these assumptions the laminate behavior can be reduced to a two-dimensional analysis of the laminate mid plane.

Using the constitutive law, the stress-strain relationship in principal natural coordinate directions (X, Y) can be given as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{5}$$

where

$$[\bar{Q}] = [T][Q][T]^T,\ [Q] = \begin{bmatrix} \dfrac{E_L}{(1-\nu_{LT}\nu_{TL})} & \dfrac{\nu_{LT}E_T}{(1-\nu_{LT}\nu_{TL})} & 0 \\ \dfrac{\nu_{TL}E_L}{(1-\nu_{LT}\nu_{TL})} & \dfrac{E_T}{(1-\nu_{LT}\nu_{TL})} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix},\quad [T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -\sin 2\theta \\ \sin^2\theta & \cos^2\theta & \sin 2\theta \\ \dfrac{\sin 2\theta}{2} & -\dfrac{\sin 2\theta}{2} & \cos 2\theta \end{bmatrix} \tag{6}$$

T is the transformation matrix and θ is the fiber orientation with respect to X axis. E and ν are Young's modulus and poison's ratio respectively of the laminae in the principle material coordinate directions, L and T.

Referring to Fig. 4, the displacements at any point in a laminate is given by

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{7}$$

where u_0, v_0 and w_0 are the displacements along the coordinate lines of a material point on X Y plane, where $Z = 0$ (mid plane). Combining the strain displacement relationships with Eq. (7)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} \partial u/\partial x \\ \partial u/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{bmatrix} = \begin{bmatrix} \partial u_0/\partial x \\ \partial u_0/\partial y \\ \partial u_0/\partial y + \partial v_0/\partial x \end{bmatrix} - Z\begin{bmatrix} \partial^2 w_0/\partial x^2 \\ \partial^2 w_0/\partial y^2 \\ 2\partial^2 w_0/\partial x\partial y \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_s^0 \end{bmatrix} - Z\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{8}$$

or

$$\{\varepsilon\} = \{\varepsilon\}^0 + Z\{\kappa\}$$

Here, ε_x^0, ε_y^0, and ε_s^0 are the mid plane strains, while κ_x, κ_y, and κ_s are the plate curvatures.

The stresses in any layer 'k' can be written as

$$[\sigma]_k = [\bar{Q}]_k[\varepsilon^0] + z[\bar{Q}]_k[\kappa] \tag{9}$$

Let h_k be the thickness of layer k, then the total thickness of the composite laminate with n layers (Fig. 4) can be written as

$$h = \sum_{k=1}^{n} h_k \tag{10}$$

Since the stresses in a laminated composite vary from ply to ply, it is convenient to define laminate force (N_x, N_y and N_s) and moment (M_x, M_y and M_s) resultants. These resultants of stresses and moments acting on a laminate cross section, provide us with a statically equivalent system of forces and moments acting at the mid plane of the laminated composite. These force and moment resultants can be written in compact form as

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} \tag{11}$$

$$A_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k(h_k - h_{k-1}); \quad B_{ij} = \frac{1}{2}\sum_{k=1}^{n} (\bar{Q}_{ij})_k(h_k^2 - h_{k-1}^2) \text{ and } D_{ij} = \frac{1}{3}\sum_{k=1}^{n} (\bar{Q}_{ij})_k(h_k^3 - h_{k-1}^3) \tag{12}$$

where i, j = 1, 2, and 6.

The matrices A, B, D are known as in-plane, coupling and bending stiffness matrices respectively.

These stiffness matrices depend on $\overline{Q}$ and $\overline{Q}$ in turn depends on the orientation angle of each ply. Hence, the force and moment results vary with the stacking sequence. One can precisely optimise the stacking sequence of the laminate to improve the structural performance.

Force resultants due to temperature rise

The forces developed due to a temperature rise, ΔT, in a symmetric and balanced laminate are given by Tsai and Hahn (1980):

$$N_x = \Delta T \sum_{k=1}^{n} (\overline{Q}_{11}\alpha_x + \overline{Q}_{12}\alpha_y)_k (h_{k+1} - h_k) = R_1 \Delta T$$

$$N_y = \Delta T \sum_{k=1}^{n} (\overline{Q}_{12}\alpha_x + \overline{Q}_{22}\alpha_y)_k (h_{k+1} - h_k) = R_2 \Delta T \tag{13}$$

$$N_{xy} = \Delta T \sum_{k=1}^{n} (\overline{Q}_{66}\alpha_{xy})_k (h_{k+1} - h_k) = R_{12} \Delta T$$

where $\overline{Q}_{ij}$ are the material stiffness coefficients of the single layer defined in Eq. (6), h_i is the i^{th} layer with respect to mid plane and α_x, α_y and α_{xy} are the thermal coefficients of the i^{th} layer. As fibers impose a mechanical restraint on the matrix in the longitudinal direction, coefficient α_x is normally small whereas the matrix is forced to expand in the transverse direction, giving larger transverse coefficient α_y.

R_1, R_{12} and R_2 are the loads due to a unit temperature rise, and their sign will decide whether the buckling is caused by an increase or decrease in the temperature. For example, Kevlar/epoxy laminates buckle under cooling, whereas graphite/epoxy laminates buckle under heating (Mathew *et al.* 1992). From Eq. (13), it is possible to obtain the quantities $N_y/N_x = R_2/R_1$ and $N_{xy}/N_x = R_{12}/R_1$ for a given stacking sequence.

Thermal buckling of simply supported laminate plate

A Simply supported laminate composite plate of rectangular shape ($a \times b$), with symmetrical stacking sequence is shown in Fig. 5. The laminated composite plate is composed of n layers, each of thickness h_k and subject to a uniform temperature variation ΔT through the thickness. The total thickness of the laminate is $h = \sum_{k=1}^{n} h_k$.

Assuming that the plate is loaded, in the x-y plane, by the forces λN_x, λN_y, and λN_{xy}, where λ is a scalar amplitude parameter, the governing differential equation for the buckling behaviour of the plate, under the assumption of the classical plate theory (Vinson and Sierakowski 1987) is

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = \lambda\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + N_{xy}\frac{\partial^2 w}{\partial x \partial y}\right) \tag{14}$$

where D_{ij} are the bending stiffness coefficients and w is the vertical displacement. The laminate will buckle into m and n half-waves in the x and y directions when the amplitude parameter reaches a value λ_b given by (Vinson and Sierakowski 1987)

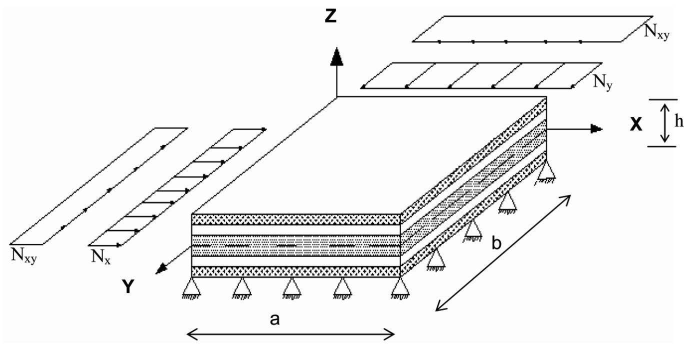

Fig. 5 A simply supported laminate composite plate

$$\frac{\lambda_b}{\pi^2} = \frac{D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4}{N_x\left(\frac{m}{a}\right)^2 + N_y\left(\frac{n}{b}\right)^2 + N_{xy}\left(\frac{mn}{ab}\right)} \tag{15}$$

derived from the displacement field

$$w(x, y) = \sum_m \sum_n A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{16}$$

that satisfies all imposed boundary conditions of the plate. The smallest value of λ_b is the critical buckling load λ_{cb} and can be evaluated from Eq. (15). Once λ_{cb} is obtained, the critical temperature rise Δt_{cr} is calculated as $\Delta t_{cr} = \lambda_{cb}/R_1$.

