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Statistical models from weigh-in-motion data

Tommy H.T. Chan†, T. J. Miao‡ and Demeke B. Ashebo‡†

Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

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Abstract. This paper aims at formulating various statistical models for the study of a ten year Weigh-in-Motion (WIM) data collected from various WIM stations in Hong Kong. In order to study the bridge live load model it is important to determine the mathematical distributions of different load affecting parameters such as gross vehicle weights, axle weights, axle spacings, average daily number of trucks etc. Each of the above parameters is analyzed by various stochastic processes in order to obtain the mathematical distributions and the Maximum Likelihood Estimation (MLE) method is adopted to calculate the statistical parameters, expected values and standard deviations from the given samples of data. The Kolmogorov-Smirnov (K-S) method of approach is used to check the suitability of the statistical model selected for the particular parameter and the Monte Carlo method is used to simulate the distributions of maximum value stochastic processes of a series of given stochastic processes. Using the statistical analysis approach the maximum value of gross vehicle weight and axle weight in bridge design life has been determined and the distribution functions of these parameters are obtained under both free-flowing traffic and dense traffic status. The maximum value of bending moments and shears for wide range of simple spans are obtained by extrapolation. It has been observed that the obtained maximum values of the gross vehicle weight and axle weight from this study are very close to their legal limitations of Hong Kong which are 42 tonnes for gross weight and 10 tonnes for axle weight.

Key words: weigh in motion; stochastic process; Monte Carlo method; gross vehicle weight; axle weight; inattentive traffic status; dense traffic status.

1. Introduction

This paper discusses the methodologies, analytical concepts and the statistical models derived from the analysis of a 10-year weigh-in-motion (WIM) database. The statistical concepts required and the terminologies adopted in the development of bridge live load models are introduced. From a database containing ten years' vehicle records, the development procedure of statistical models (Ang and Tang 1984) for the parameters related to bridge live loading models are presented in this paper.

Bridge loading models are very closely related to gross vehicle weights, axle weights, axle spacings and average daily number of trucks. If the mathematical distributions of these parameters are obtained accurately, bridge live load models can subsequently be easily formulated. WIM

† Associate Professor, Corresponding author, E-mail: cetommy@polyu.edu.hk
‡ Doctor
‡† Ph.D. Student

systems can provide a great amount of real traffic data to assist in determining these parameters. The problem is how to determine these parameters from the measured WIM data. It is impossible to determine these parameters in a load model if the large amount of WIM data is not described statistically. There are many international methods to define bridge live load models.

Heywood (1992) recommended the analysis of WIM data to determine statistical distribution of moments, shears and reactions in short span bridges. According to Heywood bridge internal force distribution models could be used as the basis of the formulation of bridge live load models. This approach can provide an accurate bridge internal force database for the accurate formulation of bridge live load models. It also provides a tool to check already developed bridge live load models. This paper includes studies for representative vehicles from the large amount of WIM data in Hong Kong.

Harman and Davenport (1976) suggested a statistical analysis of the Ontario WIM data. In their statistical study, truck types and axle spacings were selected by the Monte Carlo simulation process. The weight parameter used was the ratio of gross vehicle weight to the corresponding legal weight limit. The statistical description of bridge responses to give statistical information about the traffic was then calculated.

Nowak (1991) adopted a reliability approach for the development of a new USA bridge design loading code. The procedures, which also included in the text book by Nowak and Collins (2000), can be summarized as follows:

- Selection of representative bridges;
- Establishment of the statistical database for load resistance parameters (statistical data for resistance includes material tests, component tests and field measurement);
- Development of load resistance analysis procedure;
- Selection of the target reliability index;
- Calculation of load and resistance factors.

To define a reliable highway bridge live load model, it is necessary to define all its parameters based on the actual measured WIM data. This paper is aiming to identify such parameters and their distributions based on the Hong Kong traffic database as collected at various WIM stations. It is interesting to note that although it is obvious that a statistical approach for the analysis of WIM data should be adopted, hardly any research found in the existing literature uses this approach. That is to say, that none of these methods provides statistical representations of all related parameters of loading. This may be due to the fact that it requires multidisciplinary knowledge.

The objective of statistical analysis in this paper is to obtain the accurate mathematical descriptions of the mentioned parameters above. As a prerequisite to determine accurate bridge design loadings in Hong Kong, this study not only takes advantages of code formulation methods used internationally but also presents a new method for modeling collected WIM data using a statistical approach.

