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Non-linear time-dependent post-elastic analysis of suspended cable considering creep effect

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Abstract. In this paper, the non-linear time-dependent closed-form, discrete and combined solutions for the post-elastic response of a geometrically and physically non-linear suspended cable to a uniformly distributed load considering the creep effects, are presented. The time-dependent closed-form method for the particularly straightforward determination of a vertical uniformly distributed load applied over the entire span of a cable and the accompanying deflection at time t corresponding to the elastic limit and/or to the elastic region, post-elastic and failure range of a suspended cable is described. The actual stress-strain properties of steel cables as well as creep of cables and their rheological characteristics are considered. In this solution, applying the Irvine's theory, the direct use of experimental data, such as the actual stress-strain and strain-time properties of high-strength steel cables, is implemented. The results obtained by the closed-form solution, i.e., a load corresponding to the elastic limit, post-elastic and failure range at time t, enable the direct use in the discrete non-linear time-dependent post-elastic analysis of a suspended cable. This initial value of load is necessary for the non-linear time-dependent elastic and post-elastic discrete analysis, concerning incremental and iterative solution strategies with tangent modulus concept. At each time step, the suspended cable is analyzed under the applied load and imposed deformations originated due to creep. This combined time-dependent approach, based on the closed-form solution and on the *FEM*, allows a prediction of the required load that occurs in the post-elastic region. The application of the described methods and derived equations is illustrated by numerical examples.

Key words: suspended cable; time-dependent post-elastic analysis; creep of cable; non-linear analysis; closed-form analysis; discrete combined analysis; stress-strain diagram of cable; elastic limit; deflection equation of cable.

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1. Introduction

The evaluation of the non-linear response of a suspended cable in the post-elastic region necessitates the development of accurate and computationally efficient analytical or numerical model. An important task in the post-elastic analysis of a suspended cable is a consideration of creep strain of cable corresponding to the stress level at the elastic limit, and/or at the post-elastic region. Because, the elastic limit is not clearly defined from the stress-strain diagram of cable, one of the important problems in the analysis and design of a suspended cable in the post-elastic

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(plastic) region is therefore decision of what cumulative post-elastic strains and creep strain increments are reasonable for the whole suspended cable, concerning its structural serviceability.

Serviceability criteria referring to deformations of a suspended high strength cable are the significant factors those influence the design in the post-elastic region. Limits to retain a suspended cable under serviceability load conditions in the required behaviour interval are related to maximum values of stresses lain in the post-elastic range.

The development of advanced analytical techniques for the plastic analysis and design of classical steel structures has received significant attention recently. Many of the advanced analytical techniques used in the ultimate load analysis of steel structures have implemented concentrated plastic-hinge models, although the plastic-zone method of analysis is more computationally intensive. These methods can not be applicable in case of the cable structures and that is why the new non-traditional analytical techniques must be developed and applied. Because the elastic limit is not clearly defined from stress-strain diagram of structural ropes and strands, it is difficult to be precise about the conditions at the post-elastic response analysis of a suspended cable.

There have been published only few analytical closed-form studies on the behaviour and analysis of a suspended cable in the post-elastic region. Because of the difficulties that can arise (it may be difficult to be precise about the physical conditions in post-elastic region of stress-strain diagram due to non-existence a clearly defined yield plateau), numerical methods are by far most popular. Nevertheless, some analytical work has been attempted. Irvine (1981) and Palkowski (1998) have given comprehensive mathematical formulations for post-elastic problems of suspended cable, though their solution procedures were different. However, there is a lack of the non-linear post-elastic rheological analysis of a suspended cable considering creep of cables. So far, there is no analytical solution for the calculation of the deflection of a suspended cable depending on stress and time at the post-elastic conditions. That is why, the authors focus on these problems, and elaborating them they start with the work of Irvine (1981), which has been further complemented. Irvines convenient form of the cable equations is modified, because the effects of creep strain increments need to be incorporated in them. A development of the analytical methods for a non-linear solution of cable structures is still an active area of the research, particularly for a use of the probabilistic simulation-based reliability assessment of cable structures.

Greenberg (1970) has presented solutions based on numerical methods for the post-elastic behaviour analysis of cable networks. Jonatowski and Birnstiel (1970) have presented a numerical procedure for the elasto-plastic response analysis of suspended cables. Saafan (1970) proposed numerical solutions for cable structures with the introduction of the actual stress-strain properties of the cables. Murray and Willems (1971) based their analytical method of inelastic suspended structures on the principle of minimum total potential energy. Contro *et al.* (1975) have proposed an inelastic analysis of suspension structures by non-linear programming.

Irvine (1981) has presented analytical closed-form solutions for the post-elastic response of a flat-sag suspended cable to a point load and a uniformly distributed load. Switka (1988) derived discrete analytical model of the geometrically and physically non-linear cable structures including plastic deformations and rupture of cables. Palkowski (1998) has presented an iterative method for the calculation of suspended cables in the inelastic range. The method is based on the known stress-strain relation of the cable and the result is obtained by the method of successive approximation and by the application of the secant modulus of elasticity of the cable.

Kassimali and Parsi-Feraidoonian (1987) investigated the non-linear behaviour and the ultimate strength of cable structures considering the effects of large displacements, slackening of members and inelastic material properties. This discrete method of analysis established on an Eulerian formulation accounting the arbitrarily large joint displacements is based on fundamentals previously developed by Jonatowski and Birnstiel (1970) and Saafan (1970). The incremental relationship between member end forces and end displacements is expressed by means of the tangent stiffness matrix originally given by Tezcan (1968). A Newton-Raphson type of iteration is performed to satisfy the joint equilibrium equations. The non-linear material model of a cable structural member was proposed by Jonatowski and Birnstiel (1970). Panagiotopoulos (1976) formulated variational inequality of cable structures (networks) considering the inelastic stress-unilateral behaviour for infinitesimal incremental displacements. Contri and Schrefler (1977) performed a stability investigation of cable suspended pipelines. Schrefler *et al.* (1983) presented a unified formulation for the geometrically non-linear analysis of combined and cable structures using a total Lagrangian approach. The complementary energy principle for a cable modelled as one-dimensional continuum has been presented for large deflection analysis by Cannarozzi (1987).

Most of the recent methods of non-linear analysis of cable structures are based on the discretisation of the equilibrium equations using *FEM* and solving the resulting non-linear algebraic equations by numerical methods (Jayaraman and Knudson 1981, Kmet 1994, Levy and Spillers 1995, Buchholdt 1988, Talvik 2001, Gasparini and Gautam 2002, Kwan 2003, Gattulli *et al.* 2004, Zhou *et al.* 2004, Kim *et al.* 2004, and others). A bendable finite element for the analysis of flexible cable structures was proposed by Gosling and Korban (2001). Ivanyi and Topping (2002) presented a new graph representation for cable-membrane structures modelled using both one- and two-dimensional elements. Kanno *et al.* (2002) derived a special method for friction or friction-less analysis of non-linear elastic cable structures based on second-order cone programming. Hong *et al.* (2002) adopted the concept of artificial neural network to develop preliminary design system for cable-stayed bridges. Lefik and Schrefler (2002) presented an example of the use of an artificial neural network for parameter identifications of a theoretical elasto-plastic behaviour model of a super-conducting cable under a cyclic loading. Kanno and Ohsaki (2003) have established the minimum principle of complementary energy for cable networks involving only stress components as variables in geometrically non-linear elasticity. Al-Quassab and Nair (2003) applied the wavelet-Galerkin method to study the free vibrations of a suspended cable. Brew and Lewis (2003) proposed an efficient numerical tool, which will allow a better integration of the design/analysis/manufacture of tension membrane structures. Volokh *et al.* (2003) presented a special approach for non-linear dynamic analysis of cable structures. Wang and Xu (2003) investigated a wind-rain-induced vibration of cable. Cheng *et al.* (2004) presented an improved Monte Carlo method for the probabilistic determination of initial cable forces of cable-stayed bridges.

The common approach to these investigations is to study the cable structure as a geometrically non-linear system. However, little attention is paid to the time-dependent post-elastic analysis of a suspended cable with rheological properties.

The purpose of this paper is to present the non-linear time-dependent closed-form, discrete and/or combined solutions for the elastic and post-elastic response of a geometrically and physically non-linear suspended cable to uniformly distributed load considering the creep effects. For the time-dependent analysis of a suspended cable, the time domain is divided into a discrete number of time steps. At each time step, the cable is analysed under the corresponding stress-strain properties and imposed deformations due to applied load and due to creep. In this paper, the time-dependent closed-form method for the particularly straightforward determination of the uniformly distributed load and accompanying deflection at time t those correspond to the elastic limit and/or to the elastic region, post-elastic and failure range of a suspended cable considering the creep effects, is presented. In this solution, applying the Irvine theory, the direct use of experimental data, such as the actual stress-strain and strain-time properties of high-strength steel cables, is implemented. A suspended cable is analysed at the initial time t_0 and at time t, when creep strain $\varepsilon_c(t)$ under the corresponding stress level $\sigma(t_0)$ is defined through the experimentally obtained constitutive equations.

This approach avoided the use of an incremental procedure, in which the slope of the stress-strain curve is required at numerous points, as is commonly presented in numerical solutions. On the other hand, the results obtained by the closed-form solution, i.e., a load corresponding to the elastic region, post-elastic and failure range at time t, enable the direct use in the discrete non-linear time-dependent post-elastic analysis of a suspended cable. This initial value of load is necessary for the non-linear time-dependent elastic and post-elastic discrete analysis, concerning incremental and iterative solution strategies with tangent modulus concept. Known value of the corresponding load is divided into the required number of load increments at the studied time t. At each time step, the suspended cable is analysed under the applied load and imposed deformations originated during the previous time interval due to creep. This combined time-dependent approach, based on closed-form solution and on the *FEM*, allows the prediction of the required load that occurs in the post-elastic region.

The application of the described methods and derived equations is illustrated by numerical examples.

2. Time-dependent closed-form solution of suspended cable in the elastic and post-elastic region

At this point, it is worthwhile explaining the basic terms as post-elastic, plastic, elastic limit and ultimate strain, those are often used with some different meanings by different authors.

