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A finite element model for long-term analysis of timber-concrete composite beams

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Abstract. The paper presents a finite element model for studying timber-concrete composite beams under long-term loading. Both deformability of connection system and rheological behaviour of concrete, timber and connection are fully considered. The creep of component materials and the influence of moisture content on the creep of timber and connection, the so-called "mechano-sorptive" effect, are evaluated by means of accurate linear models. The solution is obtained by applying an effective step-by-step procedure in time, which does not require storing the whole stress history in some points in order to account for the creep behaviour. Hence the proposed method is suitable for analyses of composite beams subjected to complex loading and thermo-hygrometric histories. The possibility to accurately predict the long-term response is then shown by comparing numerical and experimental results for different tests.

Key words: composite beams; concrete; creep; finite element method; long-term behaviour; mechano-sorptive effect; rheological phenomena; shrinkage; timber; wood.

1. Introduction

The composite beam is a structural system very used in modern civil engineering. It is made of two or more parallel layers or beams connected each other so as to prevent large relative slips. A very common solution is to link two parallel beams, placed one above the other, by means of a connection system. Various materials may be used for the beams, such as concrete, steel, timber, polymeric-type, etc., as well as different types of union methods.

Since a relative slip between the bottom fibre of the upper beam and the top fibre of the lower beam is generally permitted, the connection system has to be regarded as deformable. Analytical solutions were obtained only in the case of materials with linear-elastic behaviour (Betti and Gjelsvik 1996), while numerical algorithms have to be used for non-linear or rheological behaviour if accurate solutions are needed. For non-linear collapse analyses of steel-concrete composite beams, both finite difference methods (Al-Amery and Roberts 1990, Manfredi *et al.* 1999, etc.) and finite element models (Salari *et al.* 1998, Gattesco 1999, Dall'Asta and Zona 2002, etc.) were developed. For steel-concrete composite beams subjected to sustained loads, finite element methods are generally used (Amadio and Fragiacomo 1993, Kwak and Seo 2000, Fragiacomo *et al.* 2004, etc.). Such models, based on the use of one-dimensional beam elements, take into account the deformability of connection system (except the Kwak and Seo's one) and neglect the local effects

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around the connectors. The aim is, in fact, to adequately represent the global behaviour with computational effectiveness.

For wood-based composites, such as timber-concrete and timber-timber composite beams, the aforementioned models may be generally used for short-term non-linear analysis, as long as the actual constitutive laws of timber and connection are implemented. Conversely, those models cannot be used for long-term analyses since timber is characterised by a complex rheological behaviour that also affects the response of connectors, which are embedded in timber. Mechano-sorptive effect, i.e., the increment of delayed strain under sustained load due to changes of moisture content (Toratti 1992), creep, shrinkage/swelling and the influence of moisture content on the Young's modulus much affect the timber behaviour and should not be neglected.

Few numerical approaches have been proposed thus far to solve this problem (Ceccotti and Covan 1990, Capretti 1992, Fridley *et al.* 1997). They take into account the deformability of connection but are generally based on simplified rheological models for timber and connection. However, an accurate prediction of the structural behaviour under sustained load is important. The serviceability limit states, such as the control of maximum deflection in the long-term, represent in many cases the most severe design criterion, especially for medium to long span beams subjected to heavy environmental conditions (large environmental relative humidity and temperature variations). Thus some authors are investigating this issue by means of numerical approaches based on the finite difference method (Kuhlmann and Schänzlin 2001) or on the use of the Abaqus explicit finite element code (Said *et al.* 2002).

The paper presents an accurate finite element model for long-term analyses of wood-based composites, such as timber-concrete and timber-timber composite beams. The approach is general and takes into account all phenomena affecting the structural behaviour, such as deformability of connection system and rheological behaviour of component materials. The solution can be obtained for a generic load and environmental thermo-hygrometric history. The possibility to accurately predict the time-dependent behaviour is shown by comparing numerical and experimental results. The influence on the results of some parameters such as wood diffusion coefficient, surface emissivity and rheological behaviour of connection is also discussed.

2. Geometry and kinematic hypotheses

The timber-concrete composite beam is constituted by a lower timber beam, made of solid or glued-laminated timber, and by an upper concrete slab with a steel mesh placed inside (Ceccotti 1995). Timber beam and concrete flange are linked by a connection system, which can be continuous along the beam axis, like in the case of steel lattice or plate glued to timber, or made of concentrated studs, like in the case of dowels (glued or not), screws, etc. Only shear forces are generally transmitted at the timber to concrete interface, hence the hypothesis of shear connection will be assumed herein after. In timber-timber composite beams, timber panels replace the upper concrete slab. Such a type of structure has been recently employed since the Monuments and Fine Arts Service demands that the upgrading of important old buildings be "reversible", i.e., removable, and the use of timber panels instead of the concrete slab meets this criterion.

The proposed finite element is constituted by two parallel beams placed one above the other (Fig. 1). The upper beam can be made of either concrete or timber with two possible reinforcements, while the lower beam is made of timber. In this way both cases of timber-concrete and timber-timber

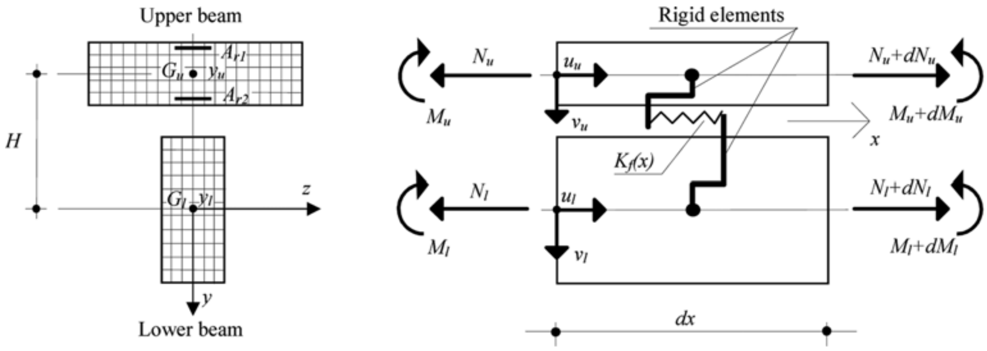

Fig. 1 The finite element used to model the composite beam

composite beams can be considered. The cross-sections of both beams are divided into horizontal and vertical fibres in order to consider different properties along the depth and the width. The connection system is modelled by linking upper and lower beams through a continuous spring system along their axis. The hypothesis of smeared connection is adopted by many authors (Amadio and Fragiacomo 1992, Salari *et al.* 1998, Dall'Asta and Zona 2002, etc.) in order to reduce the number of finite elements and obtain more efficient models (Spacone and El-Tawil 2004). This assumption is adequate also in the case of concentrated connectors, as long as their spacing is quite small with respect to the beam length. Such an assertion is supported by numerical comparisons (Fragiacomo 2000, Fragiacomo *et al.* 2004) between the model with smeared connection and models with concentrated connectors (Capretti 1992, Gattesco 1999). The local effects around studs are neglected with this type of modelling, however the global interaction between upper and lower beam can be adequately evaluated. The kinematic hypotheses are similar to those adopted by Newmark *et al.* (1951) for steel-concrete composite beams, i.e.,