2. Statistical analysis

The statistical analysis can be divided into five steps (Rohatgi 1976):

(i) WIM data collecting and handling,
(ii) The choice of stochastic processes,
(iii) Grouping the recorded WIM data,

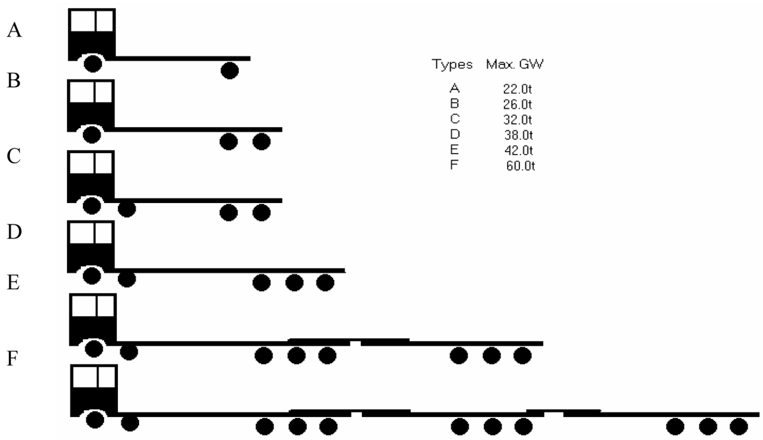

Fig. 1 Typical vehicle types in Hong Kong with legal GVW

(iv) Simulation of the selected stochastic processes, and
(v) The estimation of maximum values of the gross vehicle weight and axle weight in bridge design life.

2.1 Typical vehicle configurations in Hong Kong

Information on GVW and typical vehicle is directly available from the WIM data. After investigating the WIM data, the typical vehicle types with their legal limits specified in Hong Kong are summarized in Fig. 1.

2.2 Method of statistical analysis

Many statistical approaches (Benjamin and Cornell 1970) can be used to analyze the collected data. Two steps can be carried out.

1. To put forth several basic distribution assumptions;
2. To carry out the simulation of these assumptions.

The simulation steps can be summarized as follows:

1. Put forth statistical assumptions H_i (i = 1, 2, …);
2. Choose stochastic variables U_i and carry out statistical analysis and verify which distribution will fit the random variables;
3. Define a confidence limitation $U_{1\alpha}$, $U_{2\alpha}$ for given random variables. Their probabilities should meet the following requirement:

$$P(U_{1\alpha} < U < U_{2\alpha}) = 1 - \alpha \tag{1}$$

   Where α is a given probability level.
4. According to the statistical sample, calculate u that belongs to the random variable U;
5. When the values of u are in the area of $U_{1\alpha} < U < U_{2\alpha}$, the statistical assumptions H_i are true.

2.3 The choice of stochastic processes and the estimation of their statistical parameters

In order to calculate the statistical parameters, expected values and standard deviations, from the given samples of data, the Maximum Likelihood Estimation (MLE) method will be used in this paper.

For a random variable X, with a known probability distribution function $f_X(x)$, and the observed values x_1, x_2, ..., x_n, in a random sample of size n, the likelihood function of θ, where θ represents the set of unknown parameters, is defined as:

$$L(\theta) = \prod_{i=1}^{n} f_X(x_i|\theta) \tag{2}$$

The objective is to maximize $L(\theta)$ for the given data set. This can easily be done by taking m partial derivatives of $L(\theta)$ with respect to θ where m is the number of parameters, and equating them to zero. The MLE of parameter set θ from the solutions of the equations is then obtained. In this way the greatest probability is given to the observed set of events, provided that the true form of PDF is known.

In order to obtain the mathematical distributions and related parameters (Rohatgi 1976) of observed data, researchers have been doing much work. Lin (1990) proposed to use the following stochastic processes to analyze the related problem of highway live loads.

1) Normal Distribution (ND) $N(\mu, \sigma^2)$ (Norman 1994).

The density function can be expressed as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad (-\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0) \tag{3}$$

where, σ is the expected value and μ is the mean value.
The MLE of unknown parameters μ and σ^2

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{4}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{5}$$

where, $\bar{x}$ is the mean

$$\bar{x} = \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{6}$$

2) Weibull Distribution (WD) (Norman 1994)

Assume a variant x in the range $0 \le x \le +\infty$, which depends on two parameters, α and β. Where, α is the characteristic life parameter and β is the shape parameter. The distribution function is:

$$F(x) = 1 - \exp[-(x/\beta)^{\alpha}] \tag{7}$$

and the density function is:

$$f(x) = (\alpha x^{\alpha-1}/\beta)\exp[-(x/\beta)^{\alpha}] \tag{8}$$

According to MLE, estimates of α and β are the solutions of the simulations of the following two equations:

$$\beta = \left[(2/n)\sum_{i=1}^{n} x_i^{\alpha}\right]^{1/\hat{\alpha}} \tag{9}$$

$$\alpha = n/\left[(1/\beta)^{\alpha} * \sum_{i=1}^{n} x_i^{\hat{\alpha}} * \ln x_i - \sum_{i=1}^{n} \ln x_i\right] \tag{10}$$

3) Filtered Weibull Process (FWP) (Norman 1994)

The filtered Weibull process $\{S(t), t \in [0, T]\}$ can be expressed as follows:

$$s(t) = \sum_{n=0}^{N(t)} \omega(t; \tau, s_n) \tag{11}$$

where $\{N(t), t \in [0, T]\}$ is a Weibull stochastic process and its density function can be expressed as follows:

$$\lambda(t) = \lambda\beta t^{\beta-1}(t \geq 0; \lambda, \beta > 0) \text{ (Weibull Process)} \tag{12}$$

where β and λ are the Weibull process parameters and the Response Function is as follows:

$$\omega(t; \tau_n, S_n) = \begin{cases} s_n, t \in \tau_n \\ 0, \quad t \notin \tau_n \end{cases} \tag{13}$$

where τ_n is the acting time of the nth loading, $t_0 = 0$;

S_n (n = 1, 2,……) are independent random variances series that obey $F(X)$ distribution.