The limiting stress above which the behaviour of cable is no farther elastic is called the elastic limit. Region in the stress-strain relation, i.e., in the stress-strain diagram of the cable behind the elastic limit is called the post-elastic and/or plastic region. The main characteristic of elastic strain is reversibility. Most cables are linearly elastic after a pre-stretching is applied. The strain that does not disappear after removal of the stress is called the plastic strain. In the present paper a strain occurred in the elastic region at the time t_0 will be marked as $\varepsilon_e(t_0)$ (with corresponding stress $\sigma_e(t_0)$) and a strain in the post-elastic region, i.e., in the plastic region will be marked as $\varepsilon_p(t_0)$ (with corresponding stress $\sigma_p(t_0)$).

Steel cables stretch in effect of tensile axial stress. This extension and/or strain are of two kinds, particularly an elastic one and an inelastic one, i.e., permanent (or plastic). The elastic part of the cable strain primarily depends on elastic stretching of contained wires, so it is influenced by modulus of elasticity of the wires and the cable construction. Primarily, inelastic strain is affected by the cable pulling, when single wires are mutually pulled together or the cable core is compressed transversally. As a result of that the deformation of cables is then bigger and the modulus of elasticity smaller than in individual wires. Strain and modulus of elasticity of each steel cable are dependent on many factors, especially on the structure, i.e., on the geometry of the cable (on the number of layers of wires in a strand, on the number of strands in a cable, on number and shape of wires in a layer, on the height of wire stranding in a strand and that of strands in a cable), and further on the magnitude of loading and the number of loading cycles, on cable treatment and lubrication, etc. A pre-stretching load of about 55 percent of the breaking load is usually applied to remove constructional looseness in the cables.

In the first section of the stress-strain diagram (see Fig. 4), which is limited by the elastic limit $\varepsilon_{e,\,lim}(t_0)$, there is relatively uniform increase of strain $\varepsilon_e(t_0)$ with increasing tensile stress $\sigma_e(t_0)$, and the course of stress-strain can be considered as a linear one. The second part of the stress-strain diagram, behind the elastic limit, is characterized by a considerable increase of the post-elastic permanent strain $\varepsilon_p(t_0)$. The stress-strain curve is rising ever more slowly till rupture of the cable, when ultimate tensile strength $\sigma_u(t_0)$ with corresponding ultimate strain $\varepsilon_u(t_0)$, is achieved. Typically the elastic limit $\sigma_{e,\,lim}(t_0)$ is achieved at about 50 percent of the ultimate tensile strength $\sigma_u(t_0)$.

The closed-form static solution of a suspended cable will proceed on the following assumptions: Perfectly flexible cable, working only in tension and having zero stiffness in compression and bending will be assumed. The profile of a uniform cable hanging under its own weight between two supports is flat, so that the ratio of sag to span is 1:8 or less (hence, $d/l \leq 1/8$ is considered). Most of the suspended cables used for structural purposes are within this category. The analysis will be hold for suspended cables under vertical uniformly distributed static load, with no slackening behaviour and no unloading behaviour in plastic region. Loading is monotonically increasing, being distributed over the ground plan of the cable projection. The cable is homogeneous with a constant cross section along its entire length. The expression for a cable length is expanded into a binomial series, considering just its first two terms. All relationships for geometrical and force quantities of a suspended cable will be expressed by means of a cable horizontal force which is constant everywhere along the cable since no longitudinal load is acting.

For the time-dependent analysis of the suspended cable, the time domain is divided into a discrete number of time steps. At each time t, the structure is analysed under uniformly distributed load of intensity q, per unit span and under the imposed deformations due to the elastic, post-elastic and creep strains. The response results are compared with those obtained at the initial reference time t_0. All the approaches are fully automated through a computer program.

2.1 Response of cable to uniformly distributed load at the elastic region

Consider a suspended cable with a span l, hanging under its self-weight g_0 between two supports those are not at the same level as shown in Fig. 1. The following equation for the initial reference profile $z(x, t_0)$ of this cable at time t_0 is given by

$$z(x,\,t_0) \;=\; \frac{g_0}{2H(t_0)\cos\beta}x(l-x)+x\tan\beta \tag{1}$$

where β is the inclination of the connecting line of the suspension points of the cable with the horizontal x axis. The horizontal component of cable tension $H(t_0)$ of the cable loaded only by its self-weight g_0 per unit length at the starting initial time t_0 is generally defined as a ratio of a bending moment $M(x)$ at an investigated x cross section of the simple horizontal beam with the same span and load as the investigated cable to a sag $d(x)$ measured from the connecting line of suspension points. Hence, $H(t_0)$ can be written as $H(t_0) = M(x)/d(x)$. The horizontal component of cable tension $H(t_0)$ for a suspended cable with the mid-span sag d, hanging under its self-weight g_0 between two supports at the same level ($\beta = 0$), is given as $H(t_0) = g_0 l^2/8d$.

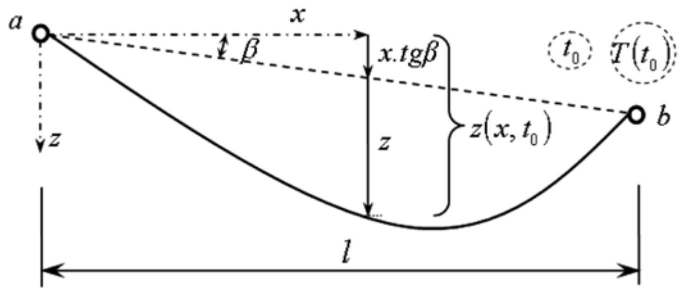


Fig. 1 Profile geometry of a suspended cable at initial time t_0

2.1.1 Deflection and cable equation as the time functions

In this section, the characteristic time-dependent equations for the non-linear elastic response of a suspended cable to a vertical uniformly distributed load q at time t are derived and presented.

Suppose that under applied vertical uniformly distributed load q and under self-weight g_0 the shear force at some cross section x along the span of a suspended cable is $Q = (ql/2)(1 - 2x/l) + (g_0 l/2\cos\beta)(1 - 2x/l)$. Then, the vertical equilibrium at a cross section of the deformed cable at time t requires that

$$((H)(t_0) + \Delta H(t))\frac{d}{dx}((z(x, t_0)) + w(x, t)) = Q + (H(t_0) + \Delta H(t))\tan\beta \tag{2}$$

where $w(x, t)$ is the additional vertical cable deflection and $\Delta H(t)$ is the increment in the horizontal component of cable tension $N(t)$ at the investigated time t. The resultant horizontal component of cable tension $N(t)$ is given as $H(t) = H(t_0) + \Delta H(t)$. The right-hand side of the equilibrium Eq. (2) is analogous to the shear force in a simple supported horizontal beam of uniform weight under the action of a uniformly distributed load (the span of the beam is the same as that of the suspended cable). Eq. (2) may be integrated directly, and after the boundary conditions have been satisfied, the additional vertical deflection $w(x, t)$ at time t is

$$w(x, t) = \frac{1}{H(t_0) + \Delta H(t)}\left\{\frac{ql}{2}x\left(1 - \frac{x}{l}\right) - \frac{\Delta H(t) g_0 l}{2H(t_0)\cos\beta}x\left(1 - \frac{x}{l}\right)\right\} \tag{3}$$

To complete the solution, $\Delta H(t)$ must be determined. Use is made of a cable equation that incorporates elastic (application of the Hooke's law), creep and temperature strain to provide a closure condition at investigated time t relating the changes in cable tension force to the changes in cable geometry when the cable is displaced from its original equilibrium profile.

If $ds(t_0)$ is the original length of the cable element and $ds(t)$ is its new length at time t, then $(ds(t_0))^2 = dx^2 + dz^2$ and $(ds(t))^2 = (dx + du(x, t))^2 + (dz + dw(x, t))^2$, where $u(x, t)$ and $w(x, t)$ are the longitudinal and vertical components of the displacements of the cable element, respectively. If the profile of the cable is flat, so that the ratio of its sag to span is 1:8 or less, the fractional change in its length, correct to the second order of small quantities, is

$$\frac{ds(t) - ds(t_0)}{ds(t_0)} = \frac{du(x, t)}{ds(t_0)}\frac{dx}{ds(t_0)} + \frac{dw(x, t)}{ds(t_0)}\frac{dz}{ds(t_0)} + \frac{1}{2}\left(\frac{dw(x, t)}{ds(t_0)}\right)^2 \tag{4}$$

In the cable element of a vertically loaded suspended cable of appreciable curvature, the longitudinal displacement component is much smaller than the transverse one. Accordingly, the axial strain $\varepsilon(t) = (ds(t) - ds(t_0))/ds(t_0)$, related to the current displacement of the cable element, can be evaluated through the following simplified expressions $\varepsilon_x(t) = \partial u(x,t)/\partial x + (1/2)(\partial w(x,t)/\partial x)^2$ and $\varepsilon_z(t) = \partial w(x,t)/\partial z$. Thus the corresponding axial strain $\varepsilon(t)$ is obtained from a solution of the quadratic equation $\varepsilon^2(t)(ds(t_0))^2 + 2\varepsilon(t)(ds(t_0))^2 - 2(\varepsilon_x(t)dx^2 + \varepsilon_z(t)dz^2) = 0$.

