Structural Engineering and Mechanics, Vol. 23, No. 4 (2006) 387-407
DOI: http://dx.doi.org/10.12989/sem.2006.23.4.387

Optimum design of plane steel frames with PR-connections using refined plastic hinge analysis and genetic algorithm

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(Received May 20, 2005, Accepted February 14, 2006)

Abstract. A Genetic Algorithm (hereinafter GA) based optimum design algorithm and program for plane steel frames with partially restrained connections is presented. The algorithm was incorporated with the refined plastic hinge analysis method, in which geometric nonlinearity was considered by using the stability functions of beam-column members and material nonlinearity was considered by using the gradual stiffness degradation model that included the effects of residual stress, moment redistribution by the occurrence of plastic hinges, partially restrained connections, and the geometric imperfection of members. In the genetic algorithm, a tournament selection method and micro-GAs were employed. The fitness function for the genetic algorithm was expressed as an unconstrained function composed of objective and penalty functions. The objective and penalty functions were expressed, respectively, as the weight of steel frames and the constraint functions which account for the requirements of load-carrying capacity, serviceability, ductility, and construction workability. To verify the appropriateness of the present method, the optimum design results of two plane steel frames with fully and partially restrained connections were compared.

Keywords: optimum design; genetic algorithm; refined plastic hinge analysis; plane steel frames with partially restrained connections.

1. Introduction

Recently, research on the optimum design of steel frames by use of genetic algorithm has been made actively. Kim (1999) and Camp *et al.* (1998) conducted optimum designs of steel algorithms in connection with the AISC-LRFD (1994) analysis method, in which the nonlinear effects of steel frames were considered with the introduction of the moment amplification factors B1 and B2 on the basis of linear elastic structural analysis. Pezeshk *et al.* (2000), Schinler (2000), and Yun and Kim

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(2005), in recognition of the problem of using linear elastic analysis results in optimum design, also conducted discrete optimum designs of plane steel frames incorporating advanced analysis methods such as geometric nonlinear analysis methods, plastic zone analysis methods, and refined plastic hinge analysis methods. Since most of the researches conducted until now, however, has been concerned with steel frames having fully restrained connections (hereinafter, FR-connections), the optimum design results do not reflect the true behavior of steel frames which have partially restrained connections (hereinafter, PR-connections). Even though Foley *et al.* (2001) conducted an optimum design of steel frames with PR-connections by using a genetic algorithm, the design incorporated the plastic zone analysis method which is inappropriate for general design purposes due to the complex numerical analysis process. In addition, there is no design method that has been clearly proposed in current design specifications for plane steel frames with PR-connections, although PR-connection affects the moment distribution and displacement of frame members and is an important factor in terms of the ultimate strength of plane steel frames.

This research presents a discrete optimum design method for plane steel frames with PR-connections. The method incorporates a genetic algorithm and a refined plastic hinge analysis method (Al-Mashary and Chen 1991, Deirelein *et al.* 1991, Liew *et al.* 1993, Attala *et al.* 1994, King and Chen 1994) by which one can estimate the nonlinear behavior of the whole steel structural system and members with FR- and/or PR-connections up to the load of ultimate limit states effectively and accurately. In the genetic algorithm, the micro-GAs (Krishnakumar 1989) in which the mutation process is not necessary and the single-point crossover probability is 1.0 and a tournament selection method were employed.

The fitness function for the genetic algorithm was expressed as an unconstrained function composed of objective and penalty functions. The objective and penalty functions were expressed respectively, as the weight of steel frames and the constraint functions that account for the requirements of load-carrying capacity, serviceability, ductility, and construction workability. In the refined plastic hinge analysis method, geometric nonlinearity was considered by using the stability functions of beam-column members and material nonlinearity was considered by using the gradual stiffness degradation model that included the effects of residual stress, moment redistribution by the occurrence of plastic hinges, PR-connections, and the geometric imperfection of members. To verify the appropriateness of the present method, the optimum design results of two plane steel frames with FR- and PR-connections were compared.

2. Refined plastic hinge analysis of steel frames

2.1 Modeling for geometric non-linearity

Beam-columns in steel frames are subject to both axial compression and bending moment. The bending moment in a beam-column consists of primary and secondary bending moments. The primary bending moment is caused by the applied end moments and/or transverse loads on the members. The secondary bending moment is caused by the axial force acting through the lateral displacement of the member relative to its chord and the axial force acting through the relative displacement of the ends of the member. The second order effects can be captured by the stability functions (Chen and Lui 1987). The force-displacement relationship, considering the geometric nonlinearity of the beam-column member subject to both axial force and bending moment as shown in Fig. 1, may be written as:

$$\begin{bmatrix} M_A \\ M_B \\ P \end{bmatrix} = \frac{EI}{L}\begin{bmatrix} S_1 & S_2 & 0 \\ S_2 & S_1 & 0 \\ 0 & 0 & A/I \end{bmatrix}\begin{bmatrix} \theta_A \\ \theta_B \\ e \end{bmatrix} \tag{1}$$

in which M_A, M_B, θ_A and θ_B are the end moments and the corresponding joint rotations at member ends A and B, respectively. P and e (positive in tension) are the axial force and displacement in the longitudinal direction of the member, and A, I, L, and E are the cross-sectional area, the moment of inertia, the length, and the modulus of elasticity of the member, respectively. S_1 and S_2 are the stability functions that account for the axial force effect on the bending stiffness of the member. The conventional stability functions may be written as:

$$S_1 = \left\{ \frac{\pi\sqrt{\rho}\sin(\pi\sqrt{\rho}) - (\pi^2\rho)\cos(\pi\sqrt{\rho})}{2 - 2\cos(\pi\sqrt{\rho}) - \pi\sqrt{\rho}\sin(\pi\sqrt{\rho})} \right. \quad \text{for} \quad P < 0 \tag{2a}$$

$$S_1 = \left\{ \frac{\pi^2\rho\cosh(\pi\sqrt{\rho}) - (\pi\sqrt{\rho})\sinh(\pi\sqrt{\rho})}{2 - 2\cosh(\pi\sqrt{\rho}) + \pi\sqrt{\rho}\sinh(\pi\sqrt{\rho})} \right. \quad \text{for} \quad P > 0 \tag{2b}$$

$$S_2 = \left\{ \frac{(\pi^2\rho) - \pi\sqrt{\rho}\sin(\pi\sqrt{\rho})}{2 - 2\cos(\pi\sqrt{\rho}) - \pi\sqrt{\rho}\sin(\pi\sqrt{\rho})} \right. \quad \text{for} \quad P < 0 \tag{2c}$$

$$S_2 = \left\{ \frac{(\pi\sqrt{\rho})\sinh(\pi\sqrt{\rho}) - (\pi^2\rho)}{2 - 2\cosh(\pi\sqrt{\rho}) + \pi\sqrt{\rho}\sinh(\pi\sqrt{\rho})} \right. \quad \text{for} \quad P > 0 \tag{2d}$$

in which $\rho = P/(\pi^2 EI/L^2)$, and P is taken as positive in tension.

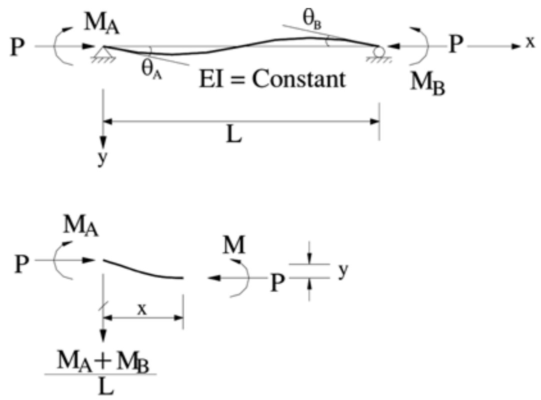


Fig. 1 Beam-column member subject to axial force and bending moment at ends

2.2 Modeling for gradual stiffness degradation

A beam-column member in a steel frame experiences inelastic behavior with a plastic hinge by bending moment or with buckling by axial force. Therefore, to reflect the inelastic behavior in a beam-column member, a force-displacement relationship which considers the stiffness degradation by residual stress, bending and axial force, geometric imperfection, and PR-connection is required.

