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Transient energy flow in ship plate and shell structures under low velocity impact

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Abstract. Structural members commonly employed in marine and off-shore structures are usually fabricated from plates and shells. Collision of this class of structures is usually modeled as plate and shell structures subjected to dynamic impact loading. The understanding of the dynamic response and energy transmission of the structures subjected to low velocity impact is useful for the efficient design of this type of structures. The transmissions of transient energy flow and dynamic transient response of these structures under low velocity impact are presented in the paper. The structural intensity approach is adopted to study the elastic transient dynamic characteristics of the plate structures under low velocity impact. The nine-node degenerated shell elements are adopted to model both the target and impactor in the dynamic impact response analysis. The structural intensity streamline representation is introduced to interpret energy flow paths for transient dynamic response of the structures. Numerical results, including contact force and transient energy flow vectors as well as structural intensity stream lines, demonstrate the efficiency of the present approach and attenuating impact effects on this type of structures.

Key words: dynamic response; finite element method; low velocity impact; plate and shell structures; structural intensity.

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1. Introduction

With the growth in the sea transportation and the threat from the possible leakage of the environmentally sensitive cargoes, there has been a pressing demand and consequently an increased interest in the study on the response and structural integrity of marine structures due to the collisions of ship to rock, iceberg, and other ship. The knowledge on the dynamic response of ships and structural components due to impact load is useful for efficient design and detailed understanding of the marine structure performance in the collision scenario.

Structural members commonly employed in ship and offshore structures are fabricated from plates. To gain insight into the global behavior of the structural performance of a ship in a collision accident, it is necessary that the behavior and performances of structural member such as plate under impact loads be assessed. In case of collision of ships, the problems can be modeled as plate and shell structures of ship hull subjected to contact-impact loading. Though, contact-impact plays a fundamental role, it is commonly ignored or simplified in view of the complexity of marine structure (Wang *et al.* 1997, Pedersen and Zhang 1998, 2000). In previous study, it is common that the contact-impact of marine structures under collision has been formulated based on the simplified formula, such as mass-spring model. This is valid only for small impact load at very low velocity collision. For common low velocity collision, usually encountered in collision or grounding of commercial ships, this simplified model is not appropriate. When a ship is exposed to a suddenly large impact load in collisions, the ship hull suffers large deformation and transient analysis of such structures is unavoidable. In order to understand the performance of the ship plate and shell under collision, the non-linear large displacement transient energy flow analysis of such structures is imperative.

The structural intensity approach can be adopted to study the dynamic response of plate impact problem. The concept of structural intensity was introduced to extend the vector acoustics approach to energy flow in structure-born sound fields (Noiseux 1970). The development of structural intensity concept by Pavic (1976) led to a growing interest in this field in the past two decades (Williams 1991). Structural intensity is the power flow per unit cross-sectional area in elastic medium and it is analogous to acoustic intensity in a fluid medium due to structural vibration. Structural intensity field indicates the magnitude and direction of transient energy flow at any point of a structure and structural intensity distribution offers full information of energy transmission paths and positions of sources and sinks of mechanical energy. Dissipative elements and mechanical modification can be used for an alteration of energy flow paths within the structure and the amount of energy injected into the structure. Therefore, it is necessary to investigate the energy flow paths for the damage detection in plates subjected to transient dynamic loads. The structural intensity approach has successfully been used in the field of plate vibration to determine major vibration energy transmission paths in such structure. Few reports on transient impact phenomenon using this approach have been published (Lee *et al.* 2003).

The paper presents the methodology for predicting dynamic response and structural intensity of ship plate under collision impact. Degenerated shell element is used to model both target and impactor. Governing equations incorporating contact-impact algorithm and nonlinear large displacement phenomenon are derived through updated Lagrangian approach. The structural intensity stream line representation is introduced to interpret energy flow paths for transient dynamic response of marine plate structures under low velocity impact. The effects of stiffeners and dissipative damper element on the energy flow and energy path in the plate are explored and discussed. Numerical results show that the method is efficient for studying elastic dynamic response and attenuating impact deformation for marine plate structures subjected to low velocity impact. The study provides a framework and basis for further impact-contact damage analysis and design of this type of structures under collision loads.

2. Governing equations of plate/shell structure under low velocity impact

2.1 Degenerated shell element for contact-impact

Liu and Swaddiwudhipong (1997) showed that the degenerated shell element offers an effective means to treat contact-impact problems. Cartesian coordinates at any point in the shell element can be defined in terms of nodal and thickness coordinates as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sum_{k=1}^{n} N_k \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} + \sum_{k=1}^{n} N_k h_k \frac{\zeta}{2} \begin{bmatrix} t_k^x \\ t_k^y \\ t_k^z \end{bmatrix} \tag{1}$$

where k denotes the node number and n is the number of nodes in the element, N_k is the element shape function defined on a surface ζ= constant, ζ is the natural coordinate in the direction perpendicular to the middle surface and h_k is the shell thickness.

