Structural Engineering and Mechanics, Vol. 25, No. 2 (2007) 147-160
DOI: http://dx.doi.org/10.12989/sem.2007.25.2.147

Fuzzy dynamic structural analysis of two-dimensional frame

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(Received October 10, 2005, Accepted August 22, 2006)

Abstract. In this paper, a dynamic analysis based on the fuzzy set theory is presented as a possible complementary tool to the classical stochastic methods for dynamic analyses. Material parameters of a structure are influenced by uncertainties and therefore they are considered to be fuzzy quantities with given distributions, that means fuzzy numbers with given membership functions. The fuzzy dynamic analysis is conducted with help of fuzzy arithmetic defined on the so-called α-cuts. The results of the analysis are also obtained in the form of fuzzy numbers, which compared to the stochastic methods is less computationaly expensive while at the same time they still provide information about the distribution of a quantity. This method is demonstrated on an analysis of a two-dimensional frame subjected to possible seismic load, where the uncertain eigenmodes and eigenfrequencies are used in the modal analysis.

Keywords: earthquake design; eigenfrequencies; eigenmodes; finite element method; fuzzy numbers.

1. Introduction

Design of concrete structures with respect to earthquake loading contains several obstacles which are mentioned in this paper and some of them are removed by application of suitable tools. First of all, there are uncertainties in concrete composition and therefore material parameters are also influenced by uncertainty.

Concrete, as a convenient building material, inherently involves uncertainty about its composition, which is difficult to be eliminated completely, however, this uncertainty can be assessed by statistical, fuzzy, or other suitable tools. In the case of concrete structures, such as frames made of reinforced concrete, it is costly to obtain a sufficient experimental data set which would yield desired statistical characteristics of material parameters. Instead, the knowledge gained with practicing engineers can be included as fuzzy numbers in the material modeling.

For design purposes, traditionally though, one may wish to conduct a statistical analysis, using the statistical characteristics of several measured events. In the case of earthquake, however, the measured data for each site of interest is not particularly dense, leaving the statistical characteristics with little relevance. On the other hand, the expected seismic load at a site can be alternatively expressed by the fuzzy sets (Zadeh 1965), which take into account the scarcity of seismic stations

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and the information about local sub-soil composition. The work presented in (Fischer *et al.* 2002) is an example of a recent effort in this field. Growing interest in description and handling with uncertainties can be documented by many journal articles. Even special issue about uncertainties in structural dynamics can be found (Journal of Sound and Vibration, vol. 288, issue 3, 2005). Application of the fuzzy set theory in description of uncertainty is still more frequent.

Described uncertainties in material parameters as well as in seismic loading require nondeterministic dynamic analysis. In this paper, an approach to dynamic analysis based on the fuzzy set theory is presented as a germane alternative to the classical stochastic dynamic analyses. The material parameters of reinforced concrete are considered to be fuzzy quantities with a given distribution, i.e., fuzzy numbers with a desired shape of the membership function (Valliapan and Pham 1993). The dynamic analysis is, then, performed with help of the fuzzy arithmetics on α-cuts. The result of such an analysis is in the form of fuzzy numbers which compared to the stochastic approach is less expensive for a usual number of α-cuts in terms of computation time and still providing an idea of distribution of the sought quantity. Description of uncertainty in seismic loading is not solved in this paper. Instead, this uncertainty is hidden in response spectra defined in standards which are applicable.

In order to further improve the computational efficiency, inspired by (Akpan *et al.* 2001), the concept of the surface response function (Bucher *et al.* 1988, Rajashekhar and Ellingwood 1993) is utilized. This approach is demonstrated in an illustrative example of a 2D frame where the effect of uncertain material parameters transpires in corresponding distributions of natural modal shapes and natural frequencies. The effect of the number of α-cuts representing the input data on quality of fuzzy results obtained by the response surface function technique is discussed. The modal superposition concept influenced by uncertainties modelled by fuzzy set theory is studied e.g. in (Moens and Vandepitte 2005) and (De Gersem *et al.* 2005). A methodology for a possible application to the seismic design is also explained.

2. Fuzzification of dynamic finite element analysis

The uncertainty, which is present in input parameters, can be tackled with help of the fuzzy set theory (Zadeh 1965). In this theory, the uncertain quantities are defined in terms of fuzzy sets. Unlike in the classical set theory, here the membership of an element to a fuzzy set also assumes the values between 0 and 1, where 0 means "does not belong" and 1 means "definitely belongs" to a fuzzy set. Usually, the fuzzy sets represent vague verbal evaluation. In cases when a fuzzy set represents a numeral, it is called a fuzzy number.