2.2.1 Residual stress

For a stub column, the stress-strain curve exhibits a smooth transition from elastic to perfectly plastic due to residual stress in the steel column. In the refined plastic hinge analysis, the stiffness degradation of a beam-column member for residual stress may be considered by the equation for E_t which is obtained from the Column Research Council (hereinafter CRC; Galambos 1998). The CRC E_t is employed because it is easier to reflect the gradual yielding process (plastification) over the cross section due to residual stress than to update the modulus of elasticity of the cross-sectional area of a beam-column member that remains elastic. The CRC E_t is given as follows:

$$E_t = 1.0E \qquad \text{for} \quad P \le 0.5P_y \tag{3a}$$

$$E_t = 4\frac{P}{P_y}E\left(1 - \frac{P}{P_y}\right) \qquad \text{for} \quad P > 0.5P_y \tag{3b}$$

in which P is the second-order axial force and P_y is the axial load at full yield.

2.2.2 Plastic hinge

In refined plastic hinge analysis, member stiffness is assumed to degrade parabolically after the member end forces exceed a specified initial yield function. When plastic strength is reached, the plastic hinge is modeled as a true hinge with an applied plastic moment. The member tangent stiffness is then adjusted to account for the presence of the plastic hinge. The incremental force-displacement relationship, with and without plastic hinges at their ends, may be written as (Liew *et al*. 1993a, 1993b):

$$\begin{bmatrix} \Delta M_A \\ \Delta M_B \end{bmatrix} = \frac{E_t I}{L}\begin{bmatrix} \eta_A\left[S_1 - \frac{S_2^2}{S_1}(1-\eta_B)\right] & \eta_A \eta_B S_2 \\ \eta_A \eta_B S_2 & \eta_B\left[S_1 - \frac{S_2^2}{S_1}(1-\eta_A)\right] \end{bmatrix}\begin{bmatrix} \Delta\theta_A \\ \Delta\theta_B \end{bmatrix} \tag{4}$$

in which the terms η_A and η_B are scalar parameters that allow for a gradual inelastic stiffness reduction of the member associated with plastification at ends A and B, respectively.

The parameter η is equal to 1.0 when the member is elastic and zero when a plastic hinge is formed at the end. The parameter is assumed to vary according to a prescribed function:

$$\eta = 1 \qquad \text{for} \quad \alpha \le 0.5 \tag{5a}$$

$$\eta = 4\alpha(1-\alpha) \qquad \text{for} \quad \alpha > 0.5 \tag{5b}$$

in which α is a force-state parameter that measures the magnitude of the axial force P and bending moment M at the member end. By adopting the cross-sectional strength equations from the AISC-

LRFD (1986, 1994, 2001), the term α may be expressed as:

$$\alpha = \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \qquad \text{for} \quad \frac{P}{P_y} \geq \frac{2}{9}\frac{M}{M_p} \tag{6a}$$

$$\alpha = \frac{P}{2P_y} + \frac{M}{M_p} \qquad \text{for} \quad \frac{P}{P_y} < \frac{2}{9}\frac{M}{M_p} \tag{6b}$$

in which M_p is the plastic moment capacity of a beam-column member.

2.2.3 Geometric imperfection

Geometric imperfections result from unavoidable errors during fabrication or erection. For structural members, the types of geometric imperfections are out-of-straightness and out-of-plumbness. These imperfections cause additional moments in a column member and further degradation of its bending stiffness. In the current study, the further reduced tangent modulus method (Kim 1996) which reduces the CRC E_t with a reduction factor 0.85 was employed to account for further stiffness degradation due to geometric imperfections. The advantage of this method over other methods is its convenience in design use because it eliminates problem of explicit imperfection modeling or equivalent notional loads. Another advantage is that it does not require the determination of the direction of geometric imperfections which are often difficult to determine in a large system.

2.2.4 PR-connection

The behavior of a connection can be presented in a moment-rotation relationship as shown in Fig. 2. Numerous moment-rotation relationships of PR-connections were proposed by various experiments, but herein, the three-parameter power model proposed by Kishi and Chen (1990) is adopted. The three parameters are the initial stiffness of connection R_{ki}, the ultimate moment capacity of connection M_u, and the shape parameter n:

$$m = \frac{\theta}{(1+\theta^n)^{1/n}} \qquad \text{for} \quad \theta > 0, m > 0 \tag{7}$$

in which $m = M/M_u$ and $\theta = \theta_r/\theta_0$ ($\theta_0 = M_u/R_{ki}$ is reference plastic rotation, θ_r is rotation

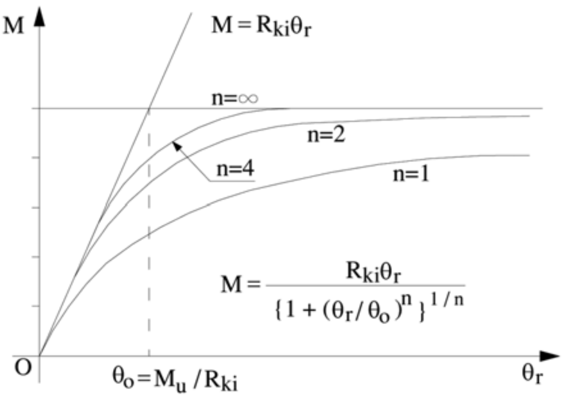


Fig. 2 Moment-rotation behavior of connection

corresponding to moment M). When a connection is loaded, the tangent stiffness of the connection R_{kt} at arbitrary rotation θ_r can be derived by simply differentiating Eq. (7) as:

$$R_{kt} = \frac{dM}{d|\theta_r|} = \frac{M_u}{\theta_0(1-\theta^n)^{1+1/n}} \tag{8}$$

For practical use of the power model, the three parameters for a given connection configuration must be determined. Herein, the practical procedures for determining the three parameters are presented for the two types of connections: top- and seat-angle connection (hereinafter, TS-connection), and top- and seat-angle connection with a double web angle (hereinafter, TS&W-connection). R_{ki} and M_u of the TS-connection derived from the assumed failure mechanism shown in Fig. 3 are as follows (Kishi and Chen 1990):

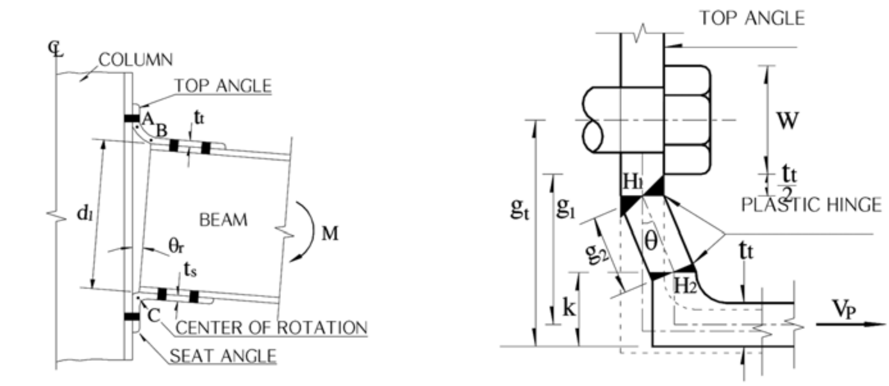
$$R_{ki} = \frac{3EI}{1+(0.78t_t^2/g_1^2)}\frac{d_1^2}{g_1^3} \tag{9}$$

$$M_u = M_{os} + M_p + V_p d_2 \tag{10}$$

in which EI is the bending stiffness of the angle's leg adjacent to the column face, d_1 is the distance between the centers of the legs of the top- and bottom-angle, t_t is the thickness of the top-angle, $g_1 = g_t - D/2 - t_t/2$ (g_t = the distance from the top-angle's heel to the center of the fastener holes in the leg adjacent to the column face, D is d_b if rivets are used as fasteners where d_b is the diameter of the fastener and W if bolts are used as fasteners where W is the diameter of the nut), M_{os} is the plastic moment capacity at point C of the seat-angle, M_p is the plastic moment capacity at point H2 of the top-angle, V_p is the plastic shear capacity, and $d_2 = d + t_s/2 + k$ (d = the beam depth, t_s = the thickness of seat-angle, k = the distance from the top-angle's heel to the toe of the fillet).

