Structural Engineering and Mechanics, Vol. 25, No. 1 (2007) 75-89 DOI: http://dx.doi.org/10.12989/sem.2007.25.1.075

Lateral force-displacement ductility relationship of non-ductile squat RC columns rehabilitated using FRP confinement

K. Galal[†]

Department of Building, Civil and Environmental Engineering, Concordia University, Montréal, Québec, Canada H3G 1M8

(Received January 3, 2006, Accepted August 9, 2006)

Abstract. Post-earthquake reconnaissance and experimental research indicate that squat reinforced concrete (RC) columns in existing buildings or bridge piers are vulnerable to non-ductile shear failure. Recently, several experimental studies were conducted to investigate upgrading the shear resistance capacity of such columns in order to modify their failure mode to ductile one. Among these upgrading methods is the use of fibre-reinforced polymer (FRP) jackets. One of the preferred analytical tools to simulate the response of frame structures to earthquake loading is the lumped plasticity macromodels due to their computational efficiency and reasonable accuracy. In these models, the columns' nonlinear response is lumped at its ends. The most important input data for such type of models is the element's lateral force-displacement backbone curve. The objective of this study is to verify an analytical method to predict the lateral force-displacement ductility relationship of axially and laterally loaded rectangular RC squat columns retrofitted with FRP composites. The predicted relationship showed good accuracy when compared with tests available in the literature.

Keywords: ductility; rehabilitation; FRP; reinforced concrete; squat columns; non-ductile; shear; forcedisplacement.

1. Introduction

Rectangular reinforced concrete (RC) columns are widely used in bridge pier design, and they make up the majority of building columns. Columns in need of strengthening and retrofit are very common. Among those are squat columns that are susceptible to shear failure during an extreme loading event such as a major earthquake. The engineering common sense has been to avoid the construction of squat columns in seismically active zones. However, there exist a large number of columns that are at risk of brittle failure modes. These columns may have been originally designed as long columns and then partial supporting non-structural elements (e.g., masonry walls) were later constructed therefore creating a squat column. Squat columns may also have been the result of recent design following current codes.

Many buildings designed according to older strength-based codes are susceptible to abrupt nonductile strength deterioration once their shear capacity is reached during an earthquake event. Squat

[†] Assistant Professor, E-mail: galal@bcee.concordia.ca

K. Galal



Fig. 1 Examples of non-ductile shear failure of squat RC columns during earthquakes (NISEE 2006)

RC columns are vulnerable to such type of failure (Fig. 1). Performance-based seismic engineering is the modern approach to earthquake resistant design. Seismic performance *(performance level)* is described by designating the maximum allowable damage state *(damage parameter)* for an identified seismic hazard *(hazard level)*. Performance levels describe the state of a structure after being subjected to a certain hazard level as: Fully operational, Operational, Life safe, Near collapse, or Collapse (FEMA 273/274 1997, Vision 2000 1995). Overall lateral deflection, ductility demand, and inter-storey drift are the most commonly used damage indices. The ductility of the column past initial steel bar yielding has become the target for good design. This approach is expected to decrease the probability of failure of the structure, and increase its energy dissipating capacity, when subjected to the design ground motions.

Fibre-reinforced polymer (FRP) composites are used to increase the shear strength of existing concrete beams and columns by wrapping or partially wrapping the members. Wrapping RC columns with carbon or glass FRP sheets was shown experimentally to increase their shear capacity and the flexural ductility without significant increase in the column's stiffness. Additional shear strength contribution is introduced by orienting the fibres normal to the axis of the member or to cross potential shear cracks. Increasing the shear strength can alter the failure mode to be relatively more ductile compared to shear failure. Shear strengthening using external FRP wraps may be provided at locations of expected plastic hinges or stress reversal for enhancing post-yield behaviour of columns subjected to seismic loads by completely wrapping the section.

The objective of this study is to verify and conduct a parametric study on an analytical model to predict the lateral force-displacement ductility relationship of axially and laterally loaded rectangular RC squat columns retrofitted with FRP composites. This relationship provides the lateral force-deformation backbone envelope which forms the main input data for lumped plasticity macromodels that are used in predicting the nonlinear hysteretic behaviour of squat RC columns.

1.1 Experimental studies

Despite the importance of the subject, only a few studies were found in the literature on the rehabilitation of squat RC columns using FRP composites.

Yoshimura *et al.* (2000) conducted an experimental study on the behaviour of squat RC columns strengthened externally by carbon fibre-reinforced polymers (CFRP). Eight $150 \times 150 \times 300$ mm specimens with no transverse ties were tested under constant gravity load and repeated lateral forces. The variables were the longitudinal reinforcement content (ratio of the area of longitudinal reinforcement to the area of concrete section, $\rho_t = 0.019$ and $\rho_t = 0.034$), the concrete compressive strength (16 and 38 MPa), and the amount of CFRP sheets (1 to 14 wraps). It was concluded that if squat RC columns, which are expected to fail in brittle shear failure mode during an earthquake, are strengthened by an *appropriate* fibre cross-sectional area, brittle shear failure would be prevented.

