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Application of inverse reliability method to estimation of cable safety factors of long span suspension bridges

Jin Cheng† and Ru-Cheng Xiao‡

Department of Bridge Engineering, Tongji University, Shanghai, 200092, China

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Abstract. An efficient and accurate algorithm is proposed to estimate cable safety factor of suspension bridges satisfying prescribed reliability levels. Uncertainties in the structure and load parameters are incorporated. The proposed algorithm integrates the concepts of the inverse reliability method and deterministic method for assessing cable safety factors of suspension bridges. The unique feature of the proposed method is that it offers a tool for cable safety assessment of suspension bridges, when the reliability level is specified as a target to be satisfied by the designer. After the accuracy and efficiency of the method are demonstrated through two numerical examples, the method is used to estimate cable safety factors of suspension bridges with span length ranging from 2000 to 5000 m. The results show that the deterministic method overestimates cable safety factor of suspension bridges because of neglecting the parameter uncertainty effects. The actual cable safety factor of suspension bridges should be estimated based on the proposed method.

Keywords: inverse reliability method; cable safety factor; suspension bridges; target reliability index; uncertainties.

1. Introduction

Main cables are the most important components of suspension bridges, and therefore ensuring their safety under different load conditions is of great importance to engineers. In the design of main cables of suspension bridges, a cable safety factor (γ) is introduced to ensure an acceptable safety margin for main cables. Traditionally, the cable safety factor is estimated using a deterministic model. The cable safety assessments is accepted or rejected depending on whether the estimated safety factor is within the acceptable value or not. Due to the estimation focus of safety factor commonly used in main cable design of suspension bridges, even when conservative safety factors are used, the what safety degree is still unknown. In practice, both the acceptable and estimated safety factors are adjusted entirely based on empiricism. Uncertainties which arise from variations in geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc.), load magnitude and distribution can not be incorporated within the estimation of cable safety factors.

Probabilistic analysis provides the tool for incorporating structural modeling uncertainties in the

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† Research Associate Professor, Corresponding author, E-mail: chengjin@tsinghua.org.cn
‡ Professor

estimation of the cable safety factor by describing the uncertainties as random variables. Matteo *et al.* (1994) presented a methodology to estimate the current safety factor for the main suspension cables. The methodology estimates the cable safety factor using ductile wire and ducile-brittle wire models, following two different approaches within each model: a Monte Carlo simulation approach and an extreme-value-distribution approach. Haight *et al.* (1997) used a Type-I extreme value distribution to compute cable safety factors for four suspension bridges. Cremona (2003) presented a probabilistic approach for cable residual strength assessment. The approach is applied to the Tancarville suspension cables by taking into account tensile test results, inspection data and weigh-in-motion records. The application of a probabilistic approach in cable safety factors has contributed to more realistic representations of the actual reliability of main cables.

Current bridge design standards have been developed to ensure structural safety by defining a target reliability index (Reid 2002). In other words, the structural reliability level is specified as a target to be satisfied by the designer. Thus, calibration of cable safety factors is needed to guarantee the specified reliability of main cables. However, the above-mentioned probabilistic methods are not appropriate for solving this problem. Therefore, there is a need for a methodology for calibrating cable safety factors satisfying prescribed reliability.

The inverse reliability method can be pursued for the calibration of cable safety factors satisfying prescribed reliability. The basic idea of the method is to determine the unknown parameter considered in the design such that a prescribed target reliability index is reached. In recent years, many efforts have been focused on the development of the method and/or application of the method in different design problems. Winterstein *et al.* (1993) utilized this method for the estimation of design loads associated with specified target reliability levels for offshore structures. An extension of the method was developed by Der Kiureghian *et al.* (1994) for general limit state functions. Li and Foschi (1998) introduced an inverse reliability method for determining design parameters, and applied it to problems of earthquake and offshore engineering. Fitzwater *et al.* (2003) applied inverse reliability methods for extreme loads on pitch- and stall-regulated wind turbines. More recently, Saranyaseontorn and Manuel (2004) extended the inverse reliability method to estimate nominal loads for the design of wind turbines against ultimate limit states.

