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Probabilistic shear-lag analysis of structures using Systematic RSM

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Abstract. In the shear-lag analysis of structures deterministic procedure is insufficient to provide complete information. Probabilistic analysis is a holistic approach for analyzing shear-lag effects considering uncertainties in structural parameters. This paper proposes an efficient and accurate algorithm to analyze shear-lag effects of structures with parameter uncertainties. The proposed algorithm integrated the advantages of the response surface method (RSM), finite element method (FEM) and Monte Carlo simulation (MCS). Uncertainties in the structural parameters can be taken into account in this algorithm. The algorithm is verified using independently generated finite element data. The proposed algorithm is then used to analyze the shear-lag effects of a simply supported beam with parameter uncertainties. The results show that the proposed algorithm based on the central composite design is the most promising one in view of its accuracy and efficiency. Finally, a parametric study was conducted to investigate the effect of each of the random variables on the statistical moment of structural stress response.

Key words: probabilistic analysis; response surface method (RSM); finite element method (FEM); Monte Carlo simulation (MCS); shear-lag effect; box-section beam.

1. Introduction

Box-section beams have been used in many civil engineering structures such as bridge decks, shear/core walls and framed tubes. When structures are subjected to loads, the distribution of the longitudinal normal stresses induced in the flanges of box-section beams is generally non-uniform, especially in a wide flange of box-section beams. The longitudinal stress in a flange near the web is

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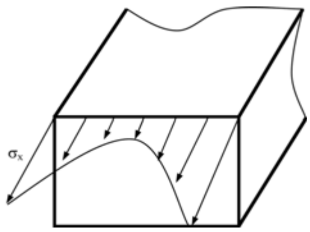

Fig. 1 Positive shear lag

much larger than that far from the web, as shown in Fig. 1. This is usually referred to as positive shear lag phenomenon. However, for some cases, the stress near the web may be much smaller than that far from the web. This is usually called the negative shear lag phenomenon.

The shear lag effects have been extensively studied (Song and Scordelis 1990, Kwan 1996, Luo and Li 2000, Luo *et al.* 2002, Lee *et al.* 2002). These studies were based on the assumption of complete determinacy of structural parameters. This is usually referred to as deterministic analysis. In reality, however, there are uncertainties in design variables. These uncertainties include geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc.), load magnitude and distribution, etc. Thus the deterministic analysis cannot provide comprehensive understanding of the characteristics of shear lag phenomena. Therefore, the shear lag behavior of structures should be studied under a probabilistic viewpoint.

Probabilistic analysis provides a tool that incorporates structural modeling uncertainties in structural analysis by describing the uncertainties as random variables. Several techniques exist to perform structural probabilistic analyses. These techniques may be divided into three categories, based on their essential philosophy, as: (1) Monte Carlo simulation (MCS), (2) Stochastic finite element method (SFEM), and (3) response surface method (RSM).

MCS is the most common and traditional method for a probabilistic analysis. Extensive reviews of the method are found in (Melchers 1999, Haldar and Mahadevan 2000). In brief, the method uses randomly generated samples of the input variables for each deterministic analysis, and estimates response statistics after numerous repetitions of the deterministic analysis (Haldar and Mahadevan 2000). The main advantages of the method are: (1) engineers with only a basic working knowledge of probability and statistics can use it (Haldar and Mahadevan 2000); (2) it always provides correct results when a very large number of simulation cycles are performed (each simulation cycle represents a deterministic analysis). However, the method has one drawback: it needs an enormously large amount of computation time.

SFEM is another method for the probabilistic analysis of structures with random system parameters. The method has been developed to study the shear-lag behavior of random box-section beam structures. Yang *et al.* (2001) used the perturbation technique with finite segment method to solve the shear-lag problem of random box-girders. In their study, the influence of each of the random variables on the overall probabilistic analysis is not investigated. On the other hand, the SFEM used in their work has drawbacks. First, it needs to derive the governing equations of the system from knowledge of the statistical properties of the system. The derivation of governing equations can present severe mathematical and numerical difficulties for complex structures. Second, the existing deterministic analysis code available to design engineers lacks the capability like the deviation of governing equations of the system. Therefore, to use the method, it is necessary to modify the existing deterministic analysis code.