2.1 Fuzzy numbers

The notion of a fuzzy number arises from the experience of the everyday life when many phenomena which can be quantified are not characterized in the terms of absolutely precise numbers. An example of a fuzzy number is shown in Fig. 1, where μ represents the membership function and a_1 and a_2 stand for two real numbers on the real axis, which represent the minimum and the maximum value for a given degree of membership. The intervals defined for a specific value of the membership function, e.g. $\alpha = 0.5$, represent the so-called α-cuts. A fuzzy number can be equally expressed by either a nominal value and a membership function on each side of the

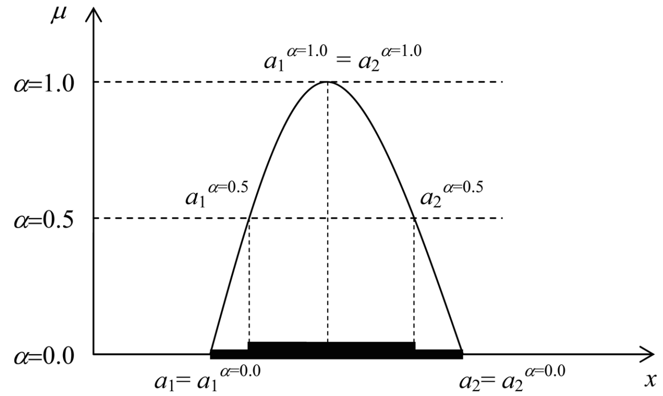

Fig. 1 Normal fuzzy number and its α-cuts

nominal value or by a set of α-cuts, i.e., intervals denoted by $[a_1^{\alpha};a_2^{\alpha}]$ where a_1^{α} denotes the minimum value and a_2^{α} denotes the maximum value for a given α-cut. More on the definitions of fuzzy numbers can be found in (Kaufman and Gupta 1985).

2.2 Fuzzy arithmetic

A fuzzy arithmetic operations depend on the definition of a fuzzy number. In the cases when fuzzy numbers are defined by a set of α-cuts, the problem of fuzzy arithmetic is reduced to the well-known arithmetic operations on intervals, which are applied to each α-cut (Kaufman and Gupta 1985). Implicitly, this means a sequence of combinations on each α-cut in order to obtain the minimum and the maximum value for each α-cut. However, in the case of a large number of α-cuts, e.g. ten α-cuts, it is necessary to solve about the same volume of computation as it is with the stochastic approaches. This fact makes the formulation merely unsolvable due to the number of all necessary arithmetic operations.

To eliminate the drawback of the formulation with a large number of α-cuts, new techniques for solving fuzzy linear equation systems have been developed, e.g. (Buckley and Qu 1991). However, these techniques are not easily applicable to robust problems, such as the fuzzy dynamic finite element analysis. Therefore, another technique for reducing the large number of combinations, originally developed for other problems, e.g. statistical analysis, should be exploited.

2.3 Response surface function

Fuzzy analyses, as well as stochastic analyses, suffer from non-occurrence of analytical solutions in the case of non-deterministic input data. This situation can be remedied by the following. Let $\tilde{\boldsymbol{x}} \in \tilde{\boldsymbol{X}}$ denote the vector of input data from the space of input data, $\tilde{\boldsymbol{X}}$, and $\tilde{\boldsymbol{y}} \in \tilde{\boldsymbol{Y}}$ denote the vector of output data from the space of output data, $\tilde{\boldsymbol{Y}}$. Both stochastic and fuzzy analyses require the knowledge of the response which can be written in the form

$$\tilde{\boldsymbol{y}} = \mathcal{F}(\tilde{\boldsymbol{x}}) \tag{1}$$

where $\mathcal{F}$ denotes the response of a system (structure) to the input data collected in the vector $\tilde{\boldsymbol{x}}$.

This represents a mapping from the space $\tilde{X}$ to the space $\tilde{Y}$. The non-occurence of analytical solution requires application of a suitable numerical method which discretizes the problem and solves it numerically. The space $\tilde{X}$ is discretized by an n-dimensional space, X, and similarly the space $\tilde{Y}$ by an m-dimensional space, Y. A stochastic analysis based on simulation methods generates thousands or millions of samples of input data (the vectors x) and then the deterministic computation follows. The fuzzy analysis based on a large number of α-cuts requires computation of all combinations of input data which also leads to thousands or millions of samples. Both approaches yield the response of a system based on a huge amount of output data (thousands or millions of the vectors y) obtained from many executions of standard (deterministic or crisp) solutions.

In order to reduce the necessary number of computation runs, the concept of a response surface function has been used many times. The basic idea of the response function is to approximate the operator $\mathcal{F}$ by a suitable function which should be as simple as possible. The function for the k-th output parameter can be written in the form

$$f^{(k)}(x) = a^{(k)} + \sum_{i=1}^{n} b_i^{(k)} x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{(k)} x_i x_j \tag{2}$$

where the superscript identifies an output parameter and n denotes the dimension of the space of the input data, X. The unknown coefficients are obtained from the least square method in the following way. Let the set of input parameters contain s samples. Each sample is located in the vector $x^{[i]}$, where the superscript identifies a sample. The standard computation gives output data, which are collected in the vectors $y^{[i]}$. The coefficients of the response function minimize the following expession

$$F^{(k)}(a^{(k)}, b_i^{(k)}, c_{ij}^{(k)}) = \sum_{l=1}^{s} (f^{(k)}(\mathbf{x}^{[l]}) - y_k^{[l]})^2 \tag{3}$$

In many cases, it is not necessary to use the quadratic terms. Considering only the linear terms then simplifies further computation.