Although the inverse reliability method founds some applications for different design problems, its application to calibration of cable safety factors of suspension bridges has not been reported. Therefore, the purpose of the current study is to apply the inverse reliability method to estimate cable safety factors of suspension bridges satisfying prescribed reliability. On this purpose, this paper first presents an efficient algorithm for the solution of inverse reliability problems. Secondly, two numerical examples are presented to demonstrate the validity and the efficiency of the proposed method. Finally, the proposed method is applied to calibration of cable safety factors for suspension bridges with span length ranging from 2000 to 5000 m.

2. Proposed method for inverse reliability analysis

The inverse reliability problem is defined as follows (Der Kiureghian *et al.* 1994): Let θ be a parameter of the limit-state function, *G*, i.e., $G(\mu,\theta) = g(x,\theta)$. We want to find θ such that the reliability index, β equals a target value, β_t. $G(\mu,\theta)$ represents the transformation of the limit state function $g(x,\theta)$ from the original space to the space of standard normal variable. μ is the vector of standard normal variables, and x represents the vector of basic random variables. θ represents a deterministic design parameter that must be selected to achieve a specified reliability. In the application example described later, the parameter θ represents the cable safety factor for suspension cables.

For a target reliability index β_t, the inverse problem can be described by

$$\text{find the parameter } \theta \text{, which minimizes } \beta_t = \|\mu\| \tag{1}$$

$$\text{subject to } G(\mu, \theta) = 0$$

As stated in Li and Foschi (1998), the above inverse reliability problem can be solved by "trial and error", using a forward reliability method like FORM and varying the parameter, θ until the reliability achieved matches with the required target. The trial and error procedure is inefficient and involves difficulties resulting from repetitive forward reliability analysis. Thus, it is desired to develop an efficient and more direct approach to determine the design parameters for specified target reliabilities. A general inverse reliability methodology is proposed in the paper, which allows the direct determination of the design parameters when the corresponding target reliabilities are given. The proposed algorithm is similar to the inverse reliability method developed by Der Kiureghian *et al.* (1994) The algorithm involves an iterative algorithm given by the following recursive formulae:

$$\mu^{k+1} = \mu^k + \lambda^k d\mu^k \tag{2}$$

$$\theta^{k+1} = \theta^k + \lambda^k d\theta^k \tag{3}$$

$$d\mu^k = -\beta_t \frac{\nabla_\mu G(\mu^k, \theta^k)}{\|\nabla_\mu G(\mu^k, \theta^k)\|} - \mu^k \tag{4}$$

$$d\theta^k = \frac{[\nabla_\mu G(\mu^k, \theta^k), \mu^k] - G(\mu^k, \theta^k) + \beta_t \|\nabla_\mu G(\mu^k, \theta^k)\|}{\nabla_\theta G(\mu^k, \theta^k)} \tag{5}$$

where μ^k is the vector of standard normal variables at the kth iteration, θ^k represents the deterministic design parameter at the kth iteration, ∇_μ is a vector of gradient operators with respect to μ, ∇_θ is a vector of gradient operators with respect to θ, [,] is the inner product of two vectors, λ^k is the step size at kth iteration, which is determined through a line search algorithm proposed by Der Kiureghian *et al.* (1994). The algorithm proceeds iteratively until convergence is achieved, i.e., when

$$\frac{(\|\mu^{k+1} - \mu^k\|^2 + |\theta^{k+1} - \theta^k|^2)^{1/2}}{(\|\mu^{k+1}\|^2 + |\theta^{k+1}|^2)^{1/2}} \le \varepsilon \tag{6}$$

where ε is a small control parameter assigned by the user. From the past authors' experience, a value of $\varepsilon = 10^{-4}$ to 10^{-3} usually provides satisfactory θ estimates.