By the effect of the increment in axial tension force $\Delta N(t)$ exerted on the cable element, the elastic element of the cable $ds(t_0)$ ($ds(t_0)$ here denotes the length at the equilibrium state under its self-weight) is extended by $\Delta ds(t) = ds(t) - ds(t_0)$. Assuming Hooke's law, the constitutive equation is usually written in the form of

$$\Delta N(t) = AE(t)\Delta\varepsilon(t) \tag{5}$$

where $E(t)$ is the modulus of elasticity of the cable at time t and A is the cross-sectional area of the cable. The change in cable strain $\Delta\varepsilon(t)$ due to increment in tension is given by $\Delta\varepsilon(t) = \varepsilon(t) - \varepsilon_0(t_0)$. The strain of cable $\varepsilon_0(t_0)$ under its self-weight has low order of magnitude and will be ignored here (i.e., $\varepsilon_0(t_0) = 0$). Consequently, considering $\varepsilon_0(t_0) = 0$ and substituting $\Delta N(t) = \Delta(H(t)ds(t_0)/dx$ into Eq. (5) leads to the following expression

$$\varepsilon(t) = \frac{\Delta ds(t)}{ds(t_0)} = \frac{ds(t) - ds(t_0)}{ds(t_0)} = \frac{\Delta H(t)}{E(t)A}\frac{ds(t_0)}{dx} \tag{6}$$

If the effect of a creep strain $\varepsilon_c(t)$ of the cable at time t and the effect of a uniform temperature difference of $\Delta T(t) = T(t) - T(t_0)$ (where $T(t_0)$ and $T(t)$ are the initial and computational temperature of the cable, respectively) are considered, terms $\varepsilon_c(t)$ and $\varepsilon_T(t) = \alpha\Delta T(t)$ need to be added to the elemental equation, where α is the coefficient of expansion. On the basis of Eq. (4) and Eq. (6), the cable equation for the element can be written as

$$\frac{\Delta H(t)\dfrac{ds(t_0)}{dx}}{E(t)A} + \varepsilon_c(t) + \alpha\Delta T(t) = \frac{du(x,t)}{ds(t_0)}\frac{dx}{ds(t_0)} + \frac{dw(x,t)}{ds(t_0)}\frac{dz}{ds(t_0)} + \frac{1}{2}\left(\frac{dw(x,t)}{ds(t_0)}\right)^2 \tag{7}$$

After multiplication of Eq. (7) by $(ds(t_0)/dx)^2$, one obtains

$$\frac{\Delta H(t)\left(\dfrac{ds(t_0)}{dx}\right)^3}{E(t)A} + \varepsilon_c(t)\left(\frac{ds(t_0)}{dx}\right)^2 + \alpha\Delta T(t)\left(\frac{ds(t_0)}{dx}\right)^2 = \frac{du(x,t)}{dx} + \frac{dw(x,t)}{dx}\frac{dz}{dx} + \frac{1}{2}\left(\frac{dw(x,t)}{dx}\right)^2 \tag{8}$$

If the effects of elastic cable deformation, assuming Hooke's law, as well as the effects of creep strain and of temperature change and the fractional change in length of the cable, correct to the second order of small quantities are considered, the cable equation follows the general integrated form as

$$\int_{t_0}^{t}\frac{\Delta H(t)L_e}{E(t)A}dt + \int_{t_0}^{t}\varepsilon_c(t)L_c dt + \int_{t_0}^{t}\alpha\Delta T(t)L_T dt$$

$$= \int_{t_0}^{t}\int_{0}^{l}\frac{du(x,t)}{dx}dx\,dt + \int_{t_0}^{t}\int_{0}^{l}\left(\frac{dw(x,t)}{dx}\frac{dz(x,t_0)}{dx}\right)dx\,dt + \frac{1}{2}\int_{t_0}^{t}\int_{0}^{l}\left(\frac{dw(x,t)}{dx}\right)^2 dx\,dt \tag{9}$$

where L_e, L_c and L_T are the members connected with the length of the non-loaded cable under its own weight g_0 at time t_0. They can be determined from the following expressions

$$L_e = \int_0^l \left(\frac{ds(t_0)}{dx}\right)^3 dx = l\left(1 + \frac{3}{2}\tan^2\beta + \frac{g_0^2 l^2}{8H^2(t_0)\cos^2\beta}\right) \quad (10)$$

$$L_c = L_T = \int_0^l \left(\frac{ds(t_0)}{dx}\right)^2 dx = \frac{l}{\cos^2\beta}\left(1 + \frac{g_0^2 l^2}{12H^2(t_0)}\right) \quad (11)$$

After respective adjustment of cable Eq. (9) for a continuous uniformly distributed load, the increment in the horizontal component of cable tension force $\Delta H(t)$ at time t, can be obtained. Substituting Eq. (1) and Eq. (3) into Eq. (9), and performing the necessary integration, the following cubic equation for $\Delta H(t)$ as a time function is found as

$$\Delta H^3(t) + \left[\frac{E(t)A}{L_e}\frac{g_0^2 l^3}{24H^2(t_0)\cos^2\beta} + 2H(t_0) + \frac{E(t)A}{L_e}\varepsilon_c(t)L_c + \frac{E(t)A}{L_e}\varepsilon_T(t)L_T\right]\Delta H^2(t)$$

$$+ \left[\frac{E(t)A}{L_e}\frac{g_0^2 l^3}{12H(t_0)\cos^2\beta} + H^2(t_0) + 2\frac{E(t)A}{L_e}\varepsilon_c(t)L_c H(t_0) + 2\frac{E(t)A}{L_e}\varepsilon_T(t)L_T H(t_0)\right]\Delta H(t)$$

$$+ \frac{E(t)A}{L_e}\left[\varepsilon_c(t)L_c H^2(t_0) + \varepsilon_T(t)L_T H^2(t_0)\right] - \frac{E(t)A}{L_e}\frac{q(t)l^3}{12\cos\beta}\left(g_0 + \frac{q(t)}{2}\cos\beta\right) = 0 \quad (12)$$

The strain increment $\varepsilon_c(t)$ of the cable at time t under non-linear creep (at constant stress level $\sigma(t_0)$) is characterised by a constitutive equation in the form of the logarithmic-exponential approximation function as

$$\varepsilon_c(t) = (a + c\ln t)(1 - e^{-bt}) \quad (13)$$

with coefficients a, b and c for the corresponding stress level σ. On the base of creep tests the concrete forms of the constitutive equation were obtained (Kmet 2004, Kmet and Holickova 2004). By statistical investigation of the resultant creep curves the optimal creep constitutive equations for steel cables were found. Values of their coefficients depend on the stress level and/or on the load of a suspended cable.

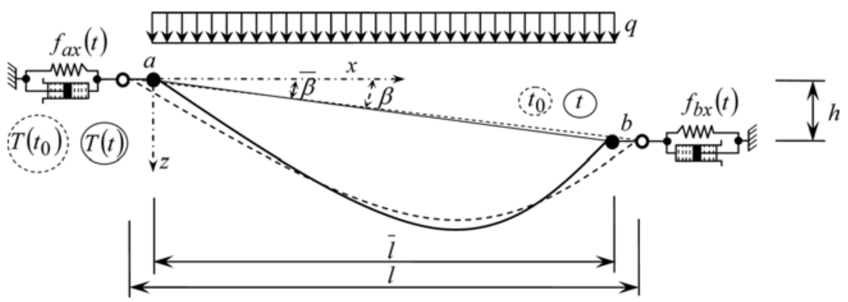


Fig. 2 Geometry and loading of a suspended cable

If horizontal supports flexibilities of $f_{ax}(t)$ and $f_{bx}(t)$ (elastic and viscoelastic yielding supports) occur at each end, respectively, as shown in Fig. 2 one may replace the axial tension stiffness $E(t)A$ of the cable by the modified stiffness at time t given by

$$\overline{E(t)A} = \frac{E(t)A}{\frac{L_e + E(t)Af_x(t)}{L_e}} \tag{14}$$

where $f_x(t) = f_{ax}(t) + f_{bx}(t)$, and then proceed as if the supports were unyielding.

In a non-linear elastic analysis of a suspended cable the increment of the horizontal component of cable tension $\Delta H(t)$ and the corresponding additional deflection $w(x, t)$ at time t are generally the dependent variables, while a uniformly distributed load q, stiffness and geometrical parameters of the cable, i.e., modulus of elasticity of the cable $E(t)$, cross-sectional area of the cable A at time t, the members connected with lengths of the cable L_e, L_c and L_T under g_0 and the horizontal component of cable tension $H(t_0)$ at time t_0 are the independent variables. However, in non-linear closed-form post-elastic analyses the situation is best reversed. In the post-elastic range the cable stress $\sigma_p(t_0)$ with the corresponding post-elastic strain $\varepsilon_p(t_0)$ at the initial time t_0 and the accompanying creep strain $\varepsilon_c(t)$ at time t are known from the stress-strain as well as from the strain-time (creep curves under corresponding stress levels) properties of the cable. Thus, the dependent variables, i.e., uniformly distributed load corresponding to the post-elastic region $q_p(t)$ and the associated deflection $w_p(x, t)$ at time t, may be unambiguously determined.

The elastic limit represents the limit of applicability of Eq. (3) and Eq. (12). The reverse approach described for the post-elastic analysis we shall also apply in the following closed-form solution for the elastic region.

2.1.2 Uniformly distributed load and associated deflection of cable corresponding to the elastic limit and/or to the elastic region as time functions

This section will be focused on developing simple time-dependent equations for determining the uniformly distributed load and also the associated deflection of the suspended cable corresponding to the elastic limit and/or to the elastic region at required times.

The cable stress $\sigma_e(t_0)$ and accompanying strain $\varepsilon_e(t_0)$ corresponding to the elastic limit and/or to the elastic region are determined from the experimentally obtained stress-strain diagram of the cable. The corresponding creep strain $\varepsilon_c(t)$ of the cable is determined from the adequate creep curve (creep strain – time diagram) or is calculated from the adequate constitutive equation of creep. It is known that the elastic limit of cables cannot be directly defined as in the case of tension tests of steels. Typically for cables, the elastic limit $\sigma_{e,lim}(t_0)$ is reached at about 50 percent of the ultimate tensile strength $\sigma_u(t_0)$.

The horizontal component of cable tension force $H_e(t_0) = H(t_0) + \Delta H_e(t_0)$ will be used in the following analyses. For the geometries under consideration $(d/l \leq 1/8)$ the cable axial tension force $N_e(t_0)$ is, at most, about 10 percent higher than its horizontal component $H_e(t_0)$. This small variation can be compensated by the decrease of the horizontal component of cable tension. For this purpose the coefficient 1,1 can be used. Therefore the horizontal component of cable tension $H_e(t_0)$ corresponding to the elastic limit and/or to the elastic region can be determined as

$$1.1H_e(t_0) = \sigma_e(t_0)A \tag{15}$$

where $\sigma_e(t_0)$ is the known stress at the elastic limit and/or from the elastic region and A is the cross-sectional area of the cable.