3. Response of structure to seismic loading

In the case of known accelerograms, the response of a structure can be obtained by numerical integration of the equation of motion

$$\boldsymbol{M}\frac{d^2\boldsymbol{d}(t)}{dt^2} + \boldsymbol{C}\frac{d\boldsymbol{d}(t)}{dt} + \boldsymbol{K}\boldsymbol{d}(t) = \boldsymbol{f}(t) \tag{4}$$

where $\boldsymbol{M}$ denotes the mass matrix, $\boldsymbol{C}$ stands for the damping matrix, $\boldsymbol{K}$ denotes the stiffness matrix, $\boldsymbol{f}(t)$ expresses the load vector, $\boldsymbol{d}(t)$ is the vector of the nodal displacements which are computed and t stands for time. Eq. (4) represents a semidiscrete problem where the spatial coordinates are discretized while the time is still assumed to be continuous (Bathe 1996).

The Newmark method is an example of very popular method for the numerical integration of Eq. (4). The numerical integration is the relatively demanding part of computation, especially if the structure is described by a finite element model with a large number of degrees of freedom. Modern personal computers with fast processors and large RAM let the engineers compute the response of relatively large structures. The supports of a structure undergo prescribed accelerations for which the appropriate inertial forces located at nodes are evaluated. The mentioned approach is elegant and robust, however it requires the knowledge of the ground accelerograms. Therefore, the backward analysis of a structure can be performed mostly after an earthquake struck. The problem occurs in the design of new structures. The engineers have to use the data for some past earthquake or they have to use some artificial accelerogram, or suitable accelerograms from past events, such as that given in (Fischer *et al.* 2002).

Prior to powerful computers, another method was used frequently. The response of a structure to seismic load was computed with the help of response spectra consisting in the mode superposition method, which transforms the original problem into a set of independent equations. Each independent equation has the same form as the equation for a single degree of freedom system. The mode superposition method requires the knowledge of the natural mode shapes and natural frequencies. The natural frequencies (eigenvalues) and the natural mode shapes (eigenvectors) of an undamped structure are obtained from the relation

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\boldsymbol{u} = \boldsymbol{0} \tag{5}$$

where the nonzero vector $\boldsymbol{u}$ is the eigenvector containing the natural mode shapes and ω stands for the natural frequency. Eq. (5) represents a generalized problem of eigenvalues. The most common method for solution of such problems is the subspace iteration (Bittnar and Šejnoha 1996).

The vibration of a structure caused by seismic excitation is decribed by the equation of motion, which has the form

$$\boldsymbol{M}\ddot{\boldsymbol{d}}(t) + \boldsymbol{C}\dot{\boldsymbol{d}}(t) + \boldsymbol{K}\boldsymbol{d}(t) = -\boldsymbol{M}\boldsymbol{s}\ddot{v}_g(t) \tag{6}$$

where $\boldsymbol{s}$ contains either horizontal or vertical displacements. Let $\boldsymbol{u}_i$ be the i-th eigenmode (eigenvector) and let $\boldsymbol{U}$ be the modal matrix which contains eigenmodes in its columns. The unknown displacements can be expressed in the form

$$\boldsymbol{d}(t) = \boldsymbol{U}\boldsymbol{v}(t) \tag{7}$$

The substitution of Eq. (7) to the equation of motion, Eq. (6), and followed by multiplication with the transposed modal matrix results in

$$\boldsymbol{U}^T\boldsymbol{M}\boldsymbol{U}\ddot{\boldsymbol{v}}(t) + \boldsymbol{U}^T\boldsymbol{C}\boldsymbol{U}\dot{\boldsymbol{v}}(t) + \boldsymbol{U}^T\boldsymbol{K}\boldsymbol{U}\boldsymbol{v}(t) = -\boldsymbol{U}^T\boldsymbol{M}\boldsymbol{s}\ddot{\boldsymbol{v}}_g(t) \tag{8}$$

All the matrices in Eq. (8) are diagonal if the proportional damping and the normalized eigenmodes are assumed. This leads to a set of independent equations

$$\ddot{v}_i(t) + 2\xi_i\omega_i\dot{v}_i(t) + \omega_i^2 v_i(t) = -f_i\ddot{v}_g(t) \tag{9}$$

where the index of an appropriate eigenmode is denoted by i.