The procedure of the proposed method is summarized in Fig. 1.

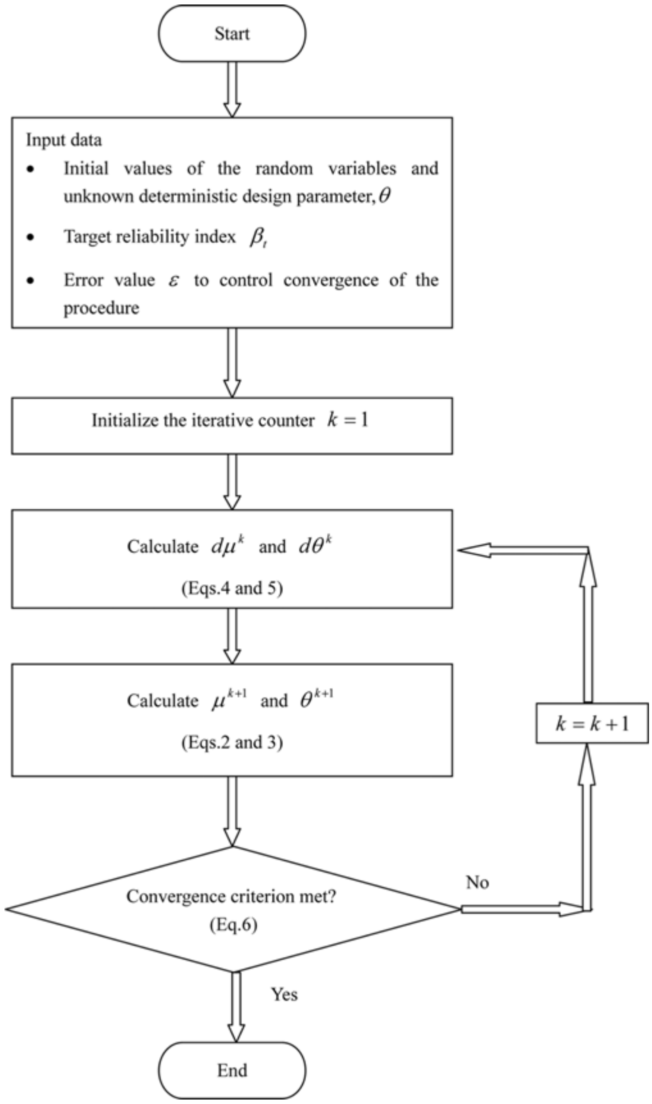

Fig. 1 Flow chart for the inverse reliability procedure

3. Verification study

The main objective here is to investigate the computation efficiency and accuracy of the proposed method for inverse reliability analysis. Two examples are presented in this section. The first example is a simple limit state function. This example has been frequently referred by many researches to verify their developed algorithms. The second example considers a general limit state function. This example demonstrates the accuracy and efficiency of the proposed method for a problem when the random variables are non-gauss variables.

Table 1 Comparisons of the value for design parameter θ (Example 1)

	Proposed method	Der Kiureghian *et al.* (1994)	Li and Foschi (1998)
θ	0.367101	0.3670	0.367183

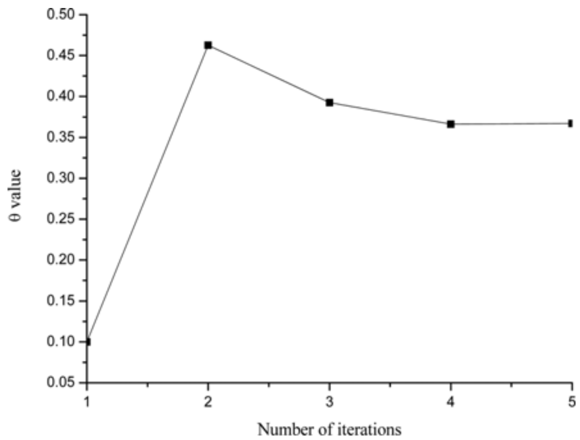

Fig. 2 Illustration of the iterative process for Example 1

3.1 Example 1: Simple limit state function

The example is taken from Der Kiureghian *et al.* (1994). The limit state function is

