Structural Engineering and Mechanics, Vol. 24, No. 5 (2006) 621-639 621
DOI: http://dx.doi.org/10.12989/sem.2006.24.5.621

Elasto-plastic stability of circular cylindrical shells subjected to axial load, varying as a power function of time

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(Received January 23, 2006, Accepted June 28, 2006)

Abstract. Stability of a cylindrical shell subject to a uniform axial compression, which is a power function of time, is examined within the framework of small strain elasto-plasticity. The material of the shell is incompressible and the effect of the elastic unloading is considered. Initially, employing the infinitesimal elastic-plastic deformation theory, the fundamental relations and Donnell type stability equations for a cylindrical shell have been obtained. Then, employing Galerkin's method, those equations have been reduced to a time dependent differential equation with variable coefficient. Finally, for two initial conditions applying a Ritz type variational method, the critical static and dynamic axial loads, the corresponding wave numbers and dynamic factor have been found. Using those results, the effects of the variations of loading parameters and the variations of power of time in the axial load expression as well as the variations of the radius to thickness ratio on the critical parameters of the shells for two initial conditions are also elucidated. Comparing results with those in the literature validates the present analysis.

Keywords: elasto-plastic stability; cylindrical shell; time dependent compressive axial load; critical parameters.

1. Introduction

Plastic stability phenomena have provided during the last four decades some of the crucial test cases for the validity of metal plasticity theories. In many of the studies, the stability problem of shells subjected to dynamic loading is considered when the strain is below the proportionality limit.

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Although, the experiments show that elastic-plastic deformation occurs in dynamic loading condition (see Wolmir 1975). In the plastic deformation zone, the constitution of any particular thin-walled structure, made by plates and shells, is not recommendable before the stability and strength problems are studied. The plasticity theory provides more correct results for the same strength of the designs and constructions.

Plastic stability of circular cylindrical shell under different loading has been systematically studied since the late 1940s due to its importance in aerospace engineering. The investigations at this time have established the sound theory describing the buckling phenomenon. Numerous works about this problem can be found in the literature, covering theoretical and experimental studies of elastic and plastic buckling under axial compression, external pressure or end torsion. The most commonly used constitutive relations of plasticity for buckling problems are deformation and incremental theories (Hill 1983). In the present work the deformation theory is used for the description of the plastic behavior of the material. Il'ishin puts the mathematical and technical formulation of elasto-plastic bending theory of plates and shells forward (Il'ishin 1947, 1948). The validity of Il'ishin theory is repeatedly proved by the experimental evidence (Bojinski and Wolmir 1962, Lee 1962). Besides, Gerard (1956, 1957) and Batterman (1965) have been studied the most important investigations about plastic buckling of cylindrical shells. Elastic and elasto-plastic stability problems of cylindrical shells under axial load are widely discussed and analyzed (for details see corresponding literature, e.g., Hill 1983, Wolmir 1967, Korolyov 1971).

There are many worthy studies about the plastic buckling of plates and shells under constant loading as: (Storakers 1975, Sobel and Newman 1980, Tvergraad 1983, Tuğcu, 1991, Giezen *et al.* 1991, Li and Reid 1992, Ore and Durban 1992, Lın and Yeh 1994, Durban 1998, Durban and Zuckerman 1999, Yeh *et al.* 1999, Mao and Lu 2001, Wang *et al.* 2001, Kosel and Bremec 2004, Wang 2004). In some of these studies, the deformation theory is also utilized.

The subject of dynamic buckling of elasto-plastic cylindrical shells under axial impact has been studied in many investigations (Coppa and Nash 1962, Florence and Goodier 1968, Zimcik and Tennyson 1980, Lee 1981, Bajenov and Lomunov 1983, Lindberg 1987, Li *et al.* 1994, Yu *et al.* 1996, Lepik 1998, Karagizova and Jones 1992, 1995, 2000, 2002, Shevchenko and Piskin 2003, Wang and Tian 2003).

The stability problem of the shells under time dependent axial compressive load has been much less studied in contrast to the buckling under axial impact load. One of the most important studies about this subject is Wolmir (1975). Wolmir (1975) studied elasto-plastic stability of cylindrical panel, which has initial imperfection under time dependent axial compressive load by using the Runge-Kutta method. Jones and dos Reis (1980) studied on the dynamic buckling of a simple elastic-plastic model. The theoretical method predicts that dynamic plastic-elastic buckling governs the response for small imperfections, while dynamic instability occurs elastically for large imperfections. Furthermore the dynamic buckling load of a model with small imperfections is larger than the corresponding static buckling.

Besides, time dependent compressive loads vary not only linearly and periodically but also as power functions depending on time (Sofiyev 2002, 2003, 2005). But, one such problem, not considered till today, is the elasto-plastic stability of the shells under time dependent a-periodic axial loads.

In this study, the aim is to investigate stability of circular cylindrical shells subjected to time dependent axial compression described by a power law function-using the small elasto-plastic deformation theory and the Galerkin and Ritz type variational methods.

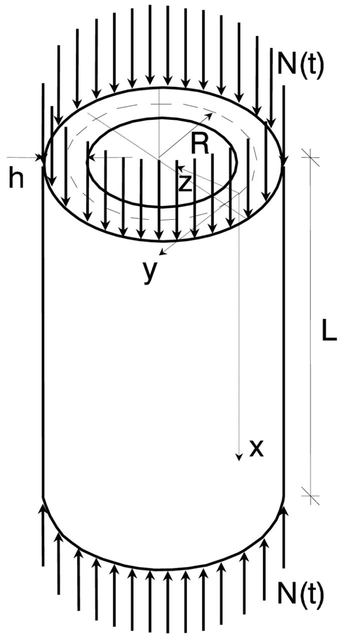

Fig. 1 Geometry and the coordinate system of a cylindrical shell

2. Fundamental relations

As shown in Fig. 1, we consider a cylindrical shell, with immovable hinged supports at the ends, length L, radius R and thickness h. The system of coordinates is selected in such a way that the origin is on the middle surface, the z axis is perpendicular to the middle surface of the shell, positive inwards, and the x and y axes are in the axial and tangential directions, respectively (Fig. 1).

The material of the shell is homogeneous isotropic, incompressible and the effect of the elastic unloading is considered. In agreement with the laws for the elasticity and plasticity of materials the stresses and strains are connected by the relations (Korolyov 1971):

$$\sigma_x = \frac{4}{3}E_s(\varepsilon_x + 0.5\varepsilon_y), \quad \sigma_y = \frac{4}{3}E_s(\varepsilon_y + 0.5\varepsilon_x), \quad \sigma_{xy} = \frac{2}{3}E_s\varepsilon_{xy} \tag{1}$$

where σ_x, σ_y, σ_{xy} and ε_x, ε_y, ε_{xy} are stress and strain components, respectively and $E_s = \sigma_i/\varepsilon_i$ is the secant modulus.