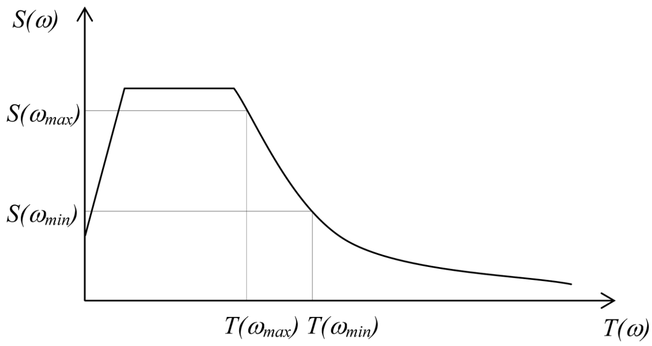

Fig. 2 Example of pseudoacceleration spectrum

The i-th maximum modal response associated with period T_i is given by

$$y(T_i)_{\max} = \frac{S(\omega_i(T_i))}{\omega_i^2(T_i)} = \frac{S(T_i)}{\left(\frac{2\pi}{T_i}\right)^2} \tag{10}$$

where ω_i is the i-th natural frequency and S is pseudo-acceleration spectrum. A typical example of a pseudoacceleration spectrum is shown in Fig. 2.

The maximum modal displacement associated with the i-th mode has the form

$$\boldsymbol{d}_i = (\boldsymbol{u}_i^T \boldsymbol{M} \boldsymbol{s} y(T_i)_{\max}) \boldsymbol{u}_i \tag{11}$$

All other quantities are obtained similarly.

The internal forces acting on the seismically loaded structure can be computed from the displacements. As was mentioned above, the displacements are expressed by

$$\boldsymbol{d} = \sum_{i=1}^{n} (y(T_i)_{\max} \boldsymbol{u}_i^T \boldsymbol{M} \boldsymbol{s}) \boldsymbol{u}_i \tag{12}$$

where n denotes the number of the used eigenmodes. The internal forces are computed through the multiplication of the displacements by the classical stiffness matrix

$$\boldsymbol{f}_{int} = \boldsymbol{K}\boldsymbol{d} \tag{13}$$

4. Fuzzification of structural dynamic analysis

The introduction of fuzziness into structural dynamic analysis is described in this section. In order to obtain clear and simple results, undamped free vibration of the single-degree-of-freedom system is considered first, followed by multi-dimensional generalization. It can be noted that graphical interpretation may constitute a major issue.

4.1 Single degree of freedom systems

Let a mass, m, be attached to a spring with a constant stiffness, k. Let the displacement of the mass be denoted by $d(t)$ and be a function of time t. The equation of motion can be written in the form of the equilibrium condition

$$m\frac{d^2d(t)}{dt^2} + kd(t) = 0 \tag{14}$$

It is an ordinary homogeneous differential equation of the second order with constant coefficients $k > 0$ and $m > 0$. The solution can be written in the form

$$d(t) = d_a\sin(\omega_0 t + \varphi) \tag{15}$$

where d_a is the amplitude of displacement, $\omega_0 = \sqrt{k/m}$ is the natural circular frequency and φ is the phase angle. The period of motion is denoted by T and can be obtained as

$$T = \frac{2\pi}{\omega_0} \tag{16}$$

For solving the equation of motion, the initial conditions must be prescribed by

$$d(0) = w \tag{17}$$

where w denotes the initial displacement, and

$$\frac{dd(0)}{dt} = v \tag{18}$$

where v is the initial velocity.

When the free vibration is commenced, the integration constants are obtained from the initial conditions in the form of the phase angle and the displacement amplitude

$$\varphi = \arctan\frac{\omega_0 w}{v} \tag{19}$$

$$d_a = \frac{\sqrt{\omega_0^2 w^2 + v^2}}{\omega_0} \tag{20}$$

The vibration of the single-degree-of-freedom system is described by the parameters of the oscillator (stiffness, k, and mass, m) and by the initial conditions (initial displacement, w, and initial velocity, v). Generally, these parameters are not known precisely and thus they can be alternatively represented by fuzzy numbers.

Two examples of fuzzy input parameters follow.

4.1.1 Fuzzy initial displacement

m, k, v are considered as crisp, not fuzzy, numbers, while w is a fuzzy number in the form $[w^-, w^+]$. For the sake of simplicity, the initial velocity is set as $v = 0$. From the conditions (17) and (18) one can obtain the amplitude of displacement $d_a = w$ and the phase angle $\varphi = \pi/2$. If the initial

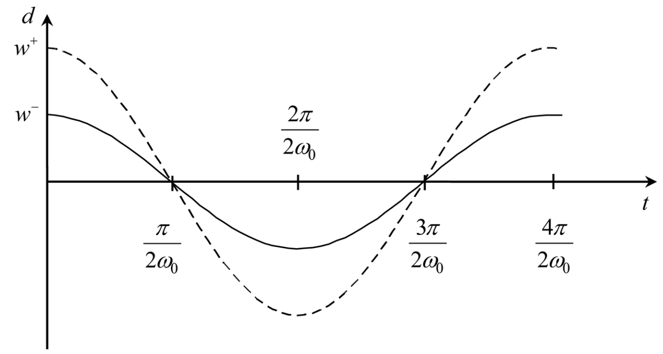

Fig. 3 Free vibration with fuzzy initial displacement

displacement is a fuzzy number, only the amplitude of the motion is changed. The period of motion, T, remains unchanged. This is shown in Fig. 3. Note that the maximum value of displacement, d^+, is defined differently in the positive and negative domains, i.e. $d^+ = w^+\sin(\omega_0 t + \pi/2)$ for $t \in (0, \pi/2\omega_0)$ and $d^+ = w^-\sin(\omega_0 t + \pi/2)$ for $t \in (\pi/2\omega_0, 3\pi/2\omega_0)$, if 2$d$ graphical interpretation of the results is used. A similar rule holds for d^-.