To overcome drawbacks of MCS and SFEM, RSM has recently been proposed to perform probabilistic analysis of structures. Yiu and Zhang (2000) used the RSM to solve the stability analysis of steel space structures with system parametric uncertainties. Shi and Xiong (2000) used the same technique for predicting the distributions of the buckling load of fiber metal laminates. Despite the fact that the RSM has been available for many years now, it has, to the author's knowledge, not yet been used for probabilistic shear-lag analyses of structures.

A RSM analysis consists of two steps: (1) selecting the function type to represent the response surface; and (2) selecting a convenient experiment design to obtain an accurate response surface. The selection of the function type is not a difficult problem. A quadratic polynomial is sufficient in many cases of engineering analysis. The main difficulty is to choose adequately an experiment design (Soares *et al.* 2002). This is because that the experiment design has an important influence on the accuracy and the cost of computing the response surface. Unfortunately, there have been no precise guidelines or theories on the selection of the grid of experimental design points in the probabilistic shear-lag analysis of structures.

The purpose of this paper is to explore the RSM for probabilistic shear-lag analysis, to investigate the effects of various experimental designs on the accuracy and the cost of computing the response surface, and to investigate the effects of each of the random variables on the overall probabilistic analysis of structures.

2. Deterministic shear-lag analysis

FEM is considered to be the most powerful and convenient numerical method for deterministic shear-lag analysis. A brief description of the FEM is presented next.

The static equilibrium equation of linear elastic structure is:

$$[K\{X\}]\{\delta\{X\}\} = \{F\{X\}\} \tag{1}$$

$$[\varepsilon\{X\}] = [B\{X\}]\{\delta\{X\}\} \tag{2}$$

$$[\sigma\{X\}] = [D\{X\}][\varepsilon\{X\}] \tag{3}$$

where $[K\{X\}]$, $[\delta\{X\}]$, and $\{F\{X\}\}$ are the global stiffness matrix, global response vector and global force vector, respectively; $[\varepsilon\{X\}]$ and $[\sigma\{X\}]$ are the strain and stress vectors, respectively; $[B\{X\}]$ and $[D\{X\}]$ are the strain-displacement matrix evaluated at the integration point and elasticity matrix, respectively. $\{X\}$ is an array of design variables. The stiffness, force, strain, stress and responses in the above equations are functions of either any design variable in $\{X\}$, or in combination, which may be Young's modulus and/or Poisson's ratio, load etc. In the present study 8-node shell elements in ANSYS (2002) are used to discretize the structure.

In design practice of box-section beams, an idea of "bending effective width" is introduced to consider the shear-lag effects. The bending effective width (B_b) can be expressed in the following form:

$$B_b = \frac{\int_B \sigma \cdot dx}{\sigma_{\max}} \tag{4}$$

where σ = top flange stress; B = width of the flange; $\sigma_{\max}$ = maximum top flange stress.

3. Stochastic shear-lag analysis

It is quite evident that for deterministic analysis, any design variable is deterministic quantity for all discretized elements of the structure. However in stochastic analysis, design variable will be randomly varying in each distcretized element of the structure, i.e., $\{X\}$ will be a random vector. If the design variable $\{X\}$ becomes uncertain in nature, $[K]$ as well as $\{\delta\}$ will be stochastic. As a result, it will make $[\varepsilon]$ random. Subsequently, any stress vector $[\sigma]$ will be obtained as a random vector. Since the stresses in the box-section beams are uncertain, the bending effective width (B_b) in Eq. (4) is also stochastic.

4. Systematic RSM for stochastic shear-lag analysis

As shown in Eq. (3), the structural response (such as maximum stress at a specific point) can not be expressed explicitly in terms of the basic random variables. To solve this problem, a systematic RSM is proposed. The proposed method is a hybrid method, consisting of conventional response surface method (RSM), finite element method (FEM) mentioned above, and Monte Carlo simulation (MCS). Some of the important components of the proposed method are briefly discussed in the following sections.

