Structural Engineering and Mechanics, Vol. 25, No. 2 (2007) 161-180
DOI: http://dx.doi.org/10.12989/sem.2007.25.2.161

A DQ nonlinear bending analysis of skew composite thin plates

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(Received November 22, 2005, Accepted August 18, 2006)

Abstract. A first endeavor is made to exploit the differential quadrature method (DQM) as a simple, accurate, and computationally efficient numerical tool for the large deformation analysis of thin laminated composite skew plates, which has very strong singularity at the obtuse vertex. The geometrical nonlinearity is modeled by using Green's strain and von Karman assumption. A recently developed DQ methodology is used to exactly implement the multiple boundary conditions at the edges of skew plates, which is a major draw back of conventional DQM. Using oblique coordinate system and the DQ methodology, a mapping-DQ discretization rule is developed to simultaneously transform and discretize the equilibrium equations and the related boundary conditions. The effects of skew angle, aspect ratio and different types of boundary conditions on the convergence and accuracy of the presented method are studied. Comparing the results with the available results from other numerical or analytical methods, it is shown that accurate results are obtained even when using only small number of grid points. Finally, numerical results for large deflection behavior of antisymmetric cross ply skew plates with different geometrical parameters and boundary conditions are presented.

Keywords: large deformation; thin laminated skew plates; differential quadrature method.

1. Introduction

Differential quadrature method is a relatively new numerical technique in structural analysis which was used successfully for different structural problems (Bert *et al.* 1988, Bert and Malik 1996, Karami and Malekzadeh 2002, 2003, Malekzadeh and Karami 2003, Karami and Malekzadeh 2004, Karami *et al.* 2003, Karami and Malekzadeh 2003, 2002, Kennedy and Simon 1967, Alwar and Ramachandra Rao 1973, 1974, Buragohain and Patodi 1978, Srinivasan and Ramachandran 1975, 1976, Srinivasan and Boby 1976, Ray *et al.* 1992, Pica *et al.* 1980, Duan and Mahendran 2003). However, most of the applications of DQM were concerned on problems with linear differential equations. Also, a major drawback of the conventional DQM is that it cannot be used in a straight forward manner for thin walled structural problems which have multiple boundary conditions for each field variable at a boundary point. An especial treatment is necessary for implementing the

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multiple boundary conditions (Bert *et al.* 1988, Bert and Malik 1996, Karami and Malekzadeh 2002, 2003, Malekzadeh and Karami 2003, Karami and Malekzadeh 2004, Karami *et al.* 2003, Karami and Malekzadeh 2003, 2002). In order to overcome this drawback, a DQ methodology was proposed by the author and his co-workers, which was only investigated for structural problems with linear differential equation (Karami and Malekzadeh 2002, 2003, Malekzadeh and Karami 2003, Karami and Malekzadeh 2004, Karami *et al.* 2003, Karami and Malekzadeh 2003, 2002). Therefore, it is essential to investigate its applicability for complexly nonlinear problems of thin structural elements. One of the complexly and practically important problem in structural analysis is the nonlinear bending of composite skew thin plates. Especially for antisymmetric composite skew plates due to bending-extensional coupling, the equilibrium equations and the related boundary conditions become more complicated than those of isotropic or orthotropic plates. Solving such a nonlinear problem clears the potential of the proposed DQM as a powerful technique for the structural analysis.

On the other hand, the research works on the nonlinear bending analysis of skew plates were restricted to isotropic and orthotropic thin skew plates (Kennedy and Simon 1967, Alwar and Ramachandra Rao 1973, 1974, Buragohain and Patodi 1978, Srinivasan and Ramachandran 1975, 1976, Srinivasan and Boby 1976, Ray *et al.* 1992). Also, skew plates with skew angle ranging from 0 to 45° with fully clamped and fully simply supported edges were considered. Nonlinear analysis of skew plates with 60° skew angle was often ignored. Nonlinear bending analyses of skew plates based on the first order shear deformation theory (FSDT) were also limited to plates with isotropic material (Pica *et al.* 1980, Duan and Mahendran 2003). Nonlinear bending analysis of antisymmetric laminated skew composite plates is not investigated yet. In using the conventional numerical method such as finite element method (FEM), a special treatment is necessary to solve skew plates with large skew angles (Duan and Mahendran 2003). The shear locking phenomenon is another drawback of conventional FEM. Some of treatments on the subjects for small deflection analysis of skew plates were reported in Duan and Mahendran (2003). The subject of this paper is not a simple matter that can be handled easily using commercially available FE programs to obtain an accurate solution.

There exist few applications of DQM for nonlinear analysis of rectangular and circular plates, which are limited to isotropic and orthotropic plates. The behavior of thin, circular, isotropic elastic plates with immovable edges was investigated by Striz *et al.* (1988). The nonlinear analysis of thin orthotropic rectangular plates was studied by Bert *et al.* (1989). Lin *et al.* (1994) employed the generalized differential quadrature to solve the problem of large deformation of thin isotropic plates under thermal loading. Chen *et al.* (2000) investigated the large deflection analysis of thin orthotropic rectangular plates. Li and Cheng (2005) studied the nonlinear free vibration analysis of orthotropic rectangular plates based on the higher order shear deformation theory of Reddy.

This work which is an extension to some previous works by the author and his co-workers (Karami *et al.* 2003, Karami and Malekzadeh 2003, 2002) is focused on geometrically nonlinear behavior of skew composite plates. The present study provides an efficient method for nonlinear analysis of antisymmetric composite skew plate bending problems that has many applications in structural engineering. Due to the high accuracy of DQM, the new solutions for antisymmetric composite skew plates can be used as benchmark.

2. The skew plate governing equations

A composite skew plate composed of perfectly bonded orthotropic layers of length a, width b and

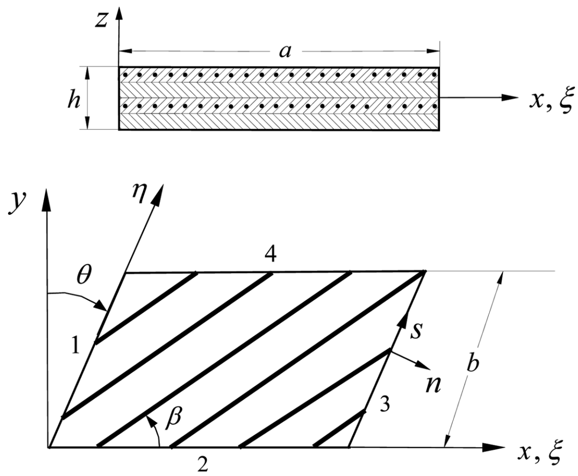

Fig. 1 The geometry of the skew composite plates

total thickness h is considered (see Fig. 1). Based on the thin plate theory (TPT), the displacement field can be expressed as Reddy (1997),

$$\bar{u}(x,y,z) = u(x,y) - z\frac{\partial w(x,y)}{\partial x},\quad \bar{v}(x,y,z) = v(x,y) - z\frac{\partial w(x,y)}{\partial y},\quad \bar{w}(x,y,z) = w(x,y) \tag{1}$$

where $(\bar{u}, \bar{v}, \bar{w})$ are the displacement components of an arbitrary point (x, y, z) of plate in the laminated skew plate, and (u, v, w) are the displacement projections on the mid-plane. By invoking the von Karman large deflection assumptions and using the Green-Lagrangian strain displacement relations, the in-plane vector of strain at an arbitrary point of the plate can be written as Reddy (1997),

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix} - z\begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = \boldsymbol{\varepsilon}_m + z\boldsymbol{\kappa} \tag{2}$$

where $\boldsymbol{\varepsilon}_m$ and $\boldsymbol{\kappa}$ are the membrane and bending components of strain, respectively,

$$\boldsymbol{\varepsilon}_m = \begin{Bmatrix} \varepsilon_{xx}^m \\ \varepsilon_{yy}^m \\ \gamma_{xy}^m \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix},\quad \boldsymbol{\kappa} = \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

For the antisymmetric cross ply laminated plates, the constitutive relationship based on the TPT can be expressed as,

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ B_{11} & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^m \\ \varepsilon_{yy}^m \\ \gamma_{xy}^m \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{3}$$

where (N_{xx}, N_{yy}, N_{xy}) and (M_{xx}, M_{yy}, M_{xy}) are the components of the in-plane force resultants and the moment ones, respectively.

