DOI: http://dx.doi.org/10.12989/sem.2007.25.1.127
Structural Engineering and Mechanics, Vol. 25, No. 1 (2007) 127-130

Technical Note

Linear vibration analysis of isotropic conical shells by discrete singular convolution (DSC)

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(Received April 12, 2005, Accepted August 3, 2006)

1. Introduction

Vibration analysis of conical shells has been of considerable research interest due to their importance in the area of structural, aerospace, chemical, submarine hulls, and mechanical applications. A number of analytical and numerical studies have been conducted on the linear and nonlinear static and dynamic analysis of conical shells (Saunders *et al.* 1960, Garnet and Kemper 1960, Irie *et al.* 1984, Tong 1993, Liew *et al.* 2005). Some selected works in this research topic includes those of Leissa (1973). More detailed information can be found in a review paper by Chang (1981). This note presents a novel computational approach, the discrete singular convolution (DSC) algorithm, for the linear free vibration analysis of conical shells.

2. Discrete singular convolution (DSC)

The discrete singular convolutions (DSC) algorithm was originally introduced by Wei (2000). Since then, applications of DSC method to various science and engineering problems have been investigated and their successes have demonstrated the potential of the method as an attractive numerical analysis technique. In the DSC method, function $f(x)$ and its derivatives with respect to x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x - x_M, x + x_M]$. This can be expressed as (Zhao and Wei 2002)

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x = x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x_i - x_k) f(x_k)\,; \quad (n = 0, 1, 2, \ldots) \tag{1}$$

where superscript n denotes the nth-order derivative with respect to x. The x_k is a set of discrete sampling points centered around the point x, σ is a regularization parameter, Δ is the grid spacing, and $2M+1$ is the computational bandwidth, which is usually smaller than the size of the computational domain.

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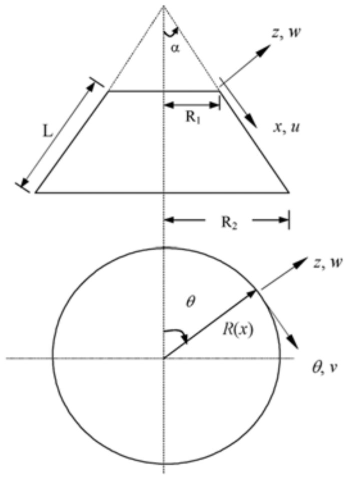

Fig. 1 Geometry and notation of conical shell

3. Formulations

Consider a thin conical shell as shown in Fig. 1. The cone semivertex angle, thickness of the shell, and cone length are denoted by α, h and L, respectively. R_1 and R_2 are the radii of the cone at its small and large edges. The conical shell is referred to a coordinate system (x, θ, z) as shown in Fig. 1. The components of the deformation of the conical shell with references to this given coordinate system are denoted by u, v, w in the x, θ and z directions, respectively. Governing equations for the linear free vibration analysis of isotropic conical shells are given;

$$L_{11}u + L_{12}v + L_{13}w - \rho h\frac{\partial^2 u}{\partial t^2} = 0 \tag{2a}$$

$$L_{21}u + L_{22}v + L_{23}w - \rho h\frac{\partial^2 v}{\partial t^2} = 0 \tag{2b}$$

$$L_{31}u + L_{32}v + L_{33}w - \rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{2c}$$

The displacement terms are taken as

$$u = U(x) \cdot \cos(n\theta) \cdot \cos(\omega t) \tag{3a}$$

$$v = V(x) \cdot \sin(n\theta) \cdot \cos(\omega t) \tag{3b}$$

$$w = W(x) \cdot \cos(n\theta) \cdot \cos(\omega t) \tag{3c}$$

Substituting Eqs. (3) into Eqs. (2), and rearranging the DSC form of the governing equations, one has the assembled form of the resulting equations as

$$[[G_{dd}]\ [G_{db}]]\begin{Bmatrix}\{U_d\}\\\{U_b\}\end{Bmatrix} - \Omega[[B_{dd}]\ [B_{db}]]\begin{Bmatrix}\{U_d\}\\\{U_b\}\end{Bmatrix} = \{0\} \tag{4}$$

where $\{U_b\}$ represents the unknown boundary grid points values, whereas, $\{U_d\}$ represent the domain grid point unknowns. The subscript b represents the degree of freedom on the boundary and subscript d represents the degree of freedom on the domain. Substituting the DSC rule into the boundary conditions at the sampling points at two boundary points leads to

$$GU = \Omega BU \tag{5}$$

where $G = G_{dd} - G_{db}G_{bb}^{-1}G_{bd}$, $B = B_{dd} - B_{db}G_{bb}^{-1}G_{bd}$ and U is the displacement vector on the domain. In this study, the following two type boundary conditions are considered.