Its maximum value is:

$$S_m = \max\{S(t), 0 \leq t \leq T\} \tag{14}$$

which can be expressed as follows (Norman 1994):

$$F_M(X) = \exp\{-\lambda T^{\beta}[1 - F(X)]\} \tag{15}$$

where λ, β are the statistical parameters of the density function of the Weibull process and T is the survey time intervals or the bridge design life (120 years).

4) Gamma Distribution (GD) (Norman 1994)

The density function of the GD $G(\alpha, \gamma)$ is:

$$f(t) = \frac{\lambda^{\alpha}}{\gamma(\alpha)} t^{\alpha-1}\exp(-\lambda t), \quad (\alpha \geq 0, \lambda > 0, \alpha > 0) \tag{16}$$

where $\gamma(\alpha)$ is the well known gamma function:

$$\gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \tag{17}$$

where the MLE of the distribution parameters α, λ can be calculated by the formulae below (Norman 1994):

$$\tilde{\alpha} = \frac{0.5000876 + 0.1648852\omega - 0.0544274\omega^2}{\omega}, \quad 0 < \omega \le 0.5772 \tag{18}$$

or,

$$\tilde{\alpha} = \frac{8.898919 + 9.059950\omega + 0.977537\omega^2}{\omega(17.79728 + 11.968477\omega + \omega^2)}, \quad 0.5772 < \omega \le 17 \tag{19}$$

$$\tilde{\lambda} = \tilde{\alpha}/\tilde{X} \tag{20}$$

where,

$$\omega = \ln\bar{X} - \left(\sum_{i=1}^{N} \ln X_i\right)/N \tag{21}$$

$$N = \sum_{i=1}^{n} n_i \tag{22}$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{n} n_i \bar{X}_i \tag{23}$$

in which n is the total days of observation, n_i is the time interval of the ith day and $\bar{X}$ is the mean value.

5) Logarithmic Normal Distribution (LND) (Norman 1994)
The density function of the LND $\ln N(\mu, \sigma^2)$ can be expressed as follows:

$$f(x) = \frac{\mathbf{1}}{\sqrt{\mathbf{2}\pi}\sigma X}\exp\left[-\frac{(\ln X - \mu)^2}{\mathbf{2}\sigma^2}\right] \tag{24}$$

The MLE of the distribution parameters can be calculated by the following formulae:

$$\hat{\mu} = \ln\bar{X} - 0.5\ln[1 + (s/\bar{X})^2] \tag{25}$$

$$\hat{\sigma} = \{\ln[1 + (s/\bar{X})^2]\}^{1/2} \tag{26}$$

where,

$$\bar{X} = \frac{1}{N}\left[\sum_{i=1}^{n} n_i \bar{x}_i\right] \tag{27}$$

$$s^2 = \frac{1}{N-1}\left[\sum_{i=1}^{n} n_i(S_i^2 + \bar{x}_i^2) - \sum_{i=1}^{n} S_i^2 - N\bar{x}^2\right] \tag{28}$$

in which,

n_i : the total number (Gross vehicle weight, axle weight or axle spacing) of the *i*th day;
$\bar{X}$: mean value
S_i : standard deviation

$$N = \sum_{i=1}^{n} n_i \tag{29}$$

6) Extreme Value Distribution – Type I (EVD) (Norman 1994)
The distribution function is:

$$F_T(X) = \exp\{-\exp[-\alpha(X-\beta)]\} \tag{30}$$

$$\frac{1}{\tilde{\alpha}} = A_n \sum_{i=1}^{n} x_i \lg\frac{i}{n} - \hat{x}_0(A_n \lg(n+1) - 1) \tag{31}$$

in which,

$$1/A_n = -\frac{\lg n!}{n} + \lg(n+1) \tag{32}$$

$$\hat{x}_0 = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{33}$$

as $\tilde{\beta}$ to, the result can be obtained from the equation:

$$e^{-\alpha\beta} = \sum_{i=1}^{N} e^{-\alpha x_i}/N \tag{34}$$

7) Inverse Gaussian Distribution (IGD) (Tweedie 1957)
The probability density function is:

$$f(x;\mu,\lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{0.5} \exp\left[-\frac{\lambda}{2\mu}\left(\frac{x}{\mu} - 2.0 + \frac{\mu}{x}\right)\right] \tag{35}$$

The MLE of μ and λ can be calculated by the following formulae (Tweedie 1957):

$$\hat{\mu} = \frac{\sum_{i=1}^{n} w_i x_i}{w_i} \tag{36}$$

$$\hat{\lambda}_0^{-1} = \frac{\sum_{i=1}^{n} w_i(x_1^{-1} - \hat{\mu}^{-1})}{n} \tag{37}$$

where,

$$\hat{\lambda}_i = \lambda_0 w_i \tag{38}$$

i = 1 to n, w_i being positive and known (Seshadri 1993).