The equivalent stress σ_i and the equivalent strain ε_i are defined by

$$\sigma_i^2 = \sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\sigma_{xy}^2 \tag{2a}$$

$$\varepsilon_i^2 = \frac{4}{9}[\varepsilon_x^2 - \varepsilon_x\varepsilon_y + \varepsilon_y^2 + 2(\varepsilon_x + \varepsilon_y)^2 + 3\varepsilon_{xy}^2] \tag{2b}$$

For the purpose of buckling analysis one should recast further the stress-strain relations (1) into a variational form. Thus, Eq. (1) gives,

$$\delta s_x = E_s \delta \varepsilon_x - (E_s - E_t)\frac{s_x}{\sigma_i}\delta \varepsilon_i \tag{3}$$

$$\delta s_y = E_s \delta \varepsilon_y - (E_s - E_t)\frac{s_y}{\sigma_i}\delta \varepsilon_i \tag{4}$$

$$\delta s_{xy} = \delta \sigma_{xy} = \frac{2}{3}E_s \delta \varepsilon_{xy} - (E_s - E_t)\frac{s_{xy}}{\sigma_i}\delta \varepsilon_i \tag{5}$$

where $E_t = d\sigma_i/d\varepsilon_i$ is the tangent modulus, δ is the symbol of variation and the following substitutions are introduced:

$$s_x = \sigma_x - 0.5\sigma_y, \quad s_y = \sigma_y - 0.5\sigma_x, \quad s_{xy} = \sigma_{xy} \tag{6a-c}$$

$$\delta s_x = \delta\sigma_x - 0.5\delta\sigma_y, \quad \delta s_y = \delta\sigma_y - 0.5\delta\sigma_x \tag{6d-e}$$

E_s and E_t are defined of relation $\sigma_i = \phi(\varepsilon_i)$, which describes the plastic behavior of the material. The function $\sigma_i = \phi(\varepsilon_i)$ is invariant to the type of the stress state and may be determined with uniaxial tension/compression experiments (See Wolmir 1967).

The following expression is obtained from (3)-(5):

$$\Pi(\sigma, \varepsilon) = \frac{\Pi(\sigma, \delta s)}{E_t} \tag{7}$$

where σ and ε are stress and strain functions and the following definitions apply:

$$\Pi(\sigma, \delta s) = \Pi(s, \delta\sigma) = \sigma_x \delta s_x + \sigma_y \delta s_y + 3\sigma_{xy}\delta s_{xy} = E_t \sigma_i \delta \varepsilon_i \tag{8}$$

$$\Pi(\sigma, \delta\varepsilon) = \sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_{xy}\delta\varepsilon_{xy} = E_t \sigma_i \delta\varepsilon_i \tag{9}$$

The variations of strain components and curvatures of the shell are obtained, respectively, employing the following approximation:

$$(\delta\varepsilon_x, \delta\varepsilon_y, \delta\gamma_{xy}) = (\delta e_x + z\chi_x, \delta e_y + z\chi_y, 2\delta e_{xy} + 2z\chi_{xy}) \tag{10}$$

where $\delta e_x, \delta e_y, \delta e_{xy}$ are the variations of the strain components on the middle surface and $\chi_x, \chi_y, \chi_{xy}$ are the variations of the middle surface curvatures of the shell.

The coordinate of the surface that separates the regions of loading and unloading is obtained from the condition that the variation of the equivalent stress or equivalent strain is equal to zero:

$$\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_{xy}\delta\varepsilon_{xy} = 0 \tag{11a}$$

Substituting of (10) into Eq. (11a) the following expression is obtained:

$$z_0 = -\frac{\Pi(\sigma, \varepsilon)}{\Pi(\sigma, \chi)} \tag{11b}$$

where

$$\Pi(\sigma, \chi) = \sigma_x\chi_x + \sigma_y\chi_y + 2\sigma_{xy}\chi_{xy} \tag{12}$$

in which χ is the curvature function.

Substitution of (7) into (11b), a result in the following expression is obtained:

$$z_0 = -\frac{\Pi(\sigma, \delta s)}{E_t\Pi(\sigma, \chi)} \tag{13}$$

In the active deformation region or for loading region ($z > z_0$), when expression (13) is considered, the following relations can be used instead of (3)-(5):

$$\delta s_x = E_s\delta\varepsilon_x - \frac{E_s - E_t}{E_t}\frac{s_x\Pi(\sigma, \delta s)}{\sigma_i^2}\frac{z_0 - z}{z_0} \tag{14}$$

$$\delta s_y = E_s\delta\varepsilon_x - \frac{E_s - E_t}{E_t}\frac{s_x\Pi(\sigma, \delta s)}{\sigma_i^2}\frac{z_0 - z}{z_0} \tag{15}$$

$$\delta s_{xy} = \frac{2}{3}E_s\delta\varepsilon_{xy} - \frac{E_s - E_t}{E_t}\frac{s_x\Pi(\sigma, \delta s)}{\sigma_i^2}\frac{z_0 - z}{z_0} \tag{16}$$

In the case of passive deformation or for elastic unloading ($z < z_0$), the variation of the stress and strain relations are written in the form:

$$\delta s_x = E\delta\varepsilon_x, \quad \delta s_y = E\delta\varepsilon_y, \quad \delta s_{xy} = \frac{2}{3}E\delta\varepsilon_{xy} \tag{17}$$

where E is the elasticity modulus.

The variations of force and moment components are obtained after Wolmir (1967) as:

$$\delta N_x - 0.5\,\delta N_y = \int_{-h/2}^{h/2}\delta s_x dz, \quad \delta N_y - 0.5\,\delta N_x = \int_{-h/2}^{h/2}\delta s_y dz, \quad \delta N_{xy} = \int_{-h/2}^{h/2}\delta s_{xy} dz \tag{18}$$

$$\delta M_x - 0.5\,\delta M_y = \int_{-h/2}^{h/2}\delta s_x z dz, \quad \delta M_y - 0.5\,\delta M_x = \int_{-h/2}^{h/2}\delta s_y z dz, \quad \delta M_{xy} = \int_{-h/2}^{h/2}\delta s_{xy} z dz \tag{19}$$

In case of stability loss, certain part of the shell material passes through the irreversible state of plastic deformation. Thus at post-buckling state, one part of the shell is loaded plastically, while the other one undergoes elastic loading. Accordingly, each integral in (18) and (19), can be regarded as additive decomposition of two integrals. The integrand in the first one is integrated from $z = -h/2$ to $z = z_0$ by using (14)-(16) instead of δs_x, δs_y, δs_{xy}. In the second integral, the expression is integrated from $z = z_0$ to $z = h/2$ by using (17) instead of δs_x, δs_y, δs_{xy}. Besides, the expression (13) is substituted in (18) and expression (11) is substituted in (19).