$$G(\mu, \theta) = \exp[-\theta(\mu_1 + 2\mu_2 + 3\mu_3)] - \mu_4 + 1.5 \tag{7}$$

where μ_1, μ_2, μ_3 and μ_4 are assumed to be independent and have a standard normal distribution. The target reliability index is $\beta_t = 2.0$. The initial values for $\theta = 0.1$, the variables $\mu = (0.2, 0.2, 0.2, 0.2)$, and a tolerance for convergence $\varepsilon = 0.001$ are considered. The results are compared with that given by Der Kiureghian *et al.* (1994) and Li and Foschi (1998) as presented in Table 1. Good agreement is observed. Fig. 2 shows that the convergence is obtained with only 5 iterations.

Table 2 Random variables and statistical data (Example 2)

	Mean	Standard deviation	Distribution type		Correlation matrix				
					x_1	x_2	x_3	x_4	x_5
x_1	2000	400	Lognormal	x_1	1.0	0.2	0.0	0.0	0.0
x_2	5	0.5	Uniform	x_2	0.2	1.0	0.0	0.0	0.0
x_3	450	90	Lognormal	x_3	0.0	0.0	1.0	0.3	0.2
x_4	1800	360	Lognormal	x_4	0.0	0.0	0.3	1.0	0.2
x_5	4.5	0.45	Uniform	x_5	0.0	0.0	0.2	0.2	1.0

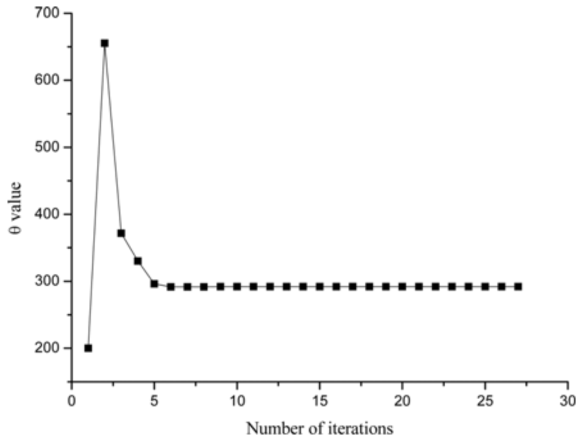

Fig. 3 Illustration of the iterative process for Example 2

3.2 Example 2: General limit state function

Let us consider the following limit state function related to five correlated random variables

$$G(x) = 1.7 - \frac{x_1}{1000 \cdot x_2} - \left(\frac{\theta}{200 \cdot x_2}\right)^2 - \frac{x_4}{1000 \cdot x_5} - \left(\frac{x_3}{200 \cdot x_5}\right)^2 \tag{8}$$

The statistics of the five random variables in this limit state function are listed in Table 2. The reliability index β_t is taken to be 2.4, initial values $\theta = 200$ and $x = (2000,5,450,1800,4.5)$, for a tolerance for convergence $\varepsilon = 0.001$ are considered. The results are shown in Fig. 3. After 27 iterations the design parameter θ is obtained by the proposed method as 292.011. To check its accuracy, a forward reliability analysis using $\theta = 292.011$ in Eq. (8) is carried out. The reliability index (2.4297) is obtained using the direct Monte-Carlo method based on 26,000 simulations. There is a very good agreement between forward reliability index and target reliability index.