Strain failure load of laminate composite plate

In order to compute the failure load of a symmetric and balanced composite laminate panel, the value of corresponding multiplier λ_{cs} must be evaluated. The principal strains in the generic i^{th} layer are related to the loads by the following relations (Le Riche and Haftka 1993).

$$\lambda N_x = A_{11}\varepsilon_x + A_{12}\varepsilon_y; \quad \lambda N_y = A_{12}\varepsilon_x + A_{22}\varepsilon_y; \quad \lambda N_{xy} = A_{66}\gamma_{xy} \tag{17}$$

and

$$\begin{aligned} \varepsilon_1^i &= \cos^2\theta_i\varepsilon_x + \sin^2\theta_i\varepsilon_y - 2\sin\theta_i\cos\theta_i\gamma_{xy} \\ \varepsilon_2^i &= \sin^2\theta_i\varepsilon_x + \cos^2\theta_i\varepsilon_y - 2\sin\theta_i\cos\theta_i\gamma_{xy} \\ \gamma_{12}^i &= \sin\theta_i\cos\theta_i\varepsilon_x - \sin\theta_i\cos\theta_i\varepsilon_y + (\cos^2\theta_i - \sin^2\theta_i)\gamma_{xy} \end{aligned} \tag{18}$$

where A_{ij} are the extensional stiffness matrix coefficients, θ_i are ply orientations, ε_i and γ_{ij} are the strains considered. It may be noted that the coupling terms (B_{ij}) are not present in Eq. (17) as the laminate is symmetric and similarly A_{16} and A_{26} terms are not present as the laminate is balanced. The strain failure load λ_{cs} correspond to the smallest load factor λ in Eq. (15) such that, in at least one of the layers, one of the principal strains reaches or exceeds its allowable value, calculated from the ultimate allowable values ε_1^{ua}, ε_2^{ua}, γ_{12}^{ua}, using a safety factor of 1.5.

Combinatorial constraints for laminate plate

The laminate is assumed to be made up of 0°, 90° and 45° plies. To enforce the symmetric condition, we consider only top half of the laminate. Similarly, balanced condition is enforced by considering stacks of two 0° and 90° plies, and +45°/−45° pair plies. Using this lamination scheme, $N/4$ ply orientations are needed to define the stacking sequence of the laminate.

In order to avoid some undesirable stiffness coupling effects in laminated composite structure, angle plies must be balanced. Apart from that, the maximum number of contiguous plies of the same orientation is constrained to four, to alleviate large scale matrix cracking and to provide damage tolerant structures (Le Riche and Haftka 1993). Therefore, the optimisation problem is the buckling load maximisation by changing their ply orientations in the stacking sequence of the plate subjected to strain constraints with the ply contiguity and angle ply balance constraints. These combinatorial constraints are usually treated in evolutionary algorithms by a penalty approach (Le Riche and Haftka 1993). However, in the present work, it is preferred to deviate from this and a correction operator is devised to convert infeasible designs (due to constraint violation) into feasible ones by correcting (or rather repairing) the laminate ply sequences. In the present case study, the constraints include strain constraints, ply contiguity and angle ply constraints. The strain constraints if violated will reduce the buckling/failure load multiplier and also will be eliminated during the annealing process. The angle ply balance constraint is automatically taken care of as only one symmetric half of the laminate is considered during the optimisation process with ply angles in stacks of two like $0°_2$, ±45°, $90°_2$ in the laminate sequence. The ply contiguity constraint however requires special attention. In order to correct the laminate sequence with more than four contiguous plies with the same orientation, the laminate sequence is scanned from the outermost ply, and if five contiguous plies of the same orientation are encountered, the innermost ply code is changed either by decrementing or incrementing the value of the code. The option of decrementing or incrementing is decided based on the quality of solution. Similarly, near plane of symmetry, if the laminate sequence encounters more than two contiguous plies of the same orientation, the code value (i.e., the value of the ply immediate to the plane of symmetry) is incremented or decremented as discussed earlier.

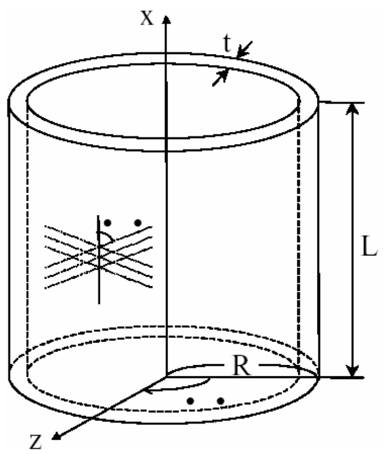

Fig. 6 Composite cylinder

3.2 Buckling of composite cylinder

The second case study considered is a composite cylinder subjected to axial compression loading. The stacking sequences are optimised using the proposed TSA algorithm to maximise the buckling loads of the composite cylinder. The configuration details of the composite cylinder are shown in Fig. 6. The practical significance of the problem is that the fuel tanks of the space shuttles are made of laminated composites and can be modelled as a composite cylindrical shell. The buckling of cylindrical shell under axial compression is of great relevance for the design of fuel tanks. Since the tanks are to be designed leak proof, an additional combinatorial constraint that the difference of ply orientations between contiguous plies should not be greater than 45 degree is imposed. This additional stacking constraint helps in preventing delamination cracks and thereby fuel leakage in the tank. The analytical solutions for buckling of composite cylinders under axial compression are given by Tasi (1966) and are as follows.

(i) Axial symmetric buckling ($m = 0$, $n = 1$)

$$\left(\frac{\overline{N}_x}{t}\right)_s = \frac{2}{Rt}\sqrt{\frac{d_{11}}{a_{22}}}\left(\sqrt{1 + \frac{b_{12}^2}{a_{22}d_{11}}} + \frac{b_{12}}{\sqrt{a_{22}d_{11}}}\right) \tag{19}$$

(ii) Non-axial symmetric buckling ($n \neq 0$)

$$\left(\frac{\overline{N}_x}{t}\right)_u = \frac{1}{Rt}\sqrt{\frac{d_{22}}{a_{11}}}\left(\Phi_1 + \frac{\left(\Phi_3 + \sqrt{\Phi_1\Phi_2 + \Phi_3^2}\right)^2}{\Phi_2}\right)(\Phi_1\Phi_2 + \Phi_3^2)^{-1/2} \tag{20}$$

where

$$\Phi_1 = \frac{a_{11}d_{11}}{a_{22}d_{22}}\mu^4 + 2\frac{d_{12} + 2d_{66}}{\sqrt{d_{11}d_{22}}}\sqrt{\frac{a_{11}d_{11}}{a_{22}d_{22}}}\mu^2 + 1 \tag{21}$$

$$\Phi_2 = \mu^4 + 2\frac{a_{12} + 0.50a_{66}}{\sqrt{a_{11}a_{22}}}\mu^2 + 1 \tag{22}$$

$$\Phi_3 = \frac{b_{12}}{a_{22}}\sqrt{\frac{a_{11}}{d_{22}}}\mu^4 + 2\frac{\{0.50(b_{11} + b_{12}) - b_{66}\}}{\sqrt{a_{22}d_{22}}}\mu^2 + \frac{b_{21}}{\sqrt{a_{11}d_{22}}} \tag{23}$$

$$\mu^2 = \frac{\lambda^2}{n^2}\sqrt{\frac{a_{22}}{a_{11}}} \tag{24}$$

$$\lambda = \frac{m\pi R}{L} \tag{25}$$

The elements of matrices $[a]$, $[b]$, and $[d]$ are defined by

$$\begin{aligned}[a] &= [A]^{-1}\\ [b] &= [B]\cdot[a]\\ [d] &= [D] - [b]\cdot[B]\end{aligned} \tag{26}$$

R is the outer radius, L is the length of the cylinder, and t is the thickness of the composite laminate. [A], [B] and [D] are the in-plane, coupling and bending stiffness matrices respectively. While m represents the half wave number of buckling mode in the axial direction, n represents the circumferential wave number of buckling mode. As mentioned earlier, the elements of B matrix vanish for the symmetric laminates.