8) Filtered Poisson Process (FPP) (Norman 1994)

The density function of the Filtered Poisson Process can then be expressed:

$$f(x) = \lambda \exp(-\lambda x) \quad x > 0 \tag{39}$$

where λ is the distribution parameter and its estimation value is 0.12.

As the loading time of a vehicle on a bridge is very short, the GVW stochastic processes on a bridge can be described by a filtered poisson process (Lin 1990). The GVW stochastic process $\{S(t),\ t \in [0, T]\}$ can be expressed as:

$$S(t) = \sum_{n=0}^{N(t)} \omega(t; \tau_n, s_n) \tag{40}$$

where,

a) $\{N(t),\ t \in [0, T]\}$ is a Poisson process of parameter λ.

b) Responding function

$$\omega(t; \tau_n, s_n) = \begin{cases} s_n, & t \in \tau_n \\ \mathbf{0}, & t \notin \tau_n \end{cases} \tag{41}$$

where,

τ_n is the loading time of the nth vehicle, $\tau_o = 0$

c) $S_n(n = 1, 2, \ldots\ldots)$ are variables following $F(x)$ which are independent of each other and $S_o = 0$. The maximum value probability distribution of the filtered poisson process can be expressed by the following formula:

$$F_M = \exp\{-\lambda T\,[1 - F(X)]\tag{42}$$

In which $F(X)$ is the stochastic process mentioned above; λ is the parameter from the poisson process which can be calculated by the MLE approach and T is the period of requirement.

2.4 Grouping the recorded WIM data

Recorded WIM data from the IRD system contain a mixture of gross vehicle weight, axle weight, axle spacing and vehicle speeds. This mixture must be divided into several particular separate domains (gross vehicle weight, axle weight, axle spacing, headway, vehicle speed, time interval etc.) in accordance with required objectives for statistical analysis. Hence, the present study uses the sub-samples mentioned above for statistical analysis. They can be generally expressed as:

$$X = \{X_i\} \tag{43}$$

Where X is the vector of a sample in a measuring time section;

$\{X_i\}$ is the ith sample in the time section and i is the number of the cell in objects of the members of the same time section and collection position.

The objects mentioned above will be analyzed statistically to obtain the mathematical distribution models of axle weight, gross vehicle weight, axle spacing and time interval. These models form a very important basis in the modeling of bridge design load models. They can provide basic information for concentrated loads, uniformly distributed loading, gross vehicle weight and axle spacing to build a bridge live loading model. Two kinds of traffic status (Miao 2001) are used. The inattentive traffic status will be used for truck loading, and the dense traffic status will be used for lane loading. With a large database of several years and many divergent analytical requirements, there is a need to divide the data into appropriate forms for analysis. This, and the desire to develop a report for bridges from the WIM data from each site, has resulted in the development of statistical parameter indices of a bridge live load models. These parameter indices, which will be discussed later, are very useful for developing bridge live load models. Stochastic processes (Miao 2001) are used to simulate the WIM data. The commonly used, MLE, approach will be used here to determine these parameters.

The WIM general data sample must be divided into several special needed domains as mentioned above. These are vehicle gross weight, axle weight, headway, vehicle speed, axle spacing and time interval respectively, because bridge live load models are closely related to these parameters.

2.5 Simulation of the selected stochastic processes

The selection of statistical models (Hahn and Samuel 1994) is the first step towards simulation of the stochastic processes. The checking method adopted in the present study is the Kolmogorov-Smirnov (K-S) approach (Hahn and Samuel 1994). It can offer a critical value followed by a given reliable parameter. The advantage of this approach is that all deviations can be obtained between every observed distribution point and theoretical distribution point.

Some commercial statistical packages can be used to check data, e.g. SPSS, SAS and SPSSX (Illinois 1999). These packages can be used to achieve some statistical objectives when studying objects which are normal or for simple problems. The stochastic process of maximum value of gross vehicle weight or axle weight in a general period is a compound stochastic process based on sub maximum values of every time sub-section. It is troublesome to use such software to check compound stochastic processes. Thus a Fortran program is developed to analyze the WIM data. It can give expected values, mean values and the K-S under a given reliable parameter.

The Monte Carlo simulation method (Kottegoda 1998) is one of the most commonly used approaches to simulate complex random variables and complex stochastic processes. The main advantage of this approach is its suitability for any problem with simple simulating procedure. The procedure is usually repeated to generate a different set of values of the variables in accordance with a specified probability distribution. In this way, a series of solutions is obtained corresponding to various sets of the random variables. The Monte Carlo method is therefore used to simulate the distributions of maximum value stochastic processes of a series of given stochastic processes.