Three displacement components (u_k, v_k, w_k) and two rotations (β_{1k}, β_{2k}) are specified at node k. The element displacement field can be expressed as:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{k=1}^{n} N_k \begin{bmatrix} u_k \\ v_k \\ w_k \end{bmatrix} + \sum_{k=1}^{n} N_k h_k \frac{\zeta}{2} \begin{bmatrix} r_k^x - s_k^x \\ r_k^y - s_k^y \\ r_k^z - s_k^z \end{bmatrix} \begin{bmatrix} \beta_{1k} \\ \beta_{2k} \end{bmatrix} \tag{2}$$

$\bar{r}_k$ and $\bar{s}_k$ are the unit vectors of local Cartesian coordinates.

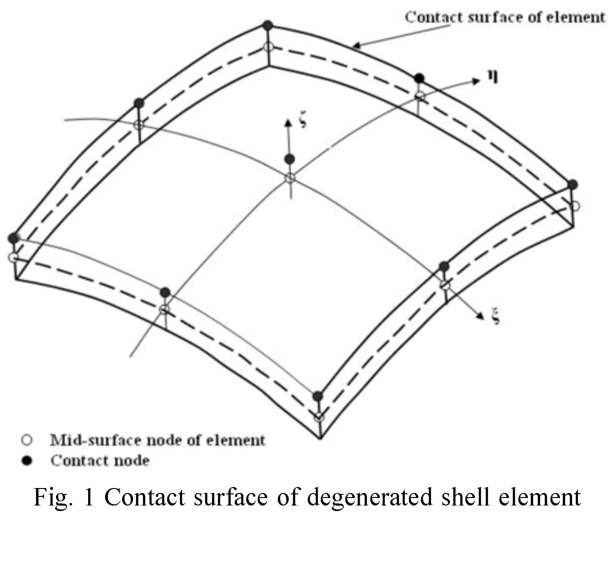


Fig. 1 Contact surface of degenerated shell element

The top and bottom surfaces of the element are defined as interior and exterior surfaces, respectively. It is assumed that only exterior surface of the shell elements has possible contact with another plate/shell structure. The pertinent features of the contact element are shown in Fig. 1. The displacements and geometry of the shell element at contact surface can be defined as follows.

$$\bar{u}^c = \begin{bmatrix} u^c \\ v^c \\ w^c \end{bmatrix} = \sum_{k=1}^{n} N_k \begin{bmatrix} u_k \\ v_k \\ w_k \end{bmatrix} + \sum_{k=1}^{n} N_k h_k \frac{1}{2} \begin{bmatrix} r_k^x - s_k^x \\ r_k^y - s_k^y \\ r_k^z - s_k^z \end{bmatrix} \begin{bmatrix} \beta_{1k} \\ \beta_{2k} \end{bmatrix} \tag{3}$$

$$\bar{x}^c = \begin{bmatrix} x^c \\ y^c \\ z^c \end{bmatrix} = \sum_{k=1}^{n} N_k \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} + \sum_{k=1}^{n} N_k h_k \frac{1}{2} \begin{bmatrix} t_k^x \\ t_k^y \\ t_k^z \end{bmatrix} \tag{4}$$

For nine-node degenerated shell element, the contact nodes on the top surface construct the contact surface. It should be noted that the contact node is dependent on the mid-surface node of the element and no additional degree of freedom is introduced into the system of equations. In this study, both the slave and master surfaces are constructed based on the mid-surface normal projection vectors.

2.2 Finite element equations

Contact-impact between two surfaces imposes the following conditions: (a) the shells cannot interpenetrate each other, (b) the normal traction across the contact interface cannot be tensile, and (c) the tractions satisfy momentum conservation on the interface. The contact force is established through the penalty constraint enforcement method. Penalty number ε is introduced to provide a linear force displacement relation in the normal direction. The incremental form of the virtual work equation for current configuration $t + \Delta t$ can be written as (Swaddiwudhipong *et al.* 2002):

$$\begin{aligned} &\int_{\Omega^t} {}_n D_{ijkl} e_{kl} \delta_n e_{ij} dV + \int_{\Omega^t} {}^n\tau_{ij} \delta_n \eta_{ij} dV + \int_{\Omega^t} {}^n\tau_{ij} \delta_n e_{ij} dV - \int_{\Omega^t} [\delta u_i (\rho^{t+\Delta t} b_i - \rho \ddot{u}_i)] dV \\ &- \int_{\Gamma_t^t} (\bar{t}_{t+\Delta t}) \delta u ds + \int_{\Gamma_c} [-\varepsilon(\mathbf{n} \otimes \mathbf{n}\ d\mathbf{u}^c + g_N^{t+\Delta t} d\mathbf{n}) + \mathbf{t}_N \cdot d(\delta \mathbf{u}^c)] d\Gamma = 0 \end{aligned} \tag{5}$$

where g_N is the gap function which controls the distance between the shell surfaces, and t_N the normal pressure on contact surface.