The axial symmetric buckling load is calculated using Eq. (19). The non-axial symmetric critical buckling load is taken as the smallest buckling load computed for all combinations of m, n (m = 1, 2, … 20, n = 1, 2,… 20). The objective function for optimal stacking sequence that yields the maximum buckling load of the composite cylinder is formulated as

$$\text{Fitness function } F = \min\left(\frac{\overline{N}_x}{t}\right) \tag{27}$$

In the analysis of the buckling load given by Eqs. (19) to (25), the bending and twisting coupling matrices D_{16} and D_{26} will be neglected. The error induced by this assumption is negligible if the following non-dimensional ratios

$$\xi = \frac{D_{16}}{(D_{11}^3 D_{22})^{1/4}} \qquad \psi = \frac{D_{26}}{(D_{22}^3 D_{11})^{1/4}} \tag{28}$$

satisfy the constraints $\xi \leq 0.20$ and $\psi \leq 0.20$. A detailed discussion of the condition and its implications is given in Nemeth (1995), where it is shown that for buckling problems, the constraints given in Eq. (28) are effective in reducing bending-twisting coupling to a negligible level. In order to satisfy this constraint, it is preferred to impose penalty on the fitness function. With this, the fitness function given in Eq. (27) can be modified as:

$$\text{Fitness function } F = \frac{\min\left(\frac{\overline{N}_x}{t}\right)}{(1 + \zeta + \chi)} \tag{29}$$

Where ζ and χ are the penalties imposed for violation of constraints given in Eq. (28) and the magnitude of ζ and χ is set equal to ξ and ψ respectively.

Combinatorial constraints for composite cylinder

The ply orientations considered for the composite cylinder are 0°, ±45°, 90°. The laminates are considered to be symmetric and balanced. Similar to the earlier numerical studies, one string of individuals represents one half of the symmetrically laminated composite cylinder. However, in order to handle the additional combinatorial constraint related to ply angle difference, we need to represent each ply with an integer. Each element in the string is an integer between 1 to 4, where 1, 2, 3, and 4 corresponds to 0°, +45°, 90° and −45° respectively.

As already discussed in the earlier case study, it is preferred to handle the combinatorial constraints of the problem during improvement through a correction operator. The correction operator will come into force whenever a particular laminate sequence generated during the evolutionary process becomes infeasible due to violation of combinatorial constraints related to ply contiguity, ply balancing and ply angle difference.

The constraint related to four contiguous plies will be handled in the same way as explained in the earlier case studies. However, ply balancing condition will not get automatically satisfied as in the case of earlier case studies because the problem representation is slightly different. Here, each ply is represented separately, while in the earlier case studies, a pair of plies of same orientation is represented together with an integer value. If the laminated plate is unbalanced there exists only a single unbalanced 45° ply. To correct this unbalance, the correction operator first attempts to replace one +45° ply by 90° or 0° ply. The 45° ply position chosen to be replaced either with 90° or 0° plies, should be the inner most 45° ply, so that constraint related to four contiguous plies is not violated. In case, if it is not possible to effect this correction, the innermost 90° or 0° ply is replaced by −45° ply.

The correction operator also helps in maintaining the ply-angle difference rule discussed earlier. This rule does not permit the ply angle difference between adjacent plies greater than 45°. The correction operator handles this constraint of ply-angle difference rule similar to the contiguous ply rule discussed in the earlier sections. The laminate sequence is scanned from outermost ply, and if the ply angle difference rule between any contiguous plies is violated, the corresponding ply angle will be incremented or decremented by a flip of a coin.

4. Numerical studies

Numerical experiments have been carried out to demonstrate the effectiveness of the proposed TSA algorithm. The performance of the algorithm is first evaluated by optimizing the stacking sequences of 48-ply and 64-ply laminate composite panels for maximisation of buckling load and comparing with the best known published results. Later, thermal buckling of composite panels is taken up as the first case study. Finally, stacking sequence optimisation of composite cylinder made of single and multiple materials is considered as second case study in this paper.

4.1 Validation and evaluation of TSA with the best known published results

The TSA algorithm presented in this paper is first evaluated by solving the numerical examples given in the literature and comparing the results with the best known published results. For this purpose, simply supported composite panels with 48 (0°, 45° and 90°) and 64 plies (0°, 45° and 90°) are considered. The plies considered here are made of graphite epoxy with thickness, t = 0.005 in. (0.0127 cm). The material properties are same as given by Le Riche and Haftka (1993) and are as follows: E_1 = 18.50 E6 psi (127.59 GPa); E_2 = 1.89E6 psi (13.03 GPa); G_{12} = 0.93E6 psi (6.41 GPa); ν_{12} = 0.30. The ultimate allowable strains are $\varepsilon_1^{ua} = 0.008$, $\varepsilon_2^{ua} = 0.029$, $\gamma_{12}^{ua} = 0.015$. For evaluation purpose, four load cases as given in Table 1 are considered. These four load cases have been solved in the literature (Le Riche and Haftka 1993, Kogiso *et al.* 1994, Soremekun *et al.* 2001) for the optimum buckling and failure loads using genetic algorithms. The buckling and failure

Table 1 Details of the load cases considered for evaluation

Load case	Number of plies	a in mm	b in mm	N_x N/cm	N_y N/cm
1	48	50.8	12.7	1.75	0.22
2	48	50.8	12.7	1.75	0.44
3	48	50.8	12.7	1.75	0.88
4	64	50.8	25.4	1.75	1.75

Table 2 Validation of the computational model for laminate composites

SNO	Stacking sequence	Load factor, λ (present work)		Load factor, λ (Le Riche and Haftka 1993)	
		Buckling load factor	Failure load factor	Buckling load factor	Failure load factor
	48 ply laminate (Load case-3)				
1	$[(90_2, \pm45_2)_2, 90_2, \pm45, 90_2, \pm45_3]_S$	9997.72	10188.33	9997.60	10187.93
2	$[\pm45, 90_4, \pm 45, 90_2, \pm45_5, 90_2, \pm45_2]_S$	9976.91	10188.33	9976.58	10187.92
	64 ply laminate (Load case-4)				
3	$[\pm45, 90_{10}, \pm45, 90_8, \pm45, 90_8]_S$	3973.12	8934.13	3973.01	8935.74
4	$[90_4, \pm45_2, 90_{16}, \pm45, 90_6]_S$	3973.12	8934.13	3973.01	8935.74
5	$[90_2, \pm45, 90_6, \pm45, 90_8, \pm45, 90_{10}]_S$	3973.12	8934.13	3973.01	8935.74
6	$[90_8, \pm45, 90_2, \pm45, 90_2, \pm45, 90_2, \pm45_6]_S$	3973.12	14205.94	3973.01	14205.18
7	$[\pm45, 90_{10}, \pm45, 90_8, \pm45, 90_8]_S$	3973.12	8934.13	3973.01	8935.74

load factors obtained using the present model are compared with the results given in Le Riche and Haftka (1993) and found to be in good agreement. The details are presented in Table 2.

The optimisation problem is to maximize the critical buckling load λ_{cb} by changing the ply orientations and subjected to strain failure constraints and the combinatorial constraints. The objective function can be stated as

$$\text{Maximise } \lambda^* = \min(\lambda_{cb}, \lambda_{cs}) \tag{30}$$

with the combinatorial constraints related to ply contiguity, ply balancing and symmetry.
where λ_{cb} is the critical buckling and λ_{cs} is the strain failure load. The design variables are the ply angles as already described earlier. The critical buckling and failure loads of the simply supported composite panels can be computed using Eqs. (15) to (18). Several optimal designs exist (multi-modal solutions) for all these load cases, which are close to optimal buckling and failure loads. The best known buckling and failure load factors for all the four load cases are compiled from the literature (Le Riche and Haftka 1993, Kogiso *et al.* 1994, Soremekun *et al.* 2001) and are given in Table 3.