Substituting $\Delta H(t) = \Delta H_e(t_0) = H_e(t_0) - H(t_0)$ into Eq. (12) and performing its necessary arrangement, the following quadratic equation for uniformly distributed load $q_e(t)$ corresponding to the elastic limit and/or to the elastic region at time t is found as

$$q_e^2(t) + 2g_0 q_e(t) - \left\{ \frac{24L_e}{E(t)Al^3}(H_e(t_0) - H(t_0))^3 + \left[\frac{g_0^2}{H^2(t_0)\cos^2\beta} + 48\frac{L_e}{E(t)Al^3}H(t_0) \right.\right.$$
$$\left. + \frac{24}{l^3}\varepsilon_c(t)L_c + \frac{24}{l^3}\varepsilon_T(t)L_T \right] \cdot (H_e(t_0) - H(t_0))^2$$
$$+ \left[\frac{2g_0^2}{H(t_0)\cos^2\beta} + 24\frac{L_e}{E(t)Al^3}H^2(t_0) + \frac{48}{l^3}\varepsilon_c(t)L_cH(t_0) + \frac{48}{l^3}\varepsilon_T(t)L_TH(t_0) \right] \cdot (H_e(t_0) - H(t_0))$$
$$\left. + \frac{24}{l^3}\varepsilon_c(t)L_cH^2(t_0) + \frac{24}{l^3}\varepsilon_T(t)L_TH^2(t_0) \right\} = 0 \tag{16}$$

Because of simplification, the following denotation for the absolute term in Eq. (16) can be accepted as

$$\Omega_e(t) = \frac{24L_e}{E(t)Al^3}(H_e(t_0) - H(t_0))^3 + \left[\frac{g_0^2}{H^2(t_0)\cos^2\beta} + 48\frac{L_e}{E(t)Al^3}H(t_0) \right.$$
$$\left. + \frac{24}{l^3}\varepsilon_c(t)L_c + \frac{24}{l^3}\varepsilon_T(t)L_T \right] \cdot (H_e(t_0) - H(t_0))^2 + \left[\frac{2g_0^2}{H(t_0)\cos^2\beta} + 24\frac{L_e}{E(t)Al^3}H^2(t_0) \right.$$
$$\left. + \frac{48}{l^3}\varepsilon_c(t)L_cH(t_0) + \frac{48}{l^3}\varepsilon_T(t)L_TH(t_0) \right] \cdot (H_e(t_0) - H(t_0)) + \frac{24}{l^3}\varepsilon_c(t)L_cH^2(t_0) + \frac{24}{l^3}\varepsilon_T(t)L_TH^2(t_0) \tag{17}$$

and the equation for $q_e(t)$ at time t is in the form as

$$q_e(t) = \sqrt{g_0^2 + \Omega_e(t)} - g_0 \tag{18}$$

From Eq. (3) the corresponding additional deflection under this load at time t is given by

$$w_e(t) = \frac{l}{2H_e(t_0)}\left\{ q_e(t) - \frac{H_e(t_0) - H(t_0)}{H(t_0)\cos\beta}g_0 \right\} x\left(1 - \frac{x}{l}\right) \tag{19}$$

These equations are not restricted to elastic limit conditions but may be used for any part of the elastic region.

2.2 Uniformly distributed load and associated deflection of cable corresponding to the post-elastic region as time functions

After the elastic limit of the cable is reached, the problem of the response becomes more difficult because it is non-linear both with respect to geometrical and also material properties.

As the load exceeds the elastic region, irreversible inelastic strains increase and the cable is characterized by a non-linear post-elastic behaviour. Because the equilibrium of forces has to be maintained, so the equations for additional deflection are of the same form as previously. Therefore Eq. (3) can be applied and the expression for the additional deflection at time t of suspended cable in the post-elastic region of its stress-strain curve is given by

$$w_p(x, t) = \frac{l}{2H_p(t_0)}\left\{q_p(t) - \frac{H_p(t_0) - H(t_0)}{H(t_0)\cos\beta}g_0\right\}x\left(1 - \frac{x}{l}\right) \tag{20}$$

The horizontal component of cable tension force corresponding to the post-elastic condition $H_p(t_0)$ is considered as

$$1.1H_p(t_0) = \sigma_p(t_0)A \tag{21}$$

where $\sigma_p(t_0)$ is the stress at the post-elastic region and A is the cross-sectional area of the cable. Cable stress $\sigma_p(t_0)$ corresponding to the post-elastic region and the accompanying strain $\varepsilon_p(t_0)$ as well as the creep strain $\varepsilon_c(t)$ are determined following the cable stress-strain properties and/or following the creep strain-time curve of the cable, respectively. Consequently, by means of the mentioned inputs, a uniformly distributed load corresponding to the post-elastic range $q_p(t)$ and the associated deflection $w_p(x, t)$ at time t may be found.

The compatibility cable equation as a time function at the post-elastic condition is changed, but not substantially. Its integrated form is given by

$$\int_{t_0}^{t}\frac{\varepsilon_p(t_0) - \varepsilon_0(t_0)}{\varepsilon_0(t_0)}\frac{H(t_0)L_e}{E(t_0)A}dt + \int_{t_0}^{t}\varepsilon_c(t)L_c dt + \int_{t_0}^{t}\varepsilon_T(t)L_T dt$$
$$= -\int_{t_0}^{t}\int_{0}^{l}\frac{d^2z(x, t_0)}{dx^2}w_p(x, t)dx dt - \frac{1}{2}\int_{t_0}^{t}\int_{0}^{l}\frac{d^2w_p(x, t)}{dx^2}w_p(x, t)dx dt \tag{22}$$

where $\varepsilon_p(t_0)$ is the strain of the cable at the post-elastic region at the time t_0 and $\varepsilon_0(t_0)$ is the strain in the cable under its self-weight in its initial unloaded profile at the time t_0.

Substituting Eqs. (1) and (20) into Eq. (22) and performing the necessary integration, the following quadratic equation for a uniformly distributed load $q_p(t)$ corresponding to the post-elastic region at time t is found

$$q_p^2(t) + 2g_0q_p(t) - 24\frac{H_p^2(t_0)}{l^3}\frac{H(t_0)L_e}{E(t_0)A}\frac{\varepsilon_p(t_0) - \varepsilon_0(t_0)}{\varepsilon_0(t_0)} - \frac{24H_p^2(t_0)}{l^3}\varepsilon_c(t)L_c$$
$$-\frac{24H_p^2(t_0)}{l^3}\varepsilon_T(t)L_T - \frac{g_0^2H_p^2(t_0)}{H^2(t_0)\cos^2\beta} + \frac{g_0^2}{\cos^2\beta} = 0 \tag{23}$$

Because of simplification, the following denotation for the absolute term in Eq. (23) can be accepted as

$$\Omega_p(t) = 24\frac{H_p^2(t_0)}{l^3}\frac{H(t_0)L_e}{E(t_0)A}\frac{\varepsilon_p(t_0)-\varepsilon_0(t_0)}{\varepsilon_0(t_0)} + \frac{24H_p^2(t_0)}{l^3}\varepsilon_c(t)L_c$$

$$+\frac{24H_p^2(t_0)}{l^3}\varepsilon_T(t)L_T + \frac{g_0^2H_p^2(t_0)}{H^2(t_0)\cos^2\beta} - \frac{g_0^2}{\cos^2\beta} \tag{24}$$

and the equation for $q_p(t)$ at time t (considering creep strain of the cable $\varepsilon_c(t)$) is in the form as

$$q_p^2(t) + 2g_0q_p(t) - \Omega_p(t) = 0 \tag{25}$$

with just one positive real root at time t given by

$$q_p(t) = \sqrt{g_0^2 + \Omega_p(t)} - g_0 \tag{26}$$

The deflection at the mid-span ($x = l/2$) of the suspended cable with supports at the same level ($\beta = 0$) in the post-elastic region at time t is given as

$$w_p(t) = \frac{l^2}{8H_p(t_0)}\left\{q_p(t) - \frac{H_p(t_0)-H(t_0)}{H(t_0)}g_0\right\} \tag{27}$$

2.3 Uniformly distributed load and associated deflection of cable corresponding to its ultimate capacity as time functions

At an ultimate condition, the additional deflection $w_u(x, t)$ at time t can be calculated from Eq. (20) in which $H_p(t_0)$ is replaced by the horizontal component of cable tension force corresponding to the ultimate condition $H_u(t_0)$ and $q_p(t)$ is replaced by $q_u(t)$. $H_u(t_0)$ is assumed as

$$1.1H_u(t_0) = \sigma_u(t_0)A \tag{28}$$

where $\sigma_u(t_0)$ is a stress at the ultimate limit and A is a cross-sectional area of the cable.

An uniformly distributed load $q_u(t)$ corresponding to the ultimate limit at time t, considering the creep strain $\varepsilon_c(t)$, can be obtained from Eq. (26) in which $q_p(t)$ is replaced by $q_u(t)$ and $\Omega_p(t)$ is replaced by $\Omega_u(t)$ where the ultimate strain $\varepsilon_u(t_0)$ instead of $\varepsilon_p(t_0)$ has to be introduced into the expression (24).

An additional deflection $w_u(t)$ at the mid-span ($x = l/2$) of the suspended cable with supports at the same level ($\beta = 0$) under the ultimate condition at time t can be calculated analogously to Eq. (27).

3. Combined time-dependent closed-form and discrete solution

Effective approach for the time-dependent geometrically and physically non-linear analysis of a suspended cable in the post-elastic region can be established on the suitable combination of the closed-form solution presented with the conventional discrete *FEM*.

Closed-form solution based on the simple one-step technique enables to frame an initial quantity of the uniformly distributed load corresponding to the individual levels, i.e., to the level at the elastic and post-elastic region. For the discrete analysis of the non-linear suspended cable this initial value is needed, due to the incremental procedure that is used for a solution. This known value of the corresponding load is divided into a required number of load increments at the studied time t.

Results obtained by closed-form solution, i.e., the load corresponding to the elastic or post-elastic region is used as input into the discrete *FEM*. This direct finding of the required load is the basic advantage of appropriate combination of mentioned methods. It should, also be noted that this combined method can cause a reduction of CPU times, which may often be desirable.

The proposed combined method allows one to assess the structural safety and serviceability of suspended cable in the post-elastic region. This method allows one to evaluate the influence of post-elastic and creep strains on the short- and long-term behaviour of the suspended cable. By this method, the stress redistributions can be evaluated and mainly the excessive deflections of a suspended cable that can occur in the post-elastic region, may be assessed.

Combined non-linear time-dependent method based on the combination of the closed-form solution with the discrete *FEM* for the behaviour analysis of the suspended cable in the post-elastic region can be briefly described by the following expressions.

The fundamental system of non-linear equilibrium equations of the suspended cable at time t can be expressed as

$$\boldsymbol{K}_T(t, \Delta)\Delta\Delta(t) = \Delta\boldsymbol{Q}(t) \tag{29}$$

where $\boldsymbol{K}_T(t, \Delta)$ is the global incremental stiffness matrix of the suspended cable; $\Delta\Delta(t)$ is the incremental vector of the nodal displacements and $\Delta\boldsymbol{Q}(t)$ is the incremental load vector.

