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Modeling and prediction of buckling behavior of compression members with variability in material and/or section properties

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Abstract. Buckling capacity of compression members may change due to inadvertent changes in the member section dimensions or material properties. This may be the result of repair, modification of section properties or degradation of the material properties. In some occasions, enhancement of buckling capacity of compression members may be achieved through splicing of plates or utilization of composite materials. It is very important for a designer to predict the buckling resistance of the compression member and the important parameters that affect its buckling strength once changes in section and/or material properties took place. This paper presents an analytical approach for determining the buckling capacity of a compression member whose geometric and/or material properties has been altered resulting in a multi-step non-uniform section. This analytical solution accommodates the changes and modifications to the material and/or section properties of the compression member due to the factors mentioned. The analytical solution provides adequate information and a methodology that is useful during the design stage as well as the repair stage of compression members. Three case studies are presented to show that the proposed analytical solution is an efficient method for predicting the buckling strength of compression members that their section and/or material properties have been altered due to splicing, coping, notching, ducting and corrosion.

Keywords: buckling analysis; stability; compression member; non-uniform columns.

1. Introduction

The optimal buckling load design of a compression member may be defined as finding the maximum value of a critical load for a given structural weight, or alternatively, minimizing the structural weight that satisfies a prescribed buckling load. In general, maximizing the buckling load capacity is an essential parameter for enhancing the structural stability. In a typical steel structural system several members may be subjected to axial forces and buckling is likely to be one of the failure modes. Besides columns, there are several other structural members or assemblies that will be subjected to compressive axial forces that may result in buckling failure. Such compression members include bracing elements, main chord of trusses, and compression flange of rolled and built-up beam-columns, air conditioning duct system, and web stiffeners of plate girders. The high

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strength to weight ratio of steel as a building material usually results in slender columns that are very susceptible to buckling. The uniformity of the cross section of a steel column is usually interrupted by splices, holes, copes, notches, connection seats, connection plates and haunches. Such change in column cross section may be a result of mechanical, structural or architectural requirements. In a harsh environment steel members need to be protected from corrosion. The non-uniform protection may result in an uneven distribution of corrosion and therefore different material and section properties will exist within a single column. This indicates that columns with non-uniform cross section are very common in structural systems and therefore their behavior need to be investigated to understand their performance. Buckling is a major design concern for steel as well as other structures. Although buckling of columns could be initiated by local, torsional or lateral buckling of the column components, however this paper is mainly concerned with flexural buckling of a column as a whole and therefore the interaction of buckling modes of various column components are not addressed.

The issue of stability of non-uniform columns under various loading has been widely discussed in the literature (Timoshenko and Gere 1961, Ku 1979, Eisenberger and Reich 1989, Arbabi and Li 1991, Lake and Mikulas 1991, Siginer 1992, Dube *et al.* 1996, Elishakoff and Rollot 1999, Gadalla and Abdalla 2004, Ermopoulos 1999, Barbero *et al.* 2000, Elishakoff 2001, Li 2001, 2002, 2003, Raftoyiannis and Ermopoulos 2005). Arbabi and Li (1991) presented a semi-analytical procedure for buckling of elastic columns with continuous or discontinuous step-varying profiles. They showed that formulas for buckling loads for members with variable profiles and different boundary conditions can be obtained in terms of the section and profile parameters. Analytical solution of buckling of a simply supported column with a piecewise constant cross section was investigated by Lake and Mikulas (1991). Parametric structural efficiency analyses were carried to determine the optimum ratio (0.7) between lengths of stiffened center section and the entire column in the buckling resistance of the column. It is concluded such column will result in material saving up to 12% relative to uniform column having the same buckling load. Siginer (1992) presented an analytical approach for buckling of column with continuous monotonic change in its flexural rigidity using Airy function. He concluded that the buckling load for any mode is less than the critical load corresponding to a column of the same length and of constant flexural rigidity whose value is the minimum along the column of variable flexural rigidity. Dube *et al.* (1996) investigated bucking of beam-columns stiffened by rings and subjected to centroidal axial conservative and follower loads. They concluded that the presence of rings does not always increase the critical bucking load of the column. Elishakoff and Rollot (1999) and Elishakoff (2001) presented new closed-form solutions for buckling of columns with variable stiffness and different boundary conditions. They demonstrated that the buckling load is dependent upon a single stiffness coefficient. Ermopoulos (1999) formulated the non-linear equilibrium equations for non-uniform columns under stepped axial loads and solved them using iterative techniques. He obtained the equivalent buckling length co-efficient and the corresponding critical loads. Barbero *et al.* (2000) studied and verified experimentally the interaction of buckling modes between local flange and global (Euler) flexural buckling for intermediate length pultruded composite columns. Li (2001, 2002, 2003) analytically solved the buckling of a multi-step non-uniform column with arbitrary distribution of flexural stiffness and also with axial load distribution. The selected functions he used seem to describe the distribution of flexural stiffness and axial forces in a typical high rise structure. Raftoyiannis and Ermopoulos (2005) studied the elastic stability of eccentrically loaded steel columns with tapered and stepped cross-section and with initial imperfection. Their study was based on the exact solution of the governing equation for buckling of columns with variable cross-section. A plasticity criterion was used to determine the material failure in the buckled configuration. The presented results can be used for the design of steel columns with tapered or stepped cross-sections.