The incremental form of the governing finite element equations can be written as

$$M^{t+\Delta t}\ddot{d} + ({}_t^tK_L + {}_t^tK_\tau + {}_t^tK_c)^{t+\Delta t}\Delta d = {}^{t+\Delta t}R - {}_t^tF - {}_t^tF_c \tag{6}$$

where M is mass matrices. $({}_t^tK_L + {}_t^tK_\tau + {}_t^tK_c)$ is the total global tangent stiffness matrices. ${}_t^tK_L$ is the linear stiffness matrix; ${}_t^tK_\tau$ is the initial stress stiffness matrix; ${}_t^tK_c$ and ${}_t^tK_c^e$ are the global and element contact stiffness matrix. ${}^{t+\Delta t}R$ is the consistent force vector at time $t + \Delta t$. ${}_t^tF$ is the equivalent nodal force vector resulting from the presence of the initial stresses in the element at time t; ${}_t^tF_c$ and ${}_t^tF_c^e$ are the equivalent global and element nodal force vector from the contribution due to contact at time t; The contact stiffness matrix ${}_t^tK_c^e$ and contact force ${}_t^tF_c$ can be obtained via the standard master and slave concept (Swaddiwudhipong *et al.* 2002). Either implicit or explicit direct time integration scheme can be adopted in the time marching scheme. The latter is usually preferred for short duration impact-contact problem. The implementation is simple and efficient but small time steps are usually required to ensure the numerical stability and the accuracy of the results.

3. Structural intensity in plates and shells

The instantaneous structural intensity component is a time dependent vector quantity indicating the change of energy density in a given infinitesimal volume. The i-component of this parameter in the time domain can be defined as (Williams 1991, Pavic 1992, Gavric and Pavic 1993):

$$I_i(t) = -\sigma_{ij}(t)v_j(t), \qquad i,j = 1,2,3 \tag{7}$$

where $\sigma_{ij}(t)$ is the component of the stress tensor at a point where subscript i represents the normal direction of the area on which the stress acts and subscript j the direction of stress. The indices $i,j = 1,2,3$ correspond to respectively the x, y, and z directions in a Cartesian coordinate system, and $v_j(t)$ the velocity in the j-direction at time t.

Pavic (1976) derived the structural intensity formulas based on plate bending theory. This pioneering work provides a big step into the field of applications of energy flow to structural problems. The general 3-D structural intensity formulations for plate were developed by Romano *et al.* (1990). For the plate mid-surface in x-y plane, the structural intensity can be expressed as (Lee *et al.* 2003):

$$\begin{aligned} I_x &= -[\dot{w}Q_x - \dot{\theta}_y M_x - \dot{\theta}_x M_{xy} + \dot{u}N_x + \dot{v}N_{xy}] \\ I_y &= -[\dot{w}Q_y - \dot{\theta}_y M_{yx} - \dot{\theta}_x M_y + \dot{u}N_{yx} + \dot{v}N_y] \end{aligned} \tag{8}$$

where M_x, M_y, M_{xy}, M_{yx}, N_{yx}, N_{xy}, Q_x, Q_y, N_x, N_y represent the two bending moments, the twisting moment, the two in-plane shear forces, the two out-of-plane shear forces and the membrane forces respectively. $\dot{u}$, $\dot{v}$, $\dot{w}$ and $\dot{\theta}_x$, $\dot{\theta}_y$ are the translational and the rotational velocities, respectively. Compared to Pavic's structural intensity formulas, two additional terms, i.e., $\dot{u}N_x + \dot{v}N_{xy}$ and $\dot{u}N_{yx} + \dot{v}N_y$, corresponding to the longitudinal and in-plane shear waves are included in Romano *et al.* (1990) formulation. The energy associated with these terms is not coupled with the bending wave energy in plates and hence the two expressions provide results with significant differences between Romano's and Pavic's formulation especially when the in-plane motion of the structure cannot be ignored.

Independent parameters of the degenerated shell element adopted herein are the three translational and two rotational nodal displacements. As no other displacement relation assumption is made, results should be more accurate than those presented by Romano *et al.* (1990), Williams (1991) and Zhang and Adin (1996). The classical plate bending theory is valid only for problems with long waves in relation to the plate thickness. It is essential that the effects of shear deformation inherently included in degenerated element be considered to ensure accurate results for sharp dynamic transient response of impact problems.