4.1 Conventional response surface method (RSM)

The response surface method (RSM), which was originally developed by Box and Wilson (1951), has been introduced in this study. The main idea of this method is to represent the structural response by using a suitable approximation function. Once the approximation function is found through such as finite element results, we can use the approximation function directly instead of deterministic finite element analysis. Performing a finite element analysis may require several minutes or hours of computation time; whereas the evaluation of a quadratic function requires only a fraction of a second. Hence, the computing time can be saved greatly. Another advantage is that the RSM makes it possible to use an existing deterministic finite element code without modifying it.

As described earlier, response surface method consists of two key elements: (1) the choice of the function type to represent the response surface; and (2) the selection of experimental sampling points (experimental design). They are briefly discussed next.

4.1.1 Representation of response surfaces

Response surfaces generally take a polynomial form. A quadratic polynomial is sufficient in many cases of engineering analysis. For the sake of simplicity, a second-order polynomial without cross terms is adopted here as

$$\hat{g}(X) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 \tag{5}$$

where $X_i (i = 1, 2, \ldots, k)$ the ith random variable; k = number of random variables; and b_0, b_i, b_{ii} and b_{ij} = unknown coefficients to be determined. The number of unknown coefficients in (5) is $p = 2k + 1$.

The polynomial can be obtained by conducting FEM analysis at the experimental sampling points and regression analysis.

4.1.2 Experiment design

Experiment design is a technique to determine the location of the sampling points. There are several versions for experiment design available in literature (Montgomery 1991, Myers 1971). Three experimental design techniques, i.e., D-optimum design, central composite design and full factorial design, are considered in this study. The coded values in the three experimental designs for k = 3 are listed in Tables 1-3, respectively. A comparison of the design point requirements by the three experimental design techniques is listed in Table 4. From this table, it can be seen that the D-optimum design requires the least number of sample points, indicating that it is the most efficient technique, and full factorial design requires the maximum number of sample points, indicating that it is the least efficient technique. The central composite design has intermediate efficiency.

Table 1 Experimental sampling points of D-optimum design in coded values for k = 3

Sample No.	X_1	X_2	X_3
1	0	0	0
2	−1	0	0
3	+1	0	0
4	0	−1	0
5	0	+1	0
6	0	0	−1
7	0	0	+1
8	+1	+1	0
9	+1	0	+1
10	0	+1	+1

Table 2 Experimental sampling points of central composite design in coded values for k = 3

Sample No.	X_1	X_2	X_3
1	0	0	0
2	−1	−1	−1
3	+1	−1	−1
4	−1	+1	−1
5	+1	+1	−1
6	−1	−1	+1
7	+1	−1	+1
8	−1	+1	+1
9	+1	+1	+1
10	$-\alpha$	0	0
11	$+\alpha$	0	0
12	0	$-\alpha$	0
13	0	$+\alpha$	0
14	0	0	$-\alpha$
15	0	0	$+\alpha$

Note: $\alpha = \sqrt[4]{2^k}$

Table 3 Experimental sampling points of full factorial design in coded values for k = 3

Sample No.	X_1	X_2	X_3
1	−1	−1	−1
2	0	−1	−1
3	1	−1	−1
4	−1	−1	0
5	0	−1	0
6	1	−1	0
7	−1	−1	1
8	0	−1	1
9	1	−1	1
10	−1	0	−1
11	0	0	−1
12	1	0	−1
13	−1	0	0
14	0	0	0
15	1	0	0
16	−1	0	1
17	0	0	1
18	1	0	1
19	−1	1	−1
20	0	1	−1
21	1	1	−1
22	−1	1	0
23	0	1	0
24	1	1	0
25	−1	1	1
26	0	1	1
27	1	1	1

Table 4 A comparison of the design point requirements by the three experimental design techniques

Number of variables	D-optimum design	Central composite design	Full factorial design
2	6	9	9
3	10	15	27
4	15	25	81
5	21	43	243
7	36	143	2187
k	$(k+1)(k+2)/2$	2^k+2k+1	3^k

4.2 Monte Carlo simulation for probabilistic results

Direct Monte Carlo simulation is the most common and traditional method for a probabilistic analysis, but its use in the shear-lag analysis of structures with parameter uncertainties may not be practical, since it takes an enormous amount time. To overcome the drawback of direct Monte Carlo simulation, we use Monte Carlo simulation together with a RSM method to obtain the probabilistic results in this paper as will be demonstrated in the numerical example.