It is not uncommon to structurally modify an installed column in order to accommodate expediently and/or correctively added piping or revised wiring. Such modifications may take many forms; among the most common are: welding brackets onto the side of a column, cutting a hole in the web of a column, or notching the flange of the column. These modifications in addition to splicing of column and stiffness degradation resulting from corrosion are very important in engineering practice.

Cope, blocks and cuts are usually needed sometimes to have uniform levels and facilitate connections. Coping and cutting may cause notches in members and therefore result in variation in stiffness. Connection seats mounted to columns and connection brackets that are welded to columns will increase the column stiffness and result in stiffness variation.

Clearly, any such modifications will affect the characteristics of the column, the most important characteristics affected is its resistance to buckling. To this end, one may wish to know how these inadvertent changes will influence the buckling capacity of the member, and, equally significantly, whether these changes are acceptable to, or anticipated by, the designer. This paper addresses these issues and presents an analytical solution for predicting the buckling strength of compression members that their section and/or material properties have been altered due to splicing, coping, notching, ducting or corrosion.

2. General formulation of buckling of multi-segment compression members

To determine the exact critical buckling load of a real column made of any number of segments of different cross sectional areas, length and material properties, as shown in Fig. 1.

$$E_k I_k y_k'' + P y_k = 0 \qquad 0 \le x \le l,\ k = 1, 2, \ldots, N \tag{1}$$

where x denotes the axial coordinate, y transverse deflection, P applied axial force, E young modulus and I moment of inertia.

The boundary conditions for the above equations are:

$$y_1(0) = 0, \quad y_N(l) = 0 \tag{2}$$

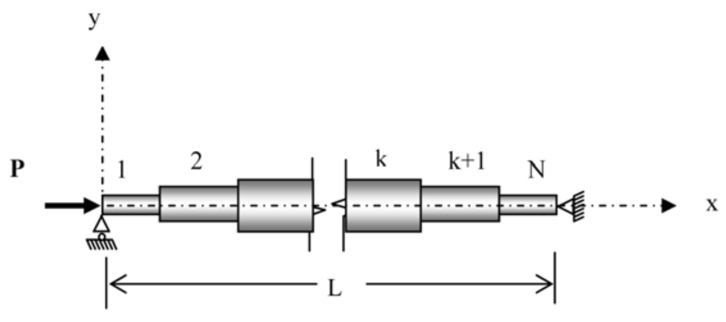


Fig. 1 A general layout of a multi-segments compression member

$$y_k(nl) = y_{k+1}(nl) \qquad 1 \le k \le N-1 \tag{3}$$

$$y_k'(nl) = y_{k+1}'(nl) \qquad 1 \le k \le N-1 \tag{4}$$

The general solution of $N + 1$ equation is given by:

$$y_k = C_k \cos a_k x + C_{k+1} \sin a_k x \qquad k = 1, 2, \ldots, N \tag{5}$$

where:

$$a_k^2 = \frac{P}{E_k I_k}$$

Eq. (5) can be expanded as follows:

$$y_1(x) = C_1 \cos a_1 x + C_2 \sin a_1 x$$
$$y_2(x) = C_3 \cos a_2 x + C_4 \sin a_2 x$$
$$\ldots\ldots$$
$$\ldots\ldots$$
$$y_k(x) = C_{2k-1} \cos a_k x + C_{2k} \sin a_k x$$
$$y_{k+1}(x) = C_{2k+1} \cos a_{k+1} x + C_{2k+2} \sin a_{k+1} x$$
$$\ldots\ldots$$
$$\ldots\ldots$$
$$y_N(x) = C_{2N-1} \cos a_N x + C_{2N} \sin a_N x$$

Let:

$$r_k = \frac{a_{k+1}}{a_k} = \sqrt{\frac{E_k I_k}{E_{k+1} I_{k+1}}}, \quad \gamma_k^2 = (a_k L)^2 = \frac{PL^2}{E_k I_k}, \quad (r_k \gamma_k)^2 = (a_{k+1} L)^2 = \frac{PL^2}{E_{k+1} I_{k+1}}$$

3. Special case

Consider a pivot ended, slender column, consisting of two uniform sections of different material properties and dimensions (E_1, I_1, L_1, E_2, I_2, L_2) and loaded axially as shown in Fig. 2.