- negligible shear strains for both component beams;
- equal vertical displacements for both component beams: $v_u = v_l = v$;
- preservation of the plane cross-sections of the single component beams;
- perfect bond between reinforcement and concrete.

These hypotheses are sufficiently accurate when studying the structural behaviour under the service load.

On the basis of these hypotheses, the strain-displacement laws for a generic point $P(x, y, z)$ of the upper (subscript u) and lower (subscript l) beam are:

$$\varepsilon_u = u_u' - y_u v'' \tag{1}$$

$$\varepsilon_l = u_l' - y_l v'' \tag{2}$$

where ε, y represent the strain and distance of the point P from the geometrical centre G of the corresponding cross-section, and u, v are the axial and vertical displacement of G, respectively. Similar equations apply for each reinforcement, as long as the quantity y_u in Eq. (1) is replaced by the appropriate distance y_{r1} or y_{r2}. For the connection system, the relative slip s_f between upper and lower beam is given by:

$$s_f = Hv' - (u_u - u_l) \tag{3}$$

where H denotes the distance between the geometrical centres G_u and G_l (Fig. 1).

3. Constitutive equations

The constitutive equations have to be written for every component materials, i.e., concrete, timber, connection system and reinforcement.

3.1 Concrete

Concrete is characterised by important rheological phenomena that affect the long-term response: creep and shrinkage. In order to account for creep, the hypothesis of linear-viscoelastic behaviour is commonly assumed. Such a hypothesis is valid if the material is subjected to compressive stresses not exceeding 40% of the short-term strength. In long-term analyses of composite beams this condition is generally satisfied, since only the quasi-permanent part of the load has to be considered for serviceability limit state verifications (CEN 1995, 1996).

The cracking phenomenon, which occurs if concrete is subjected to tensile stresses larger than the tensile strength, has been neglected. For simply supported composite beams, the most important case, the concrete flange is subjected to bending coupled with compression. Thus the slab is mainly compressed and the tensile stresses due to the service load, if any, are generally lower than the tensile strength of concrete. Even if the tensile stresses were larger, the influence of cracking on the structural response would be negligible compared to that of rheological phenomena. The cracked fibres, in fact, would be bounded to a small region at the bottom of the slab, in a zone near to the centroid fibre of the whole composite cross-section.

Let σ_c, J_c, R_c, ε_c, ε_{cs}, ε_{cT}, t_0, t and τ be the stress, creep function, relaxation function, total strain, shrinkage strain, inelastic strain due to thermal variations, initial time, final time and current time of analysis, respectively. The constitutive equation for a linear-viscoelastic material with inelastic strains is (Chiorino *et al.* 1984) the integral-type creep law (Eq. (4)) or the integral-type relaxation law (Eq. (5)):

$$\varepsilon_c(t) - \varepsilon_{cs}(t) - \varepsilon_{cT}(t) = \int_{t_0}^{t} J_c(t, \tau) d\sigma(\tau) \tag{4}$$

$$\sigma_c(t) = \int_{t_0}^{t} R_c(t, \tau) d[\varepsilon_c(\tau) - \varepsilon_{cs}(\tau) - \varepsilon_{cT}(\tau)] \tag{5}$$

A step-by-step numerical procedure can be used to solve these Volterra's integral equations (Chiorino *et al.* 1984). The time of analysis $[t_0, t]$ is divided into some steps $\Delta t_k = t_k - t_{k-1}$, with $k = 1, 2, ..., m$, and the trapezoidal rule is applied for each step. The recurrent algebraic formula approximating Eq. (4) is:

$$\Delta\sigma_{ck} = 2 \cdot \frac{\Delta\varepsilon_{ck} - \Delta\varepsilon_{csk} - \Delta\varepsilon_{cTk} - \sum_{i=1}^{k-1} \frac{\Delta\sigma_{ci}}{2} \cdot \Phi(t_k, t_i)}{J_c(t_k, t_k) + J_c(t_k, t_{k-1})} \tag{6}$$

with $$\Phi(t_k, t_i) = J_c(t_k, t_i) + J_c(t_k, t_{i-1}) - J_c(t_{k-1}, t_i) - J_c(t_{k-1}, t_{i-1}) \tag{7}$$

where $\Delta\sigma_{ck}$, $\Delta\varepsilon_{ck}$, $\Delta\varepsilon_{csk}$ and $\Delta\varepsilon_{cTk}$ represent the variations of these terms during the step k. This procedure was used for studying the long-term behaviour of steel-concrete composite beams (Amadio and Fragiacomo 1993). Being applicable for any creep function, the procedure is very

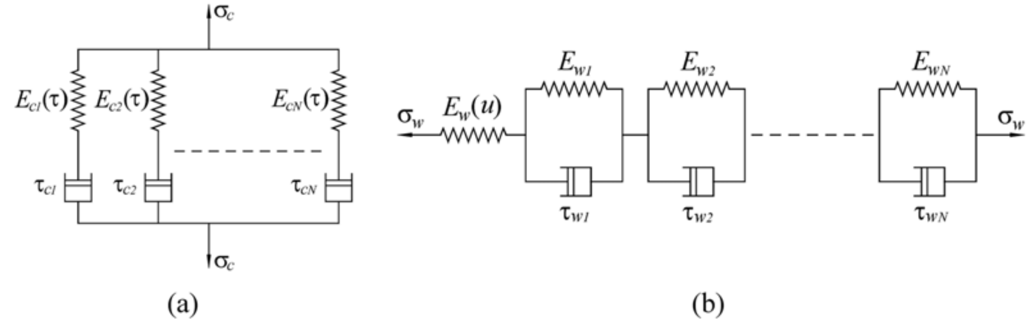

Fig. 2 (a) Maxwell's and (b) Kelvin's generalized rheological models used for concrete and timber, respectively

general but has two important drawbacks:

- the whole stress history has to be stored (terms $\Delta\sigma_{ci}$ in Eq. (6) for every step i), leading to a large amount of used computer memory;
- a heavy computational process is required, since for every step k a summation of $k-1$ products has to be carried out (Eq. (6)).