Table 5 Statistics of the random variables for a simply supported beam

Variable	Mean	Standard deviation	Distribution	Sources
E	2.842 GPa	0.2842 GPa	Normal	Yang *et al.* (2001)
μ	0.4	0.048	Normal	
P	800 N	120 N	Normal	Assumed

5. Numerical examples

To illustrate the application of the proposed method, shear-lag effect of a simply supported beam under point loads at midspan is analyzed probabilistically. The dimensions of the simply supported beam are L = 0.8 m; B = 0.16 m; H = 0.06 m; $t_f = t_w$ = 0.006 m, where L = length of the simply supported beam; B = width of the flange; H = depth of the simply supported beam; t_f = thickness of the flange; and t_w = thickness of the web. These small dimensions were chosen to save computational efforts. The other parameters such as modulus of elasticity E, Poisson's ratio μ and point load P are taken as the random variables. The mean values (μ), standard deviations (σ) and coefficients of variation (COV) of the parameters are listed in Table 5. The random variables are assumed normally distributed. As the objective of this study is to propose an efficient method for the probabilistic shear-lag analysis of structures, all random parameters in the analysis are based on arbitrary but typical values. On the other hand, since the determination of the interrelation of the random parameters is a difficult task, using the independence assumption can greatly simplify the probabilistic analysis. Therefore, all random parameters in the paper are treated as stochastically independent from each other.

5.1 Finite element modeling

The three-dimensional finite element program ANSYS (2002) was used to study the shear-lag behavior of the simply supported beam with parametric uncertainties. The simply supported beam is

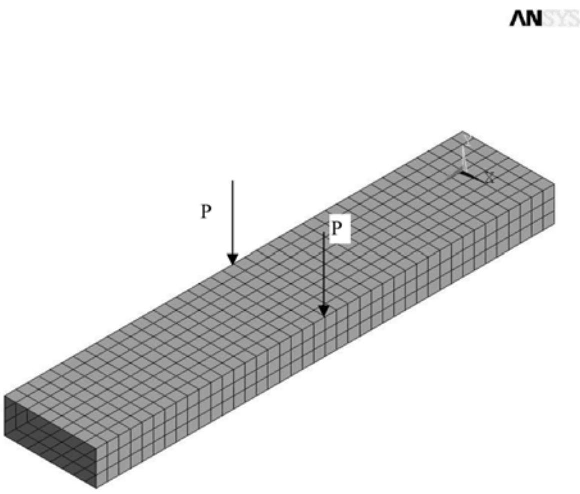

Fig. 2 Finite element model of simply supported beam

discretized with eight-node shell elements (SHELL 93). Fig. 2 shows the finite element model of simply supported beam.

5.2 Verification of the proposed method

The success of the proposed method depends on the accuracy and efficiency of the constructed responses by Eq. (5). Three constructed methods (experimental design methods) mentioned previously (D-optimum design, central composite design, designated as CCD method in the figures, and full factorial design) are considered. To check the accuracy of the constructed responses by Eq. (5), the variations of the maximum top flange stress at mid-span section constructed in Eq. (5) versus the variables, E, μ and P, are shown in Figs. 3-5, along with the 'measured' data points from the deterministic finite element analyses. From these figures, it can be seen that the response surfaces constructed by using the central composite design and the full factorial design are almost

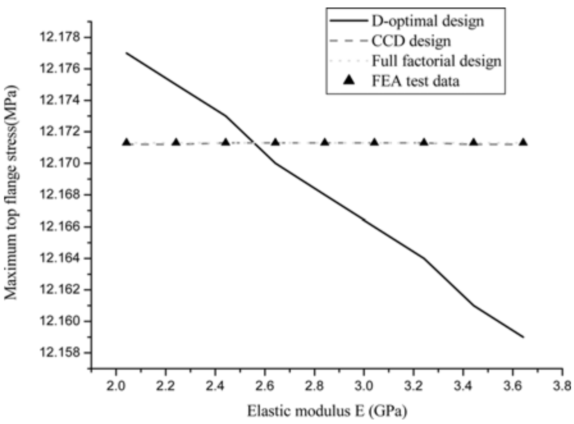

Fig. 3 Maximum top flange stress at mid-span section versus elastic modulus E (μ = 0.4, P = 800 N)