The axial load P which will cause the column to buckle elastically, according to Euler:

$$P_{cr1} = \frac{\pi^2 E_1 I_1}{L^2} \qquad (n = 1) \tag{6}$$

$$P_{cr2} = \frac{\pi^2 E_2 I_2}{L^2} \qquad (n = 0) \tag{7}$$

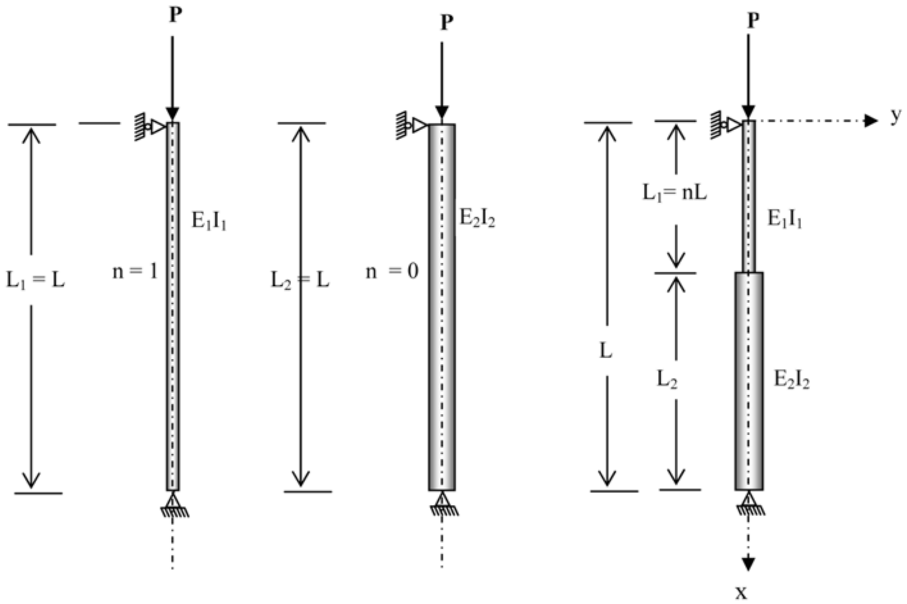


Fig. 2 Two-steps compression member with piecewise constant cross section ($0 \leq n \leq 1$)

For less extreme cases, i.e., $0 < n < 1$, the bucketing load P of the column may be analyzed by setting up the differential equations for each of the two sections of the column:

$$E_1 I_1 y_1'' + P y_1 = 0, \text{ for } 0 \leq x \leq nL \tag{8}$$

$$E_2 I_2 y_2'' + P y_2 = 0, \text{ for } nl \leq x \leq L \tag{9}$$

With the dimensionless constants, and for $k = 2$, Eqs. (5) become:

$$0 + C_3 \cos r\gamma + C_4 \sin r\gamma = 0 \tag{10}$$

$$C_2 \sin n\gamma - C_3 r \cos nr\gamma - C_4 \sin nr\gamma = 0 \tag{11}$$

$$C_2 \cos n\gamma + C_3 r \sin nr\gamma - C_4 r \cos nr\gamma = 0 \tag{12}$$

These equations have a non-trivial solution if their determinant vanishes, i.e.,

$$\begin{vmatrix} 0 & \cos r\gamma & \sin r\gamma \\ \sin n\gamma & -\cos n\gamma & -\sin nr\gamma \\ \cos n\gamma & r\sin nr\gamma & -r\cos n\gamma \end{vmatrix} = 0 \tag{13}$$

This equation yields:

$$\begin{vmatrix} 0 & 1 & \tan r\gamma \\ \tan n\gamma & -1 & -\tan nr\gamma \\ 1 & r\tan nr\gamma & -r \end{vmatrix} = 0 \tag{14}$$

Expanding and simplifying of Eq. (14), one obtains a single relationship among the column parameters a_1, a_2, n

$$r\tan n\gamma + \tan(1-n)r\gamma = 0 \tag{15}$$

Where, for convenience

$$r = \frac{a_2}{a_1} = \sqrt{\frac{E_1E_1}{E_2I_2}}, \quad \gamma^2 = (a_1L)^2 = \frac{PL^2}{E_1I_1}, \quad (r\gamma)^2 = (a_2L)^2 = \frac{PL^2}{E_2I_2} \tag{16}$$

In a sense, Eq. (15) represents a surface $\gamma = f(r, n)$ in a (r, n, γ) space, $r\gamma$ being the dimensionless buckling load. It is quite clear that:

$$f(1,0) = f(1,1) = \pi \tag{17}$$

in review of Eqs. (1) and (2).

Eqs. (15) degenerate on putting $(r, n) = (1, 1)$ or $(1, 0)$

$$\tan n\gamma + \tan(1-n)\gamma = 0 \tag{18}$$

$$\tan\gamma\tan^2 n\gamma = 0 \quad \text{for} \quad n = 1 \tag{19}$$

$$\tan\gamma = 0 \quad \gamma = i\pi, \ i = 1, 2, 3, \ldots \tag{20}$$

Table 1 shows values of $r\gamma$ for several arrays of, r and n, the combination of which is subjected to rule of practicality or common source. For instance, $r = 0$ should accompany $n = 0$; least a portion of the column be devoid of strength, i.e., as thin as a thread; when $r = 1$ and $n = 0$ (or $n = 1$) the uniform column will have a buckling load $P_{cr1} = P_{cr2}$.