These disadvantages make such procedure not applicable when a high number of temporal steps m have to be considered, such as in the case of composite beams subjected to complex environmental thermo-hygrometric histories.

To overcome these drawbacks, the relaxation function is expressed as a sum of exponential functions, which corresponds to the use of a Maxwell's generalized rheological model to represent the relaxation of concrete (Fig. 2a):

$$R_c(t,\,\tau) = \sum_{n=1}^{N} E_{cn}(\tau)\cdot e^{\left(-\frac{t-\tau}{\tau_{cn}}\right)} \tag{8}$$

Parameters $E_{cn}(\tau)$ and τ_{cn} represent, respectively, the Young's modulus at instant τ after the concrete casting and the relaxation time in the nth Maxwell's chain. These quantities are evaluated for the CEB-FIP Model Code 90 creep prediction model (CEB 1993) according to the procedure proposed by Lacidogna (1994), which is based on the use of seven Maxwell's chains. The quantities $E_{cn}(\tau)$ are expressed as functions of average environmental relative humidity RH, characteristic cylindrical compressive strength f_{ck} and notational size of member $h = 2A_c/\bar{u}$, A_c and $\bar{u}$ being, respectively, the area and perimeter of the slab cross-section exposed to atmosphere.

In order to transform the Volterra's integral Eq. (5) into an algebraic equation, the whole reference period $[t_0,\,t]$ is divided into some steps $\Delta t_k = t_k - t_{k-1}$, with $k = 1, 2, ..., m$, the time instants being measured since the concrete casting. To capture the elastic solution due to an external load applied at the instant t_q, the assumption $t_q = t_{q+1}$ is made. Now let k be a generic time step. The final recurrent algebraic equation can be achieved by substituting Eq. (8) into Eq. (5), by writing Eq. (5) for the time instants t_{k-1} and t_k, by subtracting the former equation from the latter one and by applying the trapezoidal rule (Fragiacomo 2000):

$$\Delta\sigma_{ck} = \bar{E}_{ck}(\Delta\varepsilon_{ck} - \Delta\varepsilon_{cck} - \Delta\varepsilon_{csk} - \Delta\varepsilon_{cTk}) \tag{9}$$

with:

$$\bar{E}_{ck} = \frac{1}{2}\sum_{n=1}^{N}\left[E_{cn}(t_k) + E_{cn}(t_{k-1})\cdot e^{\left(-\frac{\Delta t_k}{\tau_{cn}}\right)}\right] \tag{10}$$

$$\Delta\varepsilon_{cck} = \frac{1}{\bar{E}_{ck}}\sum_{n=1}^{N}\sigma_{cnk-1}^{hist}\left[1 - e^{\left(-\frac{\Delta t_k}{\tau_{cn}}\right)}\right] \tag{11}$$

$$\sigma_{cnk-1}^{hist} = \sigma_{cnk-2}^{hist}e^{\left(-\frac{\Delta t_{k-1}}{\tau_{cn}}\right)} + \frac{1}{2}\left[E_{cn}(t_{k-1}) + E_{cn}(t_{k-2})e^{\left(-\frac{\Delta t_{k-1}}{\tau_{cn}}\right)}\right](\Delta\varepsilon_{ck-1} - \Delta\varepsilon_{csk-1} - \Delta\varepsilon_{cTk-1}) \tag{12}$$

where:

$\Delta\sigma_{ck}$, $\Delta\varepsilon_{ck}$ and $\Delta\varepsilon_{cTk} = \alpha_{cT}[T_c(t_k) - T_c(t_{k-1})]$ are, respectively, the increment of stress, total strain and inelastic strain due to thermal variation during the step k, α_{cT} and $T_c(t_k)$ being the thermal expansion coefficient and the temperature of concrete at the time t_k;

$\Delta\varepsilon_{csk}$ is the increment of inelastic strain due to shrinkage, calculated according to the CEB-FIP M.C. 90 prediction model (CEB 1993);

$\Delta\varepsilon_{cck}$ and $\bar{E}_{ck}$ represent, respectively, an increment of creep strain and a fictitious elastic modulus taking into account creep;

σ_{cnk-1}^{hist} is a stress term that must be updated at the end of every temporal step for each Maxwell's chain n.

It can be observed by Eqs. (11) and (12) that only N parameters σ_{cnk-1}^{hist} ($N = 7$) must be updated and stored at the current time step k. In this case, the increment of creep strain is a summation of N terms for any temporal step k (Eq. (11)). Hence the drawbacks of the aforementioned step-by-step procedure (Eqs. (6) and (7)) can be overcome according to the proposed approach.

3.2 Timber

In the composite structure the lower beam is made of timber. This material may be employed also in the upper flange to replace the concrete slab. The rheological behaviour of timber is rather complex because it greatly depends on the moisture content u, which is the ratio between the mass of water content and the mass of dried timber. The quantity u varies in time and over the cross-section according to the diffusion laws, and depends on the environmental relative humidity RH and temperature T histories. The influence of T on u will be disregarded herein after since it is less important compared to the influence of RH (Toratti 1992).

Let A_w be the cross-section of the timber beam, ∂A_w its boundary and P a generic point. According to the Fick's diffusion laws, boundary and initial conditions, the problem of evaluating $u = u(y,z,t)$ for a given $RH = RH(t)$ history can be resolved on the basis of the following equations (Toratti 1992):

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left(D\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial u}{\partial z}\right) \qquad \forall P(y,z) \in A_w \tag{13}$$

$$q_u = S(u_{eq} - u) \quad \forall P(y,z) \in \partial A_w \tag{14}$$

$$u(y, z, t_0) = u_0(y, z) \quad \forall P(y, z) \in A_w \cup \partial A_w \tag{15}$$

The quantities D and S are, respectively, the diffusion coefficient and the surface emissivity, for which the values $D = 0.10368 \cdot e^{(2.28 \cdot u)}$ cm^2/day and $S = 1.1232$ cm/day can be assumed in the case of spruce timber. The quantity q_u is the moisture content flux through the boundary of the cross-section, u_0 is the moisture content distribution at the initial time of analysis t_0, and u_{eq} is the timber moisture content in equilibrium with the atmosphere given by $u_{eq} = 0.01RH/(-0.00084823RH^2 + 0.11665RH + 0.38522)$, RH being measured in %. Eqs. (13) to (15) are resolved by dividing the cross-section into cells and by using an explicit method of integration in time with a stability criterion (Incropera and De Witt 1985, Fragiacomo 2000). The values of the moisture content $u = u(y,z,t)$ are then used in the structural analysis to model the timber behaviour.