R_{ki} and M_u of the TS&W-connection derived from the assumed failure mechanism shown in Fig. 4 are as follows:

$$R_{ki} = \frac{3EI_t d_1^2}{g_1(g_1^2+0.78t_t^2)} + \frac{6EI_a d_3^2}{g_3(g_3^2+0.78t_a^2)} \tag{11}$$



(a) Deformed shape in elastic state (b) Failure mechanism in ultimate state

Fig. 3 Top- and seat-angle connection

$$M_u = M_{os} + M_{pt} + V_{pt}d_2 + 2V_{pa}d_4 \tag{12}$$

in EI_t which EI_a and are the bending stiffness of the legs adjacent to the column faces of the top-angle and web angle respectively, $d_3 = d + t_s/2$, $g_3 = g_c - W/2 - t_a/2$ (g_c = the distance from the web angle's heel to the center of fastener holes, t_a = the thickness of top-angle), M_{pt} is the ultimate moment capacity for the top-angle, V_{pt} is the shear force acting on the plastic hinge locations H_1 and H_2, V_{pa} is the resultant plastic shear force, and $d_4 = \{d_a(2V_{pu} + V_{oa})\}/\{3(V_{pu} + V_{oa})\} + l_i + t_s/2$ (d_a = the height of the web angle, V_{pu} = the shear force acting on the upper edge of the web angle, V_{oa} = the shear force acting on the lower edge of the web angle, l_i = distance from the web angle's bottom to the compression flange of the beam). Finally, the shape parameter n is obtained from the equations suggested by Kishi and Chen (1990). The equations are given in Table 1.

The incremental force-displacement relationship of the beam-column member, taking into consideration the effects of PR-connections, can be written from Eq. (4) as:

$$\begin{bmatrix} \Delta M_A \\ \Delta M_B \\ \Delta P \end{bmatrix} = \frac{E_t I}{L} \begin{bmatrix} S_{ii}^* & S_{ij}^* & 0 \\ S_{ij}^* & S_{jj}^* & 0 \\ 0 & 0 & A/I \end{bmatrix} \begin{bmatrix} \Delta\theta_A \\ \Delta\theta_B \\ \Delta e \end{bmatrix} \tag{13}$$

where

$$S_{ii}^* = \left(S_{ii} + \frac{E_t I S_{ii} S_{jj}}{L R_{ktB}} - \frac{E_t I S_{ij}^2}{L R_{ktB}}\right) / R^* \tag{14a}$$

$$S_{jj}^* = \left(S_{jj} + \frac{E_t I S_{ii} S_{jj}}{L R_{ktA}} - \frac{E_t I S_{ij}^2}{L R_{ktA}}\right) / R^* \tag{14b}$$

$$S_{ij}^* = S_{ij} / R^* \tag{14c}$$

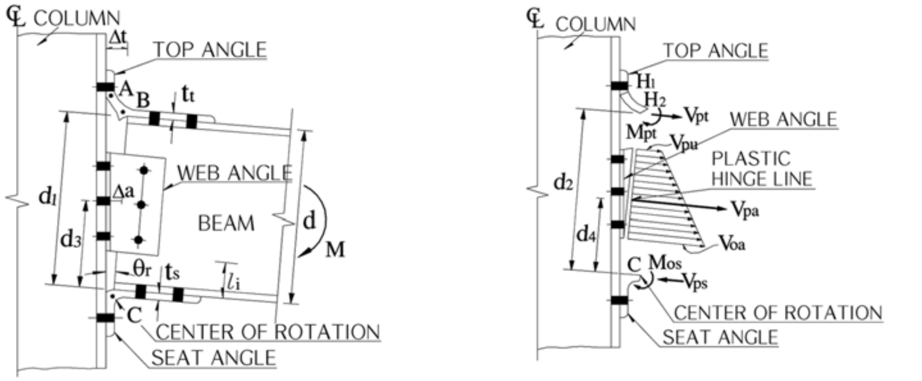
$$R^* = \left(1 + \frac{E_t I S_{ii}}{L R_{ktA}}\right)\left(1 + \frac{E_t I S_{jj}}{L R_{ktB}}\right) - \left(\frac{E_t I}{L}\right)^2 \frac{S_{ij}^2}{R_{ktA} R_{ktB}} \tag{14d}$$

in which R_{ktA} and R_{ktB} are the tangent stiffness of connections A and B respectively, and S_{ii}, S_{ij}, and S_{jj} are the coefficients defined in Eq. (4) that relate the incremental moments and rotations of beam-column member ends.

Table 1 Empirical equations for shape parameter n

Connection type	n		
TS&W	$2.003\log_{10}\theta_0 + 6.070$	for	$\log_{10}\theta_0 > -2.880$
	0.302	for	$\log_{10}\theta_0 < -2.880$
TS	$1.398\log_{10}\theta_0 + 4.631$	for	$\log_{10}\theta_0 > -2.721$
	0.827	for	$\log_{10}\theta_0 < -2.721$

TS&W: Top- and Seat-Angle Connection with Double Web Angle;
TS : Top- and Seat-Angle Connection



(a) Deformed shape in elastic state (b) Failure mechanism in ultimate state

Fig. 4 Top- and seat-angle connection with double web angle

3. Genetic algorithm and formation of optimum design problem

3.1 Genetic algorithm

Genetic algorithm is an optimization algorithm that is based on the principle of 'survival-of-the-fittest theory' and natural selection, the theory of natural evolution proposed by Darwin that only individuals well adapted to the surrounding environment and having excellent character would survive. A genetic algorithm generally forms the next generation with individuals having higher fitness than the generation's individuals (binary codes) through a searching process of reproduction, crossover, and mutation, and this process is repeatedly performed until the completion conditions are satisfied, i.e., selecting an individual having highest fitness for the whole search boundary. In the current study, the micro-genetic algorithm (Krishnakumar 1989) in which the mutation process is not necessary and the single-point crossover probability is 1.0 in a simple genetic algorithm developed by Holland (1975) is employed.

The optimum design process of the present study that the refined plastic hinge analysis is incorporated with the genetic algorithm, as shown in Fig. 5, is as follows.

① Select parameters to control a genetic algorithm suitable for a given problem, such as the dimension of the group, the number of design variables, and crossover probability. When applying the micro-genetic algorithm, the mutation process is not required and the crossover is fixed as the single-point crossover.

② Generate binary coded individuals by producing random numbers, and then make the individual populations (initial and next generations) as one desires. Because $256(=2^8)$ AISC WF- shape database numbered with decimal digit is used, the chromosome of an individual is composed of $n \times 8$ bits (n = the number of discrete design variables).

③ Convert each individual coded as a binary digit, that is, the number of design variable, into decimal digit, and then make an input database for the refined plastic hinge analysis from WF-shape database.

④ Conduct the refined plastic hinge analysis in the service and factored load states, and examine the constraint conditions concerning load-carrying capacity, serviceability, ductility, and construction workability on the basis of the analytical results.

⑤ Calculate the fitness of each individual using a fitness function consisting of an objective function and penalty functions expressed as the constraint condition equations.

⑥ Select an individual having the maximum fitness, and confirm whether it satisfies the stop conditions or not. If not, generate a next generation through reproduction and mutation, and return to step ②. A fixed number of generations was used as a convergence criterion in this study.