The simulating approach of the filtered Weibull Process can be expressed as follows:

If an observation will be ended when the *n*th vehicle loading occurs, the time intervals of adjacent loadings can be expressed as $X_1, X_2, \ldots X_n$. The appearing moment of *i*th loading is:

$$T_i = \sum_{j}^{i} X_j, \quad j = 1, 2, \ldots, n \tag{44}$$

where, $T_1, T_2, \ldots T_n$ are the appearing moments of loadings in the Weibull Process that their density

function can be expressed as $\lambda(t) = \lambda\beta t^{\beta-1}$.
where, λ, β are the statistical parameters.
The simulating statistical value will be calculated by the following formula:

$$F = \frac{[(n-1)-r]\left[(n-1)\ln T_n - \sum_{i=n-r}^{n-1} \ln T_i - (n-1-r)\ln T_{n-r}\right]}{r\left[(n-1-r)\ln T_{n-r} \sum_{i=1}^{n-1-r} \ln T_i\right]} \tag{45}$$

It is an F distribution and its statistical parameters (expected value and standard deviation) are $[2r, 2(n-1-r)]$.
where, $r = [(n-1)/2]$

The values of MLE of statistical parameters $\hat{\beta}$ and $\hat{\lambda}$ can be calculated by the following formulae.

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n} \ln(T_n/T_i)} \tag{46}$$

$$\hat{\lambda} = \frac{n}{T_n\beta} \tag{47}$$

When the acting time is very short, the complicated problems of the simulations of random processes can be carried out by a Filtered Poisson Process. As the loading time of a vehicle on a bridge is very short, the gross weight stochastic processes on a bridge can be described by a filtered poisson process. The gross weight stochastic process $\{S(t), t \in [0, T]\}$ can also be expressed by a filtered poisson process.

$$S(t) = \sum_{n=\mathbf{0}}^{N(t)} \omega(t; \tau_n, s_n) \tag{48}$$

where,

a) $\{N(t), t \in [0, T]\}$ is a poisson process of parameter λ.

b) Responding function

$$\omega(t; \tau_n, s_n) = \begin{cases} s_n, t \in \tau_n \\ \mathbf{0}, t \notin \tau_n \end{cases} \tag{49}$$

where, τ_n is the loading time of the nth vehicle, $\tau_o = 0$.

c) $S_n(n = 1, 2, \ldots\ldots)$ are variables following $F(x)$ which are independent of each other and $S_o = 0$

The maximum value probability distribution of filtered poisson process can be expressed by the following formula:

$$F_M = \exp\{-\lambda T[1 - F(X)]\} \tag{50}$$

In which $F(X)$ is the stochastic process mentioned above; λ is the parameter of poisson process, and it can be calculated by the MLE approach; T is the period of requirement.

3. Results of simulations

As mentioned previously, the WIM general data sample can be divided into several needed domains, namely vehicle gross weight, axle weight, headway, vehicle speed, axle spacing and time interval respectively.

The general specimens of every divided WIM domain assembled from 1986 to1995 are very large. It is impossible to simulate such a large specimen at any one time. Therefore, sub-samples are assembled. These sub-samples can be assembled randomly again and again. It is found that the checked samples follow several stochastic processes at the same time if they are not large enough. Thus, the samples have to be enlarged and re-simulated until only one stochastic process passes the checking instantly.

3.1 Gross weight

The gross vehicle weight samples are randomly drawn from the IRD recorded data. There are three kinds of gross vehicle weight samples.

- Drawn randomly from several continuous days' data at an observed site.
- Drawn randomly from several continuous days' data at a site for a small sample first, then another sample from a different continuous days' data at the same site drawn, to make up a bigger sample from these sub-samples.
- Drawn randomly from different sites' several continuous days' data to assemble a sample.

The above samples shall not be assembled from different year's data or the growing rate cannot be calculated. The assembly can be expressed as:

$$G = \{G_i\} \tag{51}$$

Where G is the gross weight vector of a sample in a measuring time section;

$\{G_i\}$ is the ith sample in the time section and i is the number of the cell in objects of the members of the same time section and collection position.

A sample, drawn between May 10, 1988 and May 15, 1988, contains 156,376 vehicles (including cars) from the WIM data of a site at Lung Chung Road. This raw data is plotted in Fig. 2 for the