After computations, the following expressions are obtained for the variation of the force and moment components:

$$\delta N_x = B_1(\delta e_x + 0.5\delta e_y) + D_1(\chi_x + 0.5\chi_y) - B_2\frac{\Pi(\sigma, \delta s)}{\sigma_i^2}\sigma_x \tag{20}$$

$$\delta N_y = B_1(\delta e_y + 0.5\delta e_x) + D_1(\chi_y + 0.5\chi_x) - B_2\frac{\Pi(\sigma, \delta s)}{\sigma_i^2}\sigma_y \tag{21}$$

$$\delta N_{xy} = 0.5B_1\delta e_{xy} + 0.5D_1\chi_{xy} - B_2\frac{\Pi(\sigma, \delta s)}{\sigma_i^2}\sigma_{xy} \tag{22}$$

$$\delta M_x = D_1(\delta e_x + 0.5\delta e_y) + C_2(\chi_x + 0.5\chi_y) - \tilde{C}_3\frac{\Pi(\sigma, \chi)}{\sigma_i^2}\sigma_x \tag{23}$$

$$\delta M_y = D_1(\delta e_y + 0.5\delta e_x) + C_2(\chi_y + 0.5\chi_x) - \tilde{C}_3\frac{\Pi(\sigma, \chi)}{\sigma_i^2}\sigma_y \tag{24}$$

$$\delta M_{xy} = 0.5D_1\delta e_{xy} + 0.5C_2\chi_{xy} - \tilde{C}_3\frac{\Pi(\sigma, \chi)}{\sigma_i^2}\sigma_{xy} \tag{25}$$

where the following definitions apply:

$$B_1 = \frac{2h}{3}[E + E_s - (E - E_s)\bar{z}_0], \quad B_2 = \frac{h}{4}\frac{E_s - E_t}{E_t}\frac{(1 + \bar{z}_0)^2}{\bar{z}_0} \tag{26a-b}$$

$$D_1 = \frac{h^2}{6}(E - E_s)(1 - \bar{z}_0^2), \quad C_2 = \frac{h^3}{18}[E + E_s - (E - E_s)\bar{z}_0^3] \tag{26c-d}$$

$$\tilde{C}_3 = \frac{h^3}{48}(E_s - E_t)(1 + \bar{z}_0)^2(2 - \bar{z}_0) \tag{26e}$$

in which $\bar{z}_0 = 2z_0/h$ is the dimensionless coordinate and satisfies inequality $0 \le \bar{z}_0 \le 1$.

The variations of force components are given in terms of the Airy stress function as follows:

$$(\delta N_x, \delta N_y, \delta N_{xy}) = h(\Phi_{,yy}, \Phi_{,xx}, -\Phi_{,xy}) \tag{27}$$

In the case, satisfies the following relation:

$$\Pi(\sigma, \delta s) = \Pi(s, \Phi) = s_x\frac{\partial^2\Phi}{\partial y^2} + s_y\frac{\partial^2\Phi}{\partial x^2} - 3s_{xy}\frac{\partial^2\Phi}{\partial x\partial y} \tag{28}$$

By removing the variation of strain components from (20)-(22) and expressions (27) are taken into consideration in the expressions obtained, the following expressions are obtained for the variations of strain components:

$$\delta e_x = \frac{4}{3}\frac{h}{B_1}\frac{\partial^2\Phi}{\partial y^2} - \frac{2}{3}\frac{h}{B_1}\frac{\partial^2\Phi}{\partial x^2} + \frac{D_1}{B_1}\frac{\partial^2 w}{\partial x^2} + \frac{4}{3}\frac{B_2}{B_1}\frac{\Pi(\sigma, \delta s)}{\sigma_i^2}(\sigma_x - 0.5\sigma_y) \tag{29}$$

$$\delta e_y = \frac{4}{3}\frac{h}{B_1}\frac{\partial^2\Phi}{\partial x^2} - \frac{2}{3}\frac{h}{B_1}\frac{\partial^2\Phi}{\partial y^2} + \frac{D_1}{B_1}\frac{\partial^2\Phi}{\partial y^2} + \frac{4}{3}\frac{B_2}{B_1}\frac{\Pi(\sigma, \delta s)}{\sigma_i^2}(\sigma_y - 0.5\sigma_x) \tag{30}$$

$$\delta e_{xy} = -2\frac{h}{B_1}\frac{\partial^2\Phi}{\partial x\partial y} + \frac{D_1}{B_1}\frac{\partial^2 w}{\partial x\partial y} + 2\frac{B_2}{B_1}\frac{\Pi(\sigma, \delta s)}{\sigma_i^2}\sigma_{xy} \tag{31}$$

Taking the expressions (14)-(16) into consideration in expression (13) and substituting the expressions obtained into (18) and removing $\delta e_x, \delta e_{xy}, \delta e_y$ and taking expressions (27) into consideration for variation of the moment components, the following expressions are obtained:

$$\delta M_x = h\frac{D_1}{B_1}\frac{\partial^2\Phi}{\partial y^2} - D_2\left(\frac{\partial^2 w}{\partial x^2} + 0.5\frac{\partial^2 w}{\partial y^2}\right) - D_3\frac{\Pi(\sigma, \chi)}{\sigma_i^2}\sigma_x \tag{32}$$

$$\delta M_y = h\frac{D_1}{B_1}\frac{\partial^2\Phi}{\partial x^2} - D_2\left(\frac{\partial^2 w}{\partial y^2} + 0.5\frac{\partial^2 w}{\partial x^2}\right) - D_3\frac{\Pi(\sigma, \chi)}{\sigma_i^2}\sigma_y \tag{33}$$

$$\delta M_{xy} = -h\frac{D_1}{B_1}\frac{\partial^2\Phi}{\partial x\partial y} - \frac{1}{2}D_2\frac{\partial^2 w}{\partial x\partial y} - D_3\frac{\Pi(\sigma, \delta\chi)}{\sigma_i^2}\sigma_{xy} \tag{34}$$

where the following definitions apply:

$$D_2 = C_2 - \frac{D_1^2}{B_1}, \quad D_3 = \tilde{C}_3 + \frac{D_1\tilde{B}_2}{B_1}, \quad \tilde{B}_2 = \frac{h^2}{8}(E_s - E_t)(1 + \bar{z}_0)^2 \tag{35a-c}$$