Figs. 2(a) and 2(b) show the dimensionless buckling load for different values of n and r. In Fig. 3

Table 1 $r\gamma$ for several r and n combinations

n/r	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	3.14159	3.14159	3.14159	3.14159	3.14159	3.14159	3.14159	3.14159	3.14159	3.14159
0.1	1.44982	2.82266	3.06011	3.07981	3.10845	3.12264	3.13073	3.13578	3.13919	3.14159
0.2	0.84680	1.69721	2.35572	2.70012	2.87530	2.99148	3.05697	3.0971	3.12341	3.14159
0.3	0.59755	1.17037	1.74956	2.19890	2.58117	2.77877	2.89458	3.00835	3.08642	3.14159
0.4	0.47485	0.93876	1.35642	1.78548	2.14465	2.42096	2.67053	2.87956	3.03080	3.14159
0.5	0.40484	0.80416	1.18935	1.56298	1.91063	2.22856	2.51257	2.74875	2.96800	3.14159
0.6	0.36199	0.72126	1.08445	1.42037	1.75419	2.07326	2.37421	2.65432	2.91060	3.14159
0.7	0.33583	0.67045	1.00263	1.33110	1.65450	1.97138	2.28027	2.57930	2.86708	3.14159
0.8	0.32124	0.64206	0.96497	1.28091	1.59805	1.90128	2.22545	2.53477	2.84034	3.14159
0.9	0.32124	0.63021	0.94517	1.25996	1.58411	1.88878	2.20268	2.51616	2.82916	3.14159
1.0	0.31416	0.62832	0.94248	1.25664	1.57080	1.88496	2.19911	2.51327	2.82743	3.14159

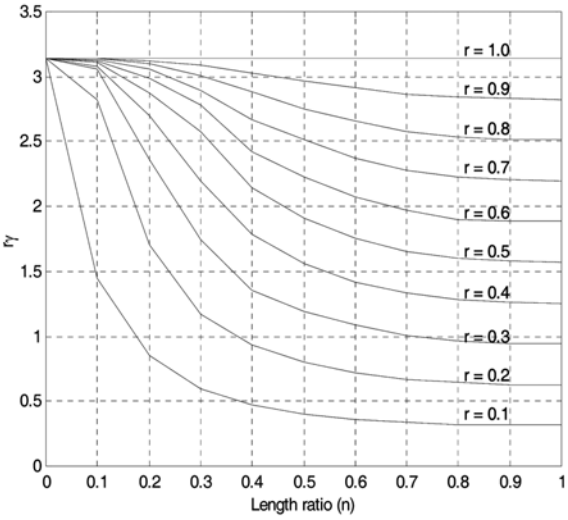


Fig. 3(a) Dimensionless buckling load over a region ($0 < r \leq 1$, $0 \leq n \leq 1$) for different values of stiffness ratio

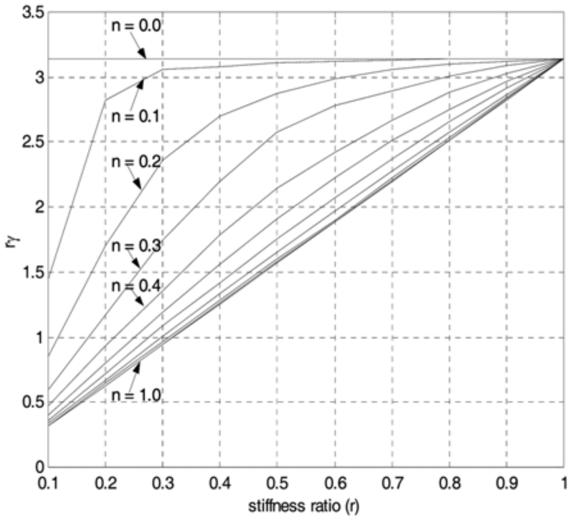
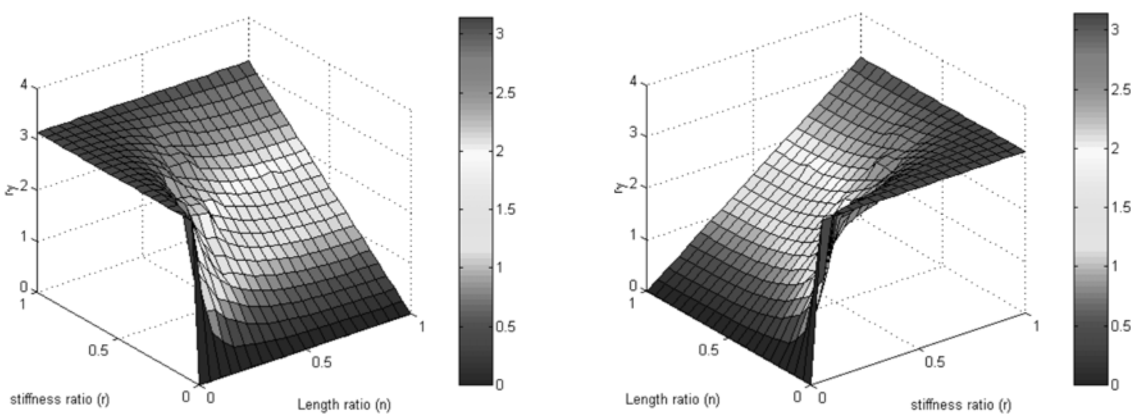