$$\textit{Simply supported edge } (S)\text{: } V = 0,\ W = 0,\ N_x = 0,\ M_x = 0 \tag{6}$$

$$\textit{Clamped edge } (C)\text{: } U = 0,\ V = 0,\ W = 0 \text{ and } W_x = 0 \tag{7}$$

4. Computed results

In this study, the numerical results are given by the dimensionless frequency parameter Ω, defined as

$$\Omega = R_2\sqrt{\frac{\rho h}{A_{11}}}\omega$$

where ω is referred to as the frequency parameter. The influence of the cone angle of the shell on the free vibration frequencies has been studied and the results are shown in Fig. 2. Five different cone angles α are taken into consideration. Results given in these figures are obtained by setting $\nu = 0.3$ and $h/R_2 = 0.01$. It is shown that the increasing value of cone angle has a small effect on the frequency parameters of the shell for uniform support conditions. In order to discuss the influences

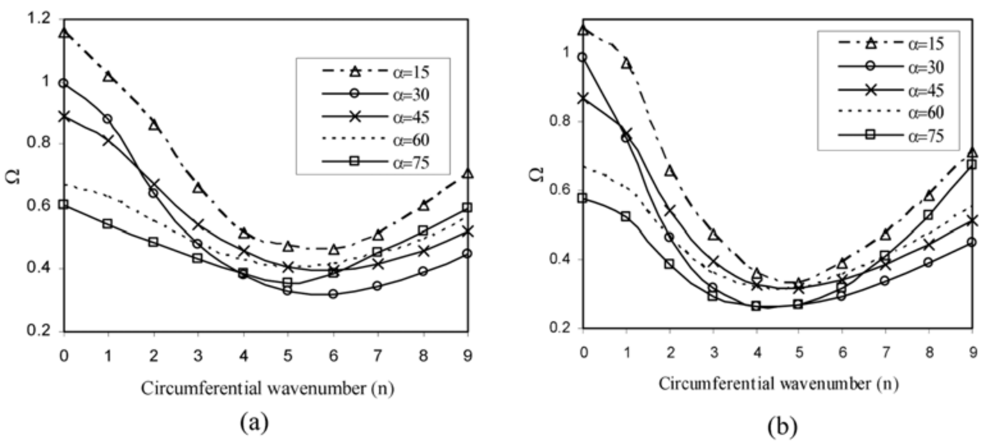

Fig. 2 Variation of frequencies for various cone angles for C-C conical shells ($h/R_2 = 0.01$); (a) $L\sin\alpha/R_2 = 0.50$, (b) $L\sin\alpha/R_2 = 0.75$

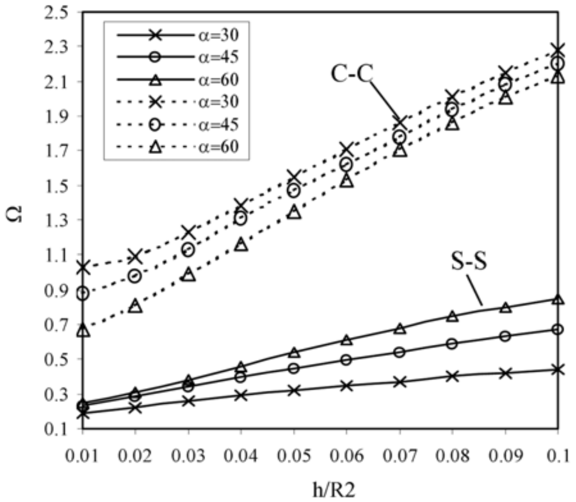

Fig. 3 Variation of frequencies for various cone angles with h/R_2 ratio ($L\sin/R_2 = 0.5$)

of h/R_2 on the frequency characteristics for C-C and S-S boundary conditions, Fig. 3 is obtained for three different cone angles, i.e., α = 30°, α = 45° and α = 60°. With the increase of ratio h/R_2, frequency parameter Ω increases rapidly. Generally, the decreasing magnitude of cone angle always decreases the frequency parameter Ω. It can be also seen that in Fig. 3, the frequencies are reduced considerably when the boundary conditions change from S-S to C-C.

Acknowledgements

The financial support of the Scientific Research Projects Unit of Akdeniz University is gratefully acknowledged.

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