The relationship of energy flow and structural intensity can be derived as follows. The flux of energy in an elastic medium can be described using a control volume approach. The flow of energy across any given closed surface is equivalent to the rate of change of the total energy inside the surface enclosing the volume (Bouthier and Bernhard 1995).

$$\int_V \left(\frac{de}{dt}\right) dV = \int_S \left(\sigma \cdot \frac{d\mathbf{u}}{dt}\right) \cdot d\mathbf{A} + \int_V (\pi_{in} - \pi_{diss}) dV \tag{9}$$

where e is the energy density inside the control volume, $\mathbf{u}$ the displacement vector of any particle on the boundary of the control volume, π_{in} the input power density or energy input per unit volume per unit time, π_{diss} the power density dissipated or energy per unit volume dissipated per unit time and dA the vector normal to the surface of the control volume for a given point on the surface. Eq. (9) states that the increase in energy inside the control volume is due to the flow of energy into the volume plus the work done by the forces on the volume. It should be noted that in transient dynamics, the energy includes both the stress work density and the kinetic energy density.

Gauss divergence theorem states that

$$\int_S \mathbf{I} \cdot d\mathbf{A} = \int_V \nabla \cdot \mathbf{I} dV \tag{10}$$

The structural intensity in Eq. (9) becomes

$$\int_V \left(\frac{de}{dt}\right) dV = \int_V (\pi_{in} - \pi_{diss} - \nabla \cdot \mathbf{I}) dV \tag{11}$$

Since the integration limits in Eq. (11) are arbitrary, the following relation in Eq. (12) is valid:

$$\frac{de}{dt} = \pi_{in} - \pi_{diss} - \nabla \cdot \mathbf{I} \tag{12}$$

Eq. (12) implies that the energy flow can be altered by varying the structural intensity of the plate.

As the structural intensity is a vector field in space, it is similar to the velocity field in the flow. Inspired by the flow net representation in fluid mechanics, the streamline distribution is adopted to depict the structural intensity transmission path. A streamline is tangent to an energy flow particle's structural intensity. If $\mathbf{r}$ is the position vector of the energy flow particle, the structural intensity streamline can be expressed as:

$$d\mathbf{r} \times \mathbf{I}(\mathbf{r}, t) = 0 \tag{13}$$

The structural intensity of the energy flow element on such the stream line is perpendicular to $\mathbf{r}$ and parallel to $d\mathbf{r}$. The cross product can be written as:

$$\begin{vmatrix} i & j & k \\ I_x & I_y & I_z \\ dx & dy & dz \end{vmatrix} = 0 \tag{14}$$

Thus the differential equation describing the structural intensity streamline is

$$\frac{dx}{I_x} + \frac{dy}{I_y} + \frac{dz}{I_z} = 0 \tag{15}$$

4. Application and numerical example

A ship hull is a complex structural assembly consisting of plating attached to frames, main supporting members and bulkheads. A good starting point for analyzing the structural performance of an entire ship hull in a collision accident is to examine how plates behave under impact collision loads. To simplify the modeling process, the effects of frame and supporting members on plate can be simulated by introducing different boundary conditions in the numerical study. Wang *et al.* (2000) pointed out that so far, most experiments focused on strength of ship's bottom in a grounding accident usually modeled the underwater rock as a rigid cone.

An example is presented herein to demonstrate the application of the power flow intensity approach. It represents the ship plate in a grounding or collision. In this example, it is assumed that a 10-mm thick square steel plate spanning 1000 mm in each direction is subjected to a cone object impact with a relative velocity of 4.0 m/s. The four sides of the steel plate are clamped. The schematic configuration of the problem is shown in Fig. 2. For the sake of simplification, the cone is treated as a rigid body which can be used to simulate the foreign hard object impact. The tip radius and the mass of the cone are assumed to be 100 mm and 1 ton, respectively. In the analysis,