Fig. 3(b) Dimensionless buckling load over a region ($0 < r \leq 1$, $0 \leq n \leq 1$) for different values of length ratio

shows a buckling surface over a region $(0 \leq r \leq 1, 0 \leq n \leq 1)$. The dimensionless buckling load is denoted by $r\gamma = \sqrt{\frac{P}{E_2 I_2}} L$. It is seen that as $r \to 0$, and $n \to 1$, $r\gamma$ gradually decreases from π to near zero. In general, $r\gamma$ seems to increase with increasing r, though not linearly. As expected, when $n = 1$, $r\gamma = r\pi$, or $\gamma = \pi$. Needless to say, the buckling capacity of the column (0, 1) is of interest to none other than a rope-trickier.



(a) (b)

Fig. 4 The buckling surface over a region ($0 < r \leq 1$, $0 \leq n \leq 1$)

4. Degenerated stability and analysis

It has been shown in Table 1 and Fig. 4 that a column with a reduced section (E_1I_1) has a buckling strength

$$r\gamma \le \pi \qquad (0 < r < 1) \tag{21}$$

and its maximum being P_{cr_2} when $r = 1$ $(n = 0, \text{ or } 1)$ a re-examination of Eq. (14), however, reveals that if:

$$r = \frac{n}{n-1} \qquad \left(n \le \frac{1}{2}\right) \tag{22}$$

Eq. (14) degenerates to:

$$(1 + r)\tan n\gamma = 0 \tag{23}$$

and the fundamental root of which is

$$n\gamma = \pi \tag{24}$$

or

$$r\gamma = \frac{\pi}{1-n} > \pi \text{ if } 0 < n \le \frac{1}{2} \tag{25}$$

Eq. (23) clearly indicates that a column where parameters r and n satisfy Eq. (22) will have a buckling strength greater than P_{cr2}; in other words, the buckling surface $\gamma = f(r, n)$ depicted in Fig. 4 has jurisdiction over all values of (r, n) except at points along a line where coordinates are $(\frac{n}{n-1}, n)$

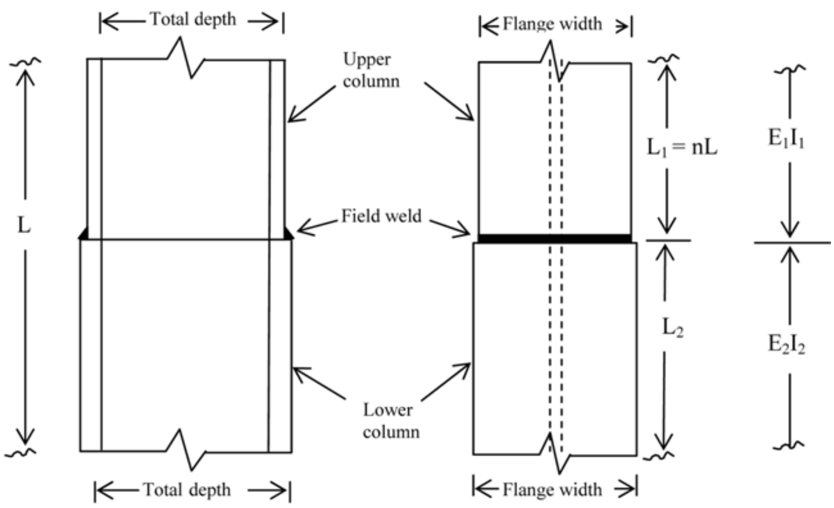


Fig. 5 Directly welded column splice

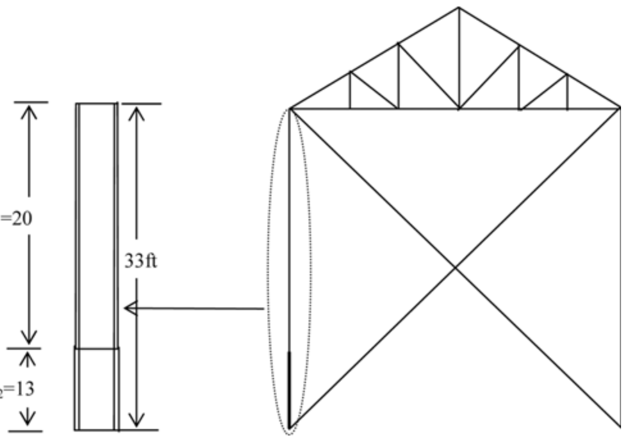


Fig. 6 A spliced column as part of an industrial building

5. Applications of multi-step columns

5.1 Case study 1: Spliced column

Bearing piles (HP-shape), Wide flanges (W-shape), tubular or box-shaped columns are commonly used in multistory steel buildings as compression members. The sizes of these columns usually change every two stories for economical reasons or when the height of the building exceeds the available column length. The change of column size usually requires splicing of two columns of different web or flange width, depth or thickness.