To evaluate the quality of the solutions obtained using the TSA algorithm, the concept of practical reliability (Le Riche and Haftka 1993) is used. Practical reliability is given by the percentage of solutions obtained using the TSA algorithm with the buckling and failure loads greater than or equal to 99.9% of the optimal solutions. The practical reliability is obtained for each load case by running 100 different instances of TSA and determines the number of final solutions that satisfy the above requirement. Another measure used in the present numerical experiments to measure the effectiveness of the TSA algorithm is the normalised price of each run (Le Riche and Haftka 1993). The price is defined as the number of function evaluations during one complete run of TSA algorithm. The normalised price is defined as the ratio of the average price of the runs to the practical reliability.

The optimal stacking sequences obtained using the proposed TSA algorithm for maximisation of buckling load and their corresponding buckling and failure load factors are shown in Table 3. In all

Table 3 Optimal stacking sequences for buckling load factor maximisation using TSA algorithm

Load case	Best known results published in literature (Le Riche and Haftka 1993, Kogiso *et al.* 1994, Soremekun *et al.* 2001)		TSA results		
	Buckling load factor	Failure load factor	Stacking sequence	Buckling load factor	Failure load factor
1	14618.12	13518.66	$[2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 3]_S$	16119.48	12513.73
	14134.76	13518.66	$[2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 3\ 2\ 2\ 2]_S$	16087.83	12513.73
	14013.71	13518.66	$[2\ 2\ 2\ 2\ 1\ 2\ 1\ 2\ 1\ 1\ 3\ 1]_S$	14437.30	13518.67
	13662.61	13518.66	$[2\ 2\ 2\ 2\ 1\ 3\ 1\ 2\ 1\ 1\ 2\ 1]_S$	14329.15	13518.67
2	12725.26	12678.77	$[2\ 2\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 2]_S$	13442.04	15779.09
	12698.40	12678.77	$[2\ 2\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 3]_S$	13441.28	14970.04
	12743.45	12678.78	$[2\ 2\ 2\ 2\ 3\ 2\ 2\ 3\ 3\ 2\ 2\ 3]_S$	13435.94	13588.68
3	9998.19	10394.81	$[3\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 2\ 2]_S$	9998.70	10403.75
	9997.60	10187.93	$[3\ 3\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 3\ 2\ 2]_S$	9995.20	10403.74
	9976.58	10187.93	$[3\ 2\ 3\ 2\ 2\ 2\ 2\ 3\ 2\ 3\ 2\ 2]_S$	9994.61	10192.33
4	3973.01	14205.18	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3]_S$	3957.22*	10733.33*
	3973.01	8935.74	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 2]_S$	3956.90*	11620.93*
	3973.01	8935.74	$[3\ 3\ 3\ 3\ 3\ 2\ 2\ 3\ 2\ 2\ 2\ 3\ 2\ 2\ 2\ 2]_S$	3977.12	14208.94
	3973.01	8935.74	$[3\ 3\ 3\ 3\ 2\ 3\ 2\ 3\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 2]_S$	3976.60	13361.90
	3973.01	14205.18	$[3\ 3\ 3\ 3\ 2\ 3\ 2\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 2\ 2]_S$	3977.12	14208.94

*Obtained with four ply contiguity constraint.

the tables, the laminate sequences are shown in a coded string form for convenience. It may be noted that '1' in coded string corresponds to '0_2^0' and similarly '2' and '3' in the string corresponds to '±45°' and '90_2^0' respectively. A close look at the results presented in Table 3 indicate that the optimal sequences obtained using the TSA algorithm for buckling load maximisation infact generate solutions which are superior to the best known bucking load factors from the literature. It can be observed that for the first load case, the optimum designs are governed by the strain failure load and not the buckling load. It is interesting to note that the optimal stacking sequences for maximizing the buckling load predominantly consists of ±45° plies. Since it is ±45° ply, more than three such plies are permissible, as the resulting stacking sequence solution do not violate the ply contiguity constraint. One can also notice that the presence of 0° plies in the stacking sequence improves the failure load and on the other hand the absence of 0° plies in the stacking sequence improves the buckling load.

Since the optimal designs for the second, third and fourth load cases given in Table 3 are governed by buckling load, the optimal stacking sequences predominantly consists of ±45° plies and limited number of 90° plies. One can notice that 0° plies are completely absent from the optimal and near optimal stacking sequences.

It may be noted that the first two stacking sequences given in Table 3 for the fourth load case (64 ply laminate) have been obtained with ply contiguity constraint. However, the results reported in the literature (Le Riche and Haftka 1993, Soremekun *et al.* 2001) are without ply contiguity constraint.

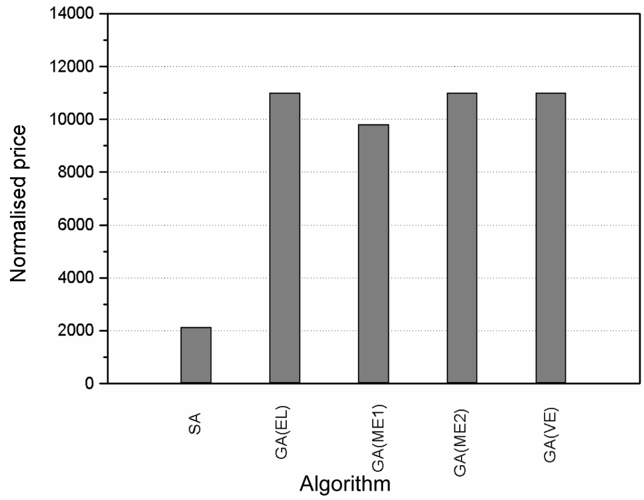

Fig. 7 Comparative performance of TSA with various GA implementations

In view of this, the buckling load factor for the 64 ply laminate (4th load case) is marginally lower than the best known values for the first two stacking sequences given in Table 3. The rest of the stacking sequences for the 64-ply laminate given in Table 3 are obtained without imposing the ply contiguity constraint and are therefore comparable with the best-known results. It should be mentioned here that, multiple stacking sequences using the proposed TSA algorithm is obtained by maintaining a separate memory archive, which stores the user specified number of best solutions, whose objective value is within 12% of the best-ever objective. These stacking sequences will be continuously updated with fitter values in the memory archive during the TSA search process.

Finally the price performance of the TSA algorithm has been evaluated by considering 64-ply composite laminate panel for which normalised price for various GA implementations is given in the literature (Soremekun *et al.* 2001). Fig. 7 shows the normalised price of various GA implementations and the TSA algorithm. It may be noted that TSA can run with various neighbourhood search options. Here, we prefer to use adaptive neighbourhood search while using TSA. It can be easily verified from Fig. 7 that the proposed algorithm is relatively faster than GA.

4.2 Optimal stacking sequences for thermal buckling

The first case study considered in this paper is thermal buckling of composite laminate panels. For this purpose, a simply supported composite panel with 48 plies (0°, ±45°, 90°) is considered to generate optimal stacking sequences for thermal buckling using the TSA algorithm. The material properties of graphite-epoxy composite lamina (AS/3501) are (http://www.composite.com) : E_1 = 20.0e06 psi (138 GPa), E_2 = 1.30e06 psi (8.96 GPa), G_{12} = 1.03e06Psi (7.10 GPa), ν_{12} = 0.3, t = 0.005 in (0.127 mm). The ultimate allowable strains are taken as ε_1^{ua} = 0.008, ε_2^{ua} = 0.029 and ν_{12}^{ua} = 0.015. The coefficients of thermal expansion, α_x = 0.03e-06C^{-1} and α_y = 28.1e-06C^{-1}. A safety factor of 1.5 has been used to reduce these values to allowable values. The plate considered in the numerical studies is simply supported on all edges, with dimensions a = 20 in (0.508 m) and b = 5 in (0.127 m). The optimisation problem considered is same as the one given in Eq. (30). In order to