Several phenomena should be taken into account when modelling the long-term behaviour of timber: creep, mechano-sorptive effect, dependence of Young's modulus on moisture content, shrinkage/swelling due to thermal and moisture content variations. Various models were developed, both linear (Ranta Maunus 1975, Toratti 1992) and non-linear (Hanhijärvi 1995a). In this paper the Toratti's model (1992), which is linear with respect to the stress, has been adopted. A good agreement with experimental bending tests can be obtained using such a model, provided that the maximum stress is less than 20% of the timber strength. This condition is generally satisfied when studying a composite beam subjected to the service load. Let now ε_w, σ_w be the total strain and stress at the time t, and $J_w(t, \tau, u)$ be the creep function of timber, given by:

$$J_w(t, \tau, u) = J_{w0}(u) + J_{wc}(t, \tau) = \frac{1}{E_w(u)} + \frac{\phi_w(t, \tau)}{E_w(u_{ref})} \tag{16}$$

$$\phi_w(t, \tau) = \left(\frac{t - \tau}{t_d}\right)^k \tag{17}$$

where $E_w(u)$ is the Young's modulus of timber dependent on moisture content, $\phi_w(t, \tau)$ is the creep coefficient, u_{ref}, t_d and k are material parameters assuming the values $u_{ref} = 0.20$, $t_d = 29500$ days and $k = 0.21$, respectively. The constitutive equation can then be written in integral form as (Toratti 1992):

$$\begin{aligned} \varepsilon_w(t) = & \int_{t_0}^{t} J_{w0}(u(\tau)) d\sigma_w(\tau) + \int_{t_0}^{t} J_{wc}(t, \tau) d\sigma_w(\tau) + \int_{t_0}^{t} \sigma_w(\tau) dJ_{w0}(u(\tau)) + \\ & + J_w^{\infty} \int_{t_0}^{t} \left\{ 1 - e^{\left[-c_w \int_{\tau}^{t} |du(\tau_1)|\right]} \right\} d\sigma_w(\tau) - \int_{t_0}^{t} b_w \varepsilon_w(\tau) du(\tau) + \int_{t_0}^{t} \alpha_{wu} du(\tau) + \int_{t_0}^{t} \alpha_{wT} dT_w(\tau) \end{aligned} \tag{18}$$

In this equation:

- the first two integrals represent the viscoelastic strain (it is the superposition integral at second member of Eq. (4));
- the third integral accounts for the dependence of the Young's modulus on the moisture content, with $J_{w0}(u) = 1/E_w(u)$ and $E_w(u) = E_{w0}(1 - k_u u)$, E_{w0} and k_u being the Young's modulus of dried timber and a material parameter equal to 1.06, respectively;
- the fourth integral represents the mechano-sorptive term, where J_w^{∞} and c_w are material parameters given by $J_w^{\infty} = 0.7 J_{w0}(u_{ref})$ and $c_w = 2.5$ for spruce timber;

- the fifth integral takes into account the dependence of shrinkage/swelling on the total strain ε_w, where b_w is a material parameter equal to 1.3 for spruce timber;
- the sixth and seventh integral represent the inelastic strains due to shrinkage/swelling and thermal expansion, where $T_w(\tau)$, α_{wT} and α_{wu} are, respectively, the temperature of timber at the time τ, the moisture and the thermal expansion coefficient.

In order to reduce the computer used memory and to speed up the computational processes, the creep coefficient is expressed as sum of six exponential functions. This corresponds to the use of a Kelvin's generalized rheological model to represent the creep behaviour of timber (Fig. 2b):

$$\phi_w(t,\tau) = \sum_{n=1}^{N} J_{wn}\left[1 - e^{\left(-\frac{t-\tau}{\tau_{wn}}\right)}\right] \tag{19}$$

Parameters J_{wn} and τ_{wn} were evaluated by Toratti (1992) in such a way to make Eq. (19) equivalent to the power-type hereditary creep coefficient given by Eq. (17). In order to transform the integral Eq. (18) into an algebraic equation, Eq. (19) is substituted in Eq. (16) and Eq. (16) in Eq. (18). The final recurrent equation is hence obtained by writing Eq. (18) for the time instants $t_{k\text{-}1}$ and t_k, by subtracting the former equation from the latter one and by applying the trapezoidal rule (Fragiacomo 2000):

$$\Delta\sigma_{wk} = \bar{E}_{wk}(\lambda_{wuk}\Delta\varepsilon_{wk} - \Delta\varepsilon_{wEuk} - \Delta\varepsilon_{wck} - \Delta\varepsilon_{wmsk} - \Delta\varepsilon'_{wuk} - \Delta\varepsilon''_{wuk} - \Delta\varepsilon_{wTk}) \tag{20}$$

with:

$$\bar{E}_{wk} = \frac{2}{2J_{w0}(u(t_k)) + J_{w0}(u_{ref})\left\{\sum_{n=1}^{N} J_{wn}\left[1 - e^{\left(-\frac{\Delta t_k}{\tau_{wn}}\right)}\right]\right\} + J_w^{\infty}[1 - e^{(-c_w|\Delta u_k|)}]} \tag{21}$$

$$\lambda_{wuk} = 1 + \frac{b_w \Delta u_k}{2} \tag{22}$$

$$\Delta\varepsilon_{wEuk} = \sigma_w(t_{k-1})[J_{w0}(u(t_k)) - J_{w0}(u(t_{k-1}))] \tag{23}$$

$$\Delta\varepsilon_{wck} = J_{w0}(u_{ref})\left\{\sum_{n=1}^{N} J_{wn}\left[1 - e^{\left(-\frac{\Delta t_k}{\tau_{wn}}\right)}\right]\sigma_{wnk-1}^{hist}\right\} \tag{24}$$

$$\Delta\varepsilon_{wmsk} = J_w^{\infty}[1 - e^{(-c_w|\Delta u_k|)}]\sigma_{wk-1}^{hist,\,m} \tag{25}$$