4.1.2 Fuzzy stiffness

m, w, v are considered as crisp numbers, while k is a fuzzy number. Let $w \neq 0$ and $v = 0$, then the displacement can be written as

$$d(t) = w\sin\left(\omega_0 t + \frac{\pi}{2}\right) = w\cos(\omega_0 t) \tag{21}$$

The natural circular frequency is defined by $\omega_0 = \sqrt{k/m}$, therefore, ω_0 is a fuzzy number in the form $[\omega_0^-, \omega_0^+] = [\sqrt{k^-/m}, \sqrt{k^+/m}]$. Two limit periods of motion can be computed as the upper bound and the lower bound in the form $[T^-, T^+] = [2\pi/\omega_0^+, 2\pi/\omega_0^-]$. The fuzziness of the stiffness coefficient leads to various periods of motion. It should be noted that after some time the fastest vibration with the shortest period overtakes the slowest vibration with the longest period. This is denoted as the limit time and it is expressed in the form

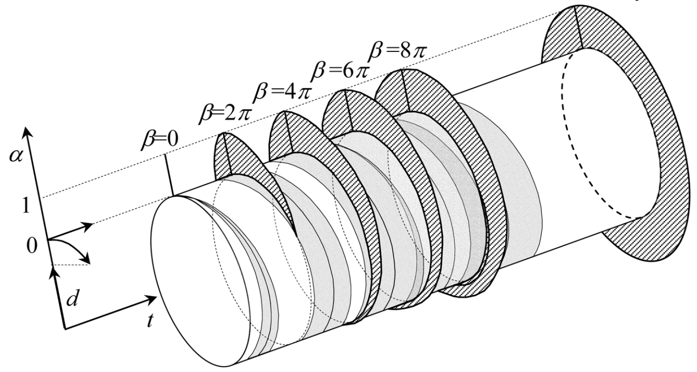

Fig. 4 Projection of solution on a cylinder

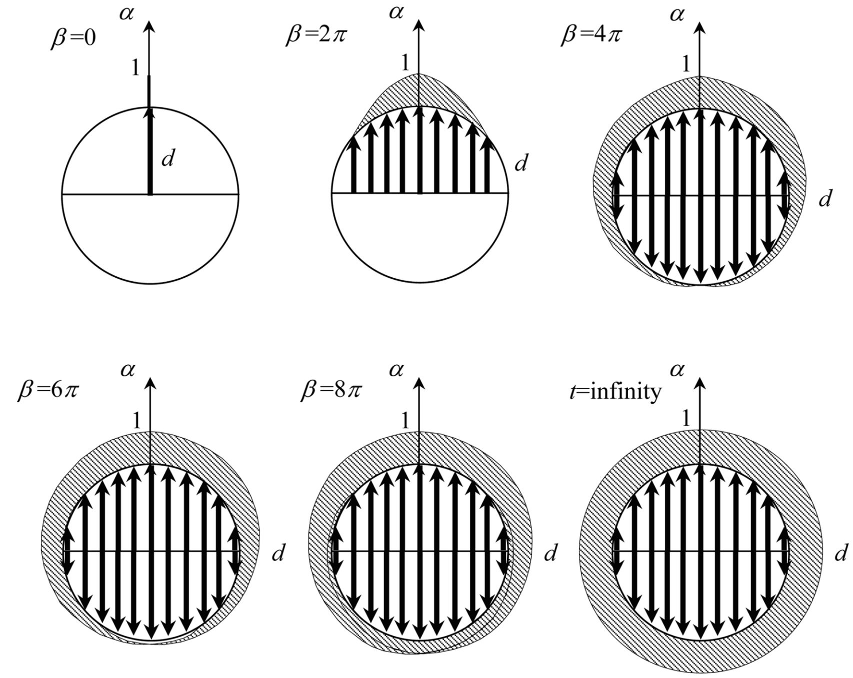

Fig. 5 Cross-sectional views for Fig. 4

$$t_{\lim} = \frac{2\pi}{\overset{+}{\omega}_0 - \overset{-}{\omega}_0} \tag{22}$$

This means that after the limit time, $t_{\lim}$, is reached the displacement cannot be clearly determined, at least not as closed interval with the zero values of the membership function for the upper and lower bound values of displacement. In other words, any value of the natural circular frequency from the range $[\overset{-}{\omega}_0, \overset{+}{\omega}_0]$ can be found for any displacement from the range $[-w, w]$ with a corresponding membership grade after the limit time is exceeded. Such quantification may be useful for setting the latest possible initiation of a controlled damping system in order to avoid collision of two structural segments, because until the limit time is reached the possible location of the segments can be estimated within some bounds, which is just enough as input data for fuzzy-logic-based controlling units. This effect is schematically described in Fig. 4 where the solution is projected with help of the angle $\beta = \omega_0 t$ on the surface of a cylinder whose radius is equal to the displacement amplitude. With progressing time the solution spreads and after reaching $t_{\lim}$ it starts overlapping itself, which is apparent in Fig. 5 (a is the membership degree). Due to the overlapping, in infinity the solution is represented by an interval $[-w, w]$ with the membership degree equal to unity for any d, which means that the displacement, d, can assume any value within the interval $[-w, w]$.