Ye et al. (2002) tested seven square 200×200 mm squat RC columns with shear span-to-depth ratios (*M*/*Vt*) ranging from 1.5 to 3.0 and strengthened with CFRP under lateral cyclic loading, where *M* is the maximum moment in a region of constant shear *V* along the element length, and *t* is the total depth of the section. The transverse steel reinforcement ratio of the specimens, ρ_t was 0.18%. Two of the specimens were fully wrapped with continuous CFRP sheets along the column height, while four were wrapped with discontinuous CFRP with different widths and spacing. The strengthened specimens had a more ductile behaviour compared to the unstrengthened ones.

Haroun *et al.* (2002) tested seven rectangular RC squat columns with transverse steel reinforcement ratio $\rho_t = 0.1\%$ under reversed cyclic load. One of the seven specimens was the control test, while the others were repaired using crack injection and a carbon fiber jacket, then retested. The shear span-to-depth ratio of the specimens was 2.0. The prepared specimens performed in a more ductile response compared to the as built one.

Ghobarah and Galal (2004) tested three square RC squat columns under cyclic lateral loads and constant axial load. Another four squat RC columns were tested by Galal *et al.* (2005). The seven columns were $305 \times 305 \times 914$ mm and were subjected to double curvature with the point of contraflexure being maintained at mid-height with a shear span-to-depth ratio of 1.5. The specimens are separated into two groups. Group 1 is designed according to the CSA (1994) code with high content of transverse reinforcement, while Group 2 was designed according to pre-1970 code, ACI (1968), with low content of transverse reinforcement. Different rehabilitation schemes using carbon-or glass- FRP were used to strengthen the squat columns.

1.2 Available analytical models

Seible *et al.* (1997) developed a design model for the jacket thickness required to obtain a targeted displacement ductility for RC columns. The design model was applied to shear retrofit of squat columns for target ductility ≥ 8 . RC columns $405 \times 610 \times 2438$ mm were tested in double bending (shear span-to-depth ratio is 2.0). The original column was expected to have a brittle shear failure, therefore it was rehabilitated using CFRP jacket. The tested column having CFRP jacket with thicknesses about half the value of that obtained by the design formulas reached a column displacement ductility of 12 which is greater than the target ductility by 50%.

ISIS Canada (2001), ACI (2002), FIB (2001) and other codes provide equations for the calculation of shear strength contribution of FRP. The equations are similar to those used for transverse stirrups contribution in a RC member. The difference in estimating the shear strength

contribution of FRP mechanism using the different approaches is approximately 10%.

Ghobarah and Galal (2004) proposed a method to predict the backbone lateral force-displacement ductility relationship for squat RC columns jacketed with FRP. The model combines the column's shear and flexural capacities to predict the backbone relationship.

In the current study, the analytical model is compared with the experimental results of eleven squat RC columns available in the literature.

2. Model summary

The nominal shear capacity, V_n , of a RC column retrofitted with FRP composites results from the contributions of four mechanisms, namely; concrete V_c , axial load V_p , transverse steel V_s , and FRP V_f .

i.e.,
$$V_n = V_c + V_p + V_s + V_f$$
 (1a)

Several design guidelines for strengthening RC structures using FRP (e.g., ISIS Canada 2001, ACI 2002, and FIB 2001) provide formulas for calculating the nominal shear capacity of RC elements according to the previous equation.

The ACI (2002) limits the total shear strength where more than one type of shear reinforcement is used to: $0.66 \sqrt{f'_{cc} bd}$, where b is the width of the column; d is the column section depth to the tensile steel; f'_{cc} is the confined concrete compressive strength such that $f'_{cc} = \beta \cdot f'_{c}$, where f'_{c} is the unconfined concrete compressive strength; and β is the confined concrete compressive strength multiplier $\beta = f'_{cc}/f'_{c}$. In the above formulation, this limit should be imposed on the sum of the contributions from transverse steel and the FRP mechanisms, $(V_s + V_f)$. Therefore, Eq. (1a) can be re-written to be:

$$V_n = V_c + V_p + \min[(V_s + V_f) \text{ and } 0.66 \sqrt{\beta f_c'} bd]$$
 (1b)

The abovementioned design guidelines assume constant shear capacity of the RC column that is independent of the displacement ductility level of the column. Priestley *et al.* (1994) and Kowalsky *et al.* (1999) illustrated that the shear capacity of a reinforced concrete column degrades with increasing displacement ductility. This is due to the decrease in the shear contribution from concrete, V_c , and axial load, V_p , mechanisms in high displacement ductility levels. The aforementioned researchers' formulas for the nominal shear capacity did not include the contribution of FRP mechanism.

Ghobarah and Galal (2004) proposed a model for predicting the shear capacity of square RC squat columns retrofitted with FRP composites at different displacement ductility levels. Fig. 2 shows the proposed shear strength envelope of an axially and laterally loaded squat column with respect to its displacement ductility. The model assumes that the column shear capacity decreases bi-linearly with the increase of the lateral displacement ductility, μ , after reaching $\mu = 2$ such that:

$$V_{\mu=4} = \frac{1}{3} (V_c + V_p) + \min[(V_s + V_f) \text{ and } 0.66 \sqrt{\beta f'_c} bd] \qquad \text{at } \mu = 4 \qquad (2a)$$

$$V_{\mu=6} = \min[(V_s + V_f) \text{ and } 0.66 \sqrt{\beta f_c' b d}]$$
 at $\mu = 6$ (2b)

78



Fig. 2 Lateral force-displacement ductility relationship (Ghobarah and Galal 2004)

$$V_r = \frac{3}{4} V_{\mu=6}$$
 (2c)

In calculating the shear strength of the FRP mechanism V_f , the model accounts for the simultaneous confinement effect of the FRP jacket on increasing the concrete strength, which in turn increases the shear strength contribution of the concrete mechanism, V_c .