Using the Eqs. (2) and (3), the equilibrium equations of an arbitrary shaped antisymmetric cross ply laminated composite plate in rectangular coordinate system and in terms of displacement components can be derived as follows Reddy (1997),

Equilibrium equations:

$$A_{11}\frac{\partial^2 u}{\partial x^2} + A_{66}\frac{\partial^2 u}{\partial y^2} + (A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y} + \left(A_{11}\frac{\partial^2 w}{\partial x^2} + A_{66}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial w}{\partial x}$$
$$+ (A_{12}+A_{66})\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y} - B_{11}\frac{\partial^3 w}{\partial x^3} = 0 \tag{4}$$

$$(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y} + A_{66}\frac{\partial^2 v}{\partial x^2} + A_{22}\frac{\partial^2 v}{\partial y^2} + \left(A_{66}\frac{\partial^2 w}{\partial x^2} + A_{22}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial w}{\partial y}$$
$$+ (A_{12}+A_{66})\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y} - B_{22}\frac{\partial^3 w}{\partial y^3} = 0 \tag{5}$$

$$B_{11}\frac{\partial^3 u}{\partial x^3} + \left(A_{11}\frac{\partial^2 w}{\partial x^2} + A_{12}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial u}{\partial x} + 2A_{66}\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x\partial y} + B_{22}\frac{\partial^3 v}{\partial y^3} + \left(A_{12}\frac{\partial^2 w}{\partial x^2} + A_{22}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial v}{\partial y}$$
$$+ 2A_{66}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\partial y} + B_{11}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x^3}\right] + B_{22}\left[\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial^3 w}{\partial y^3}\right]$$
$$- D_{11}\frac{\partial^4 w}{\partial x^4} - 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2} - D_{22}\frac{\partial^4 w}{\partial y^4} + \left\{\frac{A_{11}}{2}\left(\frac{\partial w}{\partial x}\right)^2\right.$$
$$\left. + \frac{A_{12}}{2}\left(\frac{\partial w}{\partial y}\right)^2 - B_{11}\frac{\partial^2 w}{\partial x^2}\right\}\frac{\partial^2 w}{\partial x^2} + \left\{\frac{A_{12}}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{A_{22}}{2}\left(\frac{\partial w}{\partial y}\right)^2 - B_{22}\frac{\partial^2 w}{\partial y^2}\right\}\frac{\partial^2 w}{\partial y^2}$$
$$+ 2A_{66}\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\frac{\partial^2 w}{\partial x\partial y} + q(x,y) = 0 \tag{6}$$

Eqs. (4) and (5) represent the balance of in-plane forces in x-and y-direction, respectively. Eq. (6) is resulted from the balance of transverse shear forces in z-direction and the bending equilibrium equations about x-and y-direction, respectively. Due to anisotropic nature of the plate, these equations are coupled. Also, the nonlinear terms that appear in the bending equilibrium equations are due to anisotropy. These equations are to be employed at an arbitrary point of the skew plate.

If the normal and tangent to an arbitrary edge of the skew plate is denoted by n and s, respectively (shown in Fig. 1), the boundary conditions along this edge can be classified as Reddy (1997),

Either $$u_n = 0 \text{ or } N_{nn} = 0 \tag{7}$$

Either $$u_s = 0 \text{ or } N_{ns} = 0 \tag{8}$$

Either $$w = 0 \text{ or } V_n = 0 \tag{9}$$

Either $$\frac{\partial w}{\partial n} = 0 \text{ or } M_{nn} = 0 \tag{10}$$

where,

$$u_n = n_x u + n_y v, \quad u_s = -n_y u + n_x v, \quad \frac{\partial w}{\partial n} = n_x \frac{\partial w}{\partial x} + n_y \frac{\partial w}{\partial y}$$

$$N_{ns} = (N_{yy} - N_{xx}) n_x n_y + N_{xy}(n_x^2 - n_y^2), \quad V_n = Q_n + \frac{\partial M_{ns}}{\partial s}$$

$$Q_n = \left(\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y}\right) n_x + \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y}\right) n_y, \quad M_{nn} = M_{xx} n_x^2 + 2 M_{xy} n_x n_y + M_{yy} n_y^2$$

$$M_{ns} = (M_{yy} - M_{xx}) n_x n_y + M_{xy}(n_x^2 - n_y^2)$$

The boundary conditions at any edge of the plate can be considered as the combinations of the conditions appeared in Eqs. (7)-(10).

3. DQ discretization

The DQM requires the computational domain to be rectangular and cannot be applied directly to irregular domains. To apply DQM to such problems, a coordinate transformation is necessary; that is, the irregular physical domain is transformed into a rectangular computational domain. For skew composite plates with arbitrary laminates lay up, the material points of skew plates in physical domain can be transformed into computational domain without any approximation, using the following linear transformation rules

$$x = \xi + (\sin\theta)\eta, \quad y = (\cos\theta)\eta \tag{11}$$

The transverse coordinate z in physical and computational domain is similar. Employing chain rule for the spatial derivatives and coordinate transformation in (11), the derivatives in physical domain are expressed in terms of the derivatives of space variables of computational domains. In the computational domain, the whole plate is discretized into a set of N_ξ and N_η discrete grid points in ξ-and η-direction, respectively.

In order to implement the multiple boundary conditions of thin skew plates efficiently, which is one of the most drawbacks of conventional DQM, in addition to displacement components some secondary degrees of freedom along the edges of the plates should be defined. Using the methodology proposed by Karami and Malekzadeh (2002, 2003), the degrees of freedom along the edges $\xi = 0, a$ and $\eta = 0, b$ become (u, v, w, K^ξ) and (u, v, w, K^η), respectively, in which $K^\xi = \partial^2 w/\partial\xi^2$ and $K^\eta = \partial^2 w/\partial\eta^2$. However, at the domain grid points, the degrees of freedom are (u, v, w). Using the presented degrees of freedom, the drawbacks of other methodologies can be improved, i.e.: 1. The boundary conditions will be implemented exactly at boundary grid points, 2. The equilibrium equations are satisfied at all domain grid points, and 3. The reformulations or modifications of the weighting coefficients are not necessary.

Using the transformation rules (11) and the presented degrees of freedom, efficient transformation-DQ discretization rules for the derivatives of displacement components can be obtained, which simultaneously transform the partial derivatives and discretized them in computational domain. For the in-plane components of displacement (u, v) up to the third order derivatives and for the transverse displacement w up to second order derivatives with respect to coordinate variables x and y, one has

$$\left.\frac{\partial()}{\partial x}\right|_{(\xi_i,\,\eta_j)} = \sum_{m=1}^{N_\xi} A_{im}^{\xi}()_{mj}, \quad \left.\frac{\partial^2()}{\partial x^2}\right|_{(\xi_i,\,\eta_j)} = \sum_{m=1}^{N_\xi} B_{im}^{\xi}()_{mj}, \quad \left.\frac{\partial^3()}{\partial x^3}\right|_{(\xi_i,\,\eta_j)} = \sum_{m=1}^{N_\xi} C_{im}^{\xi}()_{mj} \tag{12}$$

$$\left.\frac{\partial()}{\partial y}\right|_{(\xi_i,\,\eta_j)} = -\tan\theta\sum_{m=1}^{N_\xi} A_{im}^{\xi}()_{mj} + \sec\theta\sum_{n=1}^{N_\eta} A_{jn}^{\eta}()_{in} \tag{13}$$

$$\left.\frac{\partial^2()}{\partial x\partial y}\right|_{(\xi_i,\,\eta_j)} = -\tan\theta\sum_{m=1}^{N_\xi} B_{im}^{\xi}()_{mj} + \sec\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} A_{im}^{\xi}A_{jn}^{\eta}()_{mn} \tag{14}$$

$$\left.\frac{\partial^2()}{\partial y^2}\right|_{(\xi_i,\,\eta_j)} = \tan^2\theta\sum_{m=1}^{N_\xi} B_{im}^{\xi}()_{mj} - 2\sec\theta\tan\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} A_{im}^{\xi}A_{jn}^{\eta}()_{mn} + \sec^2\theta\sum_{n=1}^{N_\eta} B_{jn}^{\eta}()_{in} \tag{15}$$

$$\begin{aligned}\left.\frac{\partial^3()}{\partial y^3}\right|_{(\xi_i,\,\eta_j)} = &-\tan^3\theta\sum_{m=1}^{N_\xi} C_{im}^{\xi}()_{mj} + 3\sec\theta\tan^2\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} B_{im}^{\xi}A_{jn}^{\eta}()_{mn} \\ &- 3\tan\theta\sec^2\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} A_{im}^{\xi}B_{jn}^{\eta}()_{mn} + \sec^3\theta\sum_{n=1}^{N_\eta} C_{jn}^{\eta}()_{in}\end{aligned} \tag{16}$$