4. Application to long span suspension bridges

Using the proposed algorithm, the cable safety factor of single-span suspension bridges with span lengths ranging from 2,000 to 5,000 m is estimated. The basic design parameters for these bridges are listed in Table 3. The cable safety factor γ is obtained as the ratio of the cable strength (T_u) over the maximum cable tension in the main span (T_c), namely:

$$\gamma = \frac{T_u}{T_c} \tag{9}$$

For simplicity, the maximum cable tension in the main span is given as Gimsing (1997)

$$T_c = \frac{(\omega + p)l\sqrt{l^2 + f^2}}{16f} \tag{10}$$

Table 3 Basic design parameters for suspension bridges considered in this paper

l (m)	f (m)	A_c (m^2)	ω (kN/m)	p (kN/m)	Sources
2000	222	0.90	230	60	Arco and Aparicio (2001) & Xiang and Ge (2005)
2500	278	1.23	258	60	
3000	333	1.63	291	60	
3500	389	2.12	331	60	
4000	444	2.72	379	60	
5000	556	4.48	519	60	

Note: l is the main span length; f is the cable sag; A_c is the cable cross-section area; ω is the uniform dead load, including cable weight; p is the uniform live load.

Table 4 Random variables and their statistical properties

	Mean	Coefficients of variation	Distribution type	Sources
A_c	Design value of Table 3	0.05	Lognormal	Assumed
ω	Design value of Table 3	0.08	Normal	
p	Design value of Table 3	0.05	Extreme type I	
σ_c	1678 MPa	0.05	Normal	Imai (1999)

where ω and p are the uniform dead and live loads, respectively, l is the length of the main span, and f is the sag of the main cable.

The cable strength (T_u) can be expressed as

$$T_u = A_c \cdot \sigma_c \tag{11}$$

where σ_c is the rupture strength of the main cable, and A_c is the cross-section area of the main cable.

Substituting (10) and (11) into (9), γ can be written as

$$\gamma = \frac{16 \cdot f \cdot A_c \cdot \sigma_c}{(\omega + p) l \sqrt{l^2 + f^2}} \tag{12}$$

Referring to Eq. (12), several design parameters are sources of uncertainties influencing the value of cable safety factor. The major uncertainty in the estimation of cable safety factor is due to the uniform dead load, ω, the uniform live load, p, the cross-section area of the main cable, A_c, and rupture strength of the main cable, σ_c. The four parameters are taken as random variables in the subsequent study. The statistical properties of the four random variables are listed in Table 4. The four random variables are assumed to be independent unless otherwise stated. As the objective of this study is to propose an efficient for estimating cable safety factor of suspension bridges satisfying prescribed reliability levels, all random parameters in the analysis are based on arbitrary but typical values.

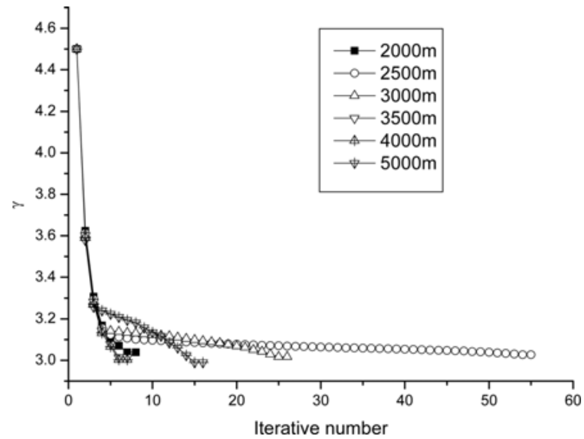

Fig. 4 Illustration of the iterative process for suspended cable safety factors with different span lengths

Referring to Eq. (12), the following limit state equation for the main cable is used:

$$G = \sigma_c - \frac{\gamma \cdot (\omega + p) l \sqrt{l^2 + f^2}}{16 f A_c} \tag{13}$$

To estimate cable safety factor using the proposed method, the target reliability level needs to be specified for the above limit state. Nowak *et al.* (1997) recommend a target component reliability index of 3.5 and a target system reliability index of 5.5 in the ultimate limit states for bridge structures. Li *et al.* (1997) suggest that the target reliability index for bridge structures should lie in an approximate range of 3.2-5.2. Hence, a target reliability index of 3.5 is considered for this study unless otherwise stated.