The linearized modified Donnell type dynamic stability and compatibility equations of the cylindrical shell are, respectively, as follows (Wolmir 1967):

$$\frac{\partial^2\delta M_x}{\partial x^2} + 2\frac{\partial^2\delta M_{xy}}{\partial x\partial y} + \frac{\partial^2\delta M_y}{\partial y^2} + \frac{\delta N_y}{R} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} + 2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y} = \rho h\frac{\partial^2 w}{\partial t^2} \tag{36}$$

$$\frac{\partial^2\delta e_x}{\partial y^2} + \frac{\partial^2\delta e_y}{\partial x^2} - 2\frac{\partial^2\delta e_{xy}}{\partial x\partial y} = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} \tag{37}$$

where N_x^0, N_y^0 and N_{xy}^0 are pre-buckling membrane forces, ρ is the density of the material and t is the time.

Finally, we combine Eqs. (27), (29)-(31) and (32)-(34) with (36)-(37) to obtain the compatibility and dynamic stability equations in the form:

$$\nabla^4\Phi + B_2'\begin{bmatrix}\frac{\sigma_x - 0.5\sigma_y}{\sigma_i^2}\frac{\partial^2\Pi(\sigma, \delta s)}{\partial y^2} + \frac{\sigma_y - 0.5\sigma_x}{\sigma_i^2}\frac{\partial^2\Pi(\sigma, \delta s)}{\partial x^2} \\ -3\frac{s_{xy}}{\sigma_i^2}\frac{\partial^2\Pi(\sigma, \delta s)}{\partial x\partial y}\end{bmatrix} = -\frac{3B_1'}{4R}\frac{\partial^2 w}{\partial x^2} \tag{38}$$

$$\nabla^4 w + \frac{D_3}{D_2}\frac{1}{\sigma_i^2}\left[\sigma_x\frac{\partial^2\Pi(\sigma, \chi)}{\partial x^2} + \sigma_{xy}\frac{\partial^2\Pi(\sigma, \chi)}{\partial x\partial y} + \sigma_y\frac{\partial^2\Pi(\sigma, \chi)}{\partial y^2}\right] -$$

$$-\frac{1}{D_2}\left(\frac{h}{R}\frac{\partial^2\Phi}{\partial x^2} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} + 2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y} - \rho h\frac{\partial^2 w}{\partial t^2}\right) = 0 \tag{39}$$

Inserting relations (12) and (28) in Eqs. (38) and (39), respectively can be obtained:

$$\nabla^4\Phi + B_2'\begin{bmatrix}\frac{s_x^2}{\sigma_i^2}\frac{\partial^4\Phi}{\partial y^4} + \frac{s_y^2}{\sigma_i^2}\frac{\partial^4\Phi}{\partial x^4} + \frac{2s_xs_y + 9s_{xy}^2}{\sigma_i^2}\frac{\partial^4\Phi}{\partial x^2\partial y^2} - \\ \frac{6s_xs_{xy}}{\sigma_i^2}\frac{\partial^4\Phi}{\partial x\partial y^3} - \frac{6s_ys_{xy}}{\sigma_i^2}\frac{\partial^4\Phi}{\partial x^3\partial y}\end{bmatrix} = -\frac{3B_1'}{4R}\frac{\partial^2 w}{\partial x^2} \tag{40}$$

$$\begin{aligned}\nabla^4 w - \frac{D_3}{D_2}\left[\frac{\sigma_x^2}{\sigma_i^2}\frac{\partial^4 w}{\partial x^4} + \frac{2\sigma_x\sigma_y + 4\sigma_{xy}^2}{\sigma_i^2}\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\sigma_y^2}{\sigma_i^2}\frac{\partial^4 w}{\partial y^4}\right] - \\ -\frac{D_3}{D_2}\left[\frac{3\sigma_{xy}\sigma_x}{\sigma_i^2}\frac{\partial^4 w}{\partial x^3\partial y} + \frac{3\sigma_{xy}\sigma_y}{\sigma_i^2}\frac{\partial^4 w}{\partial x\partial y^3}\right] - \\ -\frac{1}{D_2}\left(\frac{h}{R}\frac{\partial^2\Phi}{\partial x^2} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} + 2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y} - \rho h\frac{\partial^2 w}{\partial t^2}\right) = 0\end{aligned} \tag{41}$$

where the following definitions apply:

$$B_1' = \frac{B_1}{h} = \frac{2}{3}[E + E_s - (E - E_s)\bar{z}_0], \quad B_2' = \frac{B_2}{h} = \frac{1}{4}\frac{E_s - E_t}{E_t}\frac{(1+\bar{z}_0)^2}{\bar{z}_0} \tag{42a-b}$$

$$D_2 = C_2 - \frac{D_1^2}{B_1}, \quad D_3 = \tilde{C}_3 + \frac{D_1\tilde{B}_2}{B_1} \tag{42c-d}$$

The cylindrical shell subjected to axial compression,

$$\sigma_x = \sigma_i, \quad \sigma_{xy} = 0, \quad \sigma_y = 0 \tag{43}$$

and axial compressive load varying as a power function of time in the form (Sofiyev 2005):

$$N_x^0 = \sigma_x h = -(T_1 + T_0 t^\alpha) \tag{44}$$

where T_1 is the static axial load, T_0 is the axial loading parameter and $\alpha \geq 1$ is a positive whole number power which express the time dependence of the axial compressive load.