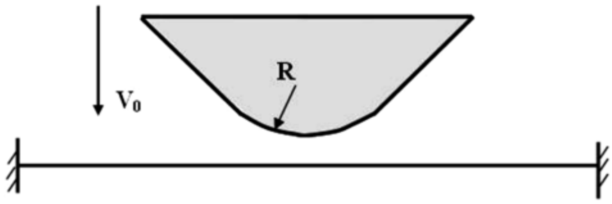
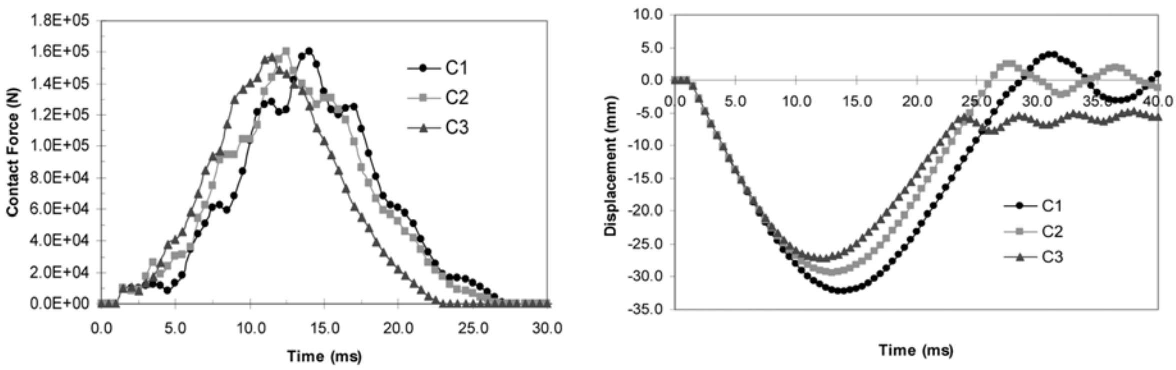


Fig. 2 Square steel plate impacted by cone with spherical nose (C1)

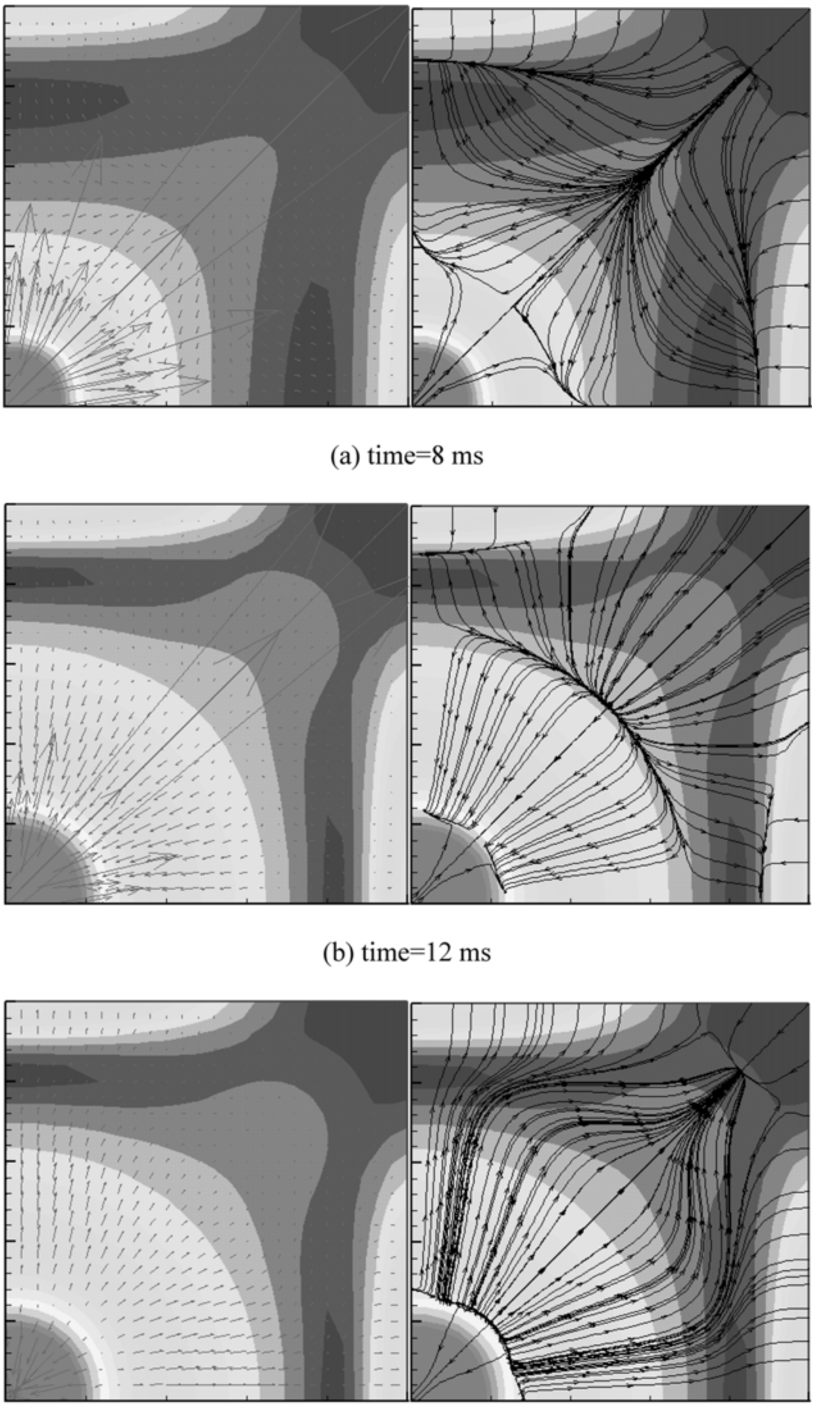
Table 1 The material properties of steel plate