Using the aforementioned degrees of freedom, the DQ-rules for the higher order derivatives (order≥3) of the transverse displacement w can be obtained as,

$$\left.\frac{\partial^3 w}{\partial x^3}\right|_{(\xi_i,\,\eta_j)} = \sum_{m=1}^{N_\xi} \bar{C}_{im}^{\xi} w_{mj} + A_{i1}^{\xi}K_{1j}^{\xi} + A_{iN_\xi}^{\xi}K_{N_\xi j}^{\xi} \tag{17}$$

$$\left.\frac{\partial^4 w}{\partial x^4}\right|_{(\xi_i,\,\eta_j)} = \sum_{m=1}^{N_\xi} \overline{D}^{\xi}_{im} w_{mj} + B^{\xi}_{i1} K^{\xi}_{1j} + B^{\xi}_{iN_\xi} K^{\xi}_{N_\xi j} \tag{18}$$

$$\left.\frac{\partial^4 w}{\partial y \partial x^3}\right|_{(\xi_i,\,\eta_j)} = -\tan\theta \sum_{m=1}^{N_\xi} \overline{D}^{\xi}_{im} w_{mj} + \sec\theta \sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} A^{\eta}_{jn} \overline{C}^{\xi}_{im} w_{mn} - \tan\theta (B^{\xi}_{i1} K^{\xi}_{1j} + B^{\xi}_{iN_\xi} K^{\xi}_{N_\xi j})$$
$$+ \sec\theta \sum_{n=1}^{N_\eta} A^{\eta}_{jn} (A^{\xi}_{i1} K^{\xi}_{1n} + A^{\xi}_{iN_\xi} K^{\xi}_{N_\xi n}) \tag{19}$$

$$\left.\frac{\partial^4 w}{\partial x^2 \partial y^2}\right|_{(\xi_i,\,\eta_j)} = \tan^2\theta \sum_{m=1}^{N_\xi} \overline{D}^{\xi}_{im} w_{mj} - \sec^2\theta \sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} (2\sin\theta A^{\eta}_{jn} \overline{C}^{\xi}_{im} w_{mn} - B^{\xi}_{im} B^{\eta}_{jn} w_{mn})$$
$$+ \tan^2\theta (B^{\xi}_{i1} K^{\xi}_{1j} + B^{\xi}_{iN_\xi} K^{\xi}_{N_\xi j}) - 2\sec\theta\tan\theta \sum_{n=1}^{N_\eta} A^{\eta}_{jn} (A^{\xi}_{i1} K^{\xi}_{1n} + A^{\xi}_{iN_\xi} K^{\xi}_{N_\xi n}) \tag{20}$$

$$\left.\frac{\partial^4 w}{\partial x \partial y^3}\right|_{(\xi_i,\,\eta_j)} = -\tan^3\theta \sum_{m=1}^{N_\xi} \overline{D}^{\xi}_{im} w_{mj} + \sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} [3\sec^2\theta\tan\theta(\sin\theta A^{\eta}_{jn} \overline{C}^{\xi}_{im} - B^{\xi}_{im} B^{\eta}_{jn})$$
$$+ \sec^3\theta A^{\xi}_{im} \overline{C}^{\eta}_{jn}] w_{mn} - \tan^3\theta \left[B^{\xi}_{i1} K^{\xi}_{1j} + B^{\xi}_{iN_\xi} K^{\xi}_{N_\xi j} - 3\csc\theta \sum_{n=1}^{N_\eta} A^{\eta}_{jn} (A^{\xi}_{i1} K^{\xi}_{1n} + A^{\xi}_{iN_\xi} K^{\xi}_{N_\xi n}) \right]$$
$$+ \sec^3\theta \sum_{m=1}^{N_\xi} A^{\xi}_{im} (A^{\eta}_{j1} K^{\eta}_{m1} + A^{\eta}_{jN_\eta} K^{\eta}_{mN_\eta}) \tag{21}$$

$$\left.\frac{\partial^3 w}{\partial y^3}\right|_{(\xi_i,\,\eta_j)} = -\tan^3\theta \sum_{m=1}^{N_\xi} \overline{C}^{\xi}_{im} w_{mj} + 3\sec^2\theta\tan\theta \sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} (\sin\theta A^{\eta}_{jn} B^{\xi}_{im} - A^{\xi}_{im} B^{\eta}_{jn}) w_{mn}$$
$$+ \sec^3\theta \sum_{n=1}^{N_\eta} \overline{C}^{\eta}_{jn} w_{in} - \tan^3\theta (A^{\xi}_{i1} K^{\xi}_{1j} + A^{\xi}_{iN_\xi} K^{\xi}_{N_\xi j}) + \sec^3\theta (A^{\eta}_{j1} K^{\eta}_{i1} + A^{\eta}_{jN_\eta} K^{\eta}_{iN_\eta}) \tag{22}$$

$$\left.\frac{\partial^4 w}{\partial y^4}\right|_{(\xi_i,\,\eta_j)} = \tan^4\theta \sum_{m=1}^{N_\xi} \overline{D}^{\xi}_{im} w_{mj} + \sec^4\theta \sum_{n=1}^{N_\eta} \overline{D}^{\eta}_{jn} w_{in} - 2\tan\theta\sec^3\theta \sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} (2A^{\xi}_{im} \overline{C}^{\eta}_{jn}$$
$$+ 2\sin^2\theta \overline{C}^{\xi}_{im} A^{\eta}_{jn} - 3\sin\theta B^{\xi}_{im} B^{\eta}_{jn}) w_{mn} + \tan^4\theta (B^{\xi}_{i1} K^{\xi}_{1j} + B^{\xi}_{iN_\xi} K^{\xi}_{N_\xi j})$$
$$- 4\sec\theta\tan^3\theta \sum_{n=1}^{N_\eta} A^{\eta}_{jn} (A^{\xi}_{i1} K^{\xi}_{1n} + A^{\xi}_{iN_\xi} K^{\xi}_{N_\xi n}) + \sec^4\theta (B^{\eta}_{j1} K^{\eta}_{i1} + B^{\eta}_{jN_\eta} K^{\eta}_{iN_\eta})$$
$$- 4\tan\theta\sec^3\theta \sum_{m=1}^{N_\xi} A^{\xi}_{im} (A^{\eta}_{j1} K^{\eta}_{m1} + A^{\eta}_{jN_\eta} K^{\eta}_{mN_\eta}) \tag{23}$$

where

$$\overline{C}_{ij}^{\xi} = \sum_{m=2}^{N_\xi - 1} A_{im}^{\xi} B_{mj}^{\xi}, \quad \overline{D}_{ij}^{\xi} = \sum_{m=2}^{N_\xi - 1} B_{im}^{\xi} B_{mj}^{\xi}, \quad \overline{C}_{ij}^{\eta} = \sum_{n=2}^{N_\eta - 1} A_{in}^{\eta} B_{nj}^{\eta}, \quad \overline{D}_{ij}^{\eta} = \sum_{n=2}^{N_\eta - 1} B_{in}^{\eta} B_{nj}^{\eta}$$

$$i = 1, \ldots, N_\xi \quad \text{and} \quad j = 1, \ldots, N_\eta$$

Using the DQ-transformation rules (12)-(23), the DQ-analogs of the governing Eqs. (4)-(6) at each grid point (ξ_i, η_j) with $i = 2, \ldots, N_\xi - 1$ and $j = 2, \ldots, N_\eta - 1$ become,

Eq. (4):

$$(A_{11} + A_{66}\tan^2\theta) \sum_{m=1}^{N_\xi} B_{im}^{\xi} u_{mj} - \sec^2\theta \left(2A_{66}\sin\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} u_{mn} - A_{66} \sum_{n=1}^{N_\eta} B_{jn}^{\eta} u_{in} \right)$$

$$+ (A_{12} + A_{66}) \left(-\tan\theta \sum_{m=1}^{N_x} B_{im}^{\xi} v_{mj} + \sec\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} v_{mn} \right) + \left(\sum_{m=1}^{N_\xi} A_{im}^{\xi} w_{mj} \right)$$

$$\left[(A_{11} + A_{66}\tan^2\theta) \sum_{m=1}^{N_\xi} B_{im}^{\xi} w_{mj} - \sec^2\theta \left(2A_{66}\sin\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} w_{mn} - A_{66} \sum_{n=1}^{N_\eta} B_{jn}^{\eta} w_{in} \right) \right]$$

$$+ \sec^2\theta (A_{12} + A_{66}) \left(-\sin\theta \sum_{m=1}^{N_\xi} A_{im}^{\xi} w_{mj} + \sum_{n=1}^{N_\eta} A_{jn}^{\eta} w_{in} \right) \left(-\sin\theta \sum_{m=1}^{N_\xi} B_{im}^{\xi} w_{mj} + \right.$$

$$\left. \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} w_{mn} \right) - B_{11} \left(\sum_{m=1}^{N_\xi} \overline{C}_{im}^{\xi} w_{mj} + A_{i1}^{\xi} K_{1j}^{\xi} + A_{iN_\xi}^{\xi} K_{N_\xi j}^{\xi} \right) = 0 \tag{24}$$