There are several types of column splices used in practice, the most common ones are: (1) flange-plated column splices; (2) directly welded flange column splices; and (3) butt-plated column splices. Example of a directly welded column splice is shown in Fig. 5. The upper column and the lower column will have different section properties (I_1 and I_2) and may be of different material properties (E_1 and E_2) resulting in a stiffness ratio ($r = E_1I_1/E_2I_2$). It is assumed that the end of the columns in the common splice are finished by milling, sawing or other suitable means to guarantee full contact bearing and transfer of axial forces between the two spliced columns AISC (2001), Gaylord (1979) and McCormac (2003).

Example of spliced wide flange columns in a typical industrial building is shown in Fig. 6. A typical hot-rolled wide flange column section is usually 20 feet in length and it needs to be spliced to provide the necessary clearance in industrial hangers of 30 feet height or more, say 33 feet. Therefore the ratio of upper column length to the total column length, in this case, is about 0.6. As an illustrative example, take the column at the ground level to be W18X311 and a column with a smaller size will be spliced on top of it. Table 2 shows different columns used in such an industrial building. Assume that the columns are made of A572 steel with modulus of elasticity E = 29000 ksi (200 GPa). The lowest column is W18X311 and the upper columns are shown in Table 2. For simplicity, the web depth of the two spliced columns is kept the same.

Fig. 7 shows stiffness, weight and critical load ratios for different spliced columns. It is observed that as the size of the upper column decreases, the critical buckling load ratio of the multi-segment column decreases as well. However, the critical buckling load ratio decent more rapidly than the

Table 2 Buckling of spliced W-shape column about weak axis (n = 0.6)

Column designation	I_{yy} (in^4)	Length ratio (n)	$r_{yy}\gamma$	Stiffness ratio (r_{yy})	Weight ratio	Buckling ratio
W18X311	795	0.0	3.141590	1.000	1.0000	1
W18X283	704	0.6	3.008716	0.941	0.9460	0.917216
W18X258	628	0.6	2.883452	0.889	0.8977	0.842432
W18X234	558	0.6	2.754545	0.838	0.8514	0.768792
W18X211	493	0.6	2.619153	0.787	0.8071	0.695074
W18X192	440	0.6	2.499881	0.744	0.7704	0.633211
W18X175	391	0.6	2.376849	0.701	0.7376	0.572417
W18X158	347	0.6	2.256250	0.660	0.7048	0.515803
W18X143	311	0.6	2.150125	0.625	0.6788	0.468421
W18X130	278	0.6	2.048340	0.592	0.6508	0.425122

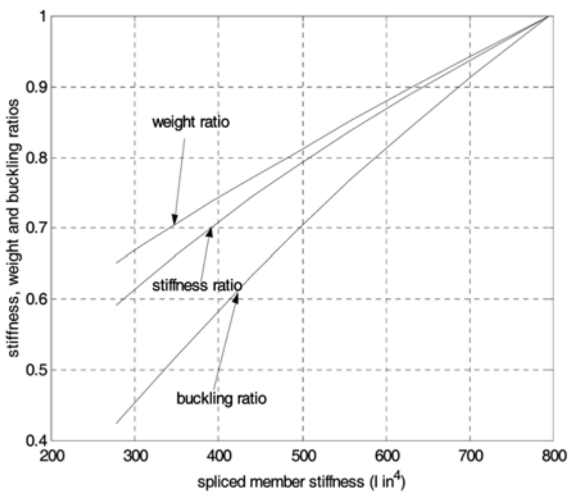


Fig. 7 Stiffness, weight and buckling load ratios of spliced W-shaped column (n = 0.6)

stiffness ratio of the multi-segment compression member and the weight ratio decent more slowly than the stiffness ratio.

5.2 Case study 2: Mechanical ducts in columns

Mechanical and service ducts sometimes require making holes and opening in columns which will result in column with different stiffness. The duct dimensions are D_1 and L_1. For practical reasons the duct length is made at most 10% of column height, i.e., n = 0.1. An idealized duct of a typical rectangular section column is shown in Fig. 8. Table 3 shows the properties of the multi-segment compression member with different sizes of mechanical ducts.