Young's modulus of elasticity (MPa)	210×10^3
Poisson's ratio	$\nu = 0.3$
Mass density (g/mm^3)	0.0078



(a) Contact force (b) Displacement at plate centre

Fig. 3 Time histories of contact force and displacement for various plate structures



(a) time=8 ms

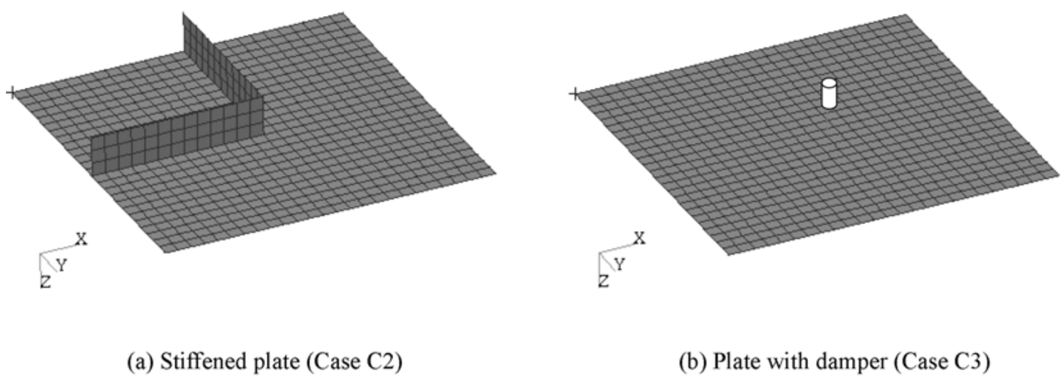
(b) time=12 ms

(c) time=16 ms

Fig. 4 Power flow vector (left), streamline distribution (right) and Von-Mises stress contours (both) at various time steps (Case C1: no modification) the relative velocity of cone object is constrained to move only vertically. The mechanical properties of the steel plate are given in Table 1.

Only a quarter of the plate is modeled in the analysis due to double symmetry. In the first case (C1), the dynamic response and power flow distribution are obtained for the square plate subjected to impact load as described earlier. The contact force and the displacement histories at the plate centre are shown in Figs. 3(a) and 3(b), respectively. The structural intensity vector, streamline distributions and Von-Mises stress contours at 8 ms, 12 ms and 16 ms after the impact are depicted in Fig. 4. The latter displayed just to demonstrate the stress wave propagation in the plate and to support the streamline presentation included in the same figure. It is observed in Fig. 4 that (i) the energy flows are transmitted from contact area to the boundaries of the plate and (ii) virtual sinks in the plate exist. This phenomenon implies that the energy flow in the impact transient case may be redirected and the deformation of the plate reduced without any increase in contact force.

Two design enforcement schemes are proposed. One is to add 20 mm thick stiffeners to the plate at the power flow virtual sink areas as shown in Fig. 5(a). The model is designated as case C2. Alternatively, dissipative element is attached to the plate according to the energy flow path as shown in Fig. 5(b) and assigned as case C3. The detailed description of all 3 cases, C1, C2 and C3 are listed in Table 2. The contact force and center displacement histories for stiffened and damping plates are included in Figs. 3(a) and (b) respectively. The maximum deformations for the latter two cases reduce significantly with no apparent increase in the maximum contact forces. Power flow vector, streamline distributions and Von-Mises stress contours at 8 ms, 12 ms and 16 ms after the impact for stiffened plate (C2) and damping plate (C3) are shown in Figs. 6 and 7 respectively. It is apparent that the energy flow from the cone contact is transmitted into the stiffener area and damper position for cases C2 and C3 respectively. The impact energy in case C3 can be effectively absorbed by the damper provided that it is properly installed at the appropriate position. The study