Eq. (5):

$$(A_{12} + A_{66}) \left(-\tan\theta \sum_{m=1}^{N_x} B_{im}^{\xi} u_{mj} + \sec\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} u_{mn} \right) + A_{66} \sum_{m=1}^{N_\xi} B_{im}^{\xi} v_{mj} +$$

$$A_{22} \left(\tan^2\theta \sum_{n=1}^{N_\xi} B_{jn}^{\xi} v_{in} - 2\tan\theta\sec\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} v_{mn} + \sec^2\theta \sum_{n=1}^{N_\eta} B_{jn}^{\eta} v_{in} \right) +$$

$$\left(-\tan\theta \sum_{m=1}^{N_\xi} A_{im}^{\xi} w_{mj} + \sec\theta \sum_{n=1}^{N_\eta} A_{jn}^{\eta} w_{in} \right) \left[(A_{66} + A_{22}\tan^2\theta) \sum_{m=1}^{N_\xi} B_{im}^{\xi} w_{mj} - \right.$$

$$\left. 2A_{22}\sec\theta\tan\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} w_{mn} + A_{22}\sec^2\theta \sum_{n=1}^{N_\eta} B_{jn}^{\eta} w_{in} \right] +$$

$$(A_{12} + A_{66}) \left(\sum_{m=1}^{N_\xi} A_{im}^{\xi} w_{mj} \right) \left(-\tan\theta \sum_{m=1}^{N_x} B_{im}^{\xi} w_{mj} + \sec\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} A_{im}^{\xi} A_{jn}^{\eta} w_{mn} \right) +$$

$$B_{22}\tan^3\theta \left[\sum_{m=1}^{N_\xi} \overline{C}_{im}^{\xi} w_{mj} - 3\cos\theta\csc^2\theta \sum_{m=1}^{N_\xi} \sum_{n=1}^{N_\eta} (\tan\theta A_{jn}^{\eta} B_{im}^{\xi} + \sec\theta A_{im}^{\xi} B_{jn}^{\eta}) w_{mn} \right.$$

$$-\csc^3\theta\sum_{n=1}^{N_\eta}\bar{C}_{jn}^{\eta}w_{in}+A_{i1}^{\xi}K_{1j}^{\xi}+A_{iN_\xi}^{\xi}K_{N_\xi j}^{\xi}-\csc^3\theta(A_{j1}^{\eta}K_{i1}^{\eta}+A_{iN_\eta}^{\eta}K_{iN_\eta}^{\eta}) = 0 \tag{25}$$

Eq. (6):

$$B_{11}\sum_{m=1}^{N_\xi}C_{im}^{\xi}u_{mj}-B_{22}\Bigg[\tan^3\theta\sum_{m=1}^{N_\xi}C_{im}^{\xi}v_{mj}+3\sec\theta\tan\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}-(\tan\theta B_{im}^{\xi}A_{jn}^{\eta}+$$

$$\sec\theta A_{im}^{\xi}B_{jn}^{\eta})v_{mn}-\sec^3\theta\sum_{n=1}^{N_\eta}C_{jn}^{\eta}v_{in}\Bigg]+B_{11}\Bigg[\left(\sum_{m=1}^{N_\xi}B_{im}^{\xi}w_{mj}\right)^2+\left(\sum_{m=1}^{N_\xi}A_{im}^{\xi}w_{mj}\right)\left(\sum_{m=1}^{N_\xi}\bar{C}_{im}^{\xi}w_{mj}\right.$$

$$\left.+A_{i1}^{\xi}K_{1j}^{\xi}+A_{iN_\xi}^{\xi}K_{N_\xi j}^{\xi})\right]+B_{22}\Bigg\{\Bigg(\tan^2\theta\sum_{m=1}^{N_\xi}B_{im}^{\xi}w_{mj}-2\sec\theta\tan\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}w_{mn}$$

$$+\sec^2\theta\sum_{n=1}^{N_\eta}B_{jn}^{\eta}w_{in}\Bigg)^2-\left(\tan\theta\sum_{m=1}^{N_\xi}A_{im}^{\xi}w_{mj}-\sec\theta\sum_{n=1}^{N_\eta}A_{jn}^{\eta}w_{mj}\right)\Bigg[-\tan^3\theta\sum_{m=1}^{N_\xi}\bar{C}_{im}^{\xi}w_{mj}$$

$$+(3\sec\theta\tan^2\theta-3\sec^2\theta\tan\theta)\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}(A_{jn}^{\eta}B_{im}^{\xi}+A_{im}^{\xi}B_{jn}^{\eta})w_{mn}+\sec^3\theta\sum_{n=1}^{N_\eta}\bar{C}_{jn}^{\eta}w_{in}$$

$$-\tan^3\theta(A_{i1}^{\xi}K_{1j}^{\xi}+A_{iN_\xi}^{\xi}K_{N_\xi j}^{\xi})+\sec^3\theta(A_{j1}^{\eta}K_{i1}^{\eta}+A_{jN_\eta}^{\eta}K_{iN_\eta}^{\eta})\Bigg]\Bigg\}-a_1\sum_{m=1}^{N_\xi}\bar{D}_{im}^{\xi}w_{mj}-$$

$$a_1(B_{i1}^{\xi}K_{1j}^{\xi}+B_{iN_\xi}^{\xi}K_{N_\xi j}^{\xi})-\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}(a_2A_{jn}^{\eta}\bar{C}_{im}^{\xi}+a_3B_{im}^{\xi}B_{jn}^{\eta}-4\tan\theta\sec^3\theta D_{22}A_{im}^{\xi}\bar{C}_{jn}^{\eta})w_{mn}$$

$$-D_{22}\sec^4\theta\Bigg[\sum_{n=1}^{N_\eta}\bar{D}_{jn}^{\eta}w_{in}+B_{j1}^{\eta}K_{i1}^{\eta}+B_{jN_\eta}^{\eta}K_{iN_\eta}^{\eta}-4\sin^3\theta\sum_{m=1}^{N_\xi}A_{im}^{\xi}(A_{j1}^{\eta}K_{m1}^{\eta}$$

$$+A_{jN_\eta}^{\eta}K_{mN_\eta}^{\eta})\Bigg]+a_4\sum_{n=1}^{N_\eta}A_{jn}^{\eta}(A_{i1}^{\xi}K_{1n}^{\xi}+A_{iN_\xi}^{\xi}K_{N_\xi n}^{\xi})+(N_{xx})_{ij}\sum_{m=1}^{N_\xi}B_{im}^{\xi}w_{mj}+$$

$$2(N_{xy})_{ij}\left(-\tan\theta\sum_{m=1}^{N_\xi}B_{im}^{\xi}w_{mj}+\sec\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}w_{mn}\right)+(N_{yy})_{ij}\Bigg(\tan^2\theta\sum_{m=1}^{N_\xi}B_{im}^{\xi}w_{mj}$$

$$-2\sec\theta\tan\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}w_{mn}+\sec^2\theta\sum_{n=1}^{N_\eta}B_{jn}^{\eta}w_{in}\Bigg) = q_{ij} \tag{26}$$

where

$$(N_{xx})_{ij} = A_{11}\left[\sum_{m=1}^{N_\xi}A_{im}^{\xi}u_{mj}+\frac{1}{2}\left(\sum_{m=1}^{N_\xi}A_{im}^{\xi}w_{mj}\right)^2\right]+A_{12}\Bigg[-\tan\theta\sum_{m=1}^{N_\xi}A_{im}^{\xi}v_{mj}+$$

$$\sec\theta\sum_{n=1}^{N_\eta}A_{jn}^{\eta}v_{in}+\frac{1}{2}\left(-\tan\theta\sum_{m=1}^{N_\xi}A_{im}^{\xi}w_{mj}+\sec\theta\sum_{n=1}^{N_\eta}A_{jn}^{\eta}w_{in}\right)^2\Bigg]-B_{11}\sum_{m=1}^{N_\xi}B_{im}^{\xi}w_{mj},$$