In the following analyses, unless stated, otherwise, the initial values for the random variables shown in Table 4 are selected to be mean value and $\gamma = 4.5$.

Fig. 4 shows that the convergence is achieved for all suspension bridges considered in this paper using the proposed algorithm. The results are confirmed by forward reliability analysis.

4.1 Effect of parameter uncertainty

One deterministic model and four random models associated with different target reliability indexes are used to investigate the effect of parameter uncertainty on cable safety factor, γ. Four different values of the target reliability index are used: 3.5, 4.0, 4.5 and 5.0. The variations of the cable safety factor with the span lengths are shown in Fig. 5 for different models. It can be seen from Fig. 5 that (1) the cable safety factors from all random models are smaller than those from deterministic model, indicating that parameter uncertainty affects the cable safety factors. In other words, neglecting the parameter uncertainties results in a significant overestimation of the cable safety factor (e.g., from 3.04 to 4.23 for a suspension bridge of 2000 m based on random model I); (2) As the target reliability index increases, the estimated cable safety factor decreases; (3) the cable safety factors vary little as the span length increases. This indicates that the span length has a minor effect on the cable safety factors. Therefore, for simplification purpose, only the cable safety factor of a suspension bridge of 2000 m or 2500 m is estimated in the subsequent study; (4) The

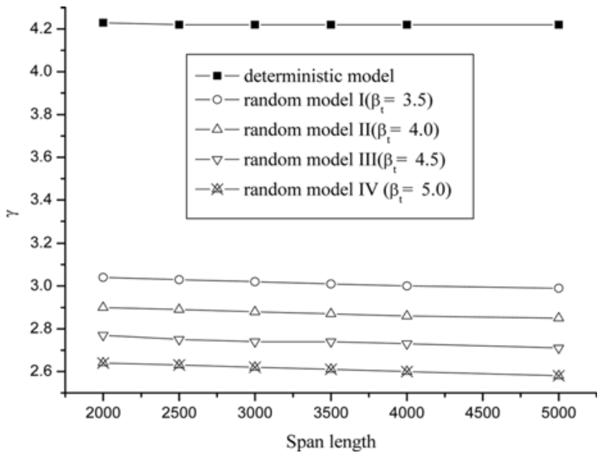

Fig. 5 Variations of the cable safety factor versus span lengths for different models

deterministic method gives higher cable safety factor of suspension bridges because of neglecting the parameter uncertainty effects. For accurate cable safety factor, it is necessary that the analysis technique incorporate the effect of structural parameters randomness. This is of special importance for accurate estimation of cable safety factor of suspension bridges, which exhibit wide dispersion in structural parameters. This problem can be solved by the proposed method. The proposed method does offer a significant improvement over the deterministic method.

4.2 Effect of different initial values of γ

Since the initial value of γ used in the proposed method is chosen arbitrarily, it is necessary to investigate the effect of initial value of γ on the estimated cable safety factor. For this purpose, four different initial values of γ are used: 2.5, 3.5, 4.0 and 4.5. The variations of the cable safety factor

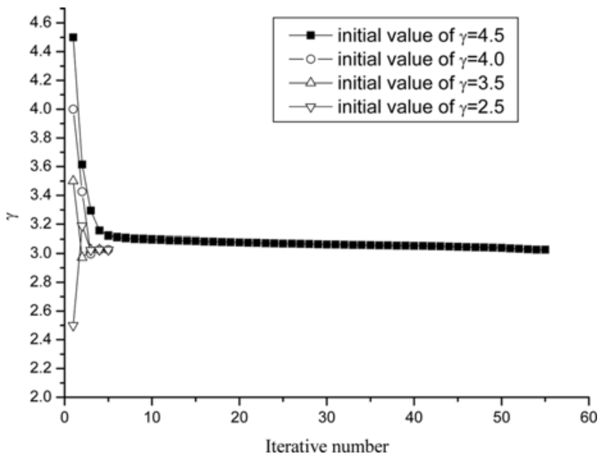

Fig. 6 Variations of the cable safety factor for a 2500 m suspension bridge versus the number of iteration for different initial values of γ