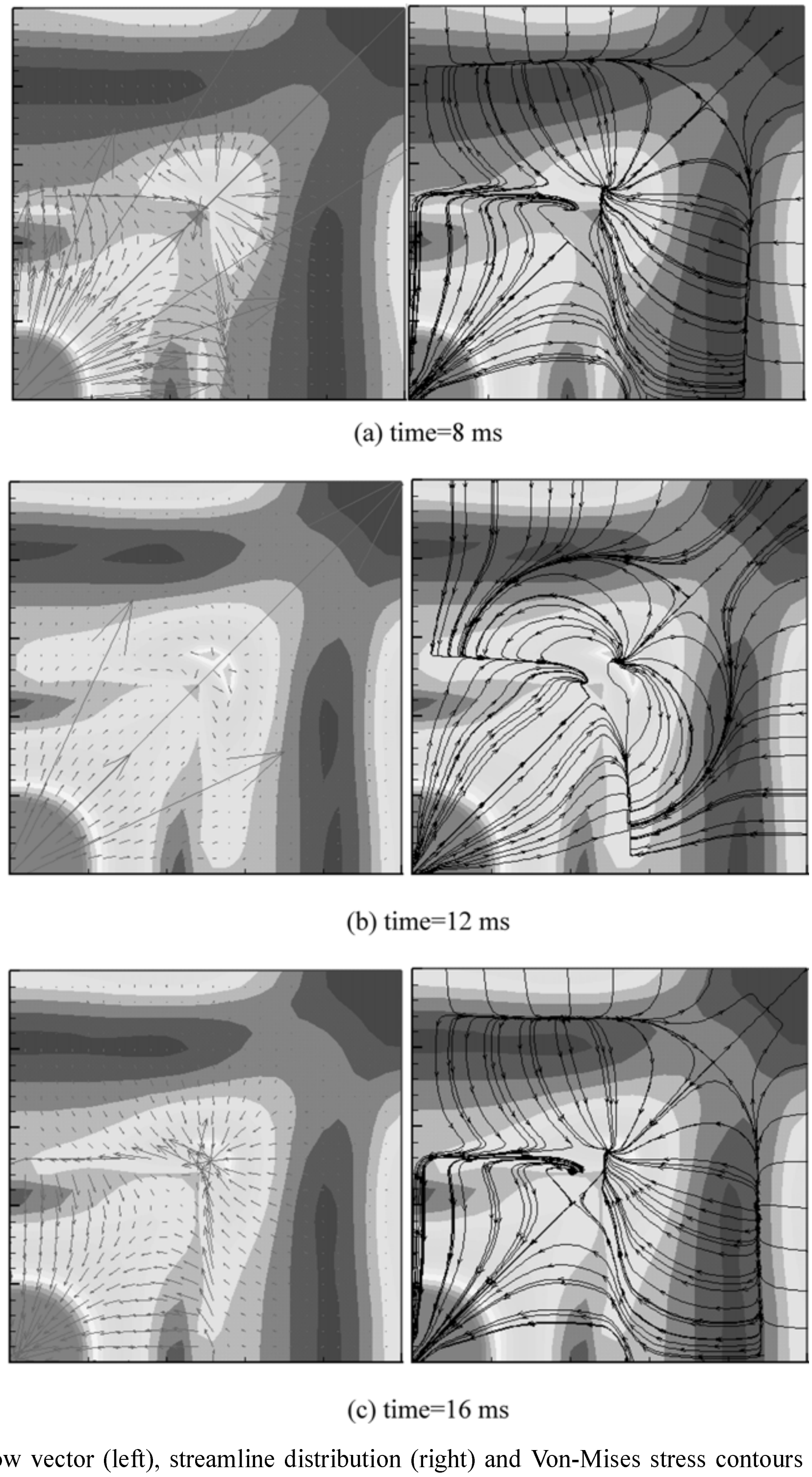


(a) Stiffened plate (Case C2) (b) Plate with damper (Case C3)

Fig. 5 Enforcement improved design of marine plate structure

Table 2 Three design cases of ship steel plate

C1	Clamped 10-mm thick square ship plate of 1000 × 1000 mm^2 without modification
C2	C1 with 20 mm thick stiffeners as depicted in Fig. 5(a)
C3	A dissipative element is attached to the square ship plate of case C1 as shown in Fig. 5(b)