3.2 Formation of optimum design problem

3.2.1 Fitness function

A fitness function is used to determine how the member combination is suitable for a given condition in the optimum design with a genetic algorithm. This study uses the fitness function suggested by Camp *et al*. (1998), and the fitness function of i individual suggested by them can be expressed as follows:

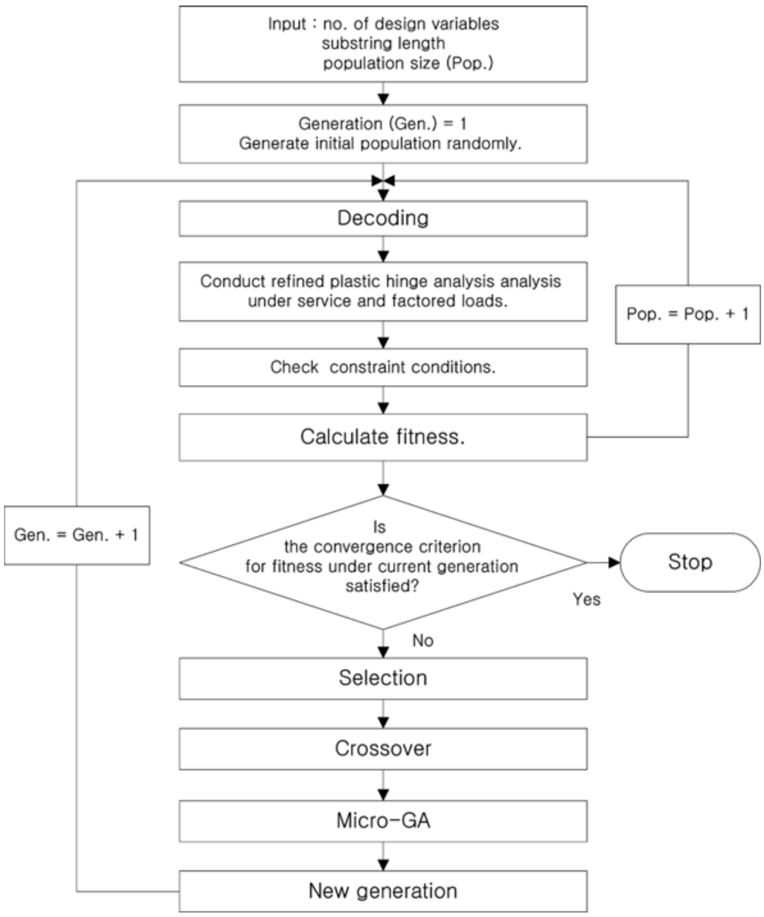


Fig. 5 Optimum design process of genetic algorithm incorporating refined plastic hinge analysis

$$F_i = \frac{P_{\max} - (P_{\min}/\xi)}{P_i - (P_{\min}/\xi)} \tag{15}$$

in which $P_{\max}$ and $P_{\min}$ are respectively the maximum and minimum values of a nonconstraint function in current generation, and ξ (= 1.01) is a modification parameter. Also, P_i is the value of a nonconstraint function of i individual, and can be expressed as follows:

$$P_i = OBJ \times \left(1 + C\sum_{t=1}^{m} \upsilon_t\right) \tag{16}$$

in which C is a constant, and m is the total number of constraint conditions. The penalty function υ_t proposed by Pezeshk *et al.* (2000) and the objective function OBJ are expressed respectively as follows:

$$OBJ = \sum_{i=1}^{N_e} A_i L_i \rho_i \tag{17}$$

$$\upsilon_t = \begin{cases} 0 & \text{if} \quad g(t) \le 0 \\ g(t) & \text{if} \quad 0 < g(t) \le 1.0 \\ g(t)^2 & \text{if} \quad g(t) > 1.0 \end{cases} \tag{18}$$

in which N_e is the total number of member, L_i, A_i and ρ_i are the length, the cross-sectional area, and the unit weight, of member respectively, and $g(t)$ is the equation of the constraint condition.

3.2.2 Constraint conditions

In this study, four kinds of constraint conditions, that is, the load-carrying capacity, serviceability, ductility, and construction workability, of steel frames were considered. To express the penalty function as a non-dimensional value, the following equation pattern for constraint conditions was taken.

$$g(i) = \frac{Calculated\ Value}{Limited\ Value} - 1.0 \tag{19}$$

If it violates a constraint condition, $g(i)$ has positive value, and if not, it has negative or zero value. Because AISC-LRFD (USCS unit, 2001) have included many factors in design equations, they are marked without unit conversion.

1) Load-carrying capacity

As constraint conditions of load-carrying capacity, the conditions of ultimate load and shear strength were considered. Since the failure of a steel frame should occur under a load state more than design load, this condition can be expressed as:

$$g(1) = \frac{1}{\lambda} - 1.0 \tag{20}$$

where the critical load factor, λ, means the ratio of ultimate load to design load when a failure mechanism is formed in a steel frame, and it is more than 1.0 when the structure fails under the load greater than its design load. Since in the refined plastic hinge analysis formulation the beam-column member is modeled as the member subjected to axial force and moments only, it is necessary to examine the shear strength of each beam-column member. The equation for the constraint condition of shear strength is expressed as:

$$g(2) = \frac{V_u}{\phi_v V_n} - 1.0 \tag{21}$$

in which V_u and ϕ_v (= 0.85) are the ultimate shear load and the shear strength reduction factor, respectively, and V_n, as the nominal shear strength, is given as:

$$V_n = 0.6F_y A_w \quad \text{for} \quad \frac{h}{t_w} \le 2.45\sqrt{\frac{E}{F_y}} \tag{22a}$$

$$V_n = 0.6F_y A_w \frac{2.45\sqrt{\frac{E}{F_y}}}{h/t_w} \quad \text{for} \quad 2.45\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \le 3.07\sqrt{\frac{E}{F_y}} \tag{22b}$$

$$V_n = \frac{4.52A_w E}{(h/t_w)^2} \quad \text{for} \quad 3.07\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \le 260 \tag{22c}$$

in which F_y and E are the yield strength and elastic modulus of WF-shaped steel, and A_w, t_w and h are the cross-sectional area, thickness of a web, and height of a web respectively.

2) Ductility

For the section of a member to reach the stage of plastic moment, a proper rotational capacity in the member section is required. For that, the member must be properly braced laterally so that lateral-torsional buckling does not occur and the member section should be compact so that local buckling does not occur. It is assumed the in refined plastic hinge analysis, a member section may be subjected to plastic moment until a failure mechanism is formed due to several plastic hinges. Therefore, when selecting steel frame members, one should select members that avoid local buckling and lateral torsional buckling. The constraint conditions of compact section are expressed as follows:

$$g(3) = \frac{b_f/2t_f}{(b_f/2t_f)_{\text{limit}}} - 1.0 \tag{23}$$

$$g(4) = \frac{h/t_w}{(h/t_w)_{\text{limit}}} - 1.0 \tag{24}$$

in which b_f, t_f, h, and t_w are respectively the flange width, flange thickness, web height, and web thickness of WF-shaped steel. In addition, the limit value of the ratio of the flange width to thickness, $(b_f/2t_f)_{\text{limit}}$, and the limit value of the ratio of the web height to thickness, $(h/t_w)_{\text{limit}}$, are as follows:

$$\left(\frac{b_f}{2t_f}\right)_{\text{limit}} = 0.38\sqrt{\frac{E}{F_y}} \tag{25a}$$

$$\left(\frac{h}{t_w}\right)_{\text{limit}} = 3.76\sqrt{\frac{E}{F_y}}\left(1 - \frac{2.75P_u}{\phi_b P_y}\right) \quad \text{for} \quad \frac{P_u}{\phi_b P_y} \le 0.125 \tag{25b}$$

$$\left(\frac{h}{t_w}\right)_{\text{limit}} = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\phi_b P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}} \qquad \text{for} \quad \frac{P_u}{\phi_b P_y} > 0.125 \tag{25c}$$

in which P_u and P_y are the required axial strength and the yield axial strength respectively, and F_y E, and ϕ_b (= 0.9) are respectively the yield stress of WF-shaped steel, the elastic modulus, and the strength reduction factor for bending.