4.2 Multi-degree of freedom systems

Eigenvectors (eigenmodes) and eigenvalues (eigenfrequencies) play an important role in the structural dynamics and therefore they are studied first. The undamped free vibration problem is described by the generalized eigenvalue problem, Eq. (5). Let stiffness or mass be a fuzzy quantity expressed by a fuzzy number. Eigenvectors and eigenfrequencies are also fuzzy and thus they are expressed by fuzzy numbers. This is true even if only one matrix entry $\boldsymbol{K}_{ij}$ or $\boldsymbol{M}_{ij}$ is fuzzy. The eigenvalues are then in the form $[\omega_i^-, \omega_i^+]$. Every component of the i-th eigenvector is in the form of fuzzy number, e.g. the j-th component has the form $[u_{i,j}^-, u_{i,j}^+]$.

The uncertainty in material parameters, which is described by the fuzzy numbers, results in the fuzzy natural frequencies and the fuzzy eigenmodes as was shown in the previous sections. The response of a structure with uncertain parameters is therefore also uncertain. The whole seismic response is then expressed by the fuzzy numbers. The difference between the crisp and the fuzzy computation of seismic response with help of the response spectrum is the fuzziness of the i-th period T_i. Therefore, the $y(T_i)_{\max}$ are also described by the fuzzy numbers. The total displacements are the results of summation of contributions from particular modes. Each mode comprises the multiplication of the fuzzy numbers used in the eigenmodes and the fuzzy number used for the maximum modal response, $y(T_i)_{\max}$.

5. Numerical example

As an example, the natural frequency analysis of a two-dimensional frame with four floors made of reinforced concrete is considered. The results of natural frequency analysis can be used in response of structures to seismic excitation, which is defined by relation (12). The overall height of the frame is 16 meters and the width is 5 + 5 meters. The dimensions of beams and columns are identical (0.5 × 0.5 m). It is assumed that the building was erected in four consecutive lifts. Each lift consists of placing concrete in three columns and in the beam which connects the upper ends of the columns. Therefore, it is further assumed that there are only four types of concrete whose composition can possibly differ. The influencing material parameters are the modulus of elasticity, E, and the density, ρ. E and ρ are fuzzy input parameters with the nominal values of 30 GPa and 2500 kg/m^3, respectively, which can change by ±10% and with a linear membership function (triangular fuzzy numbers).

For our illustrative purposes, we need 125 response surface functions to describe the first five natural vibration modes, i.e., a response surface function to express each natural frequency and the horizontal and vertical displacements at each joint (three joints on each of the four floors) for each natural mode shape. In order to obtain sufficient input and output data for calculation of the coefficients of the response surface functions it was decided to take three values (minimum, modal value, maximum) for each material parameter, E and ρ, that means $3^{2\times 4}$ (=6561) independent runs of the dynamic finite element analysis. The specific form of Eq. (2) in this example was

$$f^{(k)}(x) = b_1^{(k)}E_1 + b_2^{(k)}E_2 + b_3^{(k)}E_3 + b_4^{(k)}E_4 + b_5^{(k)}\rho_1 + b_6^{(k)}\rho_2 + b_7^{(k)}\rho_3 + b_8^{(k)}\rho_4 + a^{(k)} \qquad (20)$$

The first five normalized mode shapes are shown in Figs. 6 to 11 where the dotted lines represent all possible envelopes of mode shapes, in other words, the minimum and maximum values, which correspond to the values obtained for α-cuts (α = 0). The mode shapes obtained for possible values of input parameters (modulus of elasticity and density) are within the computed envelopes. The finite element model of this frame discretized each frame section (beam and column) by five beam elements. This is why no significant difference is distinguishable in Fig. 6. In Fig. 7, a section of