$$\sigma_{wnk-1}^{hist} = \left(\sigma_{wnk-2}^{hist} + \frac{\Delta\sigma_{wk-1}}{2}\right)e^{\left(-\frac{\Delta t_{k-1}}{\tau_{wn}}\right)} + \frac{\Delta\sigma_{wk-1}}{2} \tag{26}$$

$$\sigma_{wk-1}^{hist,\,m} = \left(\sigma_{wk-2}^{hist,\,m} + \frac{\Delta\sigma_{wk-1}}{2}\right)e^{(-c_w|\Delta u_{k-1}|)} + \frac{\Delta\sigma_{wk-1}}{2} \tag{27}$$

$$\Delta\varepsilon'_{wuk} = \alpha_{wu}\Delta u_k \tag{28}$$

$$\Delta\varepsilon''_{wuk} = -b_w \varepsilon_w(t_{k-1})\Delta u_k \tag{29}$$

$$\Delta\varepsilon_{wTk} = \alpha_{wT}[T_w(t_k) - T_w(t_{k-1})] \tag{30}$$

where:

$\Delta\sigma_{wk}$, $\Delta\varepsilon_{wk}$ and Δu_k are, respectively, the increment of stress, total strain and moisture content during the step k;

$\bar{E}_{wk}$ represents a fictitious elastic modulus that takes into account creep, mechano-sorptive effect and dependence of the Young's modulus on the moisture content;

λ_{wuk} is a multiplier of the total strain increment which accounts for the dependence of shrinkage/swelling on the total strain;

$\Delta\varepsilon_{wck}$, $\Delta\varepsilon_{wmsk}$ and $\Delta\varepsilon_{wEuk}$ are the increments of inelastic strains due to, respectively, creep, mechano-sorptive effect and variation of the Young's modulus with the moisture content;

σ_{wnk-1}^{hist} and $\sigma_{wk-1}^{hist,\,m}$ are stress terms to be updated at the end of every temporal step;

$\Delta\varepsilon''_{wuk}$ and $\Delta\varepsilon'_{wuk}$ are the components of shrinkage/swelling strain depending on and independent of the total strain, respectively;

$\Delta\varepsilon_{wTk}$ is the increment of inelastic strain due to thermal variations.

3.3 Connection

The connection system is hypothesized as smeared along the beam axis. In the case of concentrated studs with shear stiffness k_f and spacing i_f, the smeared shear stiffness K_f is given by $K_f = k_f/i_f$. Let S_f be the shear force per unit length carried by the connection system and s_f be the relative slip between concrete slab and timber beam. In the case of linear-elastic behaviour, the constitutive law would be $S_f = K_f s_f$.

Collapse push-out tests performed on several connection systems have shown that the non-linear behaviour is negligible when the shear force is low compared to the connection strength. This condition is generally satisfied in long-term verifications under the quasi-permanent part of the load. Few long-term push-out tests have been performed thus far (Bonamini *et al.* 1990, Amadio *et al.* 2001). All of those tests demonstrated that connection creeps, even more than timber, and its long-term behaviour is affected by moisture content changes. No rheological model has been proposed yet, however studies are in progress (Amadio *et al.* 2001).

Based on the remark that timber affects the behaviour of connection much more than concrete because of its larger deformability, connection is described by means of a rheological model similar to that adopted for timber. The shrinkage and creep of concrete around the connector are hence neglected being less important than the rheological phenomena at the timber to connector interface. A linear-viscoelastic model with mechano-sorptive effect is employed as constitutive law of connection:

$$s_f(t) = \int_{t_0}^{t} J_{f_0} dS_f(\tau) + \int_{t_0}^{t} J_{f_c}(t,\tau) dS_f(\tau) + J_f^{\infty} \int_{t_0}^{t} \left\{ 1 - e^{\left[-c_f \int_{\tau}^{t} |du(\tau_1)| \right]} \right\} dS_f(\tau) - \int_{t_0}^{t} b_f s_f(\tau) du(\tau) \tag{31}$$

Compared to Eq. (18), the term concerning the dependence of connection stiffness on the moisture content is missing in Eq. (31) due to the lack of experimental data, as well as the terms regarding the inelastic slips that are zero. The creep function is given by:

$$J_f(t,\tau) = J_{fo} + J_{fc}(t,\tau) = \frac{1}{K_f} + c_k \frac{1}{K_f} \sum_{n=1}^{N} J_{fn}\left[1 - e^{\left(-\frac{t-\tau}{\tau_{fn}}\right)}\right] \tag{32}$$

where c_k represents a possible creep amplification factor of connection with respect to timber. Parameters J_{fn}, τ_{fn}, J_f^{∞}, b_f, c_f and c_k should be evaluated through experimental tests. In absence of long-term push-out tests, the assumption of double creep coefficient with respect to timber ($c_k = 2$ with $J_{fn} = J_{wn}$ and $\tau_{fn} = \tau_{wn}$) can be made, as suggested by modern regulations such as Eurocode 5 (C.E.N. 1995). The other quantities concerning the mechano-sorptive effect may be conservatively assumed equal to those adopted for timber and reported in Section 3.2. The final recurrent algebraic equation can be obtained as for timber:

$$\Delta S_{fk} = \bar{K}_{fk}(\lambda_{fuk}\Delta s_{fk} - \Delta s_{fck} - \Delta s_{fmsk} - \Delta s''_{fuk}) \tag{33}$$

where each term is given by relationships analogous to those above seen for timber (Eqs. (21) to (22), (24) to (27), and (29)), as long as the quantities σ_w, E_w and ε_w are replaced by S_f, K_f and s_f.

3.4 Reinforcement

Reinforcement is considered as a linear-elastic material with possible inelastic strains due to thermal variations. The constitutive equation is:

$$\Delta\sigma_{rk} = E_r(\Delta\varepsilon_{rk} - \Delta\varepsilon_{tTk}) \tag{34}$$

where $\Delta\sigma_{rk}$, $\Delta\varepsilon_{rk}$ and $\Delta\varepsilon_{rTk} = \alpha_{rT}[T_r(t_k) - T_r(t_{k-1})]$ are the stress, total strain and inelastic strain increments, E_r is the Young's modulus, α_{rT} and $T_r(t_k)$ are the thermal expansion coefficient and temperature at instant t_k, respectively.