The transformation of the uniformly distributed load $q_p(t)$ (given by Eq. (26)), that correspond to the post-elastic region at time t, into the discrete load vector $\boldsymbol{Q}_p(t)$ of a cable finite element in the direction of vertical z axis is considered in $2D$ as

$$\boldsymbol{Q}_p(t) = \{0, q_p(t)a, 0, q_p(t)a\}^T = \left\{0, (\sqrt{g_0^2 + \Omega_p(t)} - g_0)a, 0, (\sqrt{g_0^2 + \Omega_p(t)} - g_0)a\right\}^T \tag{30}$$

where a is the corresponding loading length. Substituting Eq. (30) into Eq. (29), the following system of non-linear equilibrium equations, in the incremental combined form, is obtained

$$\boldsymbol{K}_T(t, \Delta)\Delta\Delta(t) = \Delta\boldsymbol{Q}_p(t) \tag{31}$$

The incremental nodal load vector $\Delta\boldsymbol{Q}_p(t)$ (at $2D$) of a cable finite element in the direction of vertical z axis is defined as

$$\Delta\boldsymbol{Q}_p(t) = \left\{0, \Delta((\sqrt{g_0^2 + \Omega_p(t)} - g_0)a), 0, \Delta((\sqrt{g_0^2 + \Omega_p(t)} - g_0)a)\right\}^T \tag{32}$$

Eq. (31) characterizes the strategy of the combined time-dependent closed-form and discrete solution. That enables to use directly the loading vector $\Delta\boldsymbol{Q}_p(t)$ corresponding to the level in the post-elastic region of the suspended cable obtained by the closed-form solution. This approach offers an effective tool for the time-dependent non-linear post-elastic behaviour analysis of large-span cables with the rheological properties.

An analogous approach can be applied for the elastic region of suspended cables and the following system of non-linear equilibrium equations, in the incremental combined form, can be found as

$$\boldsymbol{K}_T(t,\Delta)\Delta\Delta(t) = \Delta\boldsymbol{Q}_e(t) = \left\{0, \Delta((\sqrt{g_0^2+\Omega_e(t)}-g_0)a), 0, \Delta((\sqrt{g_0^2+\Omega_e(t)}-g_0)a)\right\}^T \tag{33}$$

where $\Omega_e(t)$ is given by Eq. (17).

Full derivation and details of the *FEM* presented can be found in Kmet (1994). The created transformation model has been implemented into the *LANSTAT* software.

It should, just be noted that the combined approach presented can also be applied by means of an arbitrary available *FEM* based software such as *COSMOS/M*, *ANSYS* etc.

4. Numerical applications and discussions of the results

The following examples are given to characterize the application of the derived theories to practical problems. Numerical illustration of the developed closed-form and combined discrete methods (as *LANSTAT* and *COSMOS/M* software are used) was carried out on the suspended cable with immovable supports shown in Fig. 3. The structural response of suspended cable is obtained using two developed analytical models.

First, the uniformly distributed load and the accompanying deflection corresponding to the elastic limit, post-elastic and failure range is calculated in a closed-form model, secondly combined *FEM* is applied. Resulting responses, i.e., deflections determined according to mentioned two models are compared.

The two suspended cables with the different properties used as the examples were selected for the investigation of their elastic and post-elastic behaviour and for the illustration of the mentioned methods. The first suspended cable analysed was the cable without the creep effect. As the second case, time-dependent behaviour of the suspended cable with the creep effect was studied.

4.1. Characteristic of suspended cable and input data

Consider a suspended cable (see Fig. 3) with the suspension points at the same level (accordingly $\beta = 0$ and $\cos\beta = 1$), with a span l = 60 m and the mid-span sag d = 6 m (so the sag to span ratio is

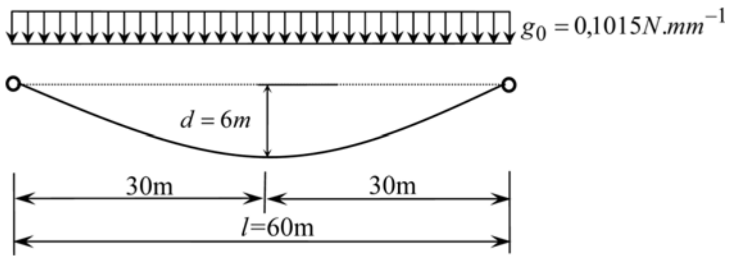


Fig. 3 Geometry and loading of the investigated suspended cable

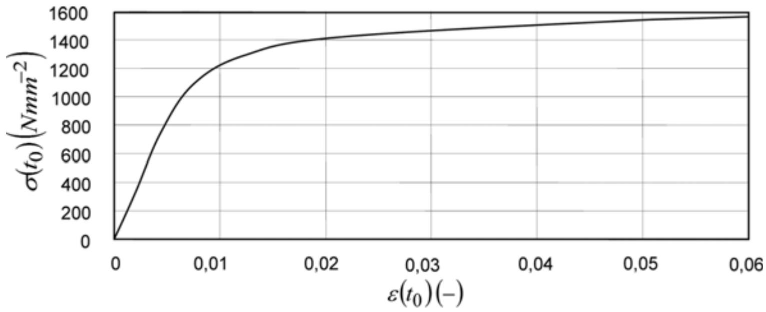


Fig. 4 Stress-strain diagram of the investigated cable

1: 10). There is a vertical uniformly distributed load q applied over the entire span. In the case of the discrete analysis, the uniformly distributed load is replaced by equivalent concentrated load forces. Thus, the point loads are applied at nodes of the suspended cable. The following properties for a steel wire spiral-strand cable with a diameter $D = 45{,}9$ mm are specified: $A = 1240$ mm^2 = 1240.10^{-6} m^2 and $g_0 = 0{,}1015$ kNm^{-1}. The minimum carrying capacity is $N_{u,\min} = 1805{,}0$ kN. The horizontal component of cable tension $H(t_0)$ of the suspended cable under its self-weight g_0 at the initial time t_0 is $H(t_0) = g_0 l^2/8d = 0{,}1015.60^2/8.6 = 7{,}6125$ kN. The corresponding strain in the cable (with its initial unloaded profile at the time t_0) under its self-weight is $\varepsilon_0(t_0) = 0{,}00003288$. The Young's modulus of elasticity of the cable is $E(t_0) = 169700$ Nmm^{-2} = $1{,}697.10^8$ kNm^{-2}.

Young's modulus affects the behaviour of suspended cables at the initial conditions and naturally also during their entire expected service-life (Lewis 2003). Therefore, in the case of the non-linear discrete analysis, at all stress levels in the post-elastic region, the corresponding instantaneous tangential values of the Young's modulus (according to the relationship $E(t) = d\sigma(t)/d\varepsilon(t)$), reflecting the stress-strain curve of the cable will be determined. In this study the stress-strain diagram of a cable as shown in Fig. 4 is considered. The following polynomial form for an approximation function of the non-linear stress-strain diagram is defined

$$\sigma(\varepsilon) = c_0 + \sum_{n=1}^{10} c_n \varepsilon^n \tag{34}$$

4.2 Time-independent analysis at the initial time t_0

4.2.1 Closed-form analysis

The length of the non-loaded cable under g_0 at time t_0 is equal to $L_e = l[1 + (g_0^2 l^2/8H^2(t_0))]$ = $60[1 + (0{,}1015^2.60^2/8.7{,}6125^2)] = 64{,}8$ m.

Results, i.e., uniformly distributed loads $q_e(t_0)$ of the suspended cable those are obtained by the closed-form solution (as Eq. (18) for the load and Eq. (19) for the deflection are used) under the corresponding elastic stresses $\sigma_e(t_0) = 84{,}85$; 169,70; 254,55; 339,40; 424,25; 509,10; 593,95 and 678,80 Nmm^{-2}, with the accompanying elastic strains $\varepsilon_e(t_0) = 0{,}0005$; 0,001; 0,0015; 0,002; 0,0025; 0,003; 0,0035 and 0,004 are shown in Fig. 5. The associated mid-span deflections $w_e(t_0)$ of the suspended cable under the corresponding strains and stresses in the elastic region are shown in Fig. 6.

The uniformly distributed loads $q_p(t_0)$ of the suspended cable obtained by the closed-form solution (as Eq. (26) for the load and Eq. (27) for the deflection are used) under the corresponding post-

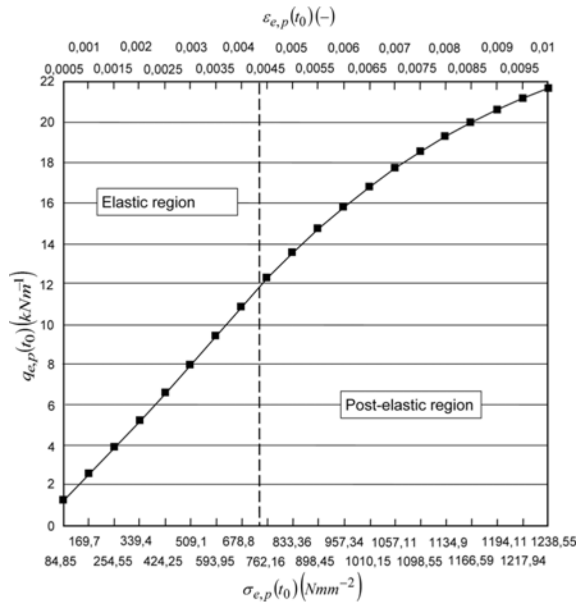


Fig. 5 Uniformly distributed load $q_{e,p}(t_0)$ of the suspended cable under the corresponding stresses $\sigma_{e,p}(t_0)$ and the accompanying strains $\varepsilon_{e,p}(t_0)$

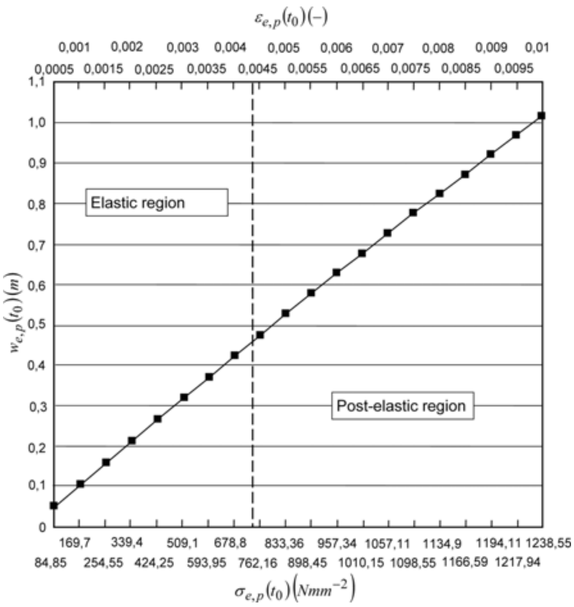


Fig. 6 The associated mid-span deflections $w_{e,p}(t_0)$ of the suspended cable under the corresponding stresses $\sigma_{e,p}(t_0)$ and the accompanying strains $\varepsilon_{e,p}(t_0)$

elastic stresses $\sigma_p(t_0) = 762{,}16$; 833,36; 898,45; 957,34; 1010,15; 1057,11; 1098,55; 1134,90; 1166,59; 1194,11; 1217,94 and 1238,55 Nmm^{-2}, with the accompanying post-elastic strains $\varepsilon_p(t_0) = 0{,}0045$; 0,005; 0,0055; 0,006; 0,0065; 0,007; 0,0075; 0,008; 0,0085; 0,009; 0,0095 and 0,01, are shown in Fig. 5. The associated mid-span deflections $w_p(t_0)$ of the suspended cable under the corresponding strains and stresses in the post-elastic region are shown in Fig. 6.