Table 4 Optimal ply sequences for 48 ply composite laminate using enumeration

SNO	Sequence	Buckling λ	Failure λ	Temperature °C	Criteria for optimisation
1	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Buckling
2	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 1]_s$	7459.31	5739.80	598.65	Buckling
3	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 2\ 3]_s$	7443.17	7565.97	635.67	Buckling
4	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 2\ 2]_s$	7440.20	8616.26	607.71	Buckling
7	$[3\ 1\ 3\ 3\ 1\ 3\ 1\ 3\ 1\ 1\ 3\ 1]_s$	5744.30	14220.40	5332.60	Failure
6	$[3\ 3\ 1\ 1\ 3\ 1\ 3\ 3\ 1\ 1\ 3\ 1]_s$	5632.76	14220.40	5332.60	Failure
5	$[3\ 1\ 3\ 3\ 1\ 1\ 3\ 1\ 3\ 1\ 1\ 3]_s$	5437.56	14220.40	5332.60	Failure
8	$[3\ 1\ 3\ 1\ 1\ 3\ 3\ 1\ 1\ 3\ 1\ 3]_s$	4935.63	14220.40	5332.60	Failure

Table 5 Optimal stacking sequences for 48 laminate ply using TSA with geometric decrement of temperature

SNO	α	Stacking sequence	Buckling λ_{cb}	Failure λ_{cs}	Temp °C	Neighbourhood search algorithm	Number of evaluations
1	0.85	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Adaptive	1720
2	0.85	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Invert	1960
3	0.85	$[3\ 3\ 2\ 3\ 2\ 2\ 3\ 2\ 2\ 1\ 2\ 2]_s$	6668.81	9068.74	420.90	Swap	1810
4	0.85	$[3\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 1\ 2\ 2\ 1]_s$	6366.63	12331.6	562.50	Permutation	1390
5	0.85	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 2\ 3]_s$	7443.18	7565.97	564.05	Swap + Invert	2020
6	0.85	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 2\ 3\ 2]_s$	7404.56	8616.26	605.57	Perm. + Invert	2080
7	0.88	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Adaptive	2130
8	0.88	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Invert	2710
9	0.88	$[3\ 3\ 2\ 3\ 2\ 2\ 3\ 2\ 2\ 1\ 2\ 2]_s$	6668.81	9068.74	420.90	Swap	1720
10	0.88	$[3\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 1\ 2\ 2\ 1]_s$	6366.63	12331.6	562.50	Permutation	1390
11	0.88	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Swap + Invert	2710
12	0.88	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 2\ 3\ 3\ 2\ 3]_s$	7371.90	7565.97	564.05	Perm. + Invert	2290
13	0.9	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Adaptive	2200
14	0.9	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Invert	3250
15	0.9	$[3\ 3\ 2\ 3\ 2\ 2\ 3\ 2\ 2\ 1\ 2\ 2]_s$	6668.81	9068.74	420.90	Swap	2290
16	0.9	$[3\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 1\ 2\ 2\ 1]_s$	6366.63	12331.6	562.50	Permutation	1600
17	0.9	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Swap + Invert	2770
18	0.9	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 2\ 3\ 3\ 2\ 3]_s$	7371.90	7515.97	564.04	Perm. + Invert	2290
19	0.99	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Adaptive	9760
20	0.99	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Invert	16990
21	0.99	$[3\ 3\ 2\ 3\ 2\ 2\ 3\ 2\ 2\ 1\ 2\ 2]_s$	6668.81	9068.74	420.90	Swap	7690
22	0.99	$[3\ 3\ 2\ 3\ 2\ 2\ 2\ 2\ 1\ 2\ 3\ 2]_s$	6389.14	906.74	420.89	Permutation	6580
23	0.99	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	Swap + Invert	10870
24	0.99	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 2\ 3\ 3\ 1\ 3]_s$	7360.15	5739.81	512.27	Perm. + Invert	9300

demonstrate the performance of the TSA algorithm for lay-up sequence optimisation, the best objective values are first identified for 48 ply laminate through an enumerative study and the optimal stacking sequences are shown in Table 4. The optimal stacking sequences obtained for 48 ply laminate employing the TSA algorithm is shown in Table 5. The optimal values presented in Table 5 are obtained using various neighbourhood search techniques discussed in the earlier sections for various values of geometric temperature decrement parameter α. A close look at the results presented in Table 5 indicates that the proposed TSA algorithm is extremely effective in capturing the optimal stacking sequences of laminate composites. This fact is clearly evident from the comparison with the best results obtained through enumerative study and given in Table 4. Further, it can be observed that the adaptive neighbourhood search technique proposed in this paper is effective in obtaining consistently optimal solutions with least number of function evaluations.

Parametric studies have been conducted on 48 ply laminate by varying the following settings in the TSA algorithm

i. The temperature TSA is controlled using arithmetic decrement instead of geometric decrement as given in Eq. (3) by varying the value of α using different neighbourhood search algorithms.
ii. Changing adaptively the number of Metropolis iterations using various values of β ranging from 1.01 to 1.10.
iii. Using single Metropolis iteration with temperature decrement as given in Eq. (4) and varying the value of γ.
iv. Alternative criteria proposed by Johnson *et al.* (1991) for computing the probability of accepting a worse move. The criteria proposed by Johnson *et al.* (1991) is $P(\Delta E) = 1 - \Delta E/t$

Table 6 Optimal stacking sequences for 48 laminate ply using TSA with various alternative settings

S.NO	Parameter settings in TSA						Neighbour-hood search algorithm	Buckling load factor	Num. of function evaluations	Remarks
	α	β	γ	M	EXP ($\Delta E/T$)	1 − ($\Delta E/T$)				
1	0.85	1.00	-	30	√	×	Adaptive	7460.99	1720	Geometric temp. decrement
2	0.25	1.00	-	30	√	×	Adaptive	7460.99	5020	Arithmetic temp. decrement
3	1.5	1.00	-	30	√	×	Adaptive	7460.99	3100	Arithmetic temp. decrement
4	2.0	1.00	-	30	√	×	Adaptive	7443.18	2350	Arithmetic temp. decrement
5	0.85	1.03	-	3	√	×	Adaptive	7460.99	5809	Adaptive iterations + geometric temp. decrement
6	0.85	1.05	-	3	√	×	Adaptive	7460.99	4160	Adaptive iterations + geometric temp. decrement
7	0.85	1.08	-	3	√	×	Adaptive	7460.99	6608	Adaptive iterations + geometric temp. decrement
8	0.85	1.00	1e-5	1	√	×	Adaptive	7460.99	5806	Single iteration + geometric temp. decrement
9	0.85	1.00	-	30	×	√	Adaptive	7320.03	2140	Alternate acceptance criteria + geometric temp. decrement
10	0.99	1.00	-	50	×	√	Adaptive	7438.53	4090	Alternate acceptance criteria + geometric temp. decrement

Table 7 Optimal ply sequences of 48 ply laminate for maximal failure load using TSA

SNO	Sequence	Buckling λ_{cb}	Failure λ_{cs}	Temp. °C	Criteria for optimisation
1	$[1\ 3\ 1\ 1\ 3\ 1\ 3\ 2\ 3\ 2\ 3\ 1]_s$	3810.35	14218.43	1417.47	Failure
2	$[2\ 3\ 1\ 1\ 3\ 1\ 1\ 3\ 2\ 3\ 1\ 3]_s$	4153.24	14218.43	1500.06	Failure
3	$[3\ 1\ 3\ 1\ 3\ 1\ 3\ 1\ 1\ 3\ 1\ 3]_s$	5130.83	14220.40	2667.74	Failure
4	$[3\ 1\ 3\ 3\ 1\ 1\ 3\ 1\ 1\ 3\ 1\ 3]_s$	5353.91	14220.41	2667.75	Failure
5	$[3\ 1\ 1\ 3\ 1\ 3\ 3\ 1\ 3\ 1\ 1\ 3]_s$	4768.32	14220.41	2667.75	Failure
6	$[3\ 1\ 1\ 3\ 3\ 1\ 3\ 1\ 3\ 1\ 3\ 1]_s$	4991.40	14220.41	2667.75	Failure

The criteria proposed by Johnson *et al.* (1991), actually approximates the exponential in the Metropolis algorithm given in Fig. 2. Extensive parametric studies have been conducted using the above four settings by using various neighbourhood search algorithms and also varying the values of the parameters like α, β, γ and M. However best stacking sequences obtained from these studies are only presented in Table 6. A close look at the results given in Table 6 indicate that the results obtained using the conventional acceptance criteria with geometric decrement operator and constant Metropolis iterations with temperature setting as suggested in Eq. (2) is effective both interms of obtaining optimal stacking sequence and minimising the function evaluations.