The constraint condition of lateral-torsional buckling is expressed as:

$$g(5) = \frac{L}{L_{\text{limit}}} - 1.0 \tag{26}$$

in which L is the member length, and L_{limit} is the unbraced length of member, L_{pd}, in plastic design. For L_{pd}, when the compression flange is equal to or larger than tension flange and is bent about primary axis of WF-shaped section, the following equation is used:

$$L_{pd} = \left[0.12 + 0.076\left(\frac{M_1}{M_2}\right)\right]\frac{E}{F_y}\gamma_y \tag{27}$$

in which (M_1/M_2) is positive when the moment causes double curvature, and is negative when it causes single curvature. γ_y (= $\sqrt{I_y/A}$) is the radius of gyration about the minor axis. In the refined plastic hinge analysis, it is assumed that out-of-plane deflection is braced in the modeling of the beam-column member. Thus, the constraint condition to prevent out-of-plane axial buckling is expressed as:

$$g(6) = \frac{P_u}{\phi_c P_{ny}} - 1.0 \tag{28}$$

in which P_u and ϕ_c (= 0.85) are the ultimate axial load and the axial strength reduction factor, respectively, and P_{ny} is the nominal axial strength expressed as:

$$P_{ny} = \left(\frac{0.877}{\lambda_c^2}\right)F_y \qquad \text{for} \quad \lambda_c > 1.5 \tag{29a}$$

$$P_{ny} = (0.658^{\lambda_c^2})F_y \qquad \text{for} \quad \lambda_c \leq 1.5 \tag{29b}$$

where, in slenderness ratio $\left(\lambda_c = \frac{KL}{r_y \pi}\sqrt{\frac{F_y}{E}},\ r_y = \sqrt{\frac{I_y}{A}}\right)$, the value of K, an effective length factor, should be 0.5 for the case that lateral displacement is prevented, but in this study, 1.0 was used for conservative design, which is for the case that lateral displacement is not prevented.

The constraint condition of the slenderness ratio is expressed as:

$$g(7) = \frac{(KL/r)}{(KL/r)_{\text{limit}}} - 1.0 \qquad \text{for} \quad \textit{compression} \tag{30a}$$

$$g(7) = \frac{(L/r)}{(L/r)_{\text{limit}}} - 1 \qquad \text{for} \quad \textit{tension} \tag{30b}$$

where r is the governing radius of gyration about the axis of buckling, and Kl/r_{limit} and l/r_{limit} is 200 and 300, respectively.

3) Serviceability

The limit value $\delta_{v,\,\mathrm{limit}}$ (= L/360, L = beam length in *cm*) of the deflection of a beam by live load, and the limit value $\delta_{h,\,\mathrm{limit}}$ (h/360, h = height of story in *cm*) of the inter-story drift of column δ_h by wind load, all suggested by ASCE Ad Hoc Committee (1986) and Ellingwood (1989), were used as the constraint conditions of displacement, which can be expressed as:

$$g(8) = \frac{\delta_v}{\delta_{v,\,\mathrm{limit}}} - 1.0 \tag{31}$$

$$g(9) = \frac{\delta_h}{\delta_{h,\,\mathrm{limit}}} - 1.0 \tag{32}$$

Additionally, AISC-LRFD proposes that we not generate a plastic hinge under a service load state when using the plastic hinge concept. The equation of the constraint condition is expressed as:

$$g(10) = \frac{1}{\lambda_{1st}} - 1.0 \tag{33}$$

in which λ_{1st} is the value obtained by dividing the load when the first hinge is generated by the service load.

4) Construction workability

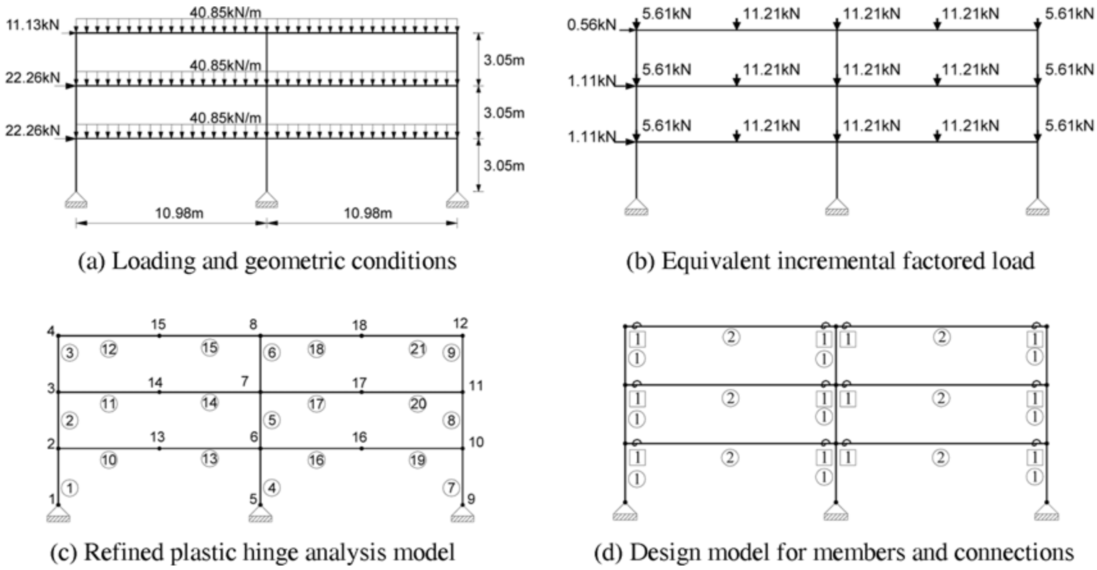
In this study, since all column members were less than 28 *cm* in depth, d_c, and the beam flange widths, d_{bf}, were less than column flange width, d_{cf}, at the joints of beam and column in the constraint conditions used in the earlier study (Pezeshk *et al.* 2001, Yun and Kim 2005) the following constraint conditions of construction workability were added for comparison of design results.

$$g(11) = \frac{28}{d_c} - 1.0 \tag{34}$$

$$g(12) = \frac{b_{bf}}{b_{cf}} - 1.0 \tag{35}$$

4. Design examples

This study proposes a discrete optimum design method for plane steel frames with PR-connections, in which a refined plastic hinge analysis method and a genetic algorithm are incorporated. The optimum design results of the plane steel frames with PR-connections were compared with the earlier results of a study (Yun and Kim 2005) in which the same plane steel frames with FR-connections were optimized by using the same genetic algorithm and refined plastic hinge analysis method. The steel members used for the design exercises were 256 WF-shaped members of A36 AISC-LRFD. The magnitude of the incremental load was set at 1/20 of the design load, the size of the design population was 50, and the crossover probability was 1.0. Since the design results may differ depending on random numbers generated in a GA-based optimum design, the optimum design was performed three times by generating three sets of random numbers.