Typically the elastic limit of cables σ_e is reached at about 50 percent of their ultimate strength σ_u. In our case, the stress $\sigma_{e,lim}(t_0) = 746{,}68$ Nmm^{-2} with the accompanying strain $\varepsilon_{e,lim}(t_0) = 0{,}0044$ at the elastic limit are reached at 51,32 percent of the ultimate strength $\sigma_u = 1455$ Nmm^{-2}.

The elastic limit (i.e., the boundary between the elastic and post-elastic region) can be obtained by the successive elastic (as Eq. (18) and Eq. (19) are used) and post-elastic (as Eq. (26) and Eq. (27) are used) solutions within the expected interval with the adequate densification of considered stresses. For this purpose the stresses $\sigma_{e,p}(t_0) = 678{,}80$; 695,77; 712,74; 729,71; 747,21; 762,16; 776,87; 791,36; 805,60; 819,60 and 833,36 Nmm^{-2}, with the accompanying strains $\varepsilon_{e,p}(t_0) = 0{,}0040$; 0,0041; 0,0042; 0,0043; 0,0044; 0,0045; 0,0046; 0,0047; 0,0048; 0,0049 and 0,005, are used. At the elastic limit (for $\sigma_{e,lim}(t_0) = 746{,}68$ Nmm^{-2} and $\varepsilon_{e,lim}(t_0) = 0{,}0044$), the uniformly distributed loads $q_e(t_0)$ and $q_p(t_0)$ of the suspended cable, as well as the associated mid-span deflections $w_e(t_0)$ and $w_p(t_0)$, obtained by the closed-form elastic and post-elastic solutions, respectively, are approximately equal, as is shown in Fig. 7.

It is obviously advisable to be conservative in use the ultimate strength σ_u. It seems reasonable to reduce σ_u by about 10 percent. Also the value of ultimate strain $\varepsilon_u(t_0)$ as measured in tension tests should not be used. Therefore values of ε_u those are equal approximately half of the test values $\varepsilon_{u,test}$ are acceptable (Greenberg 1970). Consequently, in our examples, the ultimate strength

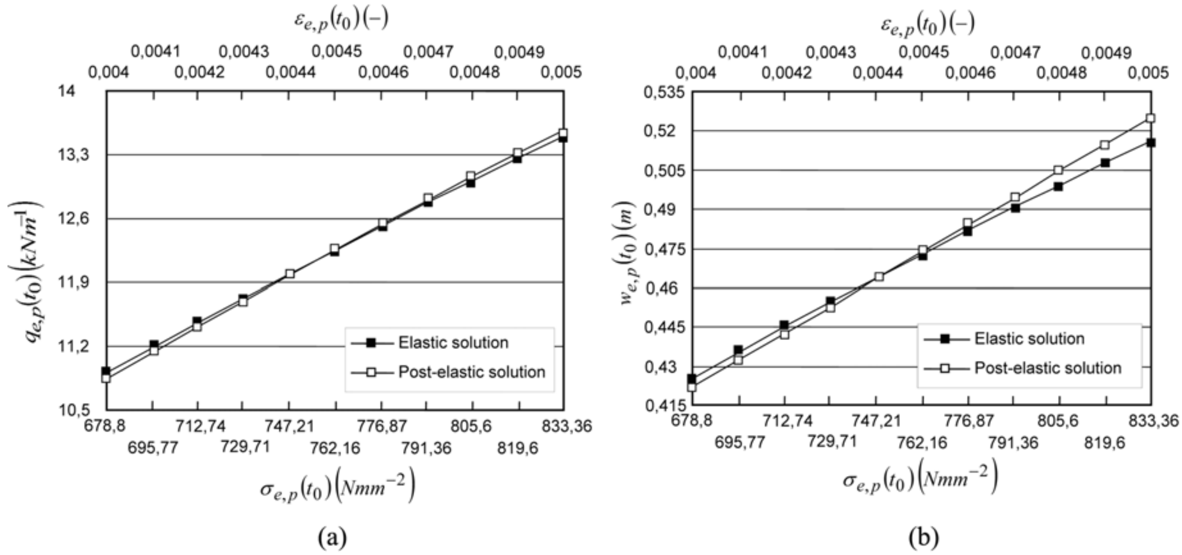


Fig. 7 The boundary of elastic and post-elastic region obtained by the elastic and post-elastic solutions. Uniformly distributed load $q_e(t_0)$ and $q_p(t_0)$ - (a), and the associated mid-span deflections $w_e(t_0)$ and $w_p(t_0)$ - (b), of the suspended cable under the corresponding stresses $\sigma_p(t_0)$ and the accompanying strains $\varepsilon_p(t_0)$

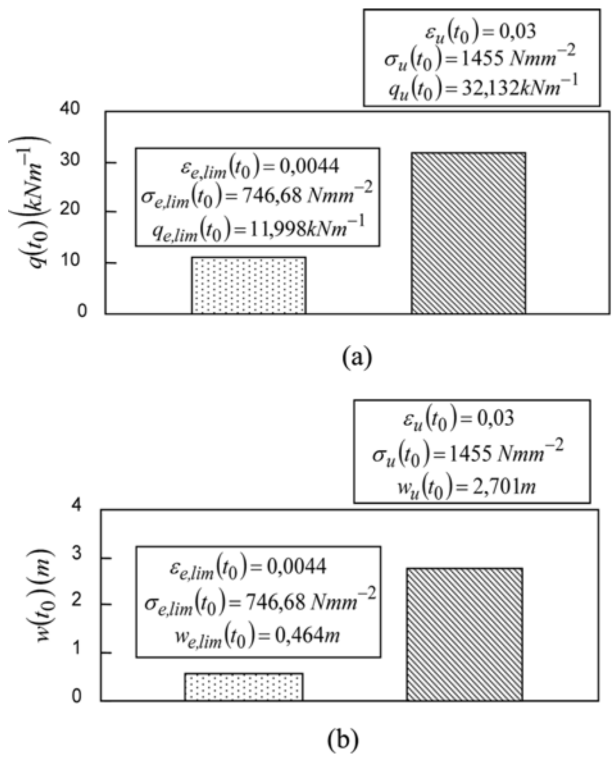


Fig. 8 Comparison of the uniformly distributed load - (a) and of the deflection - (b), at the elastic and ultimate limits

$\sigma_u(t_0) = 1455$ Nmm^{-2}, and the ultimate strain $\varepsilon_u(t_0) = 50\%\varepsilon_{u,\,test} = 0{,}5.0{,}06 = 0{,}03$ (for $\varepsilon_{u,\,test} = 0{,}06$) are considered.

The uniformly distributed load $q_u(t_0)$ and the associated mid-span deflection $w_u(t_0)$ of the suspended cable obtained by the closed-form solution under the ultimate strength $\sigma_u(t_0) =$ 1455 Nmm^{-2} and under the ultimate strain $\varepsilon_u(t_0) = 0{,}03$ are shown in Fig. 8. Comparison of the uniformly distributed loads and the associated mid-span deflections at the elastic limit ($q_{e,\,lim}(t_0)$; $w_{e,\,lim}(t_0)$) with those at the ultimate limit ($q_u(t_0)$; $w_u(t_0)$) is shown in Fig. 8, too.

For the steel typically used in cables, the ratio of strain at ultimate to strain at elastic limit is small, being of the order of 10. This may be contrasted with mild steels, which have a clearly defined yield plateau, where typically this ratio may be of the order of 100 or more.

The load causing failure of suspended cable is substantially greater than the load pertaining to the elastic limit. A suspended cable resists to applied load by changes in tension forces and geometry. Tension changes occur because of the pronounced strain-hardening characteristic of the high-strength steels used in cables, for which there is no clearly defined yield plateau. Failure will occur when the ultimate strain is reached in some part of the cable. A characteristic of flat suspended cables (as sag to span ratio of 1:8 or less is considered) is that small changes in cable length give rise to substantial changes in cable geometry. Therefore, even though the strain ratio at ultimate may be small, this behaviour together with the strain-hardening effect makes it possible for the load that causes failure to be often substantially in excess of that which just exceeds the elastic limit.

4.2.2 Combined discrete analysis

The loads corresponding to the individual strain and/or stress levels at the elastic and the post-elastic region, i.e., $q_e(t_0)$ and $q_p(t_0)$ respectively, obtained by closed-form solutions, were used in the combined discrete *FEM* analysis (as *LANSTAT* and *COSMOS/M* software are used). The known intensity of the load at the appropriate region and its direct use in the discrete non-linear analysis based on the incremental techniques belongs to the basic advantage of the combined approach. For the discrete analysis, the uniformly distributed load is replaced by the equivalent concentrated load forces, those are applied at nodes of the cable. The suspended cable was divided into 60 finite elements. Solving the non-linear problem, for every increment of the load the *Newton-Raphson* iterations were applied (at every investigated time t). The number of loading increments depends on a type of the analysis. In the case of the post-elastic analysis under the high stress levels and under the associated low values of the Young's modulus $E(\sigma, t)$, those reflect the stress-strain curve of the cable, a large number as far as of 500 increments, due to the strong non-linearity, was necessary.