The contributions of the four mechanisms to the shear strength of a rectangular RC squat column retrofitted with FRP are given by the following equations:

$$V_c = 0.3 \sqrt{f_{cc}'} A_e = 0.3 K_e \sqrt{\beta f_c'} \cdot bt$$
(3)

$$V_{p} = k_{p} \frac{Pt/2}{H} = \frac{(\zeta f_{c}' bt) \cdot t/2}{2 \cdot (M/Vt) \cdot t} = \frac{0.25 \, \zeta f_{c}'}{M/Vt} \cdot bt$$
(4)

$$V_s = \frac{A_v f_{yv} d}{s} = \rho_v f_{yv} \left(\frac{1+\gamma}{2}\right) \cdot bt$$
(5)

$$V_f = 0.95(2t_f)(\varepsilon_{fe}E_f)d = 1.9\lambda_F\varepsilon_{fe}\frac{(1+\gamma)}{2}f_{yv}\cdot bt$$
(6)

such that $\beta = \frac{f_{cc}'}{f_c'} = 2.254 \sqrt{1 + 7.94 \frac{f_1'}{f_c'}} - 2\frac{f_1'}{f_c'} - 1.254 \ge 1.0$ (Mander *et al.* 1988); $f_1' = K_e \rho_{eff} f_{yu} = K_e (\rho_v + 2\varepsilon_{fe}\lambda_F) f_{yv}$ is the effective lateral confining pressure; $K_e = A_e/bt$ is the confinement effectiveness coefficient; $\zeta = P/(f_c'bt)$ is the axial force level; M/Vt is the shear span-to-depth ratio of the column; $\rho_v = A_v/bs$ is the shear stirrups content;



Fig. 3 Properties of a squat rectangular RC column confined with FRP

$$\lambda_F = \frac{t_f}{b} \cdot \frac{E_f}{f_{yv}}$$
 is a parameter representing the FRP wrap content; and
 $\gamma = (d - d')/t$ is the ratio between longitudinal steel to the overall depth of the column *t*, as shown in Fig. 3.

where A_e is the area of effectively confined concrete core; $k_p = 1$ for columns in double curvature and 0.5 for columns in single curvature; A_v , f_{yv} , s are the total cross sectional area, yielding strength and spacing of transverse reinforcement; $2t_f$ is the total transverse design thickness of FRP sheets (i.e., for two opposite sides); ε_{fe} is the design strain for FRP: $\varepsilon_{fe} = 0.004$ for unanchored FRP sheets and $\varepsilon_{fe} = 0.006$ for anchored FRP sheets; E_f is the Young's modulus of the FRP composite material; and ρ_{eff} is the effective transformed confinement content, transforming the FRP sheets at strain ε_{fe} into an equivalent steel content having the same steel yield strength and is given by $\rho_{eff} = \rho_v + 2\varepsilon_{fe}\lambda_F$.

In the above equations, the shear strength of concrete (Eq. (3)) is in the same form given by Priestley *et al.* (1994) but the compressive strength of concrete f'_c is replaced by the confined strength of concrete f'_{cc} due to the confinement effect of ties and FRP wraps. Eqs. (4) and (5) are based on the mechanics and equilibrium of forces acting on the column. The contribution of the FRP to the shear resistance given by ACI (2002), shown in Eq. (6), is adopted in this model.

A bilinear flexural force-ductility relationship was assumed with a flexural capacity of M_u . The shear force V_{flex} that corresponds to the flexural capacity of an RC column with longitudinal reinforcement content $\rho_t = A_s/bt$, where A_s is the total area of longitudinal reinforcement, can be expressed as:

$$V_{flex} = \frac{M_u}{(M/V)} = \frac{1}{(M/Vt)} \cdot \frac{M_u}{b \cdot t^2} \cdot bt$$
(7)