(a) Loading and geometric conditions (b) Equivalent incremental factored load

(c) Refined plastic hinge analysis model (d) Design model for members and connections

Fig. 6 Three-story two-bay steel frame

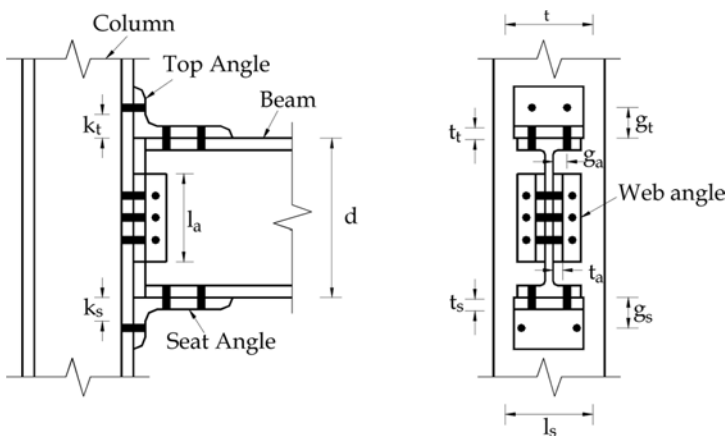
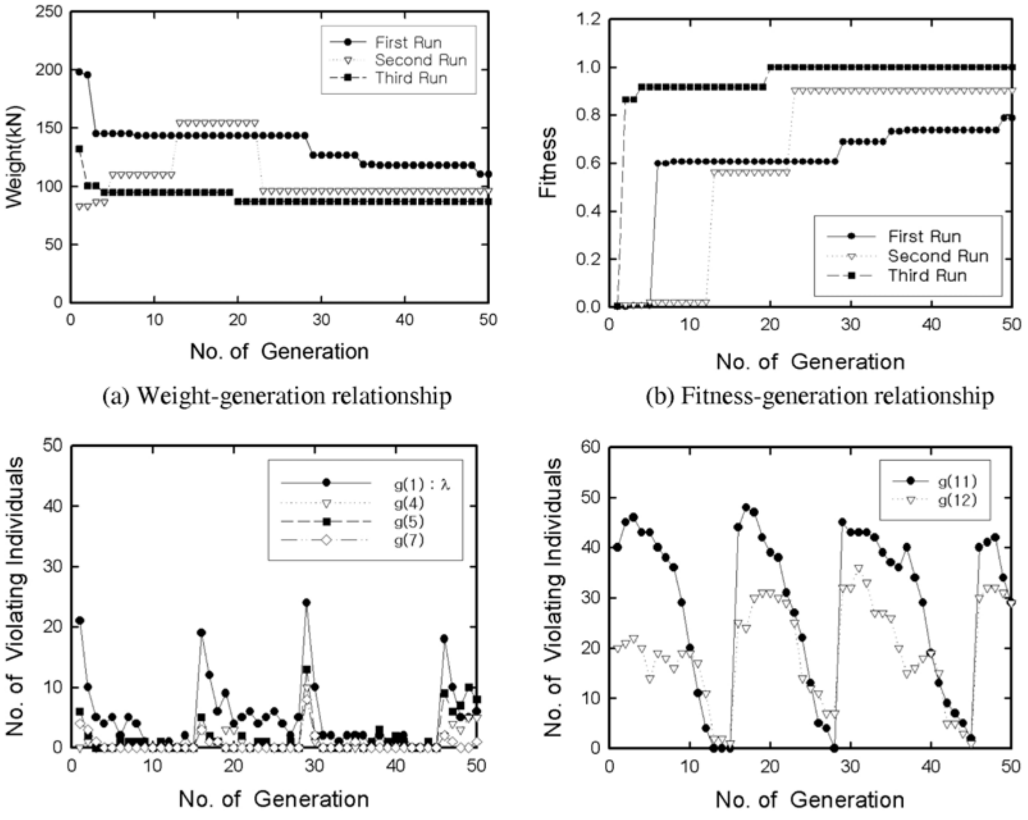


Fig. 7 Details of top- and seat-angle connection with double web angle

4.1 Three-story two-bay steel frame

The structure, shown in Fig. 6, is a model selected to evaluate the effects of connection conditions on the optimum design results. The equivalent incremental factored loads imposed on the steel frame are shown in Fig. 6(b), a model for the refined plastic hinge analysis in Fig. 6(c), and a design model indicating the connection and member types in Fig. 6(d). In the optimum design, the TS- and TS&W-connections (Figs. 3 and 4, respectively) were considered. The optimization program was coded so that the connection configurations were changed according to the discrete beams and columns selected from 256 WF-shaped members. To simplify the connection design, t_t and t_s were taken as the thickness of the beam flange, l_t and l_s as the width of the beam flange, k_t

and k_s as $2k_t$, g_t, g_s and g_a as, $2k_t$, l_a as $0.6d$, and W as 3.81 *cm* (Fig. 7). The yield stress and modulus of the elasticity of the connection material were taken as F_y = 248.1 MPa and E = 200.1 × 103 MPa, respectively. In an earlier study, it was assumed that all beams were stiffened every 1/6 point of the span, and because the loads considered in the optimum design were factored loads, the constraint conditions of ductility (lateral torsional buckling, out-of-plane buckling) were applied to the column members only and the constraint conditions of serviceability were excluded in this study.



(a) Weight-generation relationship (b) Fitness-generation relationship

(c) Number of individuals violating constraint conditions

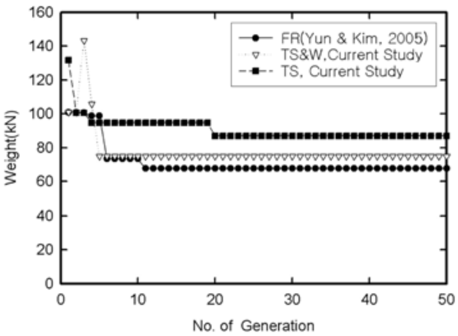
Fig. 8 Optimum design results of three-story two-bay steel frame with top- and seat-angle connections

Table 2 Optimum design results of three-story two-bay steel frame

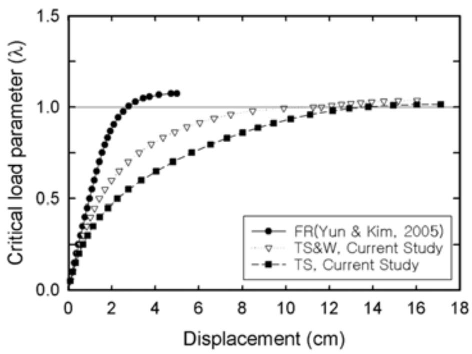
Connection type / Comparisons	Yun & Kim (2005)	Current study	
	FR	TS&W	TS
Column Group ①	W10 × 54	W10 × 54	W10 × 54
Beam Group ②	W21 × 48	W24 × 55	W21 × 68
Optimum Weight (kN)	67.7	74.8	86.9
Critical Load Factor (λ)	1.074	1.036	1.010

The optimum design results of the three-story two-bay steel frame with TS-connections are shown in Fig. 8. Figs. 8(a) and (b) show that the optimum weight and fitness decreases and increases, respectively, as the number of generations increases. The optimum weight was obtained at the third run. In the case of the TS-connection, the total weight of the steel frame increased 28.0% compared with the case of the FR-connection (Table 2). Fig. 8(c), indicating the numbers of individuals violating constraint conditions in each generation, shows that the constraint conditions of the ultimate load $g(1)$ and of the construction workability $g(11)$ and $g(12)$ have the most significant effects on the selection of the member sections.

The trench of the optimum design results of the steel frame with TS&W-connections is similar to the case of the steel frame with TS-connections. Since the rotational stiffness of the TS&W-connections, however, is stronger than that of the TS-connections and weaker than that of the FR-



(a) Weight-generation relationship



(b) Load-horizontal displacement relationship at Node 4

Fig. 9 Optimum design results of three-story two-bay steel frame with FR- and PR-connections

Table 3 Optimum design results of connection in three-story two-bay steel frame

Design elements \ Connection type	TS&W	TS
$l_t (= l_s)$ (cm)	17.80	21.00
$t_t (= t_s)$ (cm)	1.28	1.75
$k_t (= k_s)$ (cm)	2.56	3.51
d (cm)	59.94	53.59
$g_t (= g_s)$ (cm)	7.65	8.58
l_a (cm)	17.81	-
t_a (cm)	1.28	-
k_a (cm)	2.56	-
g_a (cm)	7.65	-
M_u (kN · m)	302.5	89.5
R_{ki} (MN · m/rad)	109.3	50.6
n	1.055	0.557

The diameter of nut W was fixed as 3.81 cm.; The subscripts 't', 's' and 'a' mean top angle, seat angle and web angle respectively.