Fig. 9 shows stiffness, weight and critical load ratios for different sizes of mechanical ducts on the top of the compression member. It is observed that as the size of the mechanical duct increases the critical buckling load ratio decreases. However, the decrease in the buckling ratio is very small and negligible until the size of the mechanical duct exceeds 85% of the original compression member

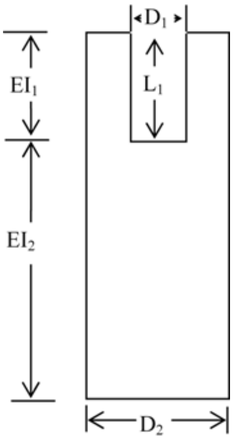


Fig. 8 Compression member with a service duct

Table 3 Buckling of a column with service duct (n = 0.1)

Case	Length ratio (n)	Duct ratio D_1/D_2	$r\gamma$	Stiffness ratio (r)	Weight ratio	Buckling ratio
C1	0.0	0	3.14159	1	1	1.000
C2	0.1	0.05	3.141587	0.99988	0.95	1.000
C3	0.1	0.10	3.141568	0.999	0.90	1.000
C4	0.1	0.15	3.141516	0.99663	0.85	1.000
C5	0.1	0.20	3.141414	0.992	0.80	1.000
C6	0.1	0.25	3.141246	0.98438	0.75	1.000
C7	0.1	0.30	3.140996	0.973	0.70	1.000
C8	0.1	0.35	3.140647	0.95713	0.65	0.999
C9	0.1	0.40	3.140126	0.936	0.60	0.999
C10	0.1	0.45	3.139462	0.90888	0.55	0.999
C11	0.1	0.50	3.1386	0.875	0.50	0.998
C12	0.1	0.55	3.137293	0.83363	0.45	0.997
C13	0.1	0.60	3.135434	0.784	0.40	0.996
C14	0.1	0.65	3.132527	0.72538	0.35	0.994
C16	0.1	0.70	3.127827	0.657	0.30	0.991
C17	0.1	0.75	3.121106	0.57813	0.25	0.987
C18	0.1	0.80	3.109836	0.488	0.20	0.980
C19	0.1	0.85	3.086724	0.38588	0.15	0.965
C20	0.1	0.90	2.942252	0.271	0.10	0.877
C21	0.1	0.95	1.396348	0.14263	0.05	0.198
C22	0.1	1	0	0	0	0.000

cross section. When the size of the duct exceeds 85% of the member cross section the decent in the critical buckling load ratio is very rapid. The stiffness changes non-linearly with the mechanical duct ratio while the weight ratio changes linearly with the mechanical duct ratio as expected.

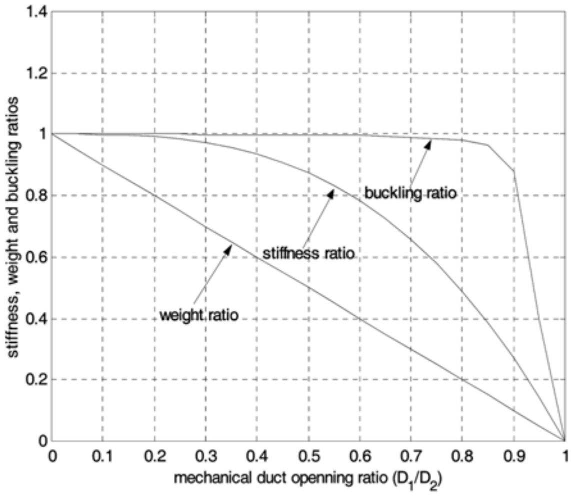


Fig. 9 Stiffness, weight and buckling load ratios of a column with service duct (n = 0.1)

5.3 Case study 3: Corroded holding frame

Holding frames of bulk carriers such as pipelines are usually under compression and they are susceptible to corrosion due to the harshness of the environment on which they exit. To extend the service life of corroded holding frames of bulk carriers a repair method using the same or different materials may need to be employed in order to regain or reinforce the buckling strength of these holding frames. In case of pitting, the buckling capacity of the holding frames will be severely affected with the increase in the degree of corrosion. Such corrosion will in turn change the uniformity of the cross section of the frame and may result in significant degradation in the buckling capacity of the column due to material and cross section variability. The main objective of this case study is to examine the common practices that are associated with repair methods of such compression members with variability in material and/or section properties. The characteristics of the repaired holding frame element should be examined to predict the viability of the repair methods on the buckling characteristics of the repaired pitted holding frame element subjected to the same axial loading condition using the proposed approach. It is assumed in this case study that pitting corrosion is regularly distributed in the upper portion (slim section) of the holding frame element that consists of two different cross section and/or material properties. For practical applications, the corroded parts were taken to be equal to 40% of the holding frame height, i.e., n = 0.4. Table 4 shows the properties of the multi-segment holding frame that originally has the same material before repair (E_1/E_2 = 1) and repaired with different materials (modulus of elasticity ratio ranges from 1 to 2) and different section properties (I_1/I_2 = 0.3, 0.4, 0.5). As shown in Table 4, the stiffness ratio ranges from 0.54772 and 1 and the buckling ratio ranges from 2.27651 to 3.14159.