Inserting expressions (43) and (44) in Eqs. (40) and (41) can be obtained:

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}\end{bmatrix}\begin{pmatrix} w \\ \Phi\end{pmatrix} = \begin{pmatrix} 0 \\ 0\end{pmatrix} \tag{45}$$

where the following definitions apply:

$$L_{11} = (D_2 - D_3)\frac{\partial^4}{\partial x^4} + 2D_2\frac{\partial^4}{\partial x^2\partial y^2} + D_2\frac{\partial^4}{\partial y^4} + (T_1 + T_0t^\alpha)\frac{\partial^2}{\partial x^2} + \rho h\frac{\partial^2}{\partial t^2} \tag{46a}$$

$$L_{12} = -\frac{h}{R}\frac{\partial^2}{\partial x^2} \tag{46b}$$

$$L_{21} = \frac{3B_1'}{4R}\frac{\partial^2}{\partial x^2} \tag{46c}$$

$$L_{22} = (1+0.25B_2')\frac{\partial^4}{\partial x^4} + (2-B_2')\frac{\partial^4}{\partial x^2 \partial y^2} + (1+B_2')\frac{\partial^4}{\partial y^4} \tag{46d}$$

Should no effect of elastic unloading take place the differential operators (46) turns into the following form:

$$\tilde{L}_{11} = \frac{E_s h^3}{9}\left[\left(\frac{1}{4}+\frac{3\varphi_{ts}}{4}\right)\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right] + (T_1 + T_0 t^{\alpha})\frac{\partial^2}{\partial x^2} + \rho h\frac{\partial^2}{\partial t^2} \tag{47a}$$

$$\tilde{L}_{12} = -\frac{h}{R}\frac{\partial^2}{\partial x^2} \tag{47b}$$

$$\tilde{L}_{21} = \frac{E_s}{R}\frac{\partial^2}{\partial x^2} \tag{47c}$$

$$\tilde{L}_{22} = \left(\frac{3}{4}+\frac{1}{4\varphi_{ts}}\right)\frac{\partial^4}{\partial x^4} + \left(3-\frac{1}{\varphi_{ts}}\right)\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{1}{\varphi_{ts}}\frac{\partial^4}{\partial y^4} \tag{47d}$$

where the following definition applies:

$$\varphi_{ts} = \frac{E_t}{E_s} \tag{48}$$

3. Solution of the governing equations

Consider a cylindrical shell with simply supported edge conditions. The solutions for Eq. (45) take the following form [37]:

$$w = \xi(t)\sin\frac{m_1 x}{R}\cos\frac{ny}{R}, \quad \Phi = \zeta(t)\sin\frac{m_1 x}{R}\cos\frac{ny}{R} \tag{49}$$

where $m_1 = m\pi R/L$, m is the half wave length in the direction of the x-axis, n is the wave number in the direction of the y-axis, $\xi(t)$ and $\zeta(t)$ are the time dependent amplitudes.

Substituting expressions (49) in the equation set (45) and applying Galerkin's method in the ranges $0 \le x \le L$ and $0 \le y \le 2\pi R$ and eliminating $\zeta(t)$, the following differential equation is obtained:

$$\frac{d^2\xi(\tau)}{d\tau^2} + \frac{t_{cr}^2}{\rho h R^4}\left\{\begin{array}{l} (D_2 - D_3)m_1^4 + 2D_2 m_1^2 n^2 + D_2 n^4 - R^2(T_1 + T_0\tau^{\alpha}t_{cr}^{\alpha})m_1^2 + \\ + \dfrac{3B_1' h R^2}{4}\dfrac{m_1^4}{[(1+0.25B_2')m_1^4 + (2-B_2')m_1^2 n^2 + (1+B_2')n^4]} \end{array}\right\}\xi(\tau) = 0 \tag{50}$$

where $t = \tau t_{cr}$, in which t_{cr} is the critical time and the dimensionless time parameter τ satisfies $0 \le \tau \le 1$.

Eq. (50) is solved for two initial conditions (Sachenkov and Baktieva 1978, Sofiyev 2005):

1) In the first approximation, the function satisfying the initial conditions $\xi_1(0) = \xi_{1,\tau}(0) = 0$ is in the following:

$$\xi(\tau) = A_1\xi_1(\tau) = A_1 e^{p\tau}\tau^2[(p+3)/(p+2)-\tau] \tag{51}$$

2) Furthermore, the curve (ξ_2, τ) has the maximum when $\tau = 1$, so in the first approximation the function satisfying the initial conditions $\xi_2(0) = \xi_{2,\tau}(1) = 0$ is in the following:

$$\xi(\tau) = A_2\xi_2(\tau) = A_2 e^{p\tau}\tau[(p+2)/(p+1)-\tau] \tag{52}$$

where the displacement amplitude $A_j (j = 1, 2)$ is found from the condition of transition to the static condition. The values of p will be determined numerically in Section 4.

Under these circumstances, displacement-time curve $(\xi - \tau)$ possesses two different characteristic regions. In the first region the inertia force acts opposite to the axial load, whereas, in the second region it changes sign at some point and enhances the axial load from that point on. Consequently, displacement amplitude ξ goes to infinity and stability loss occurs. The time at which stability loss occurred is called as critical time (t_{cr}) and the corresponding load is called dynamic critical axial load.

Applying the Ritz type variational method to Eq. (50), i.e., multiplying it by $d\xi(\tau)/d\tau$ then after integration, the following equation is obtained:

$$\left[\frac{d\xi(\tau)}{d\tau}\right]^2 + \Lambda_1[\xi(\tau)]^2 - 2\Lambda_2\int \xi(\tau)\frac{d\xi}{d\tau}\tau^\alpha d\tau = C_0 \tag{53}$$

where C_0 is the integration constant and it is assumed that the initial conditions which are taken up into consideration are equal to zero. Besides, in any points of the interval $0 < \tau < 1$, $d\xi(\tau)/d\tau$ is not equal to zero and the following definitions apply:

$$\Lambda_1 = \frac{t_{cr}^2}{\rho h R^4}\left\{\begin{array}{c}(D_2 - D_3)m_1^4 + 2D_2 m_1^2 n^2 + D_2 n^4 - R^2 T_1 m_1^2 + \\ + \dfrac{3B_1' h R^2}{4}\dfrac{m_1^4}{[(1+0.25B_2')m_1^4 + (2-B_2')m_1^2 n^2 + (1+B_2')n^4]}\end{array}\right\} \tag{54a}$$

$$\Lambda_2 = \frac{T_0 t_{cr}^{2+\alpha} m_1^2}{\rho h R} \tag{54b}$$

Available experimental data (see Lee 1962, Batterman 1965, Sobel and Newman 1980) is dominated by the axially symmetric mode (see Ore and Durban 1992), by taking this factor into consideration and substituting Eqs. (51) and (52) in (53), then after integration in $0 \le \tau \le 1$, for the Lagrange-Hamilton type functional the following expression is obtained:

$$\Gamma = A_1\left\{\frac{\Phi_1 \rho h R^4}{t_{cr}^2 m_1^2} - \Phi_2 T_0 t_{cr}^\alpha R^2 + \Phi_0\left[(D_2 - D_3)m_1^2 + \frac{0.75B_1' h R^2}{(1+0.25B_2')}\frac{1}{m_1^2} - T_1 R^2\right]\right\} \tag{55}$$

where

$$\Phi_0 = \int_0^1 [\xi(\tau)]^2 d\tau, \quad \Phi_1 = \int_0^1 \left[\frac{d\xi(\tau)}{d\tau}\right]^2 d\tau, \quad \Phi_2 = 2\int_0^1\int_0^\tau \eta^\alpha \frac{d\xi(\tau)}{d\tau}\xi(\eta)d\eta d\tau \tag{56a-c}$$