The optimal lay-up sequences for maximisation of failure load obtained employing TSA algorithm are given in Table 7 for 48 ply laminate composite panel. The results presented in the tables are obtained using adaptive neighbourhood search technique with geometric decrement of temperature with the parameter α as 0.92. A close look at the results presented in Table 7 indicates that different stacking sequences yield same values of critical failure load multiplier. One can also observe that the stacking sequences given in Table 7 yields different buckling load multiplier values and critical temperature rise values for the same failure load multiplier. The reason is that the failure load does not depend on the stacking sequence, but just depends on the thickness and the orientation of plies. Hence several optimal or near optimal stacking sequence designs for maximisation of failure load can be obtained. At the same time, buckling load depends on the stacking sequence and hence different buckling load multiplier values are obtained for the stacking sequences with the same failure load multiplier. Hence many practical optimal designs exist, for this problem where a practical design can be defined as a design giving an objective function very close to the optimal solution. The TSA algorithm is as effective as genetic algorithm in capturing the multiple optimal solutions. This is clearly evident from the multiple optimal solutions presented in Table 7. From the optimal stacking sequences for maximisation of failure load, given in Table 7, one can observe the presence of 0^o plies and at the same time, the 0^o plies are completely missing in the optimal stacking sequences obtained for maximisation of buckling load and given in Table 5. This behaviour is consistent with the results reported earlier.

Further, it can also be observed that the failure load multiplier values obtained using TSA algorithm for 48 laminate (Table 7) are comparing very well with results obtained through enumerative study and shown in Table 4. However, one can notice that the stacking sequences obtained through enumerative study are different from the TSA results. This can be explained as follows. While computing the best stacking sequences through enumerative study, the laminate sequences which have the highest failure load multiplier are compiled and best among them which are comparatively higher in buckling load multiplier and temperature rise are chosen and tabulated.

Table 8 Comparative performance of TSA and GA for 48 plies laminates

SNO	Algorithm	Stacking sequence	Buckling λ_{cb}	Failure λ_{cs}	Temp °C	Number of Function evaluations (price)
1	Enumeration	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	----
2	Simulated annealing	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	1720
3	GA without elitism	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	3400
4	GA with elitism	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	2100
5	GA with multiple elitist strategy	$[3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2\ 3\ 3\ 2]_s$	7460.99	7565.97	635.67	4300

However, in the TSA algorithm, the objective function is defined as maximisation of the failure load multiplier. Hence, TSA could not capture the laminate sequences with higher buckling load multiplier and temperature rise. In order to capture these best laminate sequences, one has to invariably resort to multi-criteria optimisation by considering all three objectives (i.e., buckling, failure and temperature) (Rama Mohan Rao *et al.* 2003) simultaneously.

Finally, comparisons have been made with genetic algorithm. For this purpose an elitist genetic algorithm (Rama Mohan Rao 2001) customised for composite lay-up sequence optimisation problems is considered. Several GA implementations like simple GA, elitist GA, and multiple elitist GA are considered for comparison purposes. In the present study, tournament selection with two-point crossover with crossover probability of 0.60 is considered. Mutation, which is a combination of simple invert and swap, is considered with probability as 0.05 for the present study. Table 8 shows the comparative performances of the TSA algorithm (with geometric temperature decrement α as 0.85 and employing adaptive neighborhood search) and genetic algorithm for 48 ply laminate composite. A close look at the results indicates that the TSA algorithm provides solutions, which are comparable with various GA implementations. However, the TSA algorithm is found to be superior in terms of computational performance, i.e., total number of function evaluations.

4.3 Optimal stacking sequences of composite cylinder

The ply orientations considered for the composite cylinder are 0°, ±45°, 90°. The laminates are considered to be symmetric and balanced. Similar to the earlier numerical studies, one string of individuals represents one half of the symmetrically laminated composite cylinder. As mentioned earlier, we need to represent each ply with an integer, in order to handle the additional

Table 9 Material properties for composite cylinder

S.No	Composites	E_L KN/mm^2	E_T KN/mm^2	ν_{LT}	G_{LT} KN/mm^2	Mass density Kg/mm^3
1	Graphite/Epoxy (AS/3501)	140.68	9.13	0.30	7.24	0.01605e-04
2	Boron/Epoxy	204.00	18.50	0.23	5.59	0.02080e-04
3	Kevlar/Epoxy	83.00	5.60	0.34	2.10	0.01000e-04
4	Glass/Epoxy (Generic S-glass-epoxy)	43.00	9.07	0.27	4.54	0.01992e-04

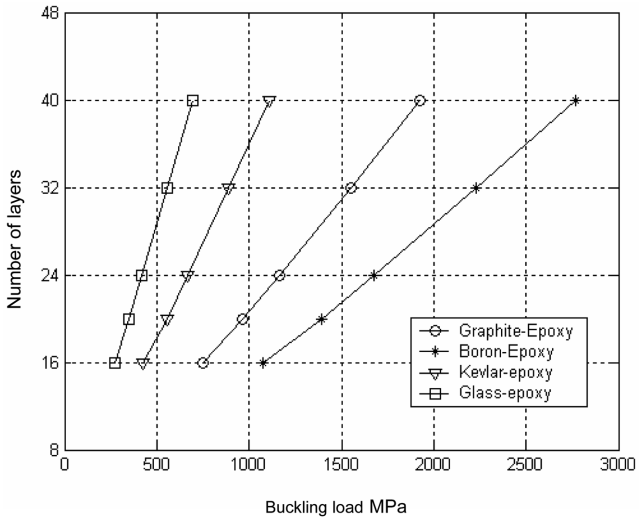

Fig. 8 Optimal buckling load of laminate composite cylinder made of different composite materials

combinatorial constraint related to ply angle difference. Each element in the string is an integer between 1 to 4, where 1, 2, 3, and 4 corresponds to 0°, +45°, 90° and −45° respectively.

Numerical studies have been carried out by considering a cylinder of L = 0.20 m, R = 0.10 m and ply thickness as 0.125 mm. In order to study the relative buckling strengths of various composite materials, the problem is solved using four different composite materials namely graphite, boron, kevlar and glass. The properties of these materials as given by Kaw (1997) are shown in Table 9. The optimisation problem is to maximize the critical buckling load by changing the ply orientations. The objective function can be stated as follows:

$$\text{Maximise } F = \min\left(\frac{\overline{N}_x}{t}\right) \tag{31}$$

Subjected to the constraints related to bending and twisting coupling terms of D matrix (Eq. (28)) and Combinatorial constraints related to ply contiguity, ply balancing, symmetry and ply angle difference.