$$(N_{yy})_{ij} = A_{12}\left[\sum_{m=1}^{N_\xi} A_{im}^{\xi}u_{mj} + \frac{1}{2}\left(\sum_{m=1}^{N_\xi} A_{im}^{\xi}w_{mj}\right)^2\right] + A_{22}\left[-\tan\theta\sum_{m=1}^{N_\xi} A_{im}^{\xi}v_{mj}\right.$$
$$\left. + \sec\theta\sum_{n=1}^{N_\eta} A_{jn}^{\eta}v_{in} + \frac{1}{2}\left(-\tan\theta\sum_{m=1}^{N_\xi} A_{im}^{\xi}w_{mj} + \sec\theta\sum_{n=1}^{N_\eta} A_{jn}^{\eta}w_{in}\right)^2\right] -$$
$$B_{22}\sec^2\theta\left(\sin^2\theta\sum_{m=1}^{N_\xi} B_{im}^{\xi}w_{mj} - 2\sin\theta\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} A_{im}^{\xi}A_{jn}^{\eta}w_{mn} + \sum_{n=1}^{N_\eta} B_{jn}^{\eta}w_{in}\right),$$

$$(N_{xy})_{ij} = A_{66}\left[-\tan\theta\sum_{m=1}^{N_\xi} A_{im}^{\xi}u_{mj} + \sec\theta\sum_{n=1}^{N_\eta} A_{jn}^{\eta}u_{in} + \sum_{m=1}^{N_\xi} A_{im}^{\xi}v_{mj} +\right.$$
$$\left.\left(-\tan\theta\sum_{m=1}^{N_\xi} A_{im}^{\xi}w_{mj} + \sec\theta\sum_{n=1}^{N_\eta} A_{jn}^{\eta}w_{in}\right)\left(\sum_{m=1}^{N_\xi} A_{im}^{\xi}w_{mj}\right)\right]$$

and

$$a_1 = D_{11} + 2(D_{12} + 2D_{66})\tan^2\theta + D_{22}\tan^4\theta$$
$$a_2 = 4(D_{12} + 2D_{66})\sec\theta\tan\theta - 4\sec\theta\tan^3\theta D_{22}$$
$$a_3 = 2(D_{12} + 2D_{66})\sec^2\theta + 6\sec^2\theta\tan^2\theta D_{22}$$
$$a_4 = 4D_{22}\sec\theta\tan^3\theta + 4(D_{12} + 2D_{66})\sec\theta\tan\theta$$

In a similar manner, the DQ analogs of different types of boundary conditions can be obtained. For the sake of brevity, the DQ analogs of the simply supported boundary condition along an arbitrary edge of the plate are obtained. The physical conditions for immovable simply supported condition are,

$$u_n = 0, \quad u_s = 0, \quad w = 0, \quad M_{nn} = 0 \tag{27}$$

The DQ analogs of the first three geometrical boundary conditions appeared in Eq. (27) are as following,

$$n_x u_{ij} + n_y v_{ij} = 0, \quad -n_y u_{ij} + n_x v_{ij} = 0, \quad w_{ij} = 0 \tag{28}$$

The DQ analogs of the natural boundary condition in Eq. (27) become

$$-(b_1 + b_2\tan^2\theta - b_3\tan^2\theta)K_{ij}^{\xi} - b_2\sum_{n=1}^{N_\eta} B_{jn}^{\eta}w_{in} - b_3\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta} A_{im}^{\xi}A_{jn}^{\eta}w_{mn} + n_x^2 B_{11}\left[\sum_{m=1}^{N_\xi} A_{im}^{\xi}u_{mj}\right.$$
$$\left.\frac{1}{2}\left(\sum_{m=1}^{N_\xi} A_{im}^{\xi}w_{mj}\right)^2\right] + n_y^2 B_{22}\left[\sum_{n=1}^{N_\eta} A_{jn}^{\eta}v_{in} + \frac{1}{2}\left(\sum_{n=1}^{N_\eta} A_{jn}^{\eta}w_{in}\right)^2\right] = 0 \tag{29}$$

where,

$$b_1 = n_x^2 D_{11} + 2n_x n_y D_{16} + n_y^2 D_{12}, \quad b_2 = n_x^2 D_{12} + 2n_x n_y D_{26} + n_y^2 D_{22}$$
$$b_3 = 2(n_x^2 D_{16} + 2n_x n_y D_{66} + n_y^2 D_{26})$$

Rearranging the discretized equilibrium equations and boundary conditions, one obtains a nonlinear system of algebraic equations which in the vector form can be written as,

$$\boldsymbol{R} = \boldsymbol{g} - \boldsymbol{q} = \boldsymbol{0} \tag{30}$$

The components of the vectors $\boldsymbol{g}$ and $\boldsymbol{q}$ are the left and right parts of the discretized equilibrium equations, i.e., Eqs. (24)-(26) and boundary conditions, for example, (28)-(29), respectively. To solve the nonlinear Eq. (30) in a systematic manner, the vector of degrees of freedom or generalized displacement vector is defined as,

$$\boldsymbol{U} = \begin{Bmatrix} \{u\} \\ \{v\} \\ \{w\} \\ \{K^{\xi}\} \\ \{K^{\eta}\} \end{Bmatrix}, \quad \{u\} = \begin{Bmatrix} u_{11} \\ u_{12} \\ \vdots \\ u_{N_\xi N_\eta} \end{Bmatrix}, \quad \{v\} = \begin{Bmatrix} v_{11} \\ v_{12} \\ \vdots \\ v_{N_x N_y} \end{Bmatrix}, \quad \{w\} = \begin{Bmatrix} w_{11} \\ w_{12} \\ \vdots \\ w_{N_\xi N_\eta} \end{Bmatrix}, \quad \{K^{\xi}\} = \begin{Bmatrix} K_{11}^{\xi} \\ \vdots \\ K_{1N_\eta}^{\xi} \\ K_{N_\xi 1}^{\xi} \\ \vdots \\ K_{N_\xi N_\eta}^{\xi} \end{Bmatrix}$$

$$\{K^{\eta}\} = [K_{11}^{\eta} \ \dots \ K_{N_\xi 1}^{\eta} \ K_{1N_\eta}^{\eta} \ \dots \ K_{N_\xi N_\eta}^{\eta}]^T \tag{31}$$

An incremental-iterative method should be used to solve the resulting nonlinear system of equations. In the present analysis, the solution algorithms are based on the Newton-Raphson method. For this purpose the load vector $\boldsymbol{q}$ is applied incrementally and for a given value of the load, the Newton-Raphson iterations are to be continued until the required accuracy is reached. A brief review of the solution algorithm is as follows. Suppose that at iteration r for the load step $\Delta \boldsymbol{f}_{i+1}$, the vector $\boldsymbol{U}_{i+1}^{r}$ gives a residual forces $\boldsymbol{R}(\boldsymbol{U}_{i+1}^{r}) \neq \boldsymbol{0}$ then an improved value of $\boldsymbol{U}_{i+1}$ is obtained by equating to zero the linearized Taylor's series expansion of $\boldsymbol{R}(\boldsymbol{U}_{i+1}^{r+1})$ in the neighborhood of $\boldsymbol{U}_{i+1}^{r}$ as

$$\boldsymbol{R}(\boldsymbol{U}_{i+1}^{r+1}) = \boldsymbol{g}(\boldsymbol{U}_{i+1}^{r}) + \boldsymbol{K}_T^{r}(\boldsymbol{U}_{i+1}^{r+1} - \boldsymbol{U}_{i+1}^{r}) - \boldsymbol{f}_{i+1} = \boldsymbol{0} \tag{32}$$

or

$$\boldsymbol{U}_{i+1}^{r+1} = \boldsymbol{U}_{i+1}^{r} + (\boldsymbol{K}_T^{r})^{-1}[\boldsymbol{f}_{i+1} - \boldsymbol{g}(\boldsymbol{U}_{i+1}^{r})] \tag{33}$$

where $\boldsymbol{K}_T^{r}$ is the tangent stiffness matrix evaluated at $\boldsymbol{U}_{i+1}^{r}$ and given by

$\vdots$

$$\boldsymbol{K}_T^r = \left.\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{U}}\right|_{\boldsymbol{U} = \boldsymbol{U}_{i+1}^r} = \begin{bmatrix} \frac{\partial g_1}{\partial U_1} & \frac{\partial g_1}{\partial U_2} & \cdots & \frac{\partial g_1}{\partial U_N} \\ \frac{\partial g_2}{\partial U_1} & \frac{\partial g_2}{\partial U_2} & \cdots & \frac{\partial g_2}{\partial U_N} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial g_N}{\partial U_1} & \frac{\partial g_N}{\partial U_2} & \cdots & \frac{\partial g_N}{\partial U_N} \end{bmatrix}_{\boldsymbol{U} = \boldsymbol{U}_{i+1}^r}$$

$N = 3N_\xi \times 3N_\eta + 2(N_\xi + N_\eta)$ is the total number of degrees of freedom and g_i $(i = 1, 2, \ldots, N)$ are the components of the vector $\boldsymbol{g}$. At each iteration the convergence is checked using a total residual criteria (Pica *et al.* 1980), i.e.,

$$\varepsilon = \frac{(\boldsymbol{R}^T\boldsymbol{R})^{1/2}}{(\boldsymbol{q}^T\boldsymbol{q})^{1/2}} < \varepsilon_o \tag{34}$$

For each load step, the strain components, the bending moments and the in-plane stress resultants can be obtained by using Eqs. (2), (3) and (12)-(15).