During an infinite time, there may be no agreement in the work done by axial force and inertia force and the minimum value of potential energy. According to this, being the minimum condition in respect of the unknown amplitude $A_j (j = 1, 2)$ of the functional Γ must support being the minimum condition in respect of wave number m_1^2 of the functional Γ. These two conditions give the following two algebraically equations dependent on t_{cr} and m_1^2:

$$\frac{\partial\Gamma}{\partial A_j} = \frac{\Phi_1 \rho h R^4}{t_{cr}^2 m_1^2} - \Phi_2 T_0 t_{cr}^\alpha R^2 + \Phi_0\left[(D_2 - D_3)m_1^2 + \frac{0.75B_1' hR^2}{(1 + 0.25B_2')}\frac{1}{m_1^2} - T_1 R^2\right] \tag{57}$$

$$\frac{\partial\Gamma}{\partial m_1^2} = \Phi_0\left[(D_2 - D_3) - \frac{0.75B_1' hR^2}{(1 + 0.25B_2')}\frac{1}{m_1^4}\right] - \frac{\Phi_1 \rho h R^4}{t_{cr}^2 m_1^4} = 0 \tag{58}$$

From Eq. (58), the following equation is obtained:

$$\frac{\Phi_1 \rho h R^4}{t_{cr}^2 m_1^2} = \Phi_0\left[(D_2 - D_3)m_1^2 - \frac{0.75B_1' hR^2}{(1 + 0.25B_2')}\frac{1}{m_1^2}\right] \tag{59}$$

Substituting expression (59) in (57), the following expression is obtained:

$$T_0 t_{cr}^\alpha = \frac{\Phi_0}{\Phi_2}\left[\frac{2(D_2 - D_3)}{R^2}m_1^2 - T_1\right] \tag{60}$$

For $T_1 = 0$ eliminating t_{cr} from Eqs. (57) and (60) one gets

$$m_1^{4(1+\alpha)/\alpha} - \delta_1 m_1^{4/\alpha} - \delta_2 = 0 \tag{61}$$

where the following definitions apply:

$$\delta_1 = \frac{R^2}{(D_2 - D_3)}\frac{0.75B_1' h}{(1 + 0.25B_2')} \tag{62a}$$

$$\delta_2(\alpha) = \frac{(0.5T_0)^{2/\alpha}\Phi_2^{2/\alpha}(\alpha)\Phi_1(\alpha)\rho h R^{(4\alpha+4)/\alpha}}{[(D_2 - D_3)\Phi_0(\alpha)]^{(2+\alpha)/\alpha}} \tag{62b}$$

For the static condition $(t_{cr} \to \infty, T_0 \to 0)$ the wave number corresponding to the critical axial load is found as

$$m_{1st} = \left[\frac{0.75B_1' hR^2}{(1 + 0.25B_2')}\frac{1}{D_2 - D_3}\right]^{1/4} \tag{63}$$

Substituting (63) in (60) and replacing $T_0 t_{cr}^\alpha \Phi_2/(h\Phi_0)$ by N_{crs}, the static critical axial load is found as

$$N_{crs} = \frac{1}{R}\left[\frac{3B_1'(D_2 - D_3)}{(1 + 0.25B_2')h}\right]^{1/2} \tag{64}$$

Should no effect of elastic unloading take place, expression (64) turns into the following form:

$$N_{crs} = \frac{2}{3}\frac{h}{R}(E_s E_t)^{1/2} \tag{65}$$

This expression is firstly obtained in Gerard (1957).

Eq. (61) solving numerically for varying α value, the value obtained is substituted within the equation, obtained by minimizing the energy functional with respect to the unknown coefficient A_j, the dynamic critical axial load $N_{crd}(\alpha)$ is found. The dynamic factor is found from the ratio of the dynamic critical axial load to the static critical axial load:

$$K_d(\alpha) = N_{crd}(\alpha)/N_{crs} \tag{66}$$

where the following definitions apply:

$$N_{crd}(\alpha) = N_0 t_{cr}^{\alpha} \tag{67}$$

$$N_0 = T_0/h \tag{68}$$

4. Numerical results

For validating the analysis, the analytical results are compared with the theoretical and experimental results. In comparison, for finding the values of E_s secant modulus and E_t tangent modulus, the method of Batterman (1965), Ore and Durban (1992) and Wang *et al.* (2001) is applied. The examples presented later were calculated with the Ramberg-Osgood elasto-plastic characteristic:

$$\varepsilon = \frac{\sigma_i}{E}\left[1 + \frac{3}{7}\left(\frac{\sigma_i}{\sigma_y}\right)^{N-1}\right] \tag{69}$$

or

$$\varepsilon = \frac{\sigma_i}{E} + K\left(\frac{\sigma_i}{E}\right)^{N} \tag{70}$$

where σ_y is yield stress, ε is the total plastic strain, (N, K) are material parameters and the following definition applies:

$$K = \frac{3}{7}\left(\frac{E}{\sigma_y}\right)^{N-1} \tag{71}$$

In Table 1 the experimental results of Lee (1962) and the calculated results with the present study are compared. The tests were performed with simple support cylindrical shells, made of Al 3003-0 with the material parameters $E = 7.0 \times 10^{10}$ (Pa), $N = 4.1$, $\sigma_y = 2.362 \times 10^7$ (Pa) or $K = 2.4811868 \times 10^{10}$. The elastic Poisson's ratio is $\nu = 0.32$. The three shells were also analyzed for axisymmetric

Table 1 Comparison of the critical static stress (MPa) with experimental and analytical results

Geometry of the shell		Lee (1962)	Ore and Durban (1992)		Mao and Lu (2001)		Present study
R/h	*L/R*	Experimental study	Deform. theory	Flow theory	Deform. theory	Flow theory	Deform. theory
9.36	4.21	96.87	88.49	162.33	89.71	165.46	88.21
19.38	4.10	78.60	74.09	122.03	74.87	124.25	72.68
29.16	4.06	64.74	67.06	103.98	67.70	106.0	64.91

Table 2 Comparisons of the dynamic factor with experimental and numerical results for elastic stability of shell under the axial compressive loads varying as a linear function of time

N_0 (MPa/sec)	Experim. study Agamirov (1990)	Numerical study Agamirov (1990)	Present study
		$K_d(1)$	
7×10^4	1.25	1.18	1.27
9×10^4	1.31	1.21	1.32
11×10^4	-	1.25	1.36
12.7×10^4	1.394	-	1.398

buckling by using deformation and flow theories of plasticity in Ore and Durban (1992) and Mao and Lu (2001). Their results are also listed in Table 1. The comparison shows that the deformation theory gives good results but the flow theory predicts much too high critical static loads. Their results obtained by using the deformation theory correspond well with those from the present study.