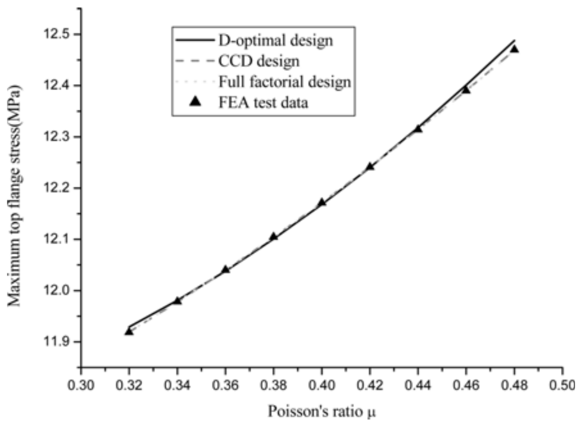

Fig. 4 Maximum top flange stress at mid-span section versus Poisson's ratio μ (E = 2.842 GPa, P = 800 N)

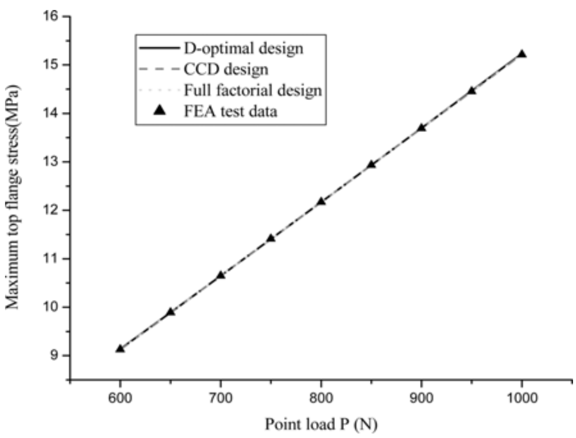

Fig. 5 Maximum top flange stress at mid-span section versus point load P (E = 2.842 GPa, μ = 0.4)

Table 6 Statistical results of the maximum top flange stress at mid-span section for different length (L) of the simply supported beam

L (m)	Statistical results		
	μ (MPa)	σ (MPa)	COV
0.4	9.6171	1.4510	0.1509
0.8	12.1794	1.8335	0.1505

Table 7 Statistical results of the bending effective width at mid-span section for different length (L) of the simply supported beam

L (m)	Statistical results		
	μ (m)	σ (m)	COV
0.4	0.05837	0.0005080	0.008703
0.8	0.08147	0.0007194	0.008830

identical and they show more favorable agreement with the finite element data than the response surface constructed using the D-optimum design. However, the design points required for full factorial design are very large compared to the central composite design, making it very inefficient computationally. The full factorial design method may not be practical for constructing the response surfaces when the number of random variables to be considered is large.

Considering both the accuracy and efficiency, the central composite design is the best choice for constructing the response surfaces. Therefore, only the central composite design using a second-order polynomial without cross terms is used to predict the statistics of the maximum top flange stress and bending effective width in the subsequent study.

5.3 Effect of length of the simply supported beam

The comparison of the means (μ), standard deviations (σ) and coefficients of variation (COV) of the maximum top flange stress and bending effective width at mid-span section for different length of the simply supported beam is listed in Tables 6 and 7, respectively. From these tables, it can be seen that: (1) the parameter uncertainties have significant effects on the variations of the maximum top flange stress. Therefore, it is important to consider parameter uncertainties in shear-lag analysis of structures if accurate stress results are obtained. It is also worth noting that the variations of bending effective width is small and can be neglected in the stochastic shear-lag analysis of structures. Therefore, only stress results are listed in the following stochastic shear-lag analysis of structures; (2) the length of the simply supported beam has a significant effect on μ and σ values, while an opposite effect is observed for COV values. As expected, the μ and σ values increase with the increase of the span length of the simply supported beam.

5.4 Effect of probability distribution type of random input variables

The means (μ), standard deviations (σ) and coefficients of variation (COV) of the maximum top flange stress at mid-span section are computed for the lognormal distributed input variables with the same mean and coefficient of variation as the normal distributed input variables listed in Table 5.