4. Numerical results

In this section, the convergence trends and accuracy of the presented algorithm are investigated through a few examples of composite skew plates. Results for large deformation analysis of skew composite plates are not available in open literature except for thin orthotropic skew plate. Also, exact or closed form solutions of nonlinear analysis of skew plates are not available yet. Therefore to validate the formulations and computer programs, the results are compared with numerical results of other methods. The governing equations of skew composite plates based on the first order shear deformation theory (FSDT) have the second order partial differential equations and hence no difficulty with applying the boundary conditions using DQM (Malekzadeh and Karami 2005). Therefore as another benchmark to validate the presented DQ methodology, the results are compared with those obtained for thin skew composite plates using a differential quadrature first shear deformation theory (DQ-FSDT) developed by the author. For this purpose a value of $h/a = 0.001$ is used in the computations.

In the entire problems considered here, the individual layers are taken to be of equal thickness and the shear correction factors are taken to be 5/6 in DQ-FSDT computations. In all solved examples ten incremental load steps are sufficient to obtain a converged solution, however to find accurate intermediate solutions, in some cases more than ten incremental load steps are used. Also, in all examples the convergence tolerance is taken to be $\varepsilon_o = 10^{-3}$. The edges of the plate are numbered from 1 to 4 as shown in Fig. 1. The first side is set at $\xi = 0$ and the other sides are numbered consecutively in the counterclockwise direction.

As a first example, to show the convergence and accuracy of the present method for composite skew plates, the large deformation analysis of orthotropic clamped skew plate considered by Alwar and Ramachandra Rao (1973) are investigated here. The material properties used by them (Alwar and Ramachandra Rao 1973) are, $E_{11}/E_{22} = 4.0$, $G_{12}/E_{22}(1 - \upsilon_{12}\upsilon_{21}) = 0.35$, $\upsilon_{12} = 0.35$. The

Table 1 Convergence of the center deflection ($W_c \times 10^3$) for orthotropic clamped skew plates with Q = 1 ($\theta = 30^{\mathrm{o}}$, $b/a = 1$)

$N_\xi = N_\eta$						
7	9	13	19	FSDT	Iyengar and Srinivasan (1968)	Alwar and Ramachandra Rao (1973)
0.4301	0.4281	0.4283	0.4283	0.4283	0.4283	0.43961

Table 2 Convergence of the results for orthotropic clamped skew plates with Q = 3200 (a/b = 1)

θ		$N_\xi = N_\eta$	W_c	$\overline{N}_{xx}$	$\overline{M}_{xx}$	$\overline{N}_{yy}$	$\overline{M}_{yy}$
30°	TPT-DQM	9	0.9198	116.0	56.13	47.36	17.87
		13	0.9195	109.1	56.69	40.78	18.32
		15	0.9195	109.1	56.71	40.96	18.32
		17	0.9195	109.1	56.72	40.97	18.32
		19	0.9195	109.1	56.72	40.97	18.32
	FSDT-DQM	19	0.9195	109.1	56.73	40.96	18.32
	Alwar and Ramachandra Rao (1973)[a]		0.94	109.0	56.8	40.9	18.2
45°	TPT-DQM	9	0.6598	63.22	47.19	35.29	21.92
		13	0.6590	62.20	46.71	30.70	22.39
		17	0.6590	62.27	46.71	30.73	22.43
		19	0.6591	62.28	46.71	30.73	22.43
	FSDT-DQM	19	0.6591	62.27	46.72	30.73	22.44
	Alwar and Ramachandra Rao (1973)[a]		0.68	62.5	47.5	31.0	22.7

[a]Data reads from graph.

convergence and accuracy of the center deflection for a skew plate with $\theta = 30^{\mathrm{o}}$ and a load parameter $Q = 1 (= qa^4/D_{22}h)$, for which the nonlinear effects are negligible, is considered. The results are compared with those of Alwar and Ramachandra Rao (1973), Iyengar and Srinivasan (1968) in Table 1. It seems that the dynamic relaxation method (DRM) used by Alwar and Ramachandra Rao (1973) gives an upper bond for the deflections. Now consider skew plates with the same material properties but with large load parameter to cause significant impact due to geometric nonlinearity. The results for the deflection, resultant bending moments about x and y axis and resultant normal membrane forces in the x and y directions at the center of the plates are presented in Table 2. This table also includes the results obtained by the DRM (Alwar and Ramachandra Rao 1973). The results are prepared for two different skew angles and for different number of grid points. Again one can see that the center deflections obtained by DQM are slightly less than those of the DRM, but still close agreement exists between the results of two methods for different parameters. From this table it is obvious that excellent agreement exists between DQ-TPT and DQ-FSDT results for central deflection and resultant bending moment and forces.

The convergences of the method for symmetric cross play laminated skew plates with two different skew angles are exhibited in Table 3. Skew plates under different load parameter and with

Table 3 Convergence of the results for [0/90/90/0] laminated CCSS skew plates ($Q = 10000$, $b/a = 1$)

θ	Method	$N_\xi = N_\eta$	W_c	$\overline{N}_{xx}$	$\overline{M}_{xx}$	$\overline{N}_{yy}$	$\overline{M}_{yy}$
30°	TPT-DQM	9	1.5490	279.2	90.23	365.8	17.77
		13	1.5484	268.2	91.68	355.9	18.11
		19	1.5476	267.9	91.91	353.4	18.15
		23	1.5473	267.8	91.95	353.3	18.15
	FSDT-DQM	23	1.5469	267.8	92.05	353.1	18.16
60°	TPT-DQM	9	0.6910	57.76	54.52	222.3	32.38
		13	0.6899	57.73	53.57	198.1	33.14
		19	0.6901	57.27	53.85	193.9	33.56
		23	0.6901	57.08	54.03	193.7	33.67
	FSDT-DQM	23	0.6901	56.67	55.31	192.7	34.52

Table 4 Convergence of the results for [0/90] CCCS cross ply laminated skew plates ($Q = 5000$, $\theta = 45°$)

	$N_\xi = N_\eta$ (TPT-DQ)				
	11	15	17	19	FSDT-DQ
W_c	1.4726	1.4670	1.4668	1.4668	1.4682
$\overline{N}_{xx}$	67.08	68.16	68.12	68.11	68.10
$\overline{M}_{xx}$	−10.74	−11.29	−11.31	−11.32	−11.23

two different skew angles are investigated. The convergence behaviors of the presented DQM for large deformation analysis of skew composite plates with antisymmetric lay up are investigated as another example. Antisymmetric cross-ply laminated skew plates are considered and the results are presented in Table 4. The material properties are, $E_{11}/E_{22} = 40$, $G_{12}/E_{22} = G_{13}/E_{22} = 0.5$, $G_{23}/E_{22} = 0.2$, $\upsilon_{12} = 0.25$. In these examples, fast convergence of the method and excellent agreement of the results with those of FSDT-DQ are evident.

In the following examples, the effects of skew angle, aspect ratio and the number of layers on the nonlinear behavior of the laminated skew plates with different boundary conditions are investigated. The material properties used are the same as those of the last solved example. Thirty grid points in each direction is used.

The effects of skew angle on the behavior of the non-dimensional center deflection, bending moment ($\overline{M}_{xx}$) and in-plane normal force ($\overline{N}_{xx}$) of symmetric composite skew plates are presented in Figs. 2-4, respectively. The results for the same parameters for fully clamped antisymmetric composite skew plates are shown in Figs. 5-7. From these figures one can see that in all cases the skew angle has significant effects on the flexural behaviors of skew plates and increasing the load parameter, the geometric nonlinearity effects are increased significantly.

The effects of different aspect ratio (b/a) on the center deflection of the antisymmetric cross-ply skew plates with a relatively acute corners are shown in Fig. 8. It is found from this figure that increasing the aspect ratio (b/a), the stiffness of the plate is decreased and the degree of hardening is increased.

Comparisons between the DQ-TPT and DQ-FSDT results for different number of layer and laminate lay up are shown in Fig. 9.