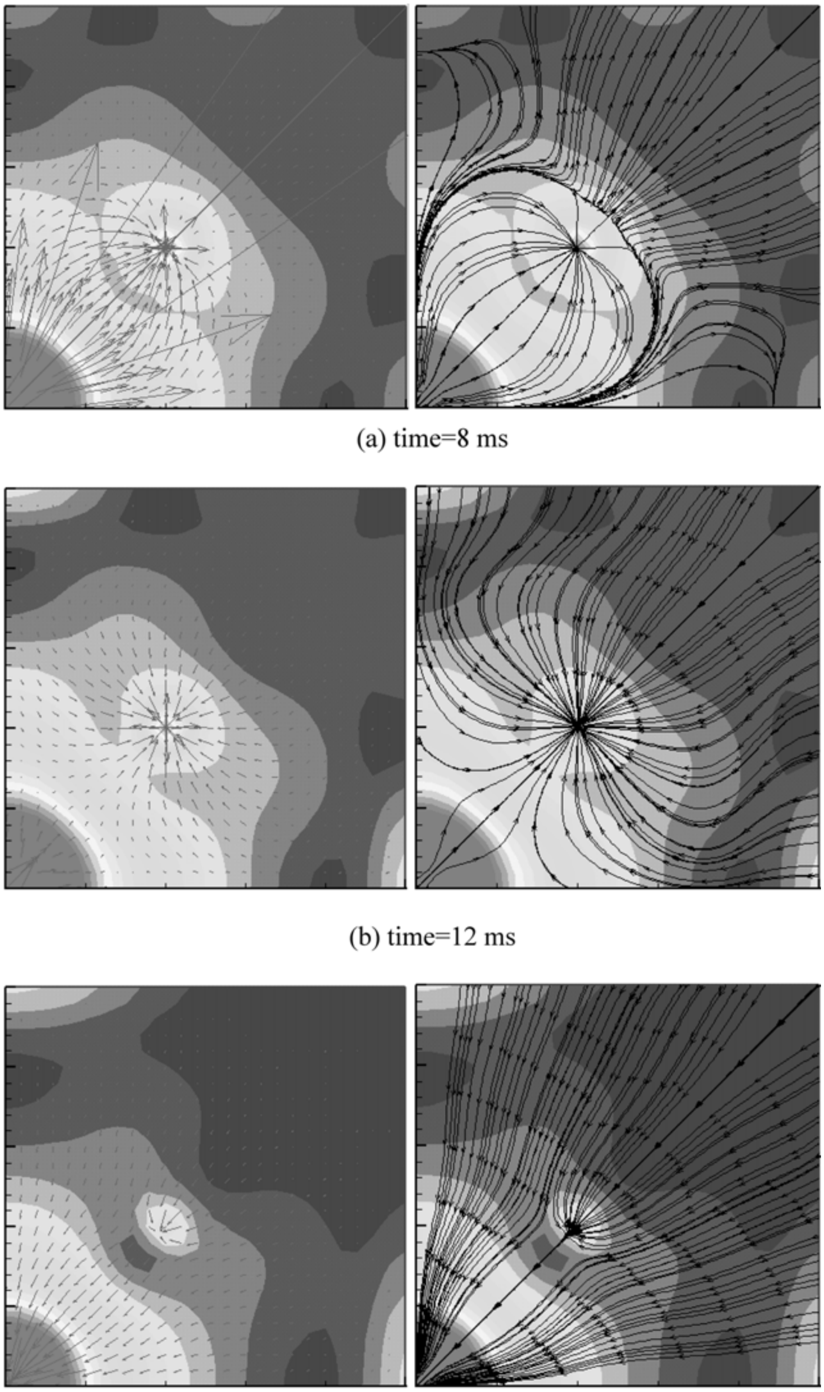


(a) time=8 ms

(b) time=12 ms

(c) time=16 ms

Fig. 6 Power flow vector (left), streamline distribution (right) and Von-Mises stress contours (both) at various time steps for stiffened plate (Case C2: with stiffened plate)



(a) time=8 ms

(b) time=12 ms

(c) time=16 ms

Fig. 7 Power flow vector (left), streamline distribution (right) and Von-Mises stress contours (both) at various time steps for plate with damper (Case C3: with damper) demonstrates that the proposed enforcements of ship plates are effective for attenuating the deformation of the plate with no significant increase in contact force. The structural intensity or energy flow may be adopted as an effective means to improve further the dynamic response of the plate subjected to low velocity impact. It should also be noted that the dissipative element which is attached to the plate in case 3 besides provides the damping effect on the vibration of the structure, also decelerate effectively the return to the original configuration of the plate. The mean line of the time histories of the displacements of the plate in case 3 mildly slope up asymptotically to the undeformed configuration.

5. Conclusions

Degenerated nine-node shell elements are employed to analyze contact-impact problems of ship collision at low velocity. Through the numerical example presented in this study where the plate is subjected to the cone impact, the collision of ship with a rock or other ship bow can be simulated by the present contact-impact approach. Based on the elastic energy flow distribution, the effective enforcement position on the plate can be identified. The results demonstrate that the structural intensity fields and structural intensity streamlines can be used to identify clearly the direction of energy flow in the structure. The method can also serve as an effective tool for elastic transient dynamic response control for ship plates provided that the power flow pattern, energy density, and structural intensity path in the ship plates can be modified. This control can be achieved effectively by providing stiffeners and/or dampers at appropriate locations. This study provides useful information for further study on elastic transient response of ship hull structures subjected to low velocity impact under collision or grounding.

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