Table 8 Comparison of the means (μ), standard deviations (σ) and coefficients of variation (COV) of the maximum top flange stress at mid-span section for normal and lognormal distributed input variables

Different types of distribution	Normal distribution			Lognormal distribution		
	μ (MPa)	σ (MPa)	COV	μ (MPa)	σ (MPa)	COV
Results	12.1794	1.8335	0.1505	12.1794	1.8341	0.1506

The computational results are listed in Table 8. As shown, the results associated with the lognormal distribution are almost the same as those associated with the normal distribution.

5.5 Sensitivity of statistics of the maximum top flange stress to random input variables

The statistics of the maximum top flange stress is influenced by the characteristics of the structural parameters used in the probabilistic analysis. To study the effect of the structural parameters on the statistics of the maximum top flange stress, a number of parametric studies have been carried out. The mean values and the coefficients of variation (COV) as listed in Table 5 are used, except for the parameters whose COV are varied over the range 0.06-0.14. Similar to the previous analyses, three random parameters are also considered.

Fig. 6 shows the mean values of the maximum top flange stress at the mid-span section versus COV. It can be seen that the mean values of the maximum top flange stress is not sensitive to the coefficients of variation of all three random parameters, E, μ and P.

Fig. 7 shows the standard deviations of the maximum top flange stress at the mid-span section versus COV. The standard deviations of the maximum top flange stress are highly influenced by the coefficients of variation of the random parameter P. The results show that the greater scatter of this parameter, the larger the standard deviations of the maximum top flange stress. This means that the accurate determination of the distribution of such a parameter is very important in obtaining accurate probabilistic results. In contrast, the coefficients of variation of the other two parameters, E and μ do not significantly affect the standard deviations of the maximum top flange stress.

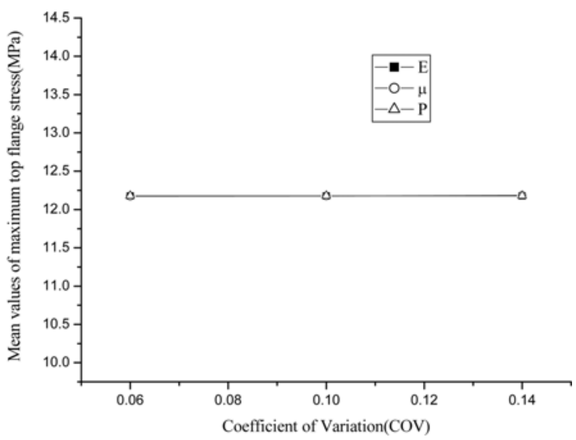

Fig. 6 Mean values of the maximum top flange stress at mid-span section versus COV

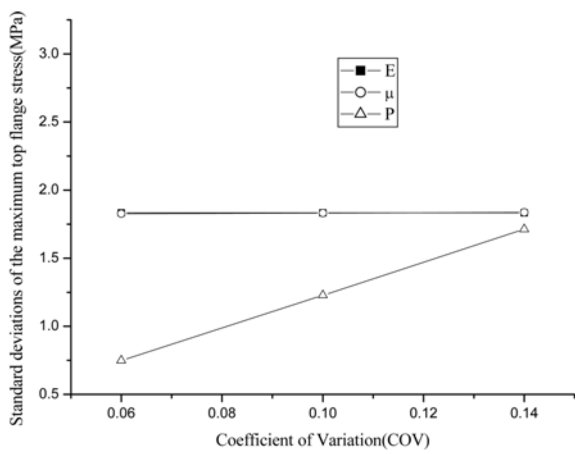

Fig. 7 Standard deviations of the maximum top flange stress at mid-span section versus COV

6. Conclusions

An efficient and accurate algorithm for the probabilistic shear-lag analysis of structures has been developed in this paper. The proposed algorithm is a hybrid method, consisting of response surface method (RSM), finite element method (FEM) and Monte Carlo simulation (MCS). Uncertainties in the structural parameters can be taken into account in this algorithm. The three unique features of the proposed algorithm are that (1) conventional RSM can be extended to analyze shear-lag effects of structures with parameter uncertainties, (2) the deterministic algorithm is extended to consider the uncertainty in the random variables and (3) it is possible to use an existing deterministic finite element code without modifying it. The algorithm is verified using independently generated finite element data.