4. Evaluation of the solving linear system

Since the component materials exhibit rheological phenomena, both strain-displacement and constitutive laws have to be written in incremental form. Let k be a generic temporal step $[t_{k-1}, t_k]$ and e a generic finite element in which the composite beam has been divided into. By rewriting Eqs. (1) to (3) in incremental form and by introducing the shape function matrixes $\mathbf{N}_u$, $\mathbf{N}_l$ and $\mathbf{N}_v$, it is possible to obtain:

$$\Delta\varepsilon_{uk} = \Delta u'_{uk} - y_u \Delta v''_k = \mathbf{N}'_u \Delta\mathbf{u}_{uk} - y_u \mathbf{N}''_v \Delta\mathbf{v}_k \tag{35}$$

$$\Delta\varepsilon_{lk} = \Delta u'_{lk} - y_l \Delta v''_k = \mathbf{N}'_l \Delta\mathbf{u}_{lk} - y_l \mathbf{N}''_v \Delta\mathbf{v}_k \tag{36}$$

$$\Delta s_{fk} = H\Delta v'_k - (\Delta u_{uk} - \Delta u_{lk}) = H\mathbf{N}'_v \Delta\mathbf{v}_k - (\mathbf{N}_u \Delta\mathbf{u}_{uk} - \mathbf{N}_l \Delta\mathbf{u}_{lk}) \tag{37}$$

where $\Delta\mathbf{u}_{uk}$, $\Delta\mathbf{u}_{lk}$ and $\Delta\mathbf{v}_k$ are the increment of nodal displacement vectors. In order to avoid locking phenomena, the shape functions employed to model the deflections and rotations ($\mathbf{N}_v$) are cubic, while the axial displacement shape functions ($\mathbf{N}_u$ and $\mathbf{N}_l$) are quadratic (Amadio and Fragiacomo 1993, Dall'Asta and Zona 2002).

The solving linear system can be obtained by substituting Eqs. (35) to (37) in Eqs. (9), (20), (33) and (34), and by applying the Principle of Virtual Work:

$$\mathbf{K}_k \Delta \mathbf{u}_k = \Delta \mathbf{f}_{q,k} + \Delta \mathbf{f}_{in,k} \tag{38}$$

where $\mathbf{K}_k$ represents the stiffness matrix, $\Delta\mathbf{u}_k$, $\Delta\mathbf{f}_{q,k}$ and $\Delta\mathbf{f}_{in,k}$ are the increment of nodal displacements, nodal equivalent forces due to external loads, and nodal equivalent forces due to inelastic strains and rheological phenomena in the component materials, respectively. The stiffness matrix and the increment of equivalent nodal force vectors of each element *e* are then assembled according to the usual finite element technique, by imposing the boundary conditions due to the presence of external restraints.

The series of time steps Δt_k assumed in the analysis has to satisfy two requirements:

- the correct approximation of the integral Eqs. (5), (18) and (31) by means of the algebraic Eqs. (9), (20) and (33);
- the correct evaluation of the moisture content and temperature variations.

The former condition implies time steps with smaller amplitude immediately after each loading instant t_q for an accurate evaluation of the corresponding creep effect (Chiorino *et al.* 1984). The latter condition implies a choice of further time instants, for example by approximating through a piecewise-linear the environmental histories $T(t)$ and $RH(t)$, and by subdividing each segment in a number of equal steps.

5. Experimental-numerical comparisons

In order to validate the proposed approach, some comparisons between numerical and experimental results are presented in this Section.

5.1 Tests performed by Hoyle et al. (1986) on timber specimens

The tests performed by Hoyle *et al.* (1986) concern simple Douglas fir timber specimens loaded in variable hygrometric conditions. Two groups of samples were loaded with constant bending moment of 1.54 kNm and subjected, in climate chamber, to cyclic relative humidity with constant temperature. Weekly and daily cycles with relative humidity in the range, respectively, 40% to 90% (Fig. 3a) and 40% to 88% (Fig. 3b) were applied on the groups. The specimens were 800 mm long with 89 × 89 mm square cross-section. The Young's modulus of dried timber was $E_{w0} = 14000$ MPa and the moisture content at the time of loading was $u_0(y, z) = 0.1083$.

The comparisons between experimental and numerical results are plotted in Fig. 3 as a ratio between total and elastic mid-span deflection versus time. The numerical solutions obtained by Toratti (1992) by employing the finite difference method are also reported. The proposed and the Toratti's solutions are nearly coincident and fit well with the experimental results. This and other comparisons carried out on timber specimens (Fragiacomo 2004) demonstrate the accuracy attainable using the proposed procedure for long-term analyses of timber beams in variable hygrometric conditions.

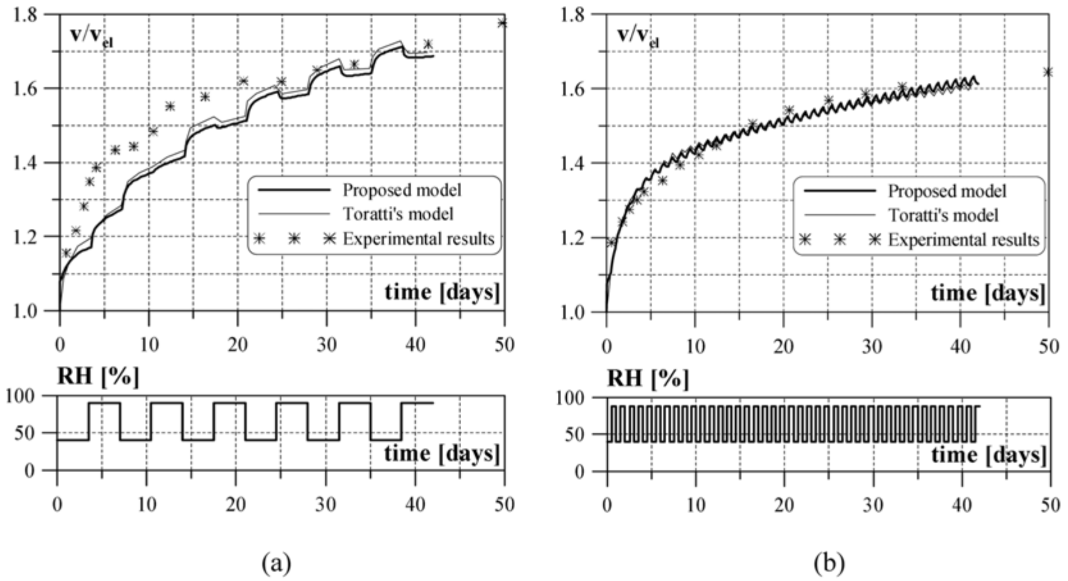

Fig. 3 Tests performed by Hoyle *et al*. (1986): (a) experimental-numerical comparison in terms of non-dimensional mid-span deflection on specimens subjected to weekly and (b) daily cycles of environmental relative humidity