The mid-span deflections $w_e(t_0)$ of the suspended cable under the corresponding elastic stresses $\sigma_e(t_0) = 84{,}85$; 169,70; 254,55; 339,40; 424,25; 509,10; 593,95 and 678,80 Nmm^{-2}, with the accompanying elastic strains $\varepsilon_e(t_0) = 0{,}0005$; 0,001; 0,0015; 0,002; 0,0025; 0,003; 0,0035 and 0,004, obtained by the closed-form and discrete solutions, are shown in Fig. 9. The mid-span deflections $w_p(t_0)$ of the suspended cable under the corresponding post-elastic stresses $\sigma_p(t_0) = 762{,}16$; 833,36; 898,45; 957,34; 1010,15; 1057,11; 1098,55; 1134,90; 1166,59; 1194,11; 1217,94 and 1238,55 Nmm^{-2}, with the accompanying post-elastic strains $\varepsilon_p(t_0) = 0{,}0045$; 0,005; 0,0055; 0,006; 0,0065; 0,007; 0,0075; 0,008; 0,0085; 0,009; 0,0095 and 0,01, obtained by the closed-form and discrete solutions, are shown in Fig. 10.

Loading and deflection of the suspended cable in the form of a graphical output of a postprocessor as the *LANSTAT* software is used, is shown in Fig. 11.

Resultant responses, i.e., deflections determined according to the mentioned models are compared.

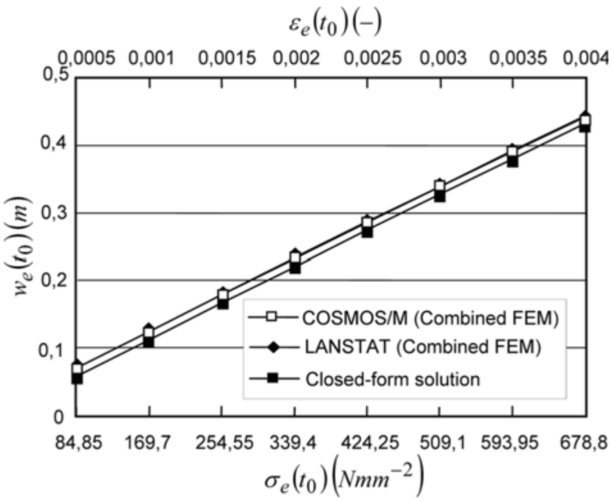


Fig. 9 The mid-span deflections $w_e(t_0)$ of the suspended cable under the corresponding elastic stresses $\sigma_e(t_0)$ and the accompanying elastic strains $\varepsilon_e(t_0)$, obtained by the closed-form and the discrete combined solutions

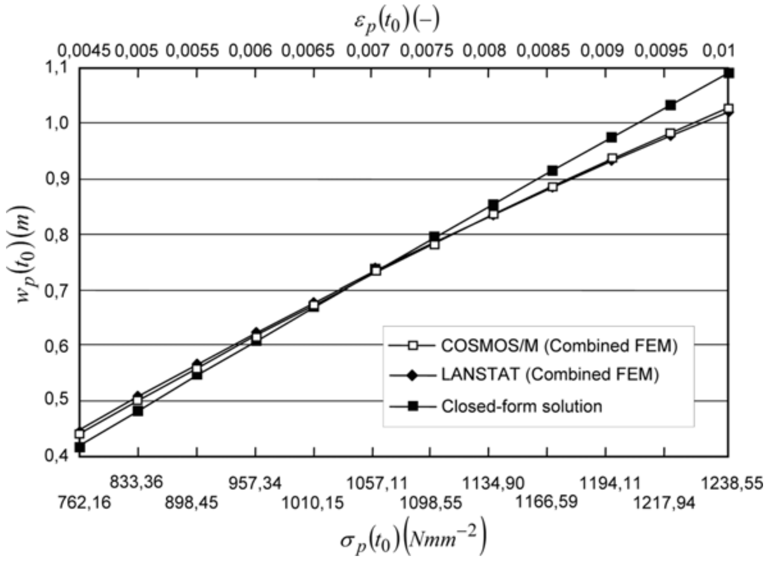
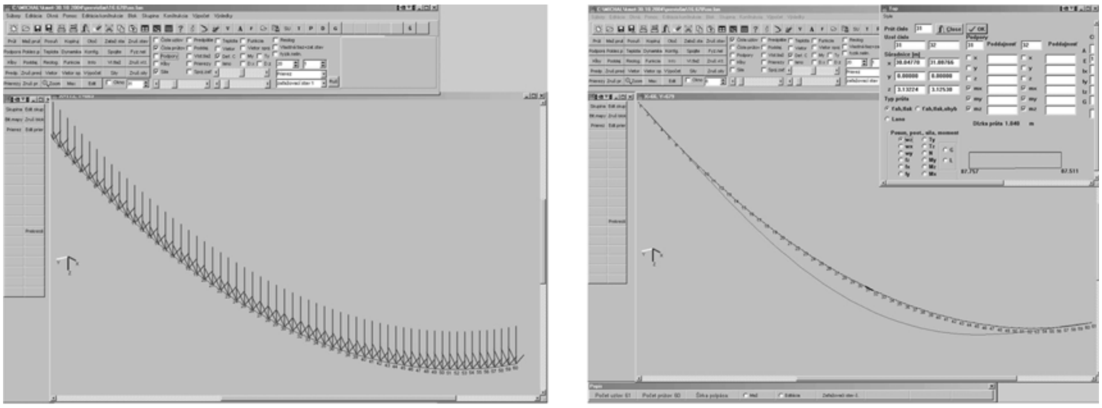


Fig. 10 The mid-span deflections $w_p(t_0)$ of the suspended cable under the corresponding post-elastic stresses $\sigma_p(t_0)$ and the accompanying strains $\varepsilon_p(t_0)$, obtained by the closed-form and the discrete combined solutions

Results obtained in the elastic region (Fig. 9) are in good agreement. There are differences between the results in the post-elastic region (see Fig. 10). These are caused by the different approaches. The discrete solution enables the actualization of the geometrical and stiffness properties of a suspended cable for every finite element during the calculation.

The presented post-elastic analyses are useful when we need to increase a utilization of the high-strength steel cables frequently used in suspended cable structures. Particularly, they are suitable for



(a) (b)

Fig. 11 Loading - (a) and deflection - (b) of the suspended cable in the form of a graphical output of a postprocessor as the *LANSTAT* software is used

making the decision: what cumulative post-elastic extension with accompanying deflection is reasonable for the whole cable. In these cases the design of suspended cables in the post-elastic region is more effective in comparison with the elastic one.

4.3 Time-dependent analysis

When the creep data of a suspended cable are available, we can realize the time-dependent response analyses by the closed-form and discrete combined model. The problem is to find the uniformly distributed load $q(t)$ (using the closed-form solution) corresponding to the elastic and post-elastic regions and the associated mid-span deflection $w(t)$ at the investigated times $t = t_0 = 0$ days (initial time), $t = 1$; 10; 100; 1000 and 10000 days. For this purpose the creep strains $\varepsilon_c(t)$ at the investigated times corresponding to the required stresses σ_e and σ_p at the elastic and post-elastic region respectively, are calculated from the logarithmic-exponential constitutive Eq. (13), that is given in the form as

$$\varepsilon_c(t) \ = \ 0{,}001(a + c\ln t)(1 - e^{-bt}) \tag{35}$$

The coefficients a, b and c in Eq. (35) for the required stress levels, as a number of percentages of the actual ultimate load carrying capacity of the cable $\sigma_u = 1455\ \mathrm{Nmm}^{-2}$ are given in Table 1. The units of time t into Eq. (35) is necessarily in days, thus the resultant $\varepsilon_c(t)$ is dimensionless.

The length of the non-loaded cable under g_0 at time t_0 is equal to

$$L_c = l[1 + (g_0^2 l^2/12H^2(t_0))] \ = 60[1 + (0{,}1015^2.60^2/12.7{,}6125^2)] = 63{,}2\ \mathrm{m}.$$

4.3.1 Elastic region

The uniformly distributed loads $q_e(t)$ of the suspended cable obtained by the closed-form solution (as Eq. (18) for the load and Eq. (19) for the deflection are used) under the corresponding elastic stresses $\sigma_e(t_0) = 84{,}85$; 169,70; 254,55; 339,40; 424,25; 509,10; 593,95 and 678,80 Nmm^{-2}, with the

Table 1 The coefficients a, b and c for the required stress levels of the logarithmic-exponential creep constitutive equation of the cable

% σ_u	a	b	c
(%)	(–)	(–)	(–)
23	0,20506	3583,71	–0,0002
25	0,225702	3257,647	0,000862
30	0,279691	2511,556	0,004134
35	0,336852	1999,831	0,00833
40	0,397113	1621,747	0,013498
45	0,460404	1322,79	0,019652
50	0,526774	1065,143	0,026767
55	0,757447	1339,503	0,061957
60	1,020605	1866,129	0,102997
65	1,317163	3278,96	0,149428
75	2,017433	3918,502	0,255023
80	2,417719	3768,74	0,315898
85	2,851148	3577,115	0,382297

accompanying elastic strains $\varepsilon_e(t_0)$ = 0,0005; 0,001; 0,0015; 0,002; 0,0025; 0,003; 0,0035 and 0,004, at the investigated times $t = t_0 = 0$ days (initial time, i.e., without creep), t = 1; 10; 100; 1000 and 10000 days, are shown in Fig. 12. These obtained loads were used in the time-dependent discrete combined *FEM* analysis (as *LANSTAT* software was used). The associated mid-span deflections $w_e(t)$ of the suspended cable at the investigated times obtained by the closed-form and discrete