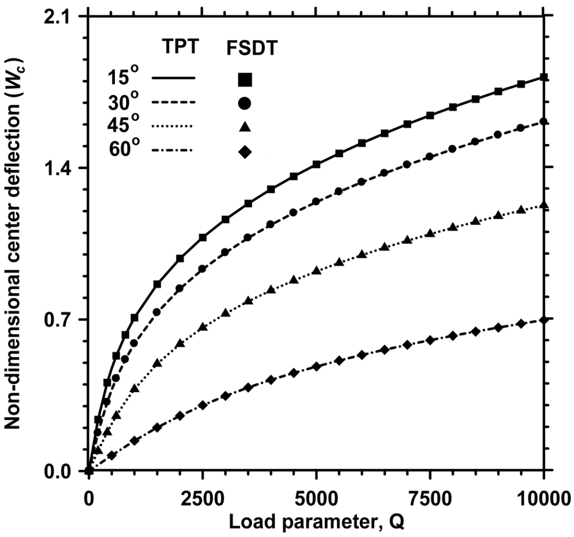

Fig. 2 The effects of skew angle on non-dimensional center deflection of [0/90/90/0] laminated SCSC skew plates

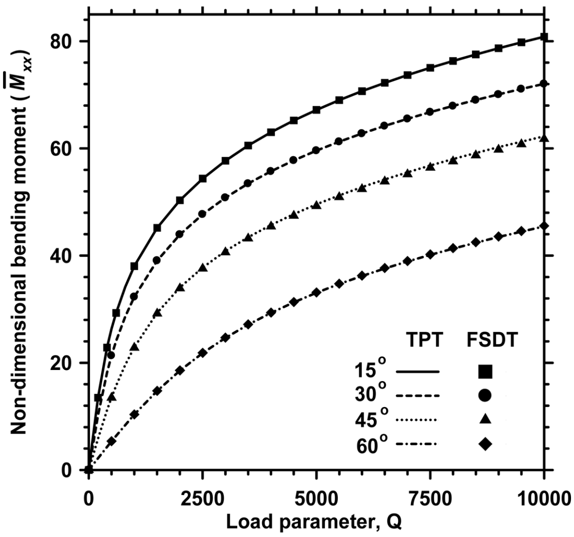

Fig. 3 The effects of skew angle on non-dimensional bending moment ($\overline{M}_{xx}$) of [0/90/90/0] laminated SCSC skew plates

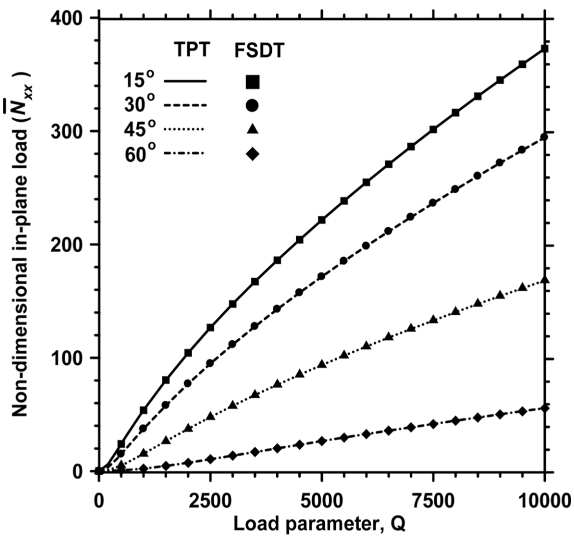

Fig. 4 The effects of skew angle on non-dimensional in-plane force ($\overline{N}_{xx}$) of [0/90/90/0] laminated SCSC skew plates

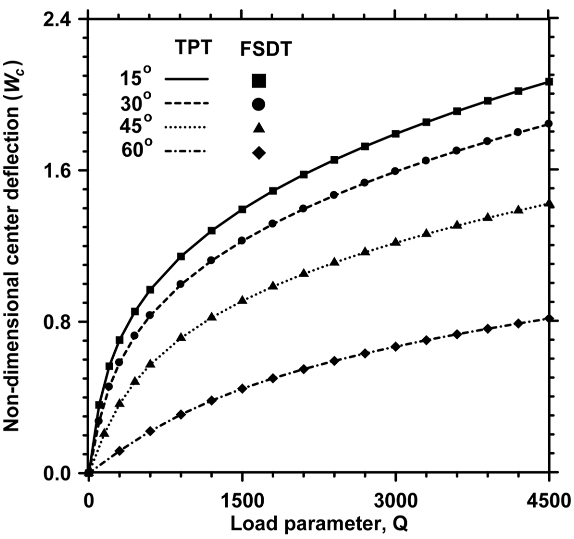

Fig. 5 The effects of skew angle on non-dimensional center deflection of [0/90] antisymmetric fully clamped skew plates

From these solved examples, it is evident that an excellent agreement exists between the non-dimensional center deflection and non-dimensional resultant stresses, i.e. bending moment and in-plane forces, of DQ-TPT and DQ-FSDT for different skew angles, aspect ratio, and boundary conditions, which is considerable for skew plates with acute corners, i.e. the case $\theta = 60^{\circ}$.

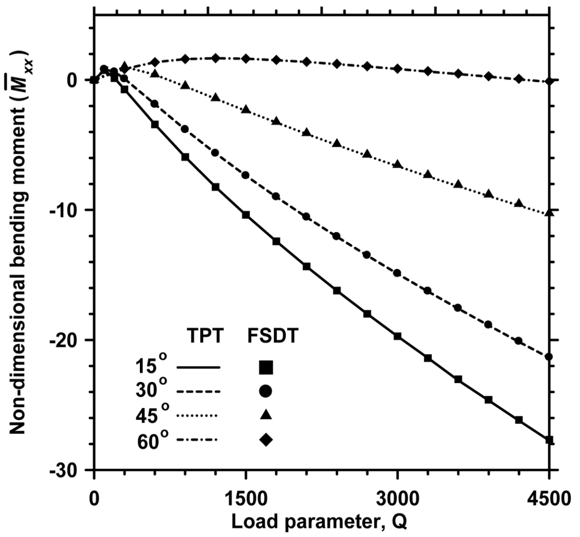

Fig. 6 The effects of skew angle on non-dimensional bending moment ($\overline{M}_{xx}$) of [0/90] laminated fully clamped skew plates

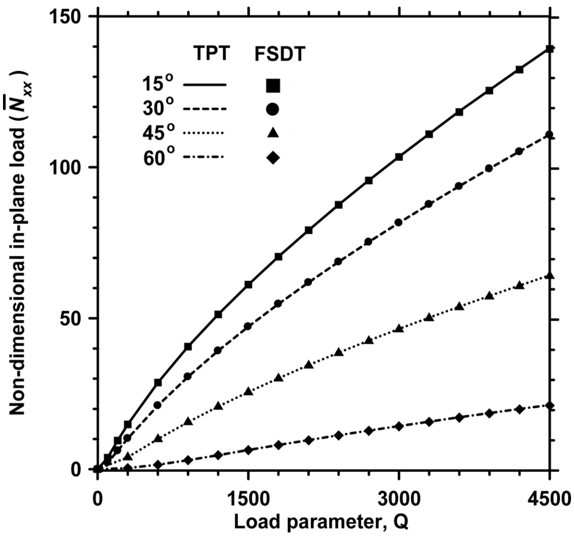

Fig. 7 The effects of skew angle on non-dimensional in-plane force ($\overline{N}_{xx}$) of [0/90] laminated fully clamped skew plates

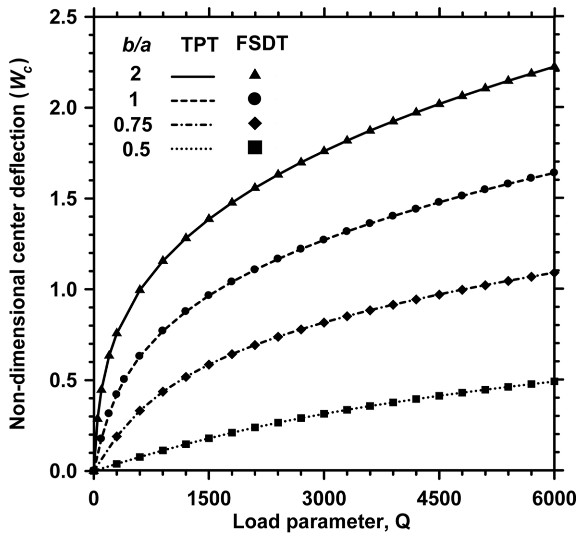

Fig. 8 The effects of aspect ratio on non-dimensional center deflection of [0/90] SCSC skew plates ($\theta = 45^{o}$)