5.2 Test performed by Bonamini et al. (1990) on composite beams

A long-term test was performed at the University of Florence on timber-concrete composite beams (Bonamini *et al*. 1990, Ceccotti and Covan 1990). The geometry and the static scheme of the specimens are displayed in Fig. 4. The timber beams were obtained by the demolition of antique floors made of oak. A polyethylene membrane was laid above the timber beam in order to prevent the moistening due to the fresh concrete just poured. The concrete slab contained two 100×100 mm steel meshes with 4 mm diameter bars placed at 20 mm from the top and at 20 mm from the bottom side. The connection system was made of 10 mm diameter dowels, which were obtained by cutting and bending reinforcing steel bars. The dowels were placed in 12 mm diameter holes drilled in the timber beam and filled with epoxy resin. Connector spacing and embedment in wood were 100 mm. The specimens were arranged in climate chamber, where the load was applied 28 days after the concrete casting and removed 347 days after the load application. The relative humidity

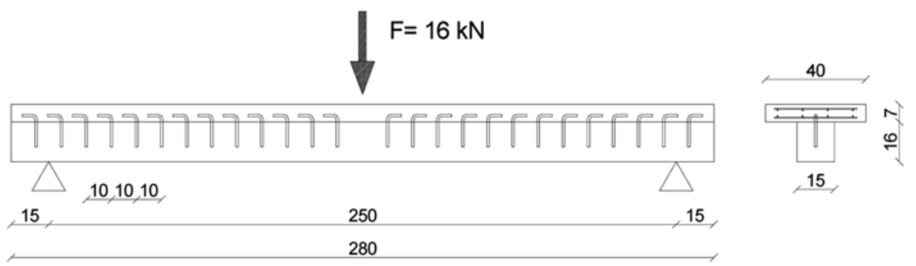

Fig. 4 Geometrical characteristics of the composite beam tested by Bonamini *et al.* (1990) (measures in cm)

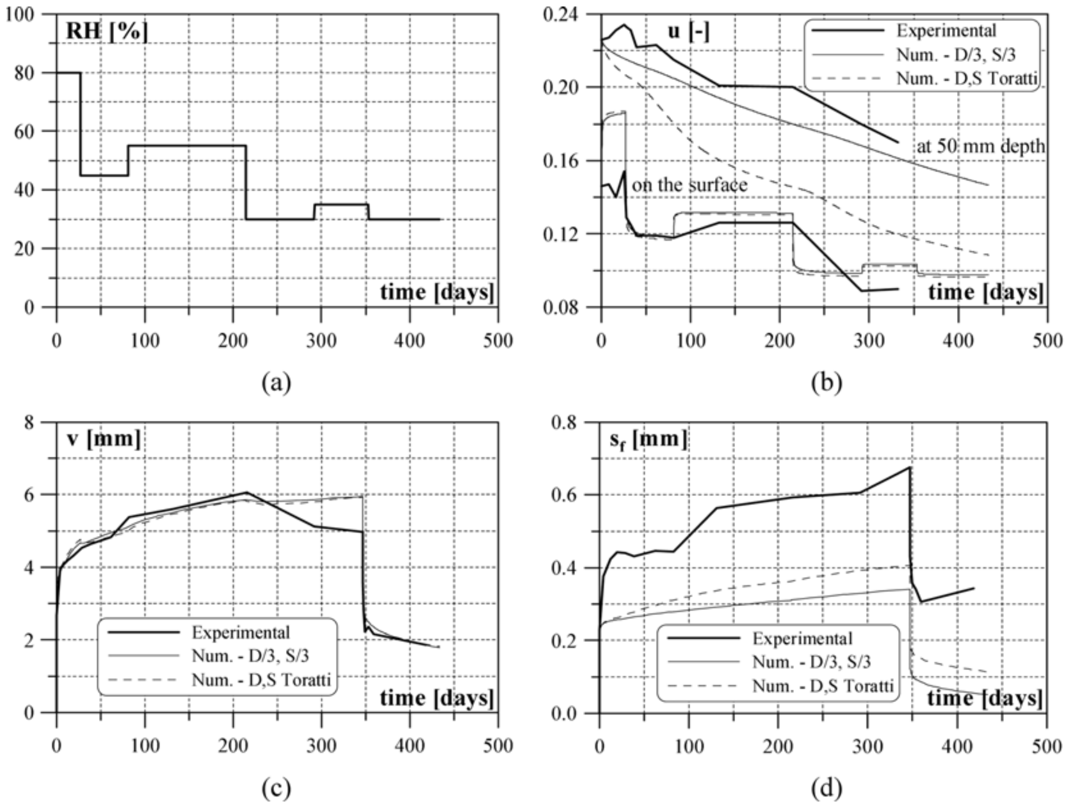

Fig. 5 Test performed by Bonamini *et al.* (1990): (a) relative humidity history of environment, (b) experimental -numerical comparisons in terms of timber moisture content on the surface and at 50 mm depth, (c) mid-span deflection and (d) slip over the supports

was changed during the test according to Fig. 5(a), while the temperature was kept constant. Different quantities were monitored: the moisture content on the surface and at the 50 mm depth in the timber beam, the mid-span deflection and the relative slip between slab and timber over the support. The experimental values averaged on three specimens are reported, respectively, in Figs. 5(b), 5(c) and 5(d), along with the numerical predictions.