	b	t	t M		$f_{co}' P$			Н	A_e		
Specimen	mm	mm	$\frac{1}{Vt}$	MPa	Pa kN ^k		k_p	mm	mm^2	K _e	
C0.5-6D13 ^[1]	150	150	1.0	28	64		1.0	300	20000	0.889	
CS20-3-15 ^[2]	200	200	2.0	26	205		0.5	400	36667	0.917	
RS-R1 ^[3]	457	610	2.0	38	625		1.0	2440	215048	0.771	
SC1 ^[4]	305	305	1.5	39	500		1.0	914	41850	0.450	
SC2 ^[4]	305	305	1.5	39	500		1.0	914	64370	0.692	
SC3 ^[4]	305	305	1.5	39	500		1.0	914	20735	0.223	
SC1R ^[5]	305	305	1.5	34	500		1.0	914	64370	0.692	
SC2R ^[5]	305	305	1.5	34	500		1.0	914	64370	0.692	
SC1U ^[5]	305	305	1.5	43	500		1.0	914	64370	0.692	
SC3R ^[5]	305	305	1.5	34	500		1.0	914	20735	0.223	
SPHI-1 ^[6]	406	610	2.0	34	500		1.0	2440	187176	0.756	
Sussimon	$2t_f$		E_f	A_{v}	S	f_{yv}		f_l'	P	f_{cc}^{\prime}	
Specimen	$2t_f$ mm	\mathcal{E}_{fe}	<i>E_f</i> GPa	A_{v} mm ²	s mm	f _{yv} MPa	Peff	f _l ' MPa	β	f _{cc} MPa	
Specimen C0.5-6D13 ^[1]	2 <i>t_f</i> mm 0.056	$arepsilon_{fe}$ 0.004	<i>E_f</i> GPa 273	A_{ν} mm ²	<i>s</i> mm	<i>f_{yv}</i> MPa (300)	ρ _{e,ff} 0.0014	<i>f</i> _l ' MPa 0.36	β 1.09	<i>f</i> _{cc} MPa 30	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2]	2t _f mm 0.056 0.333	$arepsilon_{fe}$ 0.004 0.004	<i>E_f</i> GPa 273 235	$ \frac{A_{\nu}}{\mathrm{mm}^{2}} $ 64	s mm 200	<i>f_{yv}</i> MPa (300) 345	<i>Peff</i> 0.0014 0.0061	<i>f_l</i> ′ MPa 0.36 1.94	β 1.09 1.44	<i>f_{cc}</i> MPa 30 37	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3]	2t _f mm 0.056 0.333 1.005	$arepsilon_{fe}$ 0.004 0.004 0.004	E_f GPa 273 235 226	$ \begin{array}{c} A_{\nu} \\ mm^{2} \\ \hline 64 \\ 63 \end{array} $	s mm 200 127	<i>f_{yv}</i> MPa (300) 345 276	ρ _{eff} 0.0014 0.0061 0.0083	<i>f_l</i> ' MPa 0.36 1.94 1.76	β 1.09 1.44 1.29	<i>f</i> _{cc} MPa 30 37 49	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4]	2t _f mm 0.056 0.333 1.005	ε _{je} 0.004 0.004 0.004 	<i>E_f</i> GPa 273 235 226	A_{ν} mm ² 64 63 200	s mm 200 127 65	<i>f_{yv}</i> MPa (300) 345 276 420	<i>P</i> eff 0.0014 0.0061 0.0083 0.0100	<i>f_l</i> ' MPa 0.36 1.94 1.76 1.91	β 1.09 1.44 1.29 1.30	<i>f_{cc}</i> MPa 30 37 49 51	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4] SC2 ^[4]	2t _f mm 0.056 0.333 1.005 0.990	ε _k 0.004 0.004 0.004 0.006	<i>E_f</i> GPa 273 235 226 235	A_{ν} mm ² 64 63 200 200	s mm 200 127 65 65	<i>f_{yv}</i> MPa (300) 345 276 420 420	<i>P</i> eff 0.0014 0.0061 0.0083 0.0100 0.0210	$\begin{array}{c} f_l' \\ MPa \\ 0.36 \\ 1.94 \\ 1.76 \\ 1.91 \\ 6.10 \end{array}$	β 1.09 1.44 1.29 1.30 1.81	$ f'_{cc} MPa 30 37 49 51 71 $	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4] SC2 ^[4] SC3 ^[4]	2t _f mm 0.056 0.333 1.005 0.990 0.990	ε _{fe} 0.004 0.004 0.004 0.006 0.006	$\begin{array}{c} E_f \\ \text{GPa} \\ 273 \\ 235 \\ 226 \\ \\ 235 \\ 235 \\ 235 \end{array}$	A_{ν} mm ² 64 63 200 200 200 200	s mm 200 127 65 65 65 305	$\begin{array}{c} f_{yy} \\ MPa \\ (300) \\ 345 \\ 276 \\ 420 \\ 420 \\ 420 \\ 420 \end{array}$	<i>ρ</i> eff 0.0014 0.0061 0.0083 0.0100 0.0210 0.0130	f_{l}' MPa 0.36 1.94 1.76 1.91 6.10 1.22	β 1.09 1.44 1.29 1.30 1.81 1.20	$ \begin{array}{c} f_{cc} \\ MPa \\ 30 \\ 37 \\ 49 \\ 51 \\ 71 \\ 47 \\ \end{array} $	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4] SC2 ^[4] SC3 ^[4] SC1R ^[5]	2t _f mm 0.056 0.333 1.005 0.990 0.990 2.824	ε _{fe} 0.004 0.004 0.004 0.006 0.006 0.006	$\begin{array}{c} E_f \\ GPa \\ 273 \\ 235 \\ 226 \\ \\ 235 \\ 235 \\ 235 \\ 71 \\ \end{array}$	A_{ν} mm ² 64 63 200 200 200 200 200	s mm 200 127 65 65 305 65	$\begin{array}{c} f_{yv} \\ \text{MPa} \\ (300) \\ 345 \\ 276 \\ 420 \\ 420 \\ 420 \\ 420 \\ 420 \end{array}$	<i>P</i> eff 0.0014 0.0061 0.0083 0.0100 0.0210 0.0130 0.0195	f_l' MPa 0.36 1.94 1.76 1.91 6.10 1.22 5.66	β 1.09 1.44 1.29 1.30 1.81 1.20 1.85	$ \begin{array}{c} f_{cc}\\ MPa\\ 30\\ 37\\ 49\\ 51\\ 71\\ 47\\ 63\\ \end{array} $	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4] SC2 ^[4] SC3 ^[4] SC1R ^[5] SC2R ^[5]	2t _f mm 0.056 0.333 1.005 0.990 0.990 2.824 0.660	ε _{fe} 0.004 0.004 0.004 0.006 0.006 0.006 0.004	$\begin{array}{c} E_f \\ GPa \\ 273 \\ 235 \\ 226 \\ \\ 235 \\ 235 \\ 71 \\ 235 \end{array}$	A_{ν} mm ² 64 63 200 200 200 200 200 200 200	s mm 200 127 65 65 65 305 65 65	$\begin{array}{c} f_{yv} \\ \text{MPa} \\ (300) \\ 345 \\ 276 \\ 420 \\ 420 \\ 420 \\ 420 \\ 420 \\ 420 \end{array}$	<i>P</i> eff 0.0014 0.0061 0.0083 0.0100 0.0210 0.0130 0.0195 0.0149	$\begin{array}{c} f_{l}' \\ MPa \\ 0.36 \\ 1.94 \\ 1.76 \\ 1.91 \\ 6.10 \\ 1.22 \\ 5.66 \\ 4.34 \end{array}$	β 1.09 1.44 1.29 1.30 1.81 1.20 1.85 1.69	$ f_{cc} f_$	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4] SC2 ^[4] SC3 ^[4] SC1R ^[5] SC2R ^[5] SC1U ^[5]	2t _f mm 0.056 0.333 1.005 0.990 0.990 2.824 0.660 0.990	\mathcal{E}_{fe} 0.004 0.004 0.004 0.006 0.006 0.006 0.004 0.004	$\begin{array}{c} E_f \\ \hline GPa \\ 273 \\ 235 \\ 226 \\ \\ 235 \\ 235 \\ 235 \\ 71 \\ 235 \\ 235 \\ 235 \end{array}$	A_{ν} mm ² 64 63 200 200 200 200 200 200 200 200	s mm 200 127 65 65 65 305 65 65 305	$\begin{array}{c} f_{yv} \\ MPa \\ (300) \\ 345 \\ 276 \\ 420 \\ 40 \\ 4$	ρeff 0.0014 0.0061 0.0083 0.0100 0.0210 0.0130 0.0195 0.0149 0.0174	$\begin{array}{c} f_{l}' \\ MPa \\ 0.36 \\ 1.94 \\ 1.76 \\ 1.91 \\ 6.10 \\ 1.22 \\ 5.66 \\ 4.34 \\ 5.04 \end{array}$	β 1.09 1.44 1.29 1.30 1.81 1.20 1.85 1.69 1.64	$ f_{cc} f_$	
Specimen C0.5-6D13 ^[1] CS20-3-15 ^[2] RS-R1 ^[3] SC1 ^[4] SC2 ^[4] SC3 ^[4] SC1R ^[5] SC1U ^[5] SC3R ^[5]	2t _f mm 0.056 0.333 1.005 0.990 0.990 2.824 0.660 0.990 4.236	ε _{fe} 0.004 0.004 0.004 0.006 0.006 0.006 0.004 0.004 0.004	$\begin{array}{c} E_f \\ \hline GPa \\ 273 \\ 235 \\ 226 \\ \\ 235 \\ 235 \\ 71 \\ 235 \\ 235 \\ 71 \\ 235 \\ 71 \\ \end{array}$	$ \begin{array}{c} A_{\nu} \\ mm^2 \\ \hline \\ 64 \\ 63 \\ 200 \\ 20 \\ 20 \\ 20 \\ 20 \\$	s mm 200 127 65 65 305 65 305 65 305 65	$\begin{array}{c} f_{yv} \\ \text{MPa} \\ (300) \\ 345 \\ 276 \\ 420 \\ 40 \\ 4$	ρ _{eff} 0.0014 0.0061 0.0083 0.0100 0.0210 0.0130 0.0195 0.0149 0.0174 0.0115	f_l' MPa 0.36 1.94 1.76 1.91 6.10 1.22 5.66 4.34 5.04 1.08	β 1.09 1.44 1.29 1.30 1.81 1.20 1.85 1.69 1.64 1.20	f_{cc} MPa 30 37 49 51 71 47 63 57 71 41	