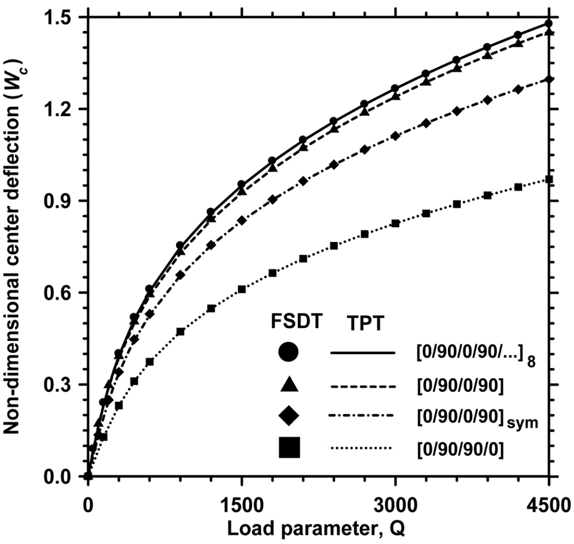

Fig. 9 The effects of number of layer and laminate lay up on non-dimensional center deflection SSSS skew plates ($\theta = 45^{o}$)

5. Conclusions

This paper explores the utility of a DQ formulation developed for the large deformation analysis of skew composite plates based on the geometrically nonlinear thin plate theory. The convergence behaviors of the method were shown for symmetric as well as antisymmetric laminated skew plates with different boundary conditions and skew angle. Good convergence is presented. The numerical results are in good agreements with those obtained by other numerical methods reported for the isotropic and orthotropic skew plates by other researchers even when only a small number of grid points are used. Comparisons between the present DQ-TPT and a DQ approach based on FSDT were performed and it was shown that the results for deflections and resultant stresses are in excellent agreements.

It can be concluded that the presented method is a convenient and efficient method for obtaining deflections as well as resultant stresses in nonlinear analysis of composite skew plates.

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Appendix A. DQ weighting coefficients

The basic idea of the differential quadrature method is that the derivative of a function, with respect to a space variable at a given sampling point, is approximated as a weighted linear sum of the sampling points in the domain of that variable. In order to illustrate the DQ approximation, consider a function $f(\xi, \eta)$ having its field on a rectangular domain $0 \le \xi \le a$ and $0 \le \eta \le b$. Let, in the given domain, the function values be known or desired on a grid of sampling points. According to DQ method, the *r*th derivative of a function $f(\xi, \eta)$ can be approximated as

$$\left.\frac{\partial^r f(\xi, \eta)}{\partial \xi^r}\right|_{(\xi, \eta) = (\xi_i, \eta_j)} = \sum_{m=1}^{N_\xi} A_{im}^{\xi(r)} f(\xi_m, \eta_j) = \sum_{m=1}^{N_\xi} A_{ij}^{\xi(r)} f_{mj} \tag{A.1}$$

$$\text{for } i = 1, 2, \ldots, N_\xi \text{ and } r = 1, 2, \ldots, N_\xi - 1$$

From this equation one can deduce that the important components of DQ approximations are weighting coefficients and the choice of sampling points. In order to determine the weighting coefficients a set of test functions should be used in Eq. (A.1). For polynomial basis functions DQ, a set of Lagrange polynomials are employed as the test functions. The weighting coefficients for the first-order derivatives in ξ-direction are thus determined as Bert and Malik (1996)

$$A_{ij}^{\xi} = \frac{1}{a}\begin{cases} \dfrac{M(\xi_i)}{(\xi_i - \xi_j)M(\xi_j)} & \text{for} \quad i \ne j \\ -\sum\limits_{\substack{i=j \\ i \ne j}}^{N_\xi} A_{ij}^{\xi} & \text{for} \quad i = j \end{cases} ; \quad i, j = 1, 2, \ldots, N_\xi \tag{A.2}$$

where,

$$M(\xi_i) = \prod_{j=1, i \ne j}^{N_\xi} (\xi_i - \xi_j)$$

The weighting coefficients of the second- and third-order derivatives can be obtained as Bert and Malik (1996), respectively,

$$[B_{ij}^{\xi}] = [A_{ij}^{\xi}][A_{ij}^{\xi}] = [A_{ij}^{\xi}]^2, \quad [C_{ij}^{\xi}] = [A_{ij}^{\xi}][A_{ij}^{\xi}][A_{ij}^{\xi}] = [A_{ij}^{\xi}]^3 \tag{A.3}$$

In a similar manner, the weighting coefficients for η-direction can be obtained.

In numerical computations, Chebyshev-Gauss-Lobatto quadrature points are used, that is Bert and Malik (1996),

$$\frac{\xi_i}{a} = \frac{1}{2}\left[1 - \cos\left[\frac{(i-1)\pi}{(N_\xi - 1)}\right]\right]; \quad \frac{\eta_j}{b} = \frac{1}{2}\left[1 - \cos\left[\frac{(j-1)\pi}{(N_\eta - 1)}\right]\right] \tag{A.4}$$

$$\text{for } i = 1, 2, \ldots, N_\xi \quad \text{and} \quad j = 1, 2, \ldots, N_\eta$$

Appendix B. Nomenclature

a	: plate dimension in ξ-direction
A_{ij}	: component of extensional stiffness of laminate
$A_{ij}^{\xi(r)}$	: weighting coefficient of the r-th order derivative in ξ-direction
A_{ij}^{ξ}, A_{ij}^{η}	: weighting coefficients of the first order derivative in ξ-and η-direction, respectively
B_{ij}^{ξ}, B_{ij}^{η}	: weighting coefficients of the second order derivative in ξ and η-direction, respectively
B_{ij}	: bending-extensional stiffness of laminate
b	: plate dimension in η-direction
D_{ij}	: bending stiffness of laminate
E_{ij}	: Young's modulus of lamina
G_{ij}	: shear modulus of lamina
$\boldsymbol{g}$	: residual forces
h	: total thickness of laminate
$\boldsymbol{f}$	: load vector
$\boldsymbol{f}_i$	: load vector at load step i
$\boldsymbol{K}_T^r$	: tangent stiffness matrix defined in Eq. (34)
M_{xx}, M_{yy}, M_{xy}	: bending moment about y and x-axis and twisting moment, respectively
$\overline{M}_{xx}$, $\overline{M}_{yy}$	: non-dimensional bending moment about y and x-axis ($=M_{xx}a^2/D_{22}h$, $M_{yy}a^2/D_{22}h$) respectively
N_ξ, N_η	: number of grid points in ξ and η-directions
N_{xx}, N_{yy}	: in-plane normal force resultant in x and y-directions
$(N_{xx})_{ij}$, $(N_{yy})_{ij}$	: discretized in-plane normal force resultant in x and y-directions
N_{xy}	: in-plane shear force resultant
$(N_{xy})_{ij}$	: discretized in-plane shear force resultant
$\overline{N}_{xx}$, $\overline{N}_{yy}$	: non-dimensional in-plane normal force resultant in x and y-directions ($=N_{xx}a^2/D_{22}$, $N_{yy}a^2/D_{22}$), respectively
n_x, n_y	: the x and y-component of unit normal vector to an arbitrary edge of the plate
q	: intensity of distributed transverse load
Q	: non-dimensional transverse load parameter ($= qa^4/D_{22}h^4$)
$\boldsymbol{R}$	: residual forces
$\boldsymbol{U}$	: the degrees of freedom vector or the generalized displacement vector
$\boldsymbol{U}_i^r$	: the degrees of freedom vector or the generalized displacement vector at iteration r of the load step i
u, v, w	: displacement component in the x, y and transverse direction of a point on mid-plane of plate, respectively
$\overline{u}$, $\overline{v}$, $\overline{w}$	: displacement component in the x, y and transverse direction of an arbitrary point of plate, respectively
V_n	: effective shear force on edge with unit normal $\vec{n}$
x, y, z	the Cartesian coordinate variables
W_c	: non-dimensional center deflection ($=w(a/2, b/2)h$)
β	: fiber orientation angle
ε	: the residual to applied force ratio

ε_o	: convergence tolerance
ε_{ij}	: strain components
ε_m	: membrane strain vector
γ_{ij}	: shear strain components
η	: oblique coordinate variable
$\boldsymbol{\kappa}$	: bending strain vector
K^{ξ}	: secondary degree of freedom along $\xi = 0$, a edges ($= \partial^2 w/\partial\xi^2$)
K^{η}	: secondary degree of freedom along $\eta = 0$, b edges ($= \partial^2 w/\partial\eta^2$)
θ	: skew angle
υ_{ij}	: Poisson's ratio of laminate
υ	: Poisson's ratio of isotropic material
ξ	: oblique coordinate variable