The diffusion of moisture content over the timber cross-section under the relative humidity history in climate chamber has been evaluated by assuming the following initial condition at the time of loading. The moisture content distribution has been regarded as constant in the 50×110 mm central core with value $u_0(y, z) = 0.226$. A linear variation between the values 0.226 and 0.146 has been considered for such a quantity in the 50 mm thick skin exposed to the atmosphere, according to measurements made during the test. Although especially the diffusion coefficient depends on the type of wood (Hanhijärvi 1995b), experimental values are available only for spruce. Thus the numerical analysis has been carried out using the values of diffusion and emissivity coefficients proposed for such a species by Toratti (Section 3.2). Furthermore, a very wide variation of both quantities has been considered so as to investigate on which amount they affect the moisture content distribution (Table 1). It has been found (Fig. 5b) that the best solution is obtained for one-third of the Toratti's values. The moisture content is almost insensitive to surface emissivity variations,

Table 1 Influence of the diffusion coefficient (D), surface emissivity (S), creep (c_k) and mechano-sorptive coefficients (J_f^∞) on the superficial (u_s) and in-depth (u_{50}) moisture content, delayed deflection ($v - v_{el}$) and slip ($s_f - s_{f,el}$) with respect to the No.1 case (D' and S' denote the values proposed by Toratti for D and S)

Case No.	D	S	c_k	J_f^∞	$\frac{u_{s,1}-u_{s,i}}{u_{s,1}}$	$\frac{u_{50,1}-u_{50,i}}{u_{50,1}}$	$\frac{v_1-v_i}{v_1-v_{el}}$	$\frac{s_{f,1}-s_{f,i}}{s_{f,1}-s_{f,el}}$
1	D'/3	S'/3	2	$0.7c_kJ_{f0}$	0.0	0.0	0.0	0.0
2	D'	S'	2	$0.7c_kJ_{f0}$	0.011	0.260	0.007	−0.415
3	10D'	S'	2	$0.7c_kJ_{f0}$	0.014	0.344	−0.036	−0.691
4	D'/10	S'	2	$0.7c_kJ_{f0}$	0.011	−0.306	−0.035	0.326
5	D'	10S'	2	$0.7c_kJ_{f0}$	0.014	0.264	0.006	−0.424
6	D'	S'/10	2	$0.7c_kJ_{f0}$	−0.029	0.218	0.008	−0.322
7	D'/3	S'/3	3	$0.7c_kJ_{f0}$	0.0	0.0	−0.112	−0.416
8	D'/3	S'/3	1	$0.7c_kJ_{f0}$	0.0	0.0	0.128	0.459
9	D'/3	S'/3	2	0	0.0	0.0	0.038	0.136

while it is markedly affected by diffusion coefficient variations (Table 1).

The rheological properties of concrete have been predicted according to the CEB-FIP Model Code 90, assuming: $RH = 50\%$ (average relative humidity during the time of testing), $f_{ck} = 20.8$ MPa and $h = 70.9$ mm. For timber, a dried Young's modulus $E_{w0} = 12145$ MPa and a moisture expansion coefficient $\alpha_{wu} = 3 \cdot 10^{-3}$ have been considered. The rheological behaviour of timber has been described by the Toratti's model, assuming the values reported in Section 3.2 except for the parameter b_w. This quantity, which accounts for the dependence of shrinkage/swelling on the total strain (Eqs. (18), (22) and (29)), was obtained by Toratti (1992) on the basis of experimental tests performed on timber specimens subjected to constant bending moment under cycles of humidity. In the composite beam, however, the timber is subjected to bending moment coupled with axial force. Since the presence of that component of shrinkage/swelling may lead to inconsistent results (Fragiacomo 2000), the assumption $b_w = 0$ has been made. For connection, on the basis of the push-out test results, the medium value of shear stiffness $k_f = 18$ kN/mm has been assumed. Due to the experimental evidence of nearly double creep coefficient with respect to timber, the Toratti's rheological model with the creep factor $c_k = 2$ has been assumed.

The numerical mid-span deflection is close to the experimental one (Fig. 5c), both as elastic ($v_{el,\,num} = 2.77$ mm and $v_{el,\,\exp} = 2.69$ mm) and delayed values. The experimental trend in time is well approximated except for the recover due to the reduction of relative humidity monitored after the 214th day, which is underestimated by the model. Less accurate is the prediction of slip ($s_{fel,\,num} = 0.19$ mm and $s_{fel,\,\exp} = 0.23$ mm), especially for the delayed values (Fig. 5d). The numerical curves follow the increasing trend of the experimental one but underestimate the values and are almost insensitive to the changes of environmental conditions. However, it has to be observed that the prediction of the slip in composite beams is a complicated issue. Furthermore, the small value of such a quantity makes a correct monitoring difficult.

The influence of local moisture distributions obtained using different emissivity and diffusion coefficients is nearly negligible on the deflection and more important on the slip (Figs. 5c, 5d and Table 1). Delayed deflection and slip are affected by the connection rheological behaviour and, most of all (Table 1), by the creep coefficient (c_k factor in Eq. (32)). These results highlight the importance of performing long-term push-out tests on the connection system in order to fully characterize the rheological behaviour.

On the whole, the comparison between experimental and numerical results may be considered as satisfactory. The moisture content can be computed once the correct values of diffusion and emissivity are known for wood. The mid-span deflection, the most important quantity in long-term serviceability verifications, can be accurately predicted even if only approximate values are available for diffusion coefficient and surface emissivity. The trend of slip can also be estimated, however with less precision.

6. Conclusions

In this paper a finite element model for long-term analysis of timber-concrete composite beams is described. The model can be used also for different wood-based composite beams, like timber-timber composite beams. Deformability of connection system and rheological behaviour of all component materials are fully considered. For concrete, the CEB-FIP M.C. (1990) creep prediction model is employed. For timber and connection, the Toratti's linear model, which includes creep and mechano-sorptive effect and depends on the local moisture content of timber, is used. The local moisture content distribution is evaluated by solving the diffusion problem over the timber cross-section for a given history of environmental relative humidity.

The rheological models are developed into a sum of exponential functions to reduce the amount of used computer memory and to speed up the computational processes. The corresponding differential equations are transformed in algebraic equations by applying the trapezoidal rule for every temporal step in which the whole time period is divided into. The stiffness matrix and the vector of equivalent nodal forces are obtained by using the Principle of Virtual Work for every temporal step. The procedure is effective for any load and environmental (temperature and relative humidity) history.

The comparison with experimental results obtained on simple and composite beams subjected to long-term loading has been satisfactory. The timber moisture content can be well estimated, as long as the right values of the diffusion coefficient and surface emissivity are employed. The mid-span deflection, the most important quantity in long-term serviceability verifications, has been accurately predicted even if only approximate values for the diffusion and emissivity were known. Larger differences have been observed on the slips, for which the numerical prediction is more complicated. Both deflection and slip are affected by the rheological behaviour of the connection system, which should be investigated by means of experimental tests. The proposed approach, therefore, represents a useful tool for analyses of timber-concrete composite beams subjected to long-term loading in variable thermo-hygrometric conditions.

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