Table 1 Parameters used to calculate the shear strength contributions from different mechanisms

^[1] Yoshimura *et al.* (2000); ^[2] Ye *et al.* (2002); ^[3] Haroun *et al.* (2002); ^[4] Ghobarah and Galal (2004); ^[5] Galal *et al.* (2005); ^[6] Seible *et al.* (1997)

Combining the shear strength envelope with the flexural capacity envelope of the column results in one of three responses; (1) Ductile behaviour (if $V_{\mu=6} > V_{flex}$); (2) Moderate ductility with shear failure (if $V_n > V_{flex} > V_{\mu=6}$); and (3) Limited ductility with brittle shear failure (if $V_{flex} > V_n$). Fig. 2 shows the combined flexure and shear response of an axially and laterally loaded RC column. The column is assumed to undergo degradation in the shear capacity from V_{flex} to V_r (residual shear strength) for the cases of moderate and limited ductilities. The degradation in strength occurs through a displacement equivalent to exactly (i.e., $\mu = 1$) or double (i.e., $\mu = 2$) the yield displacement for limited and moderate ductilities, respectively, as shown in Fig. 2.

The above equations show that the shear capacity of the four mechanisms, and hence the total shear capacity, as well as the flexural capacity of a rectangular RC squat column rehabilitated with FRP can be expressed in terms of the cross-sectional area of the column b.t.

Specimen	V_{c}	V_p	Vs	V_{f}	$\max(V_s + V_f) = 0.66 \sqrt{\beta f'} hd$	V _n	$V_{\mu=4}$	$V_{\mu=6}$	V _r
C0.5-6D13 ^[1]	33	20		8	70	61	26	8	6
CS20-3-15 ^[2]	67	26	19	51	137	163	100	69	52
RS-R1 ^[3]	452	78	71	448	1095	1048	695	519	389
SC1 ^[4]	90	83	335		372	508	394	336	252
SC2 ^[4]	162	83	336	344	438	684	520	438	329
SC3 ^[4]	43	83	71	344	357	483	399	357	268
SC1R ^[5]	153	83	335	296	414	650	492	414	310
SC2R ^[5]	146	83	335	153	395	625	472	395	297
SC1U ^[5]	162	83	365	229	439	685	521	439	329
SC3R ^[5]	40	83	71	296	334	457	375	334	251
SPHI-1 ^[6]	420	63	79	977	1038	1520	1242	1089	726

Table 2 Shear strength contributions from different mechanisms and shear capacity envelope (in kN)

^[1] Yoshimura *et al.* (2000); ^[2] Ye *et al.* (2002); ^[3] Haroun *et al.* (2002); ^[4] Ghobarah and Galal (2004); ^[5] Galal *et al.* (2005); ^[6] Seible *et al.* (1997)

3. Verifications

The main objective of the model is to analytically predict the backbone lateral force-displacement ductility response of FRP-rehabilitated squat RC columns. This objective can be assessed by examining the accuracy of the model in predicting the peak lateral resistance and the post-peak strength degradation of the squat column. Thus, in order to verify the accuracy of the model, available lateral force-deformation experimental results of eleven squat rectangular RC columns are compared with the analytical predictions of the analytical method.

Table 1 contains the parameters used to calculate the confinement effectiveness as provided by steel ties and FRP. The geometrical properties $(b, t, M/Vt, H, k_p)$, axial load (P), steel reinforcement properties (s, A_v) , FRP wraps thickness $(2t_f)$, and the mechanical properties of the materials (f'_{co}, f_{yv}, E_f) of the squat columns shown in the table are retrieved from the experimental data reported by the researchers who conducted the tests (as shown in Table 1). Table 2 contains the shear strength contribution from the various mechanisms calculated using Eqs. (3) to (6), and the total shear capacity envelope at different displacement ductility levels as shown in Fig. 2 and given in Eqs. (1b) and (2a,b,c).

Fig. 4 shows the analytical flexure and shear envelopes and the experimental response for the eleven columns. The column's analytical backbone behaviour will follow the flexure envelope until it reaches the shear capacity envelope. Subsequently, the column loses its lateral force capacity following a negative shear stiffness, until the residual shear capacity V_r is reached. Brittle or moderate-ductile behaviours are expected if the column's shear capacity is reached at displacement ductility levels less than 2 or between 2 and 6, respectively. A ductile behaviour is expected if the column's shear capacity is higher than its flexural envelope at high displacement ductility levels. In Fig. 4, the analytical lateral force-displacement ductility envelope for the eleven columns is shown as dashed curve.



Fig. 4 Comparison of analytical and experimental behaviour for the eleven test columns: ^[1] Yoshimura *et al.* (2000); ^[2] Ye *et al.* (2002); ^[3] Haroun *et al.* (2002); ^[4] Ghobarah and Galal (2004)

From the figure it can be seen that the analytical model was capable of identifying different types of behaviour of rectangular RC squat columns retrofitted with FRP. Specimens SC2 (Ghobarah and Galal 2004) and SPHI-1 (Seible *et al.* 1997) were expected to have ductile behaviour up to displacement ductility levels higher than 6. The other nine specimens experienced moderate ductility behaviour with ductility levels ranged from $\mu = 2$ to $\mu = 6$.



Fig. 4 Comparison of analytical and experimental behaviour for the eleven test columns: ^[5] Galal *et al.* (2005); ^[6] Seible *et al.* (1997) (continued)

4. Parametric study

In the current application, the displacement ductility capacity, μ_{Δ} , is defined as the ductility when the flexural capacity envelope intersects the shear capacity envelope (Fig. 2). This represents the point of formation of local mechanism, which is followed by degradation in the lateral resistance of the column.

From the formulation of the shear and flexural capacities' envelopes, it is shown that the displacement ductility capacity μ_{Δ} of rectangular RC squat columns confined with FRP depends on ten variables that control the flexure and shear envelopes' capacities. These variables are: f_c' , f_{yy} , f_y , γ , M/Vt, ρ_y , λ_F , ρ_t , K_e , and ζ .

In order to study the effect of the column's characteristics on the displacement ductility of FRPconfined RC rectangular squat columns, the displacement ductility capacity (μ_{Δ}) – FRP content (λ_F) relationship is considered. A typical RC rectangular squat column with specific properties as shown in Table 3 is considered. The properties were chosen to represent an existing RC rectangular squat column that is designed according to pre-1970 codes (ACI 1968).