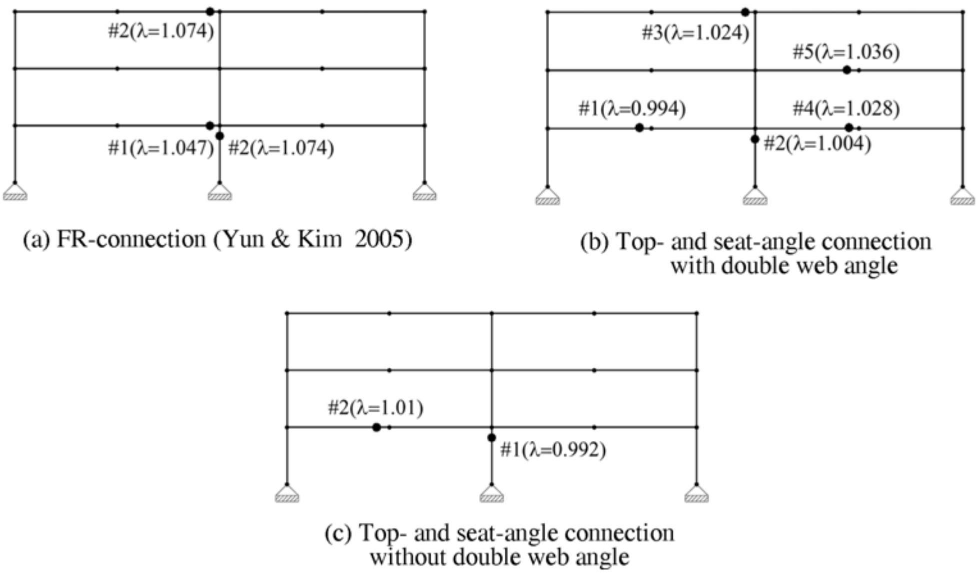
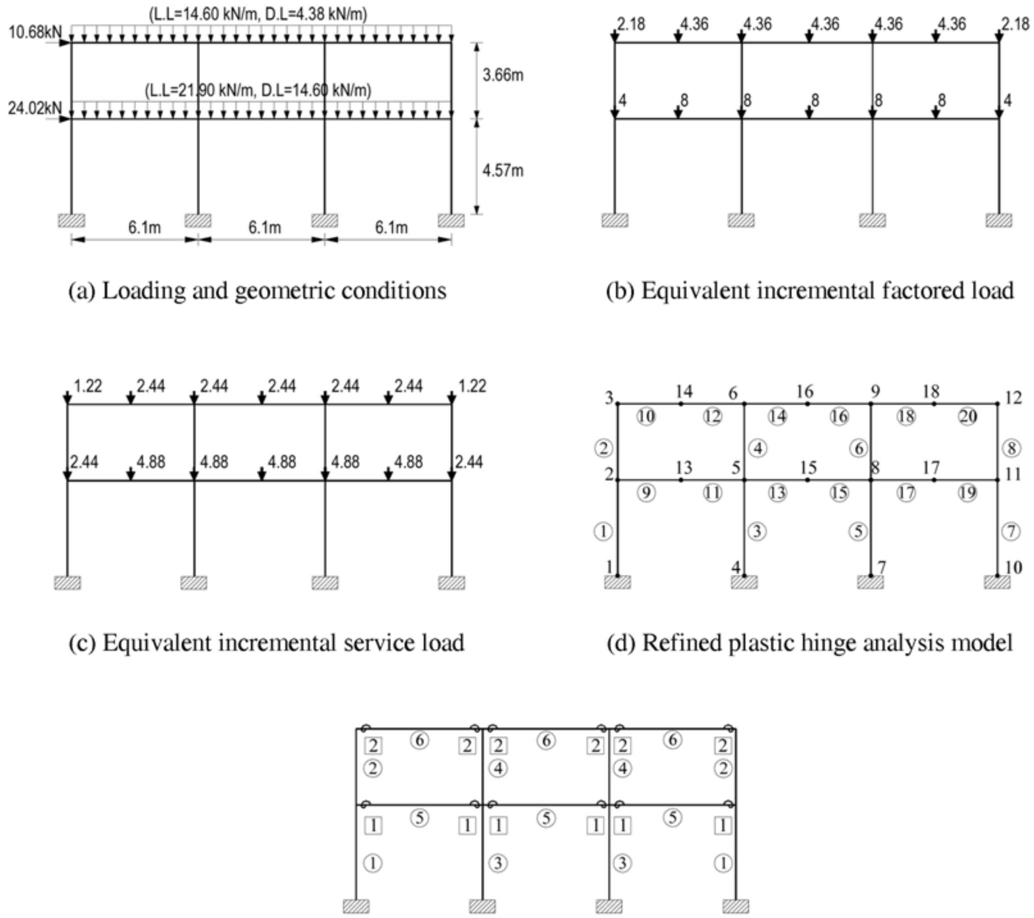


Fig. 10 Sequence of plastic hinge formation in three-story two-bay steel frame

connections, the value of the objective function (total weight of the steel frame) is between those of the TS-connections and the FR-connections. Since the critical load factor is greater than 1.0 (Table 2), the optimized steel frame with TS- and TS&W-connections can carry its design load without failure. Fig. 9(a) shows the relationship between the number of generations and the total weight of the steel frame with TS-, TS&W-, and FR-connections. Fig. 9(b) shows the relationship between the design load and the horizontal displacement of Node 4 of the optimized steel frame, indicating that the horizontal displacement of the steel frame with PR-connections is greater than that of the steel frame with FR-connections. The optimum design results regarding member sections and connection configurations are shown in Tables 2 and 3. Fig. 10 shows the sequence of plastic hinge formation in the steel frame with FR- and PR-connections, indicating that the sequence is different according to the connection type.

4.2 Two-story three-bay steel frame

A two-story three-bay steel frame with different types of connections, shown in Fig. 11(a), was optimized. The equivalent incremental factored and service loads imposed on the steel frame are shown in Figs. 11(b) and 11(c), respectively. A model for the refined plastic hinge analysis is shown in Fig. 11(d), as is a design model indicating connection and member types in Fig. 11(e). The constraint conditions of serviceability, $g(8)$ and $g(9)$, were included in this example. The optimization process of the fitness and total weight of the steel frame with different types of connections are shown in Figs. 12(a) and 12(b). The relationship between the design load and the vertical deflection of Node 15 of the optimized steel frame, indicating that the vertical deflection of the steel frame with PR-connections is greater than that of the steel frame with FR-connections regardless of heavier beam and column sizes, is shown in Fig. 12(c). The optimum design results regarding member sections and connection configurations are shown in Tables 4 and 5. The total



(a) Loading and geometric conditions (b) Equivalent incremental factored load

(c) Equivalent incremental service load (d) Refined plastic hinge analysis model

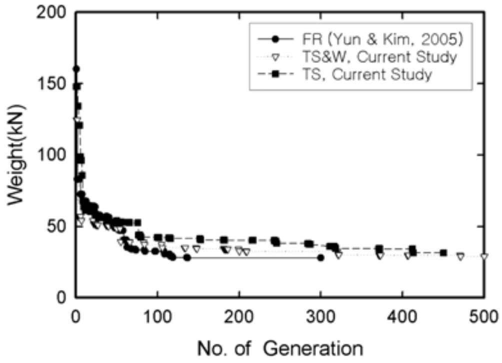
(e) Design model for members and connections

Fig. 11 Two-story three-bay steel frame

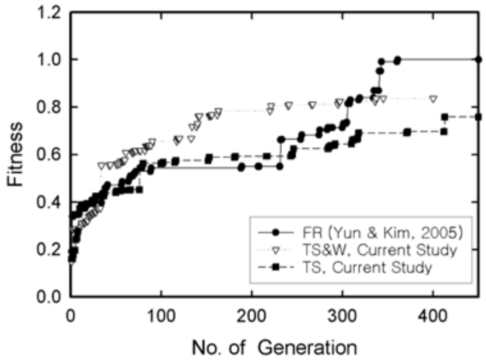
weight of the steel frame with TS- and TS&W-connections is 28% and 10.5% greater than that of the steel frame with FR-connections, respectively. The increasing rate of the beam size rather than that of the column size has more effect on the increase of the weight of frames. This indicates that the decrease of the moment redistribution capacity of beams due to PR-connections, inducing the increase of bending moment in beams, requires heavier beams.