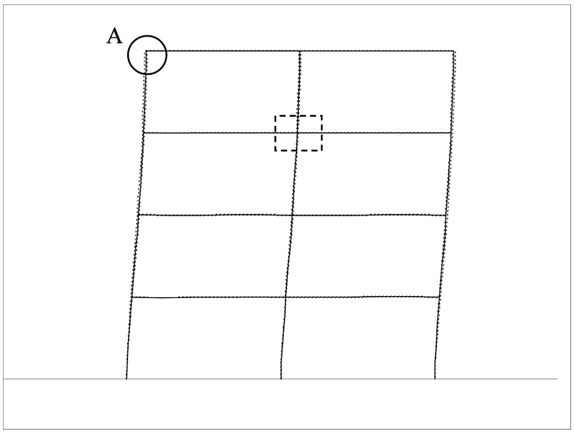

Fig. 6 Mode shape 1

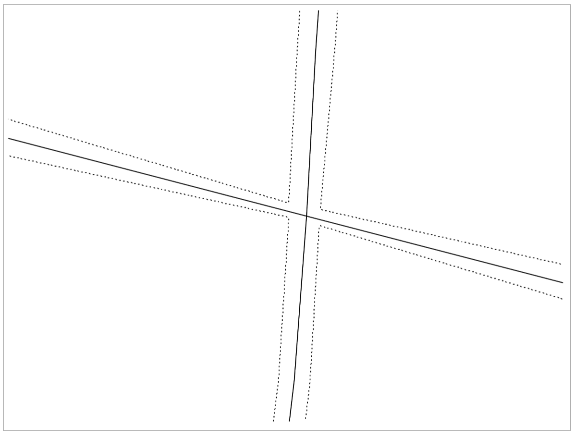

Fig. 7 Enlarged section of mode shape 1

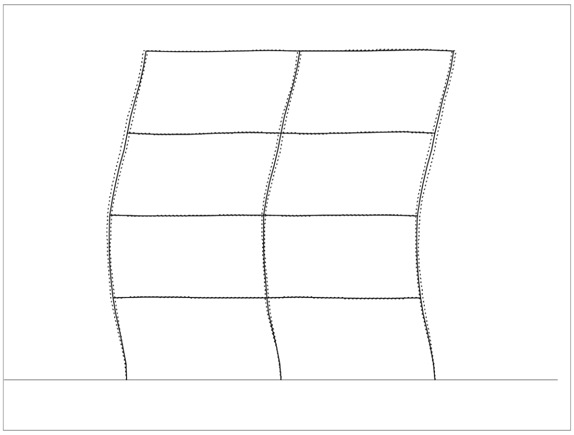

Fig. 8 Mode shape 2

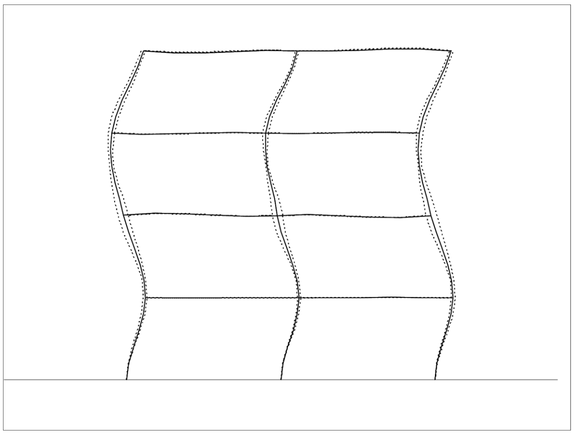

Fig. 9 Mode shape 3

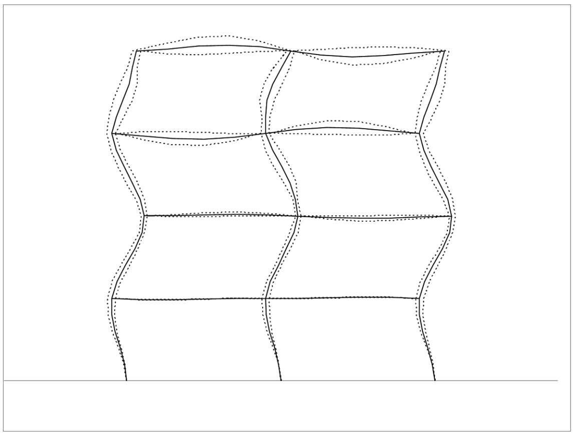

Fig. 10 Mode shape 4

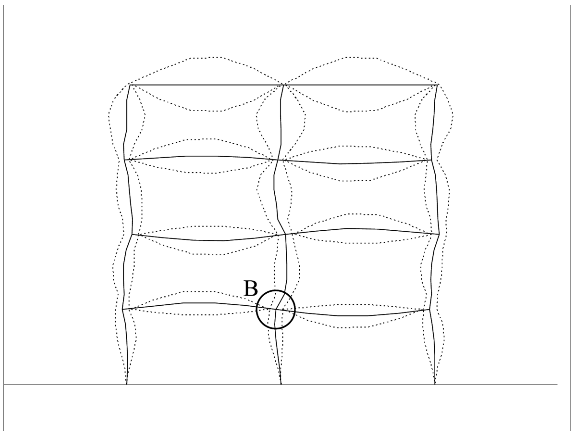

Fig. 11 Mode shape 5

the frame is enlarged and the vertical displacements are 1000 times increased compared to the horizontal displacements so that one can see the distribution of possible displacements of the frame. The distribution of the first five natural frequencies is shown in Fig. 9. It was observed that the response function gives very good results for dominant displacements (at point A in Fig. 6, which is the top left-hand joint) in lower natural mode shapes, which are important for the seismic design. As for the vertical displacements, which do not play an important role in the seismic design (at point B in Fig. 11, which is the intermediate joint of the first floor), the response function could not fit the proper shape of the membership function, which is evidenced in Fig. 13, where the solid line was obtained by the fuzzy calculation while the dashed line was obtained with help of the response surface function.