The covered range of FRP content, intended to be used in the column rehabilitation, is up to $\lambda_F = 2$, which is equivalent to 6 layers of Carbon FRP (with laminate thickness = 0.165 mm, b = 300 mm, $E_f = 240$ GPa, $f_{yy} = 400$ MPa), or 14 layers of Glass FRP (with laminate thickness = 0.25 mm, b = 300 mm, $E_f = 70$ GPa, $f_{yy} = 400$ MPa).

Fig. 5 shows the effect of the shear span-to-depth ratio M/Vt on $\mu_{\Delta}-\lambda_F$ relationship. From the figure, it can be seen that reducing the shear span-to-depth ratio of squat RC columns reduces its displacement ductility capacity and increases the required FRP content of the FRP confinement jacket. On the other hand, for a targeted displacement ductility capacity $\mu_{\Delta} = 5$ for the studied column, decreasing the shear span-to-depth ratio from 2.5 to 1.0 increases the required FRP content parameter from $\lambda_F \approx 0$ to $\lambda_F = 0.68$ and 1.0 for anchored and unanchored FRP jackets, respectively. This emphasizes the importance of identifying the expected shear span of a RC column due to its

				-		-	-		
Parameter	f_c'	f_{yv}	f_y	γ	M/Vt	$ ho_v$	$ ho_t$	K_e	ζ
	MPa	MPa	MPa	(d/t)		(A_v/bs)	(A_s/bt)		$P/f_c'bt$
Value	40	400	400	0.7	1.5	0.4%	1%	0.6	0.2

Table 3 Properties of the studied rectangular RC squat column for parametric study



Fig. 5 Effect of shear span-to-depth ratio on the displacement ductility-FRP content relationship for anchored and unanchored FRP-rehabilitated squat RC columns

impact on the required thickness of the FRP jacket, especially in the case of captive columns (for example, those created due to window openings in partially masonry-infilled frames and at the top and bottom ends of the columns in the case of masonry-infilled RC frames).

Fig. 6 shows the effect of transverse reinforcement content ρ_v on μ_{Δ} - λ_F relationship. As expected, increasing the transverse reinforcement content increases the ductility and reduces the required FRP content. This is attributed to the increase in the contribution of the transverse reinforcement mechanism and thus the shear capacity. For the studied column, anchoring the FRP jacket to the column reduces the required content of FRP by about 35% for a transverse reinforcement content range of 0.2% to 0.8%.

Fig. 7 shows the effect of the confinement coefficient K_e on $\mu_{\Delta}-\lambda_F$ relationship. For the studied column, increasing the confinement coefficient increases the displacement ductility capacity and



Fig. 6 Effect of transverse reinforcement content on the displacement ductility-FRP content relationship for anchored and unanchored FRP-rehabilitated squat RC columns



Fig. 7 Effect of the confinement coefficient K_e on the displacement ductility-FRP content relationship for anchored and unanchored FRP-rehabilitated squat RC columns



Fig. 8 Comparison between analytical and experimental lateral force-displacement ductility relationship for specimens C0.5-6D13^[1] (Yoshimura *et al.* 2000) and SPHI-1^[6] (Seible *et al.* 1997)

reduces the required FRP content. This effect diminishes as the targeted displacement ductility capacity reaches $\mu_{\Delta} = 6$ (due to the assumption of constant shear capacity beyond $\mu = 6$). On the other hand, a column with anchored FRP jacket will have higher displacement ductility capacity and less required FRP content compared to unanchored one. The effect of changing the transverse reinforcement content ρ_v to 0.2% and 0.4% for $K_e = 0.2$ is shown on the same figure, where it can be seen that decreasing ρ_v decreases the displacement ductility capacity and increases the required content of FRP.

5. Applications

The studied analytical model predicts the monotonic lateral force-displacement ductility backbone relationship of squat rectangular RC columns confined with FRP. Sometimes, the displacement ductility levels -and their corresponding shear capacities- are not reached experimentally due to the early failure of the specimen. Failure of the specimen could result from the formation of a mechanism due to the rupture of longitudinal bars, transverse reinforcement or FRP, crushing of concrete, pullout of longitudinal tensile bars, buckling of longitudinal compression bars, and the strength and stiffness degradation due to cyclic loading. Such detailed behaviour could be captured using nonlinear hysteretic models.

There are different techniques for modeling the nonlinear behaviour of RC elements. Among these, the lumped plasticity macromodels assume that failure occurs at the column ends due to the formation of plastic hinge. The main input data for such models is the backbone lateral force-displacement relationship, which could be predicted using the method evaluated earlier.

The predicted force-deformation relationships were used as input data for a lumped plasticity macromodel developed by Galal and Ghobarah (2003) to predict the hysteretic response of the test columns under constant axial load and increasing cyclic displacements. The macromodel accounts for axial force-moment and axial force-shear force interaction, bond-slip of tensile bars and buckling of compression bars. The model accommodates flexural response by quadri-linear force-deformation relationship and shear response by shear strength-deformation relationship. The displacement can be obtained by multiplying the displacement ductility by the yield displacement.