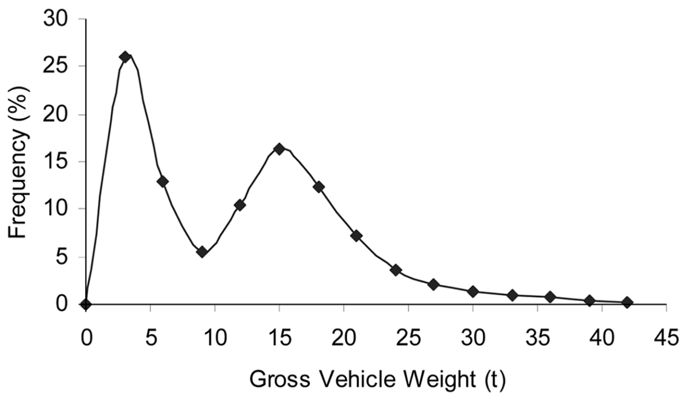

Fig. 2 Distribution of gross vehicle weight

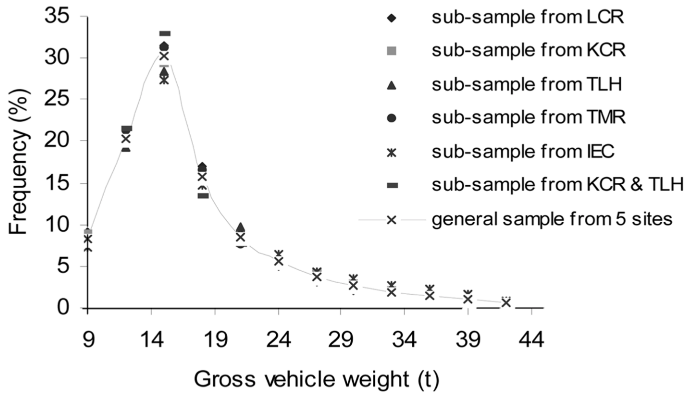

Fig. 3 Gross vehicle weight in 5 sites

Table 1 Gross vehicle weight distributions from 7 sites

Gross weight (*t*)	LCR (%)	KCR (%)	TLH (%)	TMR (%)	IEC (%)	KCR & TLH (%)	General (%)
9	9.11	8.84	8.32	7.86	7.30	8.42	8.31
12	19.89	19.95	19.3	21.32	20.38	21.45	20.38
15	31.54	29.48	28.37	31.21	27.33	32.94	30.15
18	17.06	15.73	16.21	16.65	14.76	13.45	15.64
21	8.89	8.57	9.64	7.68	8.69	7.73	8.53
24	4.97	5.63	5.96	5.25	6.38	5.08	5.55
27	3.06	4.10	3.91	3.46	4.27	3.95	3.79
30	2.04	2.91	2.55	2.82	3.59	2.73	2.77
33	1.42	1.99	2.20	1.45	2.70	1.92	1.95
36	1.00	1.25	1.60	1.00	2.20	1.18	1.37
39	0.68	0.93	1.13	0.70	1.60	0.67	0.95
42	0.34	0.62	0.81	0.60	0.80	0.48	0.61

distribution of gross vehicle weights via their volumes. It can be seen that there is more than one peak on the distribution curve. For the purpose of the maximum value of gross vehicle weight, the first peak is not the question of our research. For this reason, those gross vehicle weights less than 9.0 tonnes are deleted in the simulation procedures. After light vehicles are deleted, new samples can be re-assembled. The distribution functions of this kind of sample will then be simulated. Fig. 3 shows the seven gross vehicle weight samples from different measuring sites. They are assembled randomly. Detailed information is presented in Table 1.

To simulate the distribution of gross vehicle weight, a total number of 317,367 vehicles are studied. The details of the simulation are presented in Table 2.

It can be seen that the distribution of gross vehicle weight obeys the Inverse Gaussian Distribution (IGD). Its statistical parameters are:

Expected value = 20.04   Standard deviation = 28.30

Table 2 Distribution of gross vehicle weights of Hong Kong WIM data

Site	Volume	ND	LND	EV-ID	WD	GD	IGD
LCR	30,641	6.52656 10.35654 0.122561*	0.67257 0.45465 0.121422*	1.50604 4.56375 0.121355*	0.70609 5.22442 0.134911	0.73664 0.13885 0.122825	20.41036 28.05843 0.121407*
KCR	35,345	6.73576 10.83117 0.151421	0.54735 0.46291 0.121477*	1.34606 4.71342 0.162398	0.70633 5.32014 0.138295	0.73617 0.11052 0.136391	19.73358 29.41424 0.121430*
TLH	41,364	6.55201 10.70342 0.152535	0.57637 0.49549 0.121653	1.40153 4.72014 0.160721	0.70633 5.24578 0.130214	0.72254 0.10141 0.131247	20.14740 28.46732 0.121306*
TMR	40,817	6.52423 10.78954 0.151700	0.58251 0.47124 0.121542	1.37433 4.80146 0.155727	0.68527 5.24551 0.131925	0.74247 0.11548 0.131214	20.68114 28.38452 0.121217*
IEC	36,184	6.88547 10.82578 0.155245	0.57703 0.47569 0.121624	1.51603 4.67192 0.142756	0.71623 5.25257 0.137056	0.74330 0.17852 0.131914	20.34462 28.42570 0.121388*
KCR & TLH	52,072	6.65264 10.20875 0.140371	0.49872 0.52049 0.121599	1.56941 4.69980 0.145701	0.74102 5.33212 0.139010	0.76031 0.11851 0.138207	20.43183 28.25809 0.121493*
General	80,944	6.66666 10.63808 0.151322	0.54477 0.46443 0.121589	1.35614 4.70679 0.162357	0.69610 5.21242 0.137791	0.72634 0.10895 0.130682	20.04368 28.30258 0.121129*

Note: The first two lines are statistical parameters; the last line is the observed value.
The symbol * means passing the K-S checking. The critical value = 0.121537.