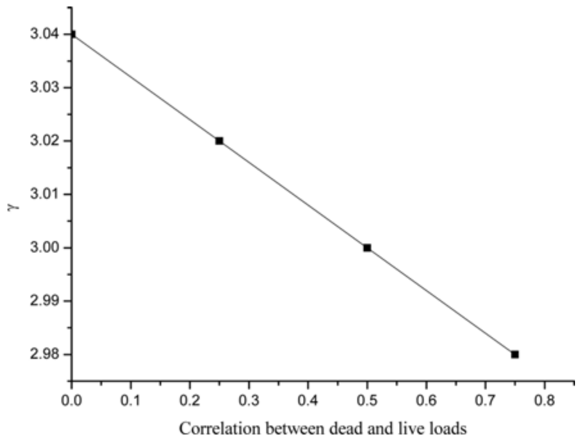

Fig. 7 Correlation influence between dead and live loads on the cable safety factor for a 2000 m suspension bridge

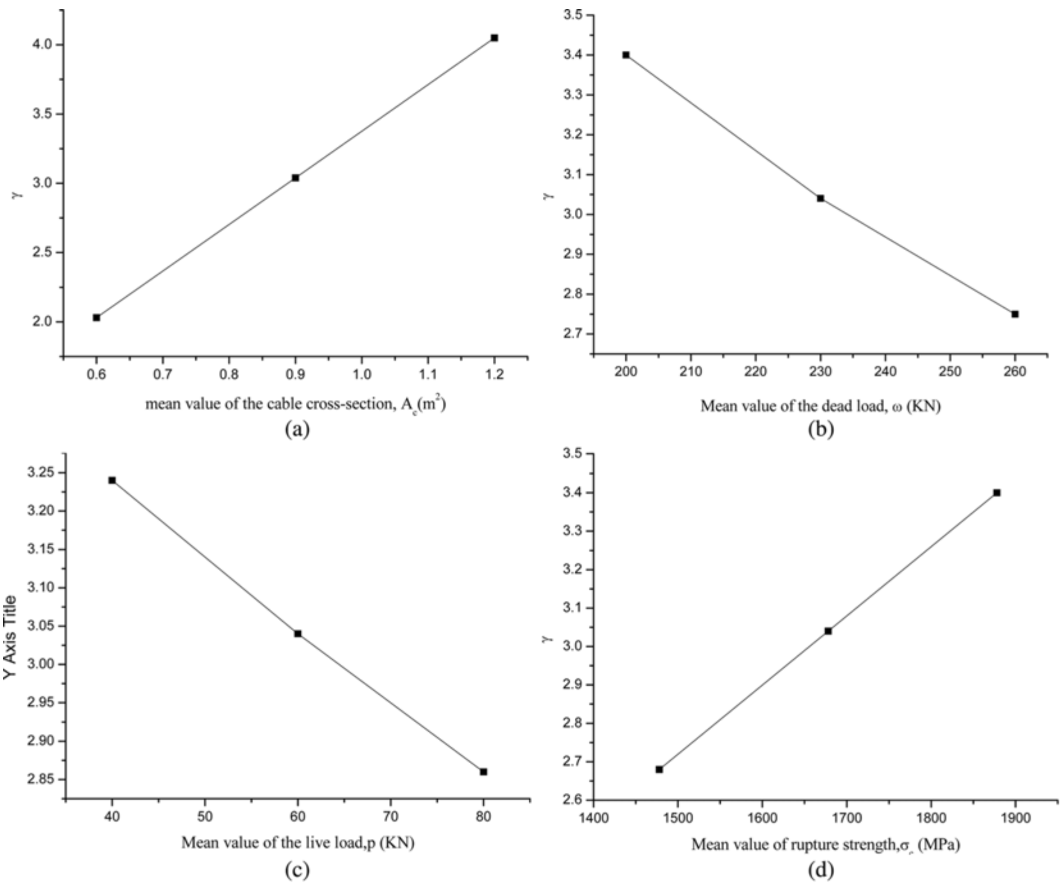

Fig. 8 Influence of the mean values for the different random variables on the cable safety factor for a 2000 m suspension bridge: (a) cable cross-section area, (b) dead load, (c) live load, and (d) rupture strength