5. Conclusions

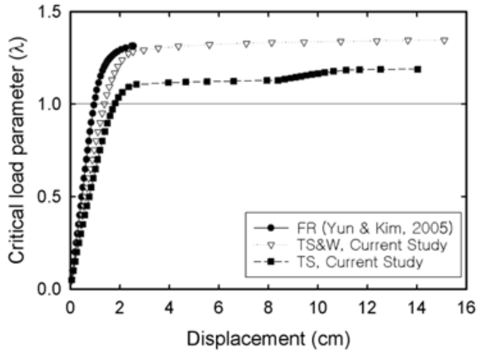
In this study, a discrete optimum design method for plane steel frames with PR-connections was developed. A genetic algorithm and a refined plastic hinge analysis method, one of the second-order inelastic analysis methods of plane steel frames, were incorporated in the present method. The objective function was expressed as the weight of the steel frame and the constraint functions



(a) Weight-generation relationship



(b) Fitness-generation relationship



(c) Load-vertical displacement relationship at Node 15

Fig. 12 Optimum design results of two-story three-bay steel frame with FR- and PR-connections

Table 4 Optimum design results of two-story three-bay steel frame

Comparison \ Connection type	Yun & Kim (2005)	Current study	
	FR	TS&W	TS
Column Group ①	W8× 31	W14× 30	W14× 34
Column Group ②	W8× 31	W5× 16	W10× 22
Column Group ③	W12× 40	W12× 35	W14× 30
Column Group ④	W8× 28	W5× 16	W12× 26
Beam Group ⑤	W16× 26	W18× 35	W16× 36
Beam Group ⑥	W12× 19	W16× 26	W14× 30
Optimum Weight (kN)	27.91	28.46	31.35
Critical Load Factor (λ)	1.136	1.345	1.186

accounting for the requirements of load-carrying capacity, serviceability, ductility, and construction workability were employed. The present method, capable of considering the geometric and material nonlinearity of the plane steel frames with different types of connections, can be effectively utilized in design practice.

Table 5 Optimum design results of connection in two-story three-bay steel frame

Design elements \ Connection type	TS&W		TS	
	Group1	Group2	Group1	Group2
$l_t (= l_s)$ (cm)	15.24	13.97	17.75	17.17
$t_t (= t_s)$ (cm)	1.07	0.86	1.09	0.99
$k_t (= k_s)$ (cm)	2.13	1.72	2.18	1.98
d (cm)	40.64	39.88	40.39	35.05
$g_t (= g_s)$ (cm)	7.21	6.81	7.26	7.06
l_a (cm)	26.97	23.93	-	-
t_a (cm)	0.76	0.64	-	-
k_a (cm)	1.52	1.27	-	-
g_a (cm)	6.60	6.35	-	-
M_u (kNm)	97.8	55.9	44.6	30.7
R_{ki} (MNm/rad)	24.9	11.7	17.2	10.3
n	1.266	1.406	0.892	1.012

The diameter of nut W was fixed as 3.81 cm.

Since an emphasis in this study has been put on developing an efficient and practical discrete optimum design method for plane steel frames with different types of connections rather than on the development of genetic algorithm techniques, there may be an ineffective aspect concerning the optimization time and local optimization technique in the genetic algorithm compared with recent genetic algorithm techniques. Therefore, genetic algorithm techniques need to be further studied.

References

Ad Hoc Committee on Serviceability (1986), "Structural serviceability: A critical appraisal and research needs", *J. Struct. Eng.,* ASCE, **112**(12), 2646-2664.

AISC-LRFD (1986, 1994, 2001), *Load and Resistance Factor Design Specifications for Structural Steel Buildings*, 1st-3rd Eds., American Institute of Steel Construction, Chicago, IL.

Al-Mashary, F. and Chen, W.F. (1991), "Simplified second-order inelastic analysis for steel frames", *J. Ins. Struct. Eng.*, **59**, 395-399.

Attala, M.N., Deierlein, G.G. and McGuire, W. (1994), "Spread of plasticity: Quasi-plastic-hinge approach", *J. Struct. Eng.,* ASCE, **120**(8), 2451-2473.

Camp, C., Pezeshk, S. and Cao, G. (1998), "Optimized design of two-dimensional structures using a genetic algorithm", *J. Struct. Eng.,* ASCE, **124**(5), 551-559.

Chen, W.F. and Lui, E.M. (1986), *Structural Stability-Theory and Implementation*, Elsevier, New York, NY.

Deirelein, G.G., Zhao, Y. and McGuire, W. (1991), "A discrete model for gradual plastification and nonlinear connection response in three-dimensional framed structures", *Annual Technical Session Preceedings*, Structural Stability Research Council, Lehigh University, Bethlehem, PA., 423-432.

Ellingwood, B. (1989), "Serviceability guidelines for steel structures", *Eng. J.*, **26**(1), 1-8.

Foley, C.M., Schinler, D. and Voss, M.S. (2001), "Optimized design of fully and partially restrained steel frames using advanced analysis and objected-oriented evolutionary computation", Research Report No. MUE-001-2001, Department of Civil and Environmental Engineering, Marquette University, Milwaukee, WI.

Galambos, T.V., Ed. (1988), *Guide to Stability Design Criteria for Metal Structures*, 4th Ed., Wiley, New York, NY.

Holland, J.H. (1975), *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, MI.

Kim, K.H. (1999), "Discrete optimum design of steel frame structures based on genetic algorithm and artificial neural network", Ph.D Dissertation, Dept. of Civil Engineering, Kyungpook National University, Daegu, Korea.

Kim, S.E. (1996), "Practical advanced analysis for steel frame design", Ph.D Dissertation, School of Civil Engineering, Purdue University, West Lafayette, IN.

King, W.S. and Chen, W.F. (1994), "Practical second-order inelastic analysis of semi-rigid frames", *J. Struct. Eng.,* ASCE, **120**(7), 2156-2175.

Kishi, N. and Chen, W.F. (1990), "Moment-rotation relations of semi-rigid connections with angles", *J. Struct. Eng.,* ASCE, **116**(7), 1813-1834.

Krishnakumar, K. (1989), "Micro-genetic algorithms for stationary and non-stationary function optimization", *The Int. Society for Optical Engineering*, Intelligent Control and Adaptive Systems, 289-296.

Liew, J.Y.R., White, D.W. and Chen, W.F. (1993a), "Second-order refined plastic hinge analysis for frame design: Part I", *J. Struct. Eng.,* ASCE, **119**(11), 3196-3216.

Liew, J.Y.R., White, D.W. and Chen, W.F. (1993b), "Second-order refined plastic hinge analysis for frame design: Part II", *J. Struct. Eng.,* ASCE, **119**(11), 3217-3237.

Pezeshk, S., Camp, C.V. and Chen, D. (2000), "Design of nonlinear framed structures using genetic optimization", *J. Struct. Eng.,* ASCE, **126**(3), 382-388.

Schinler, D. (2000), "Design of partially restrained steel frames using advanced analysis and objected-oriented evolutionary algorithm", MS Thesis, Marquette University, Milwaukee, WI.

Yun, Y.M. and Kim, B.H. (2005), "Optimum design of plane steel frame structures using second-order inelastic analysis and a genetic algorithm", *J. Struct. Eng.,* ASCE, **131**(12), 1820-1831.