The composite cylinder problem is solved for different materials given in Table 9. The optimal buckling loads obtained using the proposed TSA algorithm for various composite materials (given in Table 9) are shown in Fig. 8. A close look at the results given in Fig. 8 indicates that the composite cylinder made of boron-epoxy has the highest buckling strength followed by graphite-epoxy and kevlar-epoxy. The composite cylinder made of Glass-epoxy has the least buckling strength. kevlar is approximately 1.5 times as strong as Glass and similarly the Boron is twice as strong as kevlar and about 1.4 times of graphite-epoxy. The optimal ply sequences obtained for graphite-epoxy composite cylindrical shell using the proposed TSA are compared with GA (Rama Mohan Rao *et al.* 2001) and are presented in Table 10. It can be easily verified from the results presented in Table 10 that the optimal sequences obtained for buckling load maximisation of cylindrical shell are far more superior than genetic algorithm. However, in contrast to earlier studies on composite plates for maximisation of buckling loads, one can observe the presence of 0° plies in the optimal stacking sequence designs for maximisation of buckling load of composite cylinders,

Table 10 Optimal laminate sequences for buckling load maximisation of a graphite-epoxy composite cylindrical shell using TSA and GA

	TSA			Genetic Algorithm	
	Stacking sequence	Layers	Buckling load MPa	Stacking sequence	Buckling load MPa
1	$[2\ 1\ 4\ 4\ 3\ 3\ 2\ 3]_S$	16	751.70	$[1\ 4\ 3\ 4\ 3\ 2\ 3\ 2]_S$	715.40
2	$[4\ 4\ 1\ 2\ 2\ 3\ 4\ 3\ 2\ 3]_S$	20	966.98	$[4\ 1\ 4\ 4\ 3\ 2\ 2\ 3\ 3\ 2]_S$	958.19
3	$[2\ 2\ 1\ 4\ 4\ 3\ 4\ 4\ 3\ 2\ 2\ 3]_S$	24	1164.09	$[4\ 4\ 1\ 2\ 2\ 3\ 4\ 4\ 3\ 2\ 2\ 1]_S$	1114.07
4	$[4\ 4\ 1\ 4\ 4\ 1\ 2\ 2\ 3\ 2\ 2\ 3\ 4\ 3\ 2\ 3]_S$	32	1548.89	$[4\ 4\ 1\ 4\ 1\ 1\ 2\ 3\ 4\ 3\ 2\ 3\ 2\ 3\ 2\ 3]_S$	1485.94
5	$[2\ 2\ 1\ 2\ 2\ 1\ 4\ 4\ 3\ 4\ 4\ 3\ 4\ 4\ 3\ 2\ 3\ 3\ 2\ 3]_S$	40	1927.79	$[1\ 1\ 4\ 4\ 1\ 4\ 1\ 2\ 3\ 4\ 1\ 4\ 3\ 2\ 3\ 2\ 2\ 3\ 2\ 3]_S$	1841.75

obtained using both GA and TSA. This is basically due to the additional combinatorial constraint related to ply angle difference imposed on the optimisation problem and also it's implementation in the proposed correction operator. Further, another interesting feature, one can observe from the stacking sequences given in Table 10 is that there is identical number of 0°, ±45° and 90° plies in each of the stacking sequences obtained using GA and TSA for 16 ply and 20 ply laminate composite cylinders and near identical numbers for 24 ply and 36 ply composite cylinders. Still the stacking sequences obtained using TSA are superior to GA. From this one can conclude that TSA with adaptive neighbourhood search algorithm has better exploratory search characteristics when compared to GA.

4.4 Optimal design of composite cylinder made of two materials

This numerical study is concerned with the design of a hybrid laminate composite cylinder for optimum layer thickness, cost and ply angle sequence. Hybrid laminates incorporating two or more fibre composite materials in their construction can offer improved designs and better tailoring capabilities, as it is possible to combine the desirable properties of the two materials. In the present example, we considered two different materials. One is graphite-epoxy, which is expensive, but has high stiffness properties and the other is the glass-epoxy, which is not as stiff but is relatively cheaper.

Numerical studies have been carried out by considering a cylinder of L = 0.20 m, R = 0.10 m and ply thickness as 0.127 mm. The material properties of the two composite materials are given in Table 9. The stiffness-to-weight ratio of graphite-epoxy is about four times higher than that of glass-epoxy. However, it is also more expensive, with a cost per unit weight is 8 times higher than that of glass-epoxy. The ply orientations are considered as 0, ±45° and 90°. The laminates are considered to be symmetric and balanced.

Similar to the earlier case study, one string of individuals represents one half of the symmetrically laminated composite plate. Each element in the string is an integer between 0 and 8, where 0 represents a pair of empty plies, 1, 2, 3 and 4 represents 0°, +45°, 90° and −45° plies of graphite epoxy. Similarly 5, 6, 7 and 8 represents 0°, +45°, 90° and −45° plies of glass epoxy. We have to introduce 0 to represent empty plies in the string, as the number of individuals in a string is constant where as the number of plies in a laminate is not.

The objective functions for weight and cost minimisation can be stated as follows:

$$\text{Minimise, Weight} = 2\pi RL\ (\rho_c h + \rho_g(h - h_c)) \tag{32}$$

$$\text{Minimise, Cost} = 2\pi RL\ (\rho_c h\ C_c + \rho_g(h - h_c)\ C_g) \tag{33}$$

subjected to Constraint related to (i) minimum buckling load (ii) bending and twisting coupling terms of D matrix (Eq. (28)) (iii) ply contiguity, ply balancing, symmetry and ply angle difference where ρ_c is material density of the graphite-epoxy layers, ρ_g is the material density of glass-epoxy layers, h is the total thickness of the laminate and h_c is the thickness of the graphite-epoxy plies in the laminate. R is the radius of the mid surface of the composite cylinder and L is the length of the cylinder. C_c and C_g are the cost factors of graphite-epoxy and glass-epoxy respectively.

First, an attempt has been made to optimise with a single objective and later multi-objective situation is considered. Table 11 shows the optimal weight of the hybrid laminate composite cylinder with a constraint on maximum buckling load. Similar to the earlier case study, the laminate stacking sequences in all the tables (i.e., Table 11 to Table 13) are shown in coded string form. A close look at the Table 11 indicates that the minimum weight of the laminate sequence is 2.00 N for the buckling load constraint of 350 MPa. The minimum weight increases with the increase in the value set as a buckling load constraint. It can be observed that the composite laminate panel consists of only graphite-epoxy plies, as they are relatively lighter than the glass-epoxy plies.

Next the cost minimisation of the hybrid laminate is attempted with buckling constraint. Table 12 shows the optimal laminate sequences for minimum cost with various buckling constraints. A close

Table 11 Optimal ply sequences of bi-material composite laminate cylinder for minimal weight with constraint on buckling load

SNO	Buckling load constraint MPa	Sequence	Number of plies	Cost	Weight (N)	Buckling load MPa
1	350	$[2\ 1\ 4\ 3]_s$	8	1.63	2.00	372.66
2	425	$[1\ 1\ 2\ 3\ 4\ ]_s$	10	2.04	2.50	437.70
3	550	$[4\ 4\ 1\ 2\ 2\ 3]_s$	12	2.44	2.99	574.75
4	600	$[2\ 3\ 2\ 1\ 4\ 3\ 4]_s$	14	2.84	3.49	613.21
5	750	$[2\ 1\ 4\ 4\ 3\ 3\ 2\ 3]_s$	16	3.25	3.98	751.70

Table 12 Optimal ply sequences of bi-material composite laminate cylinder for minimal cost with constraint on buckling load

SNO	Buckling load constraint MPa	Sequence	Number of plies	Cost	Weight (N)	Buckling load MPa
1	250	$[5\ 8\ 8\ 7\ 6\ 7\ 6\ 7]_s$	16	0.50	4.94	272.64
2	300	$[4\ 8\ 7\ 6\ 7\ 6]_s$	12	0.72	3.60	312.12
3	400	$[4\ 4\ 5\ 6\ 7\ 6\ 7]_s$	14	1.13	4.09	415.56
4	550	$[1\ 4\ 3\ 2\ 7\ 8\ 5\ 6]_s$	16	1.875	4.46	556.41
5	600	$[2\ 2\ 3\ 4\ 3\ 7\ 8\ 7]_s$	16	2.22	4.34	614.79
6	700	$[1\ 2\ 2\ 3\ 4\ 3\ 4\ 5]_s$	16	2.90	4.10	708.19

Table 13 Trade-off solutions of bi-material laminate composite for multi-objective optimization of both weight and cost using TSA with constraint on buckling