The proposed algorithm has been applied to the probabilistic shear-lag analysis of a simply supported beam. Three experimental design methods (D-optimum design, central composite design and full factorial design) are considered in the proposed algorithm. The results show that the proposed algorithm based on the central composite design is the most promising one in view of its accuracy and efficiency. The proposed algorithm can obtain more information about the maximum top flange stress and bending effective width than the commonly used deterministic method, and it provides an improved understanding of shear-lag effects of structures with parameter uncertainties. The parameter uncertainties have significant effects on the variations of the maximum top flange stress. Therefore, it is important to consider parameter uncertainties in shear-lag analysis of structures if accurate stress results are to be obtained. It is also worth noting that the variations of bending effective width are small and can be neglected in the stochastic shear-lag analysis of structures.

A parametric study was conducted to investigate the effect of each of the random variables on the statistical moment of the maximum top flange stress. The results indicate that the mean values of the maximum top flange stress is not sensitive to the coefficients of variation of all random parameters, E, μ and P. However, the standard deviations of the maximum top flange stress are highly influenced by the coefficients of variation of the random parameter P. The standard deviations of the maximum top flange stress increases significantly with the increase of the coefficient of variation of the random parameter P. In contrast, the coefficients of variation of the other two parameters, E and μ do not significantly affect the standard deviations of the maximum top flange stress.

The method developed in the current research provides a convenient alternative way to analyze shear-lag effects of structures with parameter uncertainties. While the method used in this study focuses on the solution of positive shear-lag problem of structures with parameter uncertainties, it can easily be extended to evaluate negative shear-lag effects of structures with parameter uncertainties.

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References

ANSYS (2002), ANSYS, Inc., Canonsburg, PA.

Box, G.E.P. and Wilson, K.B. (1951), "On the experimental attainment of optimum conditions", *J. of the Royal Statistical Society, Series B*, **13**.

Haldar, Achintya and Mahadevan, Sankaran (2000), *Reliability Assessment Using Stochastic Finite Element Analysis*, John Wiley & Sons, New York.

Kwan, A.K.H. (1996), "Shear lag in shear/core walls", *J. Struct. Eng.*, **122**(9), 1097-1104.

Lee, S.C., Yoo, C.H. and Yoon, D.Y. (2002), "Analysis of shear lag anomaly in box girders", *J. Struct. Eng.*, **128**(11), 1379-1386.

Luo, Q.Z. and Li, Q.S. (2000), "Shear lag of thin-walled curved box girder bridges", *J. Eng. Mech.*, **126**(10), 1111-1114.

Luo, Q.Z., Li, Q.S. and Tang, J. (2002), "Shear lag in box girder bridges", *J. Bridge Eng.*, **7**(5), 308-313.

Melchers, Robert E. (1999), *Structural Reliability Analysis and Prediction*, John Wiley & Sons, New York.

Montgomery, D.C. (1991), *Design and Analysis of Experiments*, John Wiley & Sons, New York.

Myers, R.C. (1971), *Response Surface Methodology*, Allyn and Bacon, Inc., Boston.

Shi, G. and Xiong, Y. (2000), "Probabilistic buckling analysis of fiber metal laminates under shear loading condition", *Advances in Engineering Software*, **31**, 519-527.

Soares, R.C., Mohamed, A., Venturini, W.S. and Lemaire, M. (2002), "Reliability analysis of non-linear reinforced concrete frames using the response surface method", *Reliability Engineering and System Safety*, **75**, 1-16.

Song, Qi-gen and Scordelis, Alexander C (1990), "Shear-lag analysis of T-, I-, and box beams", *J. Struct. Eng.*, **116**(5), 1290-1305

Yang, L.F., Leung, A.Y.T. and Li, Q.S. (2001), "The stochastic finite segment in the analysis of the shear-lag effect on box-girders", *Eng. Struct.*, **23**, 1461-1468.

Yiu, Fang and Zhang, Qilin (2000), "Stability analysis of steel space structures with system parametric uncertainties", *Communications in Numerical Method in Engineering*, **16**, 267-273.