3.2 Axle weight

The principle of assembling the samples of axle weight is the same as for that of the gross vehicle weight. Axle weights that belong to those vehicles whose gross weights are less than 9.0 tonnes will not be assembled into the checked samples.

$$P = \{P_i\} \tag{52}$$

Where P is the axle weight vector of a sample in a measuring time section;
$\{P_i\}$ is the ith sample in the time section and i is the number of the cell in objects of the member of the same time section and collection position.

To simulate the distribution of axle weight, a total number of 696,181 axles have been studied. The distributions are plotted in Fig. 4 and the results are presented in Table 3.

It can be seen from Table 3 that axle weight obeys the Inverse Gaussian Distribution. The statistical parameters are as follows:

Expected value = 4.25 Standard deviation = 7.05

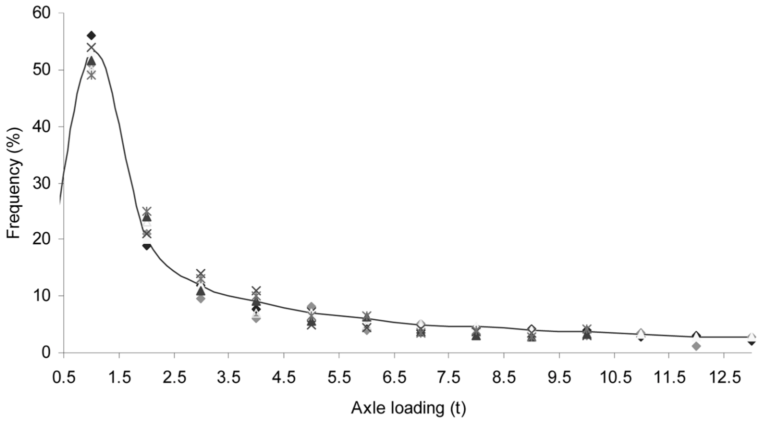

Fig. 4 Axle weight distribution of HK WIM data

Table 3 Distribution of axle weight from Hong Kong WIM data

Site	Volume	ND	LND	EV-ID	WD	GD	IGD
LCR	67,413	3.06571 3.23829 0.113473	0.45474 0.26083 0.100878	2.70253 0.94212 0.100359*	1.05044 1.98097 0.101953	0.95254 0.35421 0.103212	4.32490 7.16554 0.085358*
KCR	75,159	3.01543 3.56800 0.155227	0.49802 0.22187 0.100491	2.68255 0.93589 0.100243*	1.05790 1.96281 0.102711	0.98557 0.36204 0.100716	4.57024 7.00343 0.098543*
TLH	66,164	3.01644 3.44772 0.133289	0.48808 0.26101 0.100417	2.70267 0.98531 0.108342	1.01193 1.98001 0.105171	0.98330 0.36024 0.101602	4.33224 7.13756 0.089264*
TMR	72,753	3.05227 3.67110 0.144215	0.46533 0.26108 0.100919	2.70265 0.97147 0.104432	1.07292 1.97813 0.107147	0.99003 0.36040 0.100791	4.21329 7.23654 0.094952*
IEC	62,475	3.06144 3.63042 0.154531	0.45889 0.26284 0.100497	2.88626 0.98352 0.102473	1.07591 1.98472 0.107316	0.95374 0.33306 0.100694	4.37703 7.14162 0.094557*
KCR & TLH	112,074	3.02156 3.43852 0.162875	0.49580 0.20378 0.109349	2.76552 1.08324 0.102541	1.07697 1.99133 0.101752	0.99371 0.30654 0.101016	4.20383 7.13562 0.093683*
General	240,143	3.00625 3.46738 0.163425	0.48518 0.21681 0.100399	2.71062 0.99315 0.105243	1.05790 1.96281 0.102711	0.96393 0.32064 0.100876	4.25037 7.05367 0.097364*

Note: The first two lines are statistical parameters; the last line is the observed value.
The symbol * means passing the K-S checking. The critical value = 0.100375.

3.3 Axle spacing

The domains for axle spacing are assembled by those vehicles with their gross weights greater than 9.0 tonnes and are specified as follows:

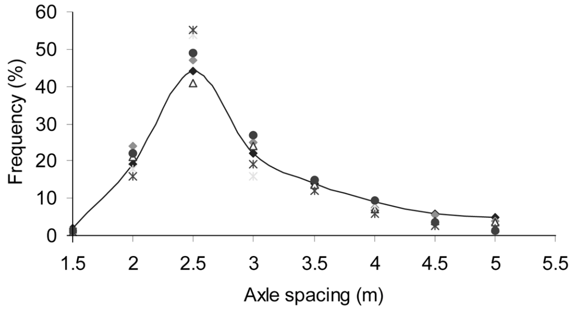

Fig. 5 Axle spacing distribution of HK WIM data

Table 4 Axle spacing simulation results

Distributions	Expected value	Standard deviation	Observed critical value
ND	3.08455	1.07073	0.131572
LND	0.40626	1.04388	0.120826*
GD	5.25546	1.07038	0.154237
WD	1.01450	3.08954	0.132855
EV-1D	1.07549	0.97742	0.131824
IGD	1.00000	0.25000	0.145994

*means passing Kolmogorov-Smirnov checking, theoretical critical value = 0.121433.

$$S = \{S_i\} \tag{53}$$

Where S is the axle spacing vector of a sample in a measuring time section;

$\{S_i\}$ is the ith sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position.