β	Stacking sequence	Number of plies	Weight (N)	Cost	Buckling load Mpa
0.0	$[4\ 4\ 1\ 2\ 3\ 6\ 7\ 8\ 7\ 8\ 5\ 6\ 7\ 6\ 5\ ]_s$	30	8.59	2.64	925.26
0.10	$[1\ 1\ 2\ 4\ 3\ 6\ 5\ 8\ 8\ 7\ 6\ 7\ 8\ 7\ 6]_S$	30	8.59	2.64	900.49
0.20	$[4\ 4\ 3\ 2\ 1\ 2\ 6\ 7\ 6\ 7\ 8\ 8\ 7]_S$	26	7.26	2.86	917.91
0.30	$[2\ 1\ 4\ 1\ 4\ 3\ 2\ 6\ 7\ 7\ 8\ 7]_S$	24	6.54	3.14	923.73
0.40	$[2\ 2\ 3\ 4\ 1\ 4\ 4\ 5\ 6\ 6\ 7\ 8]_S$	24	6.54	3.14	910.99
0.50	$[2\ 1\ 4\ 1\ 4\ 3\ 2\ 3\ 6\ 7\ 8]_s$	22	5.81	3.42	905.12
0.60	$[4\ 1\ 4\ 1\ 2\ 2\ 3\ 4\ 7\ 6\ 7]_S$	22	5.81	3.42	922.22
0.70	$[1\ 2\ 1\ 4\ 4\ 3\ 2\ 3\ 4\ 5\ 6]_S$	22	5.69	3.76	947.72
0.80	$[2\ 2\ 1\ 4\ 3\ 4\ 1\ 3\ 3\ 6\ 8]_S$	22	5.69	3.76	914.33
0.90	$[2\ 2\ 1\ 4\ 4\ 3\ 4\ 3\ 3\ 2]_S$	20	4.96	4.05	964.99
1	$[4\ 1\ 4\ 4\ 3\ 2\ 2\ 3\ 2\ 3]_S$	20	4.96	4.05	960.22

look at the ply sequences given in the table indicate that the outer layers of the laminates are of graphite-epoxy whose stiffness is much higher than the glass-epoxy plies and core layers are of glass-epoxy plies. It is quite understandable as the graphite-epoxy plies are much expensive when compared to glass-epoxy plies.

Finally, laminate ply sequence optimisation of hybrid composite panels has been attempted for simultaneous optimisation of both weight and cost. The weighted sum method has been employed to solve the multi-criteria optimisation problem, where two objective functions are combined into one overall objective function and is given as:

$$\text{Objective function} = \beta \times \text{Weight} + (1 - \beta)\ \text{Cost} \tag{34}$$

where β is the weight factor. The optimisation problem is to minimise the weighted sum of the total weight and cost of the hybrid composite laminate cylinder as given in Eq. (34) subjected to the constraints given in Eqs. (32) and (33).

Table 13 summarizes trade-off results obtained for various values of β. The results furnished in the table include the stacking sequences, number of plies and their corresponding weight, cost, buckling load. The buckling constraint is set for this problem as 900 MPa. A close look at the results indicates that, the minimum weight of the hybrid laminate is 4.96 N. It is obtained when all plies are made of graphite-epoxy. The corresponding cost is 4.05. The minimum cost is found to be 2.64, which is about 65% of the cost of stacking sequence corresponding to the optimal weight. The optimum ply sequence corresponding to the minimum cost consists of major number of glass-epoxy plies. It's weight is found to be 8.59 N which is approximately 73% heavier than the optimum weight laminate sequence.

Fig. 9 shows the Pareto front obtained by employing the TSA algorithm with weighted sum approach of multi-criteria optimisation. The solid line shows the Pareto trade-off curve generated through the points corresponding to the optimal values obtained for different values of β. This Pareto front is the set of all the non-dominated solutions, which corresponds to lower envelope of all the feasible design points in the cost/weight plane. This confirms the effectiveness of the TSA

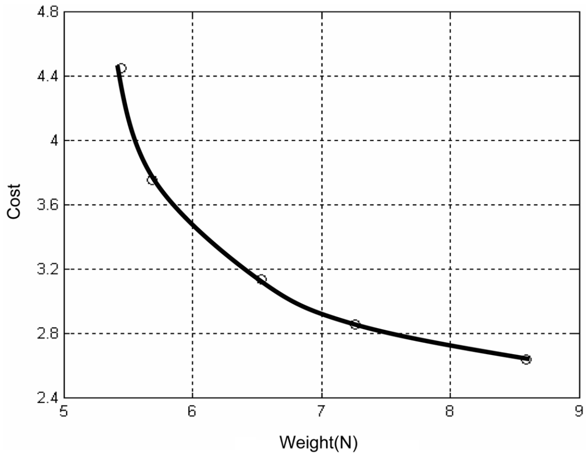

Fig. 9 Pareto optimum trade-off solutions obtained using TSA for the bi-material composite laminate cylinder

algorithm in obtaining optimal solutions for simultaneous cost/weight minimisation. The Pareto trade-off curve can be used by the designer to determine the optimal configurations for his problem. The final choice of the best design will depend on additional information that will enable him to evaluate all the points on the Pareto curve and prioritise these values depending on the application on hand.

5. Conclusions

In this paper a tabu embedded simulated annealing (TSA) algorithm has been presented and applied for stacking sequence optimisation of laminate composite structures. A new technique for temperature initialization is proposed and found to be effective for composite laminate design. Apart from this, several parameter settings like temperature decrement techniques, convergence criteria, alternative solution acceptance criteria in Metropolis algorithm, adaptive iteration scheme and finally several neighbourhood search techniques have been examined and their relative performances have been evaluated. The computational performance of the algorithm is enhanced by cache-fetch implementation. Two case studies have been conducted to evaluate the proposed TSA algorithm for stacking sequence optimisation of composite laminates. The first one is the thermal buckling of composite laminate panels. The second case study is buckling load optimisation of composite cylinder. Following are some of the conclusions based on the studies carried out in this paper

i. Among all the neighbourhood search techniques presented, adaptive neighbourhood search technique proposed in this paper for combinatorial optimisation found to be effective in obtaining optimal stacking sequences and also maintaining relatively higher computational efficiency.
ii. The geometric temperature decrement operator is found to be superior for stacking sequence optimisation of laminate composite panels and value of α ranging from 0.88 to 0.92 can be employed to obtain optimal sequences without compromising on the computational performance.
iii. Convergence of adaptive iteration scheme in the Metropolis algorithm found to be too slow

when compared to the constant number of iterations.

iv. TSA algorithm is as effective as genetic algorithm in obtaining multiple near optimal solutions as is evident from the studies presented in Tables 3 and 7. Further, numerical studies indicate that the optimal ply sequences obtained using simulated annealing are either comparable or superior to most of the GA implementations. The computational performance of SA is also found to be relatively superior for the problems solved in the paper.

v. It can be observed from the optimal stacking sequences presented in this paper that the stacking sequences with maximum buckling load are dominated by 45^o plies and 0^o plies are either minimum or completely absent. This indicates that 45^o plies have major influence in improving the buckling characteristics of the composite panels.

vi. Numerical studies on buckling strength of composite cylinder indicate that boron and graphite are more superior when compared to kevlar and glass. Buckling strength of boron-epoxy is approximately four times higher when compared to glass-epoxy.

vii. Tabu search (TS) and simulated annealing diversify the search in order to escape from local optima in contrasting styles. While TS works in a deterministic way so that its exploring trail is fixed, SA does so in a non-deterministic way and its exploring trails vary in different tries. The proposed meta-heuristic algorithm called TSA has been devised by synthesizing the exploring features of both SA and TS in order to enhance the diversification mechanism. The exploring characteristics of the TSA algorithm are clearly evident from the results given in Table 10.

viii. The proposed TSA algorithm is employed to solve a multi-objective optimisation problem of a hybrid laminate composite by simultaneously optimizing weight and cost using weighted aggregating approach. Numerical studies indicate that the trade-off solutions (Pareto-front) can be obtained for the multi-objective problem employing the proposed TSA algorithm. When cost is a primary consideration glass-epoxy can be used and when weight is a primary consideration, graphite-epoxy can be preferred. Compromise designs can easily be selected from the trade-off solutions.

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