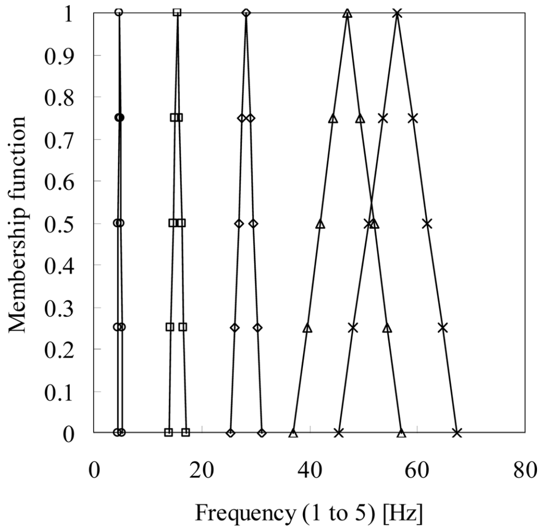

Fig. 12 Distribution of natural frequencies

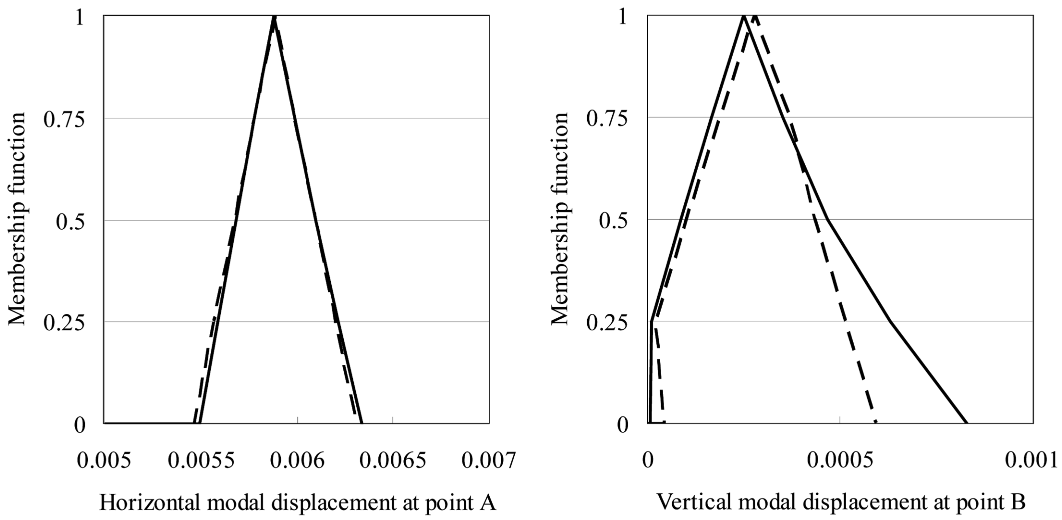

Fig. 13 Distribution of two displacements [m]

6. Possible applications

In the design of earthquake resistant structures, it is essential not to neglect any uncertainty as it may lead to an erroneous conclusion due to the dynamic simulation which may amplify such uncertainty beyond all limits. For those reasons it seems reasonable to express uncertain numerical data in terms of the fuzzy numbers and use them as such in analyses to cover all possible solutions.

In the previous sections, an approach to natural vibration analysis was shown which provides input data for further analyses considering, e.g. earthquake induced vibrations, where the uncertain seismic load is expressed by fuzzified response spectrum, whose values would be assessed by an expert in the form of fuzzy numbers. The methods based on the response-spectrum require only maximum values obtained for each natural mode for evaluation of excited vibration. Therefore, it is desirable to verify whether these values can be satisfactorily expressed by the response surface functions which were obtained only by the combinations of material parameters with three values (minimum, modal value, maximum). The resulting response surface function was also obtained for five values, corresponding to the α-cut values with α equal to 0, 0.5 and 1, however, that already meant $5^{2\times4}$ (=390,625) independent runs of the dynamic finite element analysis. The improvement was negligible, and compared with the computational effort it proved truly unnecessary.

7. Conclusions

This work contains the first step of the fuzzy earthquake design, which is the natural vibration analysis. Once the natural mode shapes are acquired it is relatively easy to compute the response of a system to earthquake induced excitation. The fuzzy numbers allow for the experts' opinions about the subsoil composition and other influencing factors to enter the right-hand-side vector of the governing equation. It was shown that the fuzzy dynamic finite element method can be satisfactorily supplemented with the response surface function concept which considerably increases the computational efficiency.

It was also hinted that input and output data collected through the combinations of only three values (minimum, modal value, maximum) for all varying material parameters provided response surface functions with the errors up to 5% from the true fuzzy results for the dominant and hence imporant responses.

Acknowledgements

This work was financially supported by the Czech Science Foundation, project no. 103/04/1320, which is gratefully acknowledged.

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