Fig. 8 shows the comparison between the analytical and experimental hysteretic forcedisplacement ductility relationship for specimens C0.5-6D13 (Yoshimura *et al.* 2000) and SPHI-1 (Seible *et al.* 1997) as an illustration. There is a good correlation between the analytical and experimental results, which indicates that the proposed approach provides reliable input data for the lumped plasticity macromodels.

6. Conclusions

A method for the calculation of the shear strength capacity-lateral displacement ductility relationship of squat rectangular RC columns jacketed with FRP is verified. The method provides formulas for determining the contribution of the concrete, steel, axial load, and FRP mechanisms to the total shear strength of the column. The total of these mechanisms varies with the increase in the lateral displacement ductility. The column's lateral force-displacement ductility backbone curve is predicted by combining the column's shear and flexural capacities. The analytical predictions were compared to the experimental behaviour of eleven test columns available in the literature. A parametric study on the effect of column characteristics on the displacement ductility capacity of a squat rectangular RC column confined with FRP is studied. Finally, the incorporation of the studied method in nonlinear modeling applications using lumped plasticity macromodels is evaluated. From the study, the following is concluded:

- 1- The analytical method was capable of predicting the lateral force-displacement ductility curve with good accuracy and was also capable of correctly describing the columns' expected type of behaviour to be either ductile, moderately ductile, or brittle.
- 2- For a specific FRP content, the displacement ductility capacity of an FRP-confined rectangular RC squat column increases with the increase of the column's shear span-to-depth ratio, or the increase of the transverse reinforcement content, or the increase of the confinement coefficient K_e value.
- 3- Nonlinear modeling using lumped plasticity macromodels with input data based on the predicted lateral force-displacement ductility backbone curve can reliably predict the mode of failure of the retrofitted RC squat columns.

There is limited experimental data available in the published literature that accounts for the postpeak shear degradation behaviour with moderate ductility of rectangular RC squat columns retrofitted with FRP. For this reason, further refinement of the model may be carried out when additional experimental data become available.

88

References

ACI (1968), "Manual of concrete practice", Part 2, Committee 318-1R-68, American Concrete Institute, Detroit.

- ACI (2002), "Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures", *Committee 440 Report 440.2R-02*, American Concrete Institute, Detroit.
- CSA (1994), "Design of concrete structures", *Standard A23.3*, Canadian Standards Association, Rexdale, Ontario.
- FEMA 273/274 (1997), "NEHRP guidelines for the seismic rehabilitation for buildings", *Federal Emergency Management Agency*, Washington D.C.
- FIB (2001), "Externally bonded FRP reinforcement for RC structures", *Technical Report Bulletin No. 14*, Fédération Internationale du Béton, France.
- Galal, K., Arafa, A. and Ghobarah, A. (2005), "Retrofit of RC square short columns", J. Eng. Struct., 27(5), 801-813.
- Galal, K. and Ghobarah, A. (2003), "Flexural and shear hysteretic behaviour of reinforced concrete columns with variable axial load", J. Eng. Struct., 25(11), 1353-1367.
- Ghobarah, A. and Galal, K. (2004), "Seismic rehabilitation of short rectangular RC columns", J. Earthq. Eng., **8**(1), 45-68.
- Haroun, M.A., Mosallam, A.S., Feng, M.Q. and Elsanadedy, H.M. (2002), "Composite jackets for the seismic retrofit and repair of bridge columns", 7th U.S. Nat. Conf. Earthquake Eng., Oakland, California.
- ISIS Canada (2001), "Strengthening reinforced concrete structures with externally-bonded fibre reinforced polymers (FRPs)", *Design Manual*, University of Manitoba.
- Kowalsky, M.J., Priestley, M.J.N. and Seible, F. (1999), "Shear and flexural behavior of lightweight concrete bridge columns in seismic regions", ACI Struct. J., 96(1), 136-148.
- Mander J.B., Priestley, M.J.N. and Park, R. (1988), "Theoretical stress-strain model for confined concrete", J. Struct. Eng., ASCE, 114(8), 1804-1826.
- NISEE (2006), http://nisee.berkeley.edu/elibrary, Images, Shear Column, Retrieved Jan. 2006.
- Priestley, M.J.N., Verma, R. and Xiao, Y. (1994), "Seismic shear strength of reinforced concrete columns", J. Struct. Eng., ASCE, 120(8), 2310-2329.
- Seible, F., Priestley, M.J.N., Hegemier, G.A. and Innamorato, D. (1997), "Seismic retrofit of RC columns with continuous carbon fibre jackets", J. Compos. Constr., ASCE, 1(2), 52-62.
- Vision 2000 committee (1995), "Performance based seismic engineering of buildings", Str. Eng. Assoc. of California (SEAOC), Sacramento, California.
- Ye, L., Yue, Q., Zhao, S. and Li, Q. (2002), "Shear strength of reinforced concrete columns strengthened with carbon-fiber-reinforced plastic sheet", J. Struct. Eng., 128(12), 1527-1534.
- Yoshimura, K., Kikuchi, K., Kuroki, M., Ozawa, K. and Masuda, Y. (2000), "Experimental study on seismic behavior of R/C short columns strengthened by carbon fiber sheets", *Composite and Hybrid Structures: Proc. of the 6th ASCCS Int. Conf. on Steel-Concrete Composite Structures*, Los Angeles, California, 927-934.