In addition to validate the analysis, the results of the elastic stability for cylindrical shells under the axial compressive loads varying as a linear function of time are compared with the experimental and numerical results of Agamirov (1990) (see Table 2). The comparisons were carried out for the following material properties and shell parameters: $E = 7.75 \times 10^4$ MPa, $\nu = 0.3$, $L/R = 2.2$, $R/h = 180$. The comparisons show that the present results correspond well to those in the literature.

In Tables 3 and 4, calculations were performed for simply supported Al 3003-0 and ST 304 shells. Al 3003-0 shells characterized by Eq. (69) with the material parameters: $E = 7.0 \times 10^{10}$ Pa, $\nu = 0.32$, $\rho = 2.77 \times 10^3$ kg/m^3, $K = 2.4811868 \times 10^{10}$ and $N = 4.1$, which are the same as those in the papers of Lee (1962), and Ore and Durban (1992). ST 304 shells characterized by Eq. (69) with the material parameters: $E = 2.2147 \times 10^{11}$ Pa, $\nu = 0.27$, $\rho = 8.5888 \times 10^3$ kg/m^3, $K = 4.74 \times 10^{23}$ and $N = 8.64$, which are the same as those in the paper of Sobel and Newman (1980). In all computations, it is assumed that $\bar{z}_0 = 1$, i.e., the effect of the elastic unloading is not taken into consideration.

In Table 3 are given the variations of the dynamic critical load and dynamic factor for the elastic and elasto-plastic stability of cylindrical shells with radius to thickness ratios. As in the elastic stability, the value of dynamic critical load is higher than the value of the static critical load in elasto-plastic stability. In the elasto-plastic stability, the values of the dynamic critical load are considerably lower than the corresponding dynamic critical load in the elastic stability. With an increase of the ratio *R/h*, the values of the dynamic critical load in the elastic and elasto-plastic stability of the shell decrease, however, the values of the dynamic factor increase. When the ratio of

Table 3 Variations of the dynamic critical load and dynamic factor for elastic and elasto-plastic stability of cylindrical shells with R/h ($T_0 = 0.5 \times 10^9$ Pa × m/sec, $R/L = 0.25$)

	Elastic stability	Elasto-plastic stability	Elastic stability	Elasto-plastic stability
	$N_{crd}(1)$ (MPa)		K_d	
	$\xi_1(0)=0,\ \xi_{1,\tau}(0)=0,\ \xi_1(\tau)=e^{p\tau}\tau^2\ [(p+3)/(p+2)-\tau]$			
R/h	Al 3003-0			
50	883.02($p=65$)	89.48($p=5$)	1.042	1.612
75	605.67($p=38$)	87.51($p=4$)	1.072	1.785
100	468.47($p=26$)	86.51($p=3$)	1.106	1.937
R/h	ST 304			
50	2708($p=138$)	245.74($p=11$)	1.020	1.270
75	1831($p=80$)	236.34($p=9$)	1.034	1.335
100	1394($p=55$)	229.56($p=7$)	1.050	1.392
	$\xi_2(0)=0,\ \xi_{2,\tau}(1)=0,\ \xi_2(\tau)=e^{p\tau}\tau\ [(p+2)/(p+1)-\tau]$			
R/h	Al 3003-0			
50	883.02($p=66$)	89.50($p=6$)	1.042	1.612
75	605.68($p=39$)	87.48($p=5$)	1.072	1.785
100	468.48($p=27$)	86.65($p=4$)	1.106	1.941
R/h	ST 304			
50	2709($p=138$)	246.17($p=13$)	1.020	1.272
75	1831($p=79$)	236.37($p=10$)	1.034	1.335
100	1394($p=56$)	229.79($p=9$)	1.050	1.393

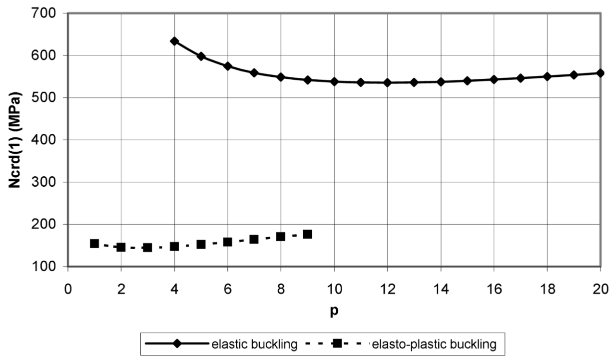

Fig. 2 Variations of the dynamic axial loads with p for elastic and elasto-plastic stability of shells ($T_0 = 2 \times 10^9$ (N/m × s); $R/L = 0.3$, $R/h = 100$)

R/h is increased, the values of the dynamic critical load are decreased in a more acute way in the elastic stability, but this decrease is less in the elasto-plastic stability. Furthermore, when the ratio of R/h is increased, the values of the dynamic critical factors are increased more slowly in the elastic stability, but this increase is more important in the elasto-plastic stability. The increase of the ratio R/h affects more the dynamic factor in the elasto-plastic stability and this effect is even increased in aluminum shells. For both approach functions, convenient critical parameters get equal values, approximately.

In Fig. 2, variations of the values of dynamic critical axial load for elastic and elasto-plastic stability of AL 3003-0 shells under the axial compressive loads varying as a linear function of time corresponding to varying values of p for the approximation function $\xi_1(\tau)$ are given. Dynamic critical axial load values are minimal according to the wave numbers and versus to different values of p. The minimum point of the curve is selected the minimum value of dynamic critical axial load according to p.

Table 4 shows the variation of the critical parameters for elastic and elasto-plastic stability of Al 3003-0 and ST 304 shells with different power of time α. The numerical analysis for the ST 304 and AL 3003-0 shells show that the loading parameter varies approximately by the following values to become the loading dynamic:

a) When the axial load varies linearly depending on time (in N/m × s), it must be in $10^8 \le T_0 < 10^{11}$, b) When the axial load varies quadratic depending on time (in N/m × s^2), it must be in $10^{10} \le T_0 \le 10^{14}$, c). When the axial load varies cubically depending on time (in N/m × s^3), it must be in $10^{13} \le T_0 \le 10^{18}$.