Fig. 10 shows the variability in stiffness ratio and buckling ratio with material modulus of elasticity ratio for different sizes of repaired sections that are located on the top portion of the holding frame. In order to repair the top 40% of a holding frame having moment of inertia ratio I_1/I_2 = 0.5 and retain the same buckling ratio, a material with a modulus of elasticity of 25% higher than the original material and a moment of inertia of 20% lower is needed as indicated by the arrow in Fig. 10. Again, in order to repair the top 40% of a holding frame having moment of inertia ratio

Table 4 Buckling of a repaired holding frame with variation in material and cross section properties (n = 0.4)

Case	E_1/E_2	I_1/I_2	Stiffness ratio (r)	Buckling ratio
no corrosion	**1**	**0.3**	**0.54772**	**2.27651**
repaired	1.1	0.3	0.57446	2.35039
repaired	1.2	0.3	0.6	2.42096
repaired	1.3	0.3	0.62450	2.48210
repaired	1.4	0.3	0.64807	2.54093
repaired	1.5	0.3	0.67082	2.59771
repaired	1.6	0.3	0.69282	2.65261
repaired	1.7	0.3	0.71414	2.7008
repaired	1.8	0.3	0.73485	2.74338
repaired	1.9	0.3	0.75498	2.78545
repaired	2	0.3	0.77460	2.82647
no corrosion	**1**	**0.4**	**0.63256**	**2.50222**
repaired	1.1	0.4	0.66332	2.5790
repaired	1.2	0.4	0.69282	2.65261
repaired	1.3	0.4	0.72111	2.71466
repaired	1.4	0.4	0.74833	2.7716
repaired	1.5	0.4	0.77460	2.8265
repaired	1.6	0.4	0.80000	2.87956
repaired	1.7	0.4	0.82462	2.91680
repaired	1.8	0.4	0.84853	2.9530
repaired	1.9	0.4	0.87178	2.98812
no corrosion	**1**	**0.5**	**0.70711**	**2.6854**
repaired	1.1	0.5	0.74162	2.75753
repaired	1.2	0.5	0.7746	2.82647
repaired	1.3	0.5	0.80623	2.8890
repaired	1.4	0.5	0.83666	2.9350
repaired	1.5	0.5	0.86603	2.97942
repaired	1.6	0.5	0.89443	3.0224
repaired	1.7	0.5	0.92195	3.05512
repaired	1.8	0.5	0.94868	3.08473
repaired	1.9	0.5	0.97468	3.11354
repaired	2	0.5	1.00000	3.14159

of I_1/I_2 = 0.4, and retain the same buckling ratio, a material with a modulus of elasticity of 33.4% higher than the original material and a cross sectional moment of inertia of 25% lower is needed as indicated by the arrow in Fig. 10. Clearly, there are many alternatives for enhancing the buckling capacity of a holding frame during repair using different combination of material properties (modulus of elasticity ratios) and section properties (moment of inertia ratios). Generalization of Fig. 10 gives the designer a tool for quickly figuring out the desired combination and the trade off between the material and cross section variability during the repairing stage. Such tool helps the designer in making an informed and quick decision depending on the material availability and the space constraints.

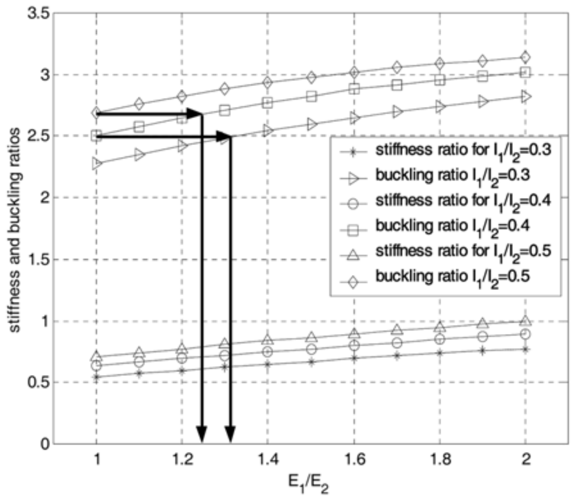


Fig. 10 Stiffness and buckling load ratios of a column with section and material variability (n = 0.4)

6. Conclusions

Analytical modeling and solution for buckling of non-uniform compression members under different configurations is developed. The derived analytical solution represents a class of closed form solution for buckling of non-uniform multi-step compression members. From the derived analytical solution and the case studies presented, it is concluded that:

1. The buckling capacity of the modified compression member is not linearly related to the change in section or material properties.
2. The buckling strength of the stepped column sometime surpasses the Eulerian buckling load of comparable compression member value. However, this seeming enhancement in buckling capability of the compression member of certain special configurations may be just a fleeting phenomenon between stable modes of equilibrium and its practicability may be illusive.
3. The buckling capacity of compression members that their section and/or material have been altered due modification, maintenance or repairing can differ drastically from the original buckling load of the compression member.
4. The degenerated compression members after modification in their section/material properties are immune to the fundamental buckling load.
5. The result can be used by designers and analyst to assess the buckling strength of columns with changes in their cross section and/or material properties.
6. A tool for quickly figuring out the desired combination and the trade off between the material and cross section variability during the repairing stage of corroded columns was developed.

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