with the iteration number are shown in Fig. 6 for different initial values of γ. It can be seen that the iteration number increases as the difference between the initial and accurate values of γ increases. However, convergence and accuracy are achieved regardless of the initial value of γ. The results indicate that the initial value of γ could have a major effect on the rate of convergence of the proposed algorithm. The accuracy of the proposed algorithm is not influenced by the initial value of γ. Therefore, the proposed algorithm can be used to estimate the cable safety factor of suspension bridges when target reliability level is specified for the limit state considered in the design.

4.3 Effect of correlation between dead and live loads

By changing the correlation between dead and live loads, the cable safety factor is computed using the proposed algorithm. The results are plotted in Fig. 7. It can be seen that the cable safety factor decreases slightly as the correlation between dead and live loads increases.

4.4 Effect of mean value of random variables

Keeping the coefficients of all random variables unchanged, only the mean value of all random variables varies, and the estimated cable safety factors are compared in Fig. 8. As the mean values of random variables A_c and σ_c increase, the estimated cable safety factors increase. However, an opposite effect is observed for the random variables ω and p. As the mean values of random variables ω and p decrease, the estimated cable safety factors increase.

5. Conclusions

An efficient and accurate algorithm has been proposed to estimate cable safety factor of suspension bridges satisfying prescribed reliability levels. Uncertainties in the structure and load parameters have been incorporated. The proposed algorithm integrates the concepts of the inverse reliability method and deterministic method for assessing cable safety factors of suspension bridges. Two numerical examples are presented to demonstrate the accuracy and the efficiency of the proposed method.

As an application of the proposed algorithm, the cable safety factor of single-span suspension bridges with span lengths ranging from 2,000 to 5,000 m is estimated. The effects of various parameters on the estimated cable safety factor of suspension bridges are investigated, and the following conclusions can be drawn:

(1) Neglecting the parameter uncertainty effects results in a significant overestimation of the cable safety factor of suspension bridges. The actual cable safety factor of suspension bridges should be estimated based on the proposed method.
(2) The target reliability index has a significant influence on the cable safety factor of suspension bridges. As the target reliability index increases, the estimated cable safety factor decreases.
(3) The span length has a minor effect on the cable safety factors of suspension bridges; the correlation between dead and live loads has a similar effect on the cable safety factors of suspension bridges.
(4) The initial value of cable safety factor could have a major effect on the rate of convergence of the proposed algorithm. The iteration number increases as the difference between the initial

and accurate values of cable safety factor increases. However, the accuracy of the proposed algorithm is not influenced by the initial value for cable safety factor. Therefore, the proposed algorithm can be used to estimate the cable safety factor of suspension bridges when target reliability level is specified for the limit state considered in the design.

(5) The mean values of random variables have significant effect on the cable safety factor of suspension bridges. The estimated cable safety factors increase with the increase in the mean value of the area and rupture strength of the main cable. However, the estimated cable safety factors decrease with the increase in the mean value of the dead and live loads.

This work provided an improved understanding of the cable safety assessment of suspension bridges. However, the deterministic method for assessing cable safety factors used in the proposed method is simple and geometric nonlinearity of structure (the stiffening effect of the tension force in the cable) is not considered in the proposed method. The results reported herein are limited to the particular cases presented. They are only valid in the context of the stated assumption. It should also be noted that the application of the proposed approach is not restricted to the cable safety assessment of suspension bridges. Broaden application of the proposed method is being explored.

Acknowledgments

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