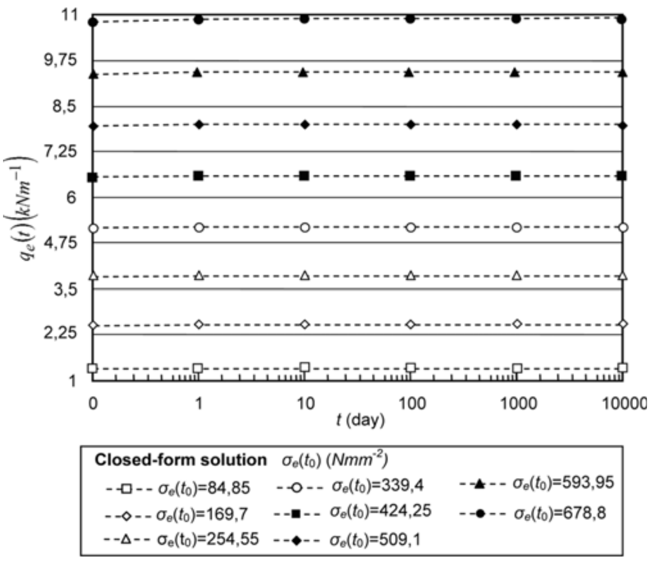


Fig. 12 The uniformly distributed loads $q_e(t)$ of the suspended cable under the corresponding elastic stresses $\sigma_e(t_0)$ at the investigated times t

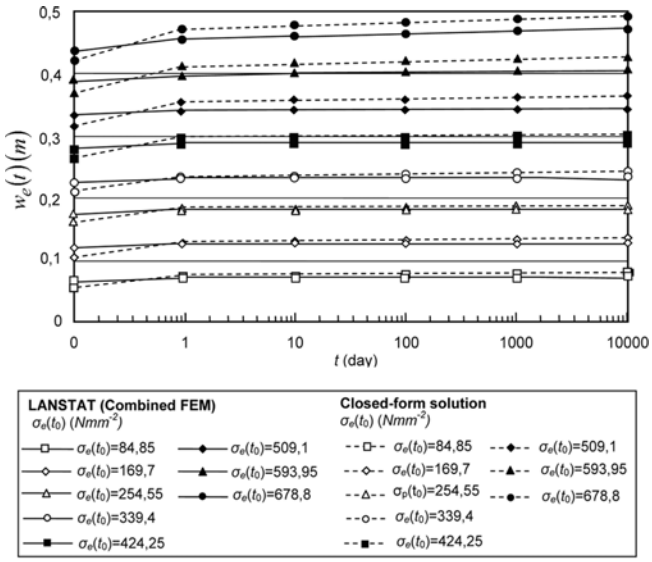


Fig. 13 The associated mid-span deflections $w_e(t)$ of the suspended cable under the corresponding elastic stresses $\sigma_e(t_0)$ at the investigated times t, obtained by the closed-form and the discrete combined solution

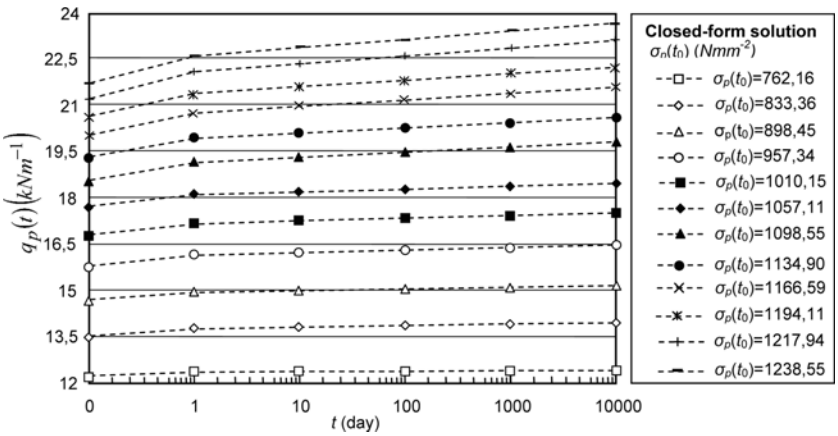


Fig. 14 The uniformly distributed loads $q_p(t)$ of the suspended cable under the corresponding post-elastic stresses $\sigma_p(t_0)$ at the investigated times t

solutions are shown in Fig. 13. Resultant deflections obtained by the present closed-form solution are little greater than those obtained by non-linear *FEM*.

4.3.2 Post-elastic region

The resultant uniformly distributed loads $q_p(t)$ of the suspended cable obtained by the closed-form solution (as Eq. (26) for the load and Eq. (27) for the deflection are used) under the corresponding post-elastic stresses $\sigma_p(t_0) = 762{,}16$; 833,36; 898,45; 957,34; 1010,15; 1057,11; 1098,55; 1134,90; 1166,59; 1194,11; 1217,94 and 1238,55 Nmm^{-2}, with the accompanying post-elastic strains $\varepsilon_p(t_0) = 0{,}0045$; 0,005; 0,0055; 0,006; 0,0065; 0,007; 0,0075; 0,008; 0,0085; 0,009; 0,0095 and 0,01,

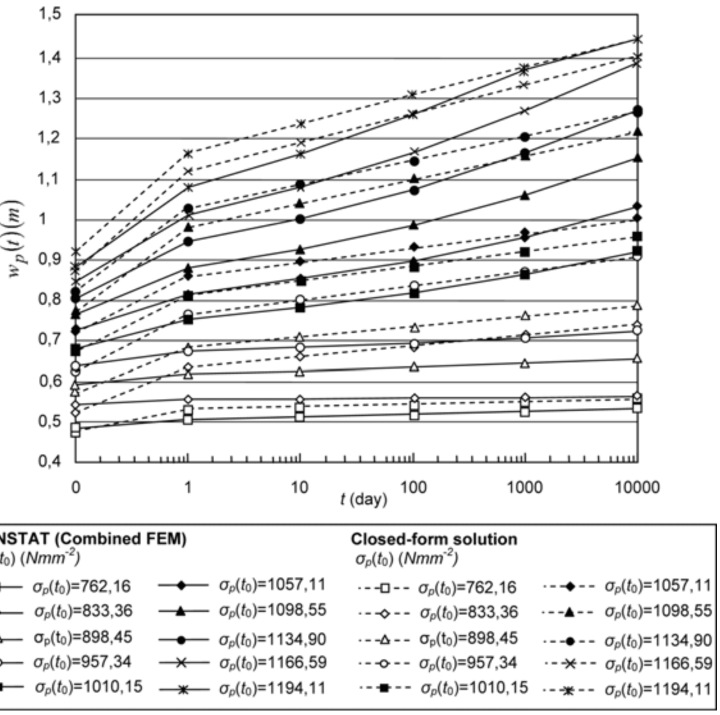


Fig. 15 The associated mid-span deflections $w_p(t)$ of the suspended cable under the corresponding post-elastic stresses $\sigma_p(t_0)$ at the investigated times t, obtained through the closed-form and the discrete combined solution

at the investigated times $t = t_0 = 0$ days (initial time, i.e., without creep), $t = 1$; 10; 100; 1000 and 10000 days, are shown in Fig. 14. These obtained loads were used in the time-dependent discrete combined *FEM* analysis (as *LANSTAT* software was used). The associated mid-span deflections $w_p(t)$ of the suspended cable at the investigated times obtained by the closed-form and discrete solution are shown in Fig. 15.

The cable deflections increase under the influence of the creep strain increments as time increases. Their magnitudes increase as the respective stress levels and/or load levels increase.

Results demonstrate that there are differences between the responses of the investigated suspended cable at the post-elastic region obtained by the closed-form and discrete *FEM* analysis (Fig. 15). Greater resultant deflections are obtained by the closed-form solution.

The reason of the mentioned differences in the results can be explained as follows. Discrete method enables more realistic sublime consideration of physically non-linear material properties (as a multi-linear stress-strain relationship with updated current modulus of elasticity is used) and geometrically non-linear structural behaviour (as tension stiffening is performed) of suspended cable in the post-elastic region. Whereas, the closed-form solution is based on known final quantities of cable forces (those were determined approximately) and post-elastic strains, the discrete method with incremental and iterative solution strategies can simulate post-elastic phenomena of cables such as changes in the material properties of the cable (modification of modulus of elasticity) as well as changes in the geometry and tension of the cable members (hence, the tangential stiffness matrix is updated for any displaced form of the suspended cable).

5. Conclusions

Time-dependent non-linear closed-form and discrete combined solutions have been presented for the elastic and post-elastic response of a flat-sag suspended cable to a uniformly distributed load. Creep of the cable was considered.

In the closed-form analysis, by treating the additional tension force as the independent variable, rather than the applied load, the actual stress-strain and strain-time (creep curves under the applied constant stresses) properties of the cable could be used directly. By the present analytical approach, the post-elastic stress with accompanying strain and the corresponding creep strain at the investigated times need only be determined by a function of the stress-strain and of the strain-time properties of the particular cable in order to obtain a unique solution for the load and associated deflection at a studied time.

Discrete combined analysis of a suspended cable in the post-elastic region, based on the *FEM* with an initial framework of the corresponding loading vector, that is defined by the closed-form solution, has been presented. For the discrete analysis of the non-linear suspended cable this initial value is needed, concerning the incremental procedure that is used for a solution. The proposed combined approach allows one to make in two steps quick and clear elastic and post-elastic behaviour analysis of suspended cable as follows. First, by the closed-form robust solution one accomplishes to allocate and directly specify an intensity of the vertically distributed load corresponding to the elastic or post-elastic region. Thereon, the improved analysis can be done by the discrete *FEM*. Thus the *CPU* times can be economized. The more realistic consideration of cable material properties and time-dependent modelling of structural behaviour, taking into account the creep effects allows one to improve the investigation of geometrically and parametrically non-linear suspended cables in the post-elastic region.

The application of the described methods and derived equations was illustrated by numerical examples. The obtained results confirm the correctness of the derived equations and techniques as well as their physical importance.

Many national and international specifications for design of structures with steel cable components are based on the *Partial Safety Factor Method.* Factors of safety vary, but the working elastic stresses are usually used for a rope or strand in the cable structures. In these cases the suspended cable will never enter the post-elastic region, nor should it.

The presented post-elastic analyses are useful when one needs to increase a utilization of the high-strength steel cables used in the suspended cable structures and to decide what post-elastic extension with accompanying deflection is reasonable for the whole cable.

The closed-form model is useful when one needs to perform a time-dependent analysis of the suspended cable in the post-elastic region and to use the obtained results as the input data for the discrete time-dependent post-elastic analysis. Also in the cases, when it is needful to compare the results of other forms of analysis. The closed-form concept leads to a reliable understanding of the significance of individual stress and/or strain levels those affect the resultant time-dependent behaviour of the suspended cables in the post-elastic region.

It is believed, that the presented solutions will lead to an improved analysis of the time-dependent response of suspended cables with rheological properties in the post-elastic region.

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