Table 4 The variation of the critical parameters for elastic and elasto-plastic stability of Al 3003-0 and ST 304 shells with different power of time ($R/h = 75$, $R/L = 0.25$)

	$\xi_2(0)=0,\ \xi_{2,\tau}(1)=0,\ \xi_2(\tau)=e^{p\tau}\tau\ [(p+2)/(p+1)-\tau]$								
	$\alpha = 1$			$\alpha = 2$			$\alpha = 3$		
	$T_0 \times 10^{-8}$N/(m × sec)			$T_0 \times 10^{-11}$N/(m × sec^2)			$T_0 \times 10^{-15}$N/(m × sec^3)		
	5	50	500	5	50	500	5	50	500
	Aluminum shells (Al 3003-0)								
	Elastic stability								
$N_{crd}(\alpha) \times 10^{-6}$ (Pa)	605.68 ($p = 39$)	751.29 ($p = 10$)	1378 ($p = 3$)	696.22 ($p = 25$)	855.12 ($p = 13$)	1219.55 ($p = 7$)	952.1 ($p = 12$)	1248.1 ($p = 11$)	1794.5 ($p = 8$)
$K_d(\alpha)$	1.072	1.333	2.439	1.233	1.514	2.159	1.686	2.210	3.177
	Elasto-plastic stability								
$N_{crd}(\alpha) \times 10^{-6}$ (Pa)	87.48 ($p = 5$)	208.95 ($p = 2.4$)	633.06 ($p = 2.1$)	110.87 ($p = 7$)	191.3 ($p = 4$)	378.85 ($p = 3.5$)	204.34 ($p = 6$)	334.79 ($p = 5$)	575.75 ($p = 5$)
$K_d(\alpha)$	1.785	4.263	12.92	2.262	3.916	7.729	4.169	6.83	11.75
	Steel shells (ST 304)								
	Elastic stability								
$N_{crd}(\alpha) \times 10^{-6}$ (Pa)	1830.7 ($p = 82$)	2047.4 ($p = 19$)	3013.5 ($p = 5$)	2050.1 ($p = 37$)	2385 ($p = 19$)	3142.1 ($p = 10$)	2696.5 ($p = 20$)	3388.5 ($p = 10$)	4651.8 ($p = 9$)
$K_d(\alpha)$	1.034	1.156	1.702	1.158	1.347	1.775	1.523	1.914	2.627
	Elasto-plastic stability								
$N_{crd}(\alpha) \times 10^{-6}$ (Pa)	236.46 ($p = 10$)	435.17 ($p = 3$)	1197.1 ($p = 2.1$)	314.95 ($p = 10$)	491.4 ($p = 6$)	903.94 ($p = 4$)	567.16 ($p = 7$)	889.8 ($p = 6$)	1490.5 ($p = 5$)
$K_d(\alpha)$	1.335	2.457	6.759	1.778	2.775	5.104	3.203	5.024	8.416

For the case that the loading law does not vary and the axial loading parameter T_0 increases, the dynamic axial critical load and the dynamic factor values increase and the values of p parameter versus to the minimum dynamic axial critical load decrease. When T_0 is increased, the values of the dynamic critical load and dynamic factor in the elasto-plastic stability change more acute than the values in the elastic stability. Consequently, when the loading law changes, the values of the loading parameter change as well.

5. Conclusions

The stability of a cylindrical shell subject to a uniform axial compression, which is a power function of time, is examined within the framework of small strain elasto-plasticity. The material of the shell is incompressible and the effect of the elastic unloading is taken into consideration. Initially, employing small elasto-plastic deformation theory, the fundamental relations and Donnell type stability equation of a cylindrical shell have been obtained. Then, employing the Galerkin's method, those equations have been reduced to a time dependent differential equation with a variable coefficient. Then, for two initial conditions applying a Ritz type variational method, the critical static and dynamic axial loads as well as the dynamic factor have been found. Finally, carrying out some computations, the effects of the variation of the power of time in the axial load expressions, of the varying loading parameter and of the varying radius to the thickness ratios on the critical parameters of the shells for two initial conditions have been studied and the results obtained were presented in the form of graphs and tables.

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Notation

B_j, C_j, D_j	: Coefficients defined in Eqs. (26a-e) and (35a-c)
E	: Elasticity modulus
E_s, E_t	: Secant and tangent moduli, respectively
h	: Thickness of the shell
K_d	: Dynamic factor
L	: Length of the cylindrical shell
L_{ij}, $\tilde{L}_{ij}$	: Differential operators defined in Eqs. (46a-d) and (47a-d), respectively
M_x, M_y, M_{xy}	: Moments per unit length of the cross-section of the shell
m	: Half wave number in the axial direction
m_{1st}	: Wave number corresponding to static critical axial load
n	: Wave number in the circumferential direction
Ox, Oy, Oz	: Coordinate axes with the origin on the middle surface of the shell
N, K	: Material parameters
N_{crs}, N_{crd}	: Static and dynamic critical axial loads, respectively
C_0	: Integration constant
T_0, T_1	: Axial loading parameter and static axial load, respectively
R	: Radius of the cylindrical shell
N_x, N_y, N_{xy}	: Forces per unit length of the cross-section of the shell
N_x^0, N_y^0, N_{xy}^0	: Pre-buckling membrane forces
t, t_{cr}	: Time and critical time, respectively
w	: Displacement of the middle surface in the inwards normal direction
z_0	: Coordinate of the surface that separates the regions of elastic and plastic deformation
$\bar{z}_0$	: $= 2z_0/h$ dimensionless coordinate
α	: Power of time in the axial compression
δ	: Symbol of variation
ϕ	: Function
χ	: Curvature function
χ_x, χ_y, χ_{xy}	: Middle surface curvatures
e_x, e_y, e_{xy}	: Strain components on the middle surface of the shell
ε_x, ε_y, ε_{xy}	: Strain components
ε_i	: Equivalent strain
ε	: Strain function
ν	: Elastic Poisson's ratio
τ	: Dimensionless time parameter
ρ	: Density of the material

φ_{ts}	: E_t/E_s
σ_x, σ_y, σ_{xy}	: Stress components
σ, σ_i	: Stress function and equivalent stress, respectively
$\xi(t)$, $\zeta(t)$	: Time dependent amplitudes
Φ	: Stress function
$\Pi(\sigma)$	: Function defined in Eqs. (8), (9) and (11)
Λ_j	: Coefficients defined in Eqs. (54a-b)
Γ	: Functional defined in Eq. (55)