A general sample of 125,954 elements assembled from 5 sites is simulated and plotted in Fig. 5. The results are presented in Table 4.

The results show that axle spacing obeys lognormal distribution. The statistical parameters are as follows:

Expected value = 0.41 Standard deviation = 1.04

3.4 Headway

The samples for headway drawn randomly from WIM data are simulated. They can be expressed as:

$$H = \{H_i\} \tag{54}$$

Where H is the headway vector of a sample in a measuring time section;

$\{H_i\}$ is the ith sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position.

There are 145,256 vehicles in the samples. The distributions of headway are plotted in Fig. 6 and the results are presented in Table 5.

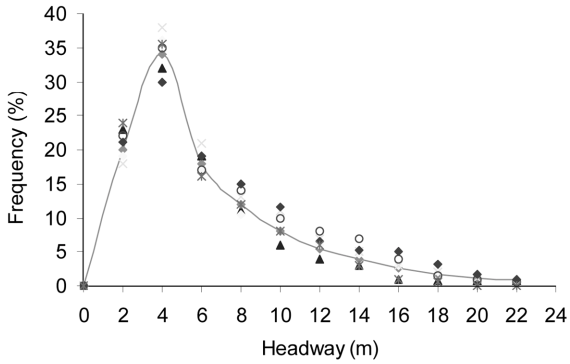

Fig. 6 The distribution of headway of Hong Kong WIM data

Table 5 Headway simulation results

Distributions	Expected value	Standard deviation	Observed critical value
ND	145.2290	152.878	0.153624
LND	4.9747	1.13675	0.126144*
GD	1.2393	0.00853	0.153209
WD	0.2481	103.6010	0.156028
EV-1D	37.7140	85.0992	0.137794
IGD	38.3655	98.9774	0.149341

*means passing Kolmogorov-Smirnov checking, theoretical critical value = 0.131025.

The results show that the headway of Hong Kong WIM data obeys a lognormal distribution. The expected value and standard deviation are as follows:

Expected value = 4.9747   Standard deviation = 1.13675

3.5 Time interval

The time interval samples are divided into two types. One is inattentive status and the other is dense status. According to the T.D.M.P, the vehicles whose time intervals are less than 2 seconds belong to the dense status whilst the remainder belong to inattentive status. They are generally expressed as:

$$T = \{T_i\} \tag{55}$$

Where T is the time interval vector of a sample in a measuring time section;
$\{T_i\}$ is the ith sample in the time section and i is the number of the cell in objects belonging to the same time section and collection position.

3.5.1 Inattentive status

This kind of status will be used for lane loading in the future. It is unnecessary to simulate it, because the IRD system uses the second for the smallest unit to record the time intervals.

3.5.2 Dense status

In the samples for time intervals of dense status, there are 8,386 elements drawn. The results are presented in Table 6.

The results show that the time interval of Hong Kong WIM data obeys the Gamma Distribution. The statistical parameters are as follows:

Expected value = 1.00879   Standard deviation = 0.12176

Table 6 Time interval simulation results

Distributions	Expected value	Standard deviation	Observation critical value
ND	8.93468	13.77943	0.153421
LD	0.39904	2.11032	0.122520
GD	1.00879	0.12176	0.121429*
WD	0.58480	7.11172	0.143427
EV-1D	2.19833	5.34510	0.132331
IGD	1.00000	0.50000	0.167024

*means passing Kolmogorov-Smirnov checking, theoretical critical value = 0.121733.

3.6 Summary of results

The samples of vehicle gross weight, axle weight, axle spacing, headway and time interval are assembled between 1986 and 1996 and are simulated one by one. The results show that Hong Kong traffic can be expressed by either the Inverse Gaussian Distribution, a Lognormal Distribution or Gamma Distribution. The results are summarized in Table 7.

These mathematical distributions of studied elements will be useful for the estimation of maximum values in bridge design life.

Table 7 Simulation results of Hong Kong WIM data

Parameters & Distribution	Gross weight (*t*)	Axle weight (*t*)	Axle spacing (*m*)	Time interval (*s*)	Headway (*m*)
Expected values	20.04	4.25	0.41	1.01	4.83
Standard deviation	28.30	7.05	1.04	0.12	1.12
Type of distribution	IGD	IGD	LD	GD	LD

3.7 The estimation of maximum value in bridge design life

Two approaches can be used to obtain the maximum value of gross vehicle weight and axle weight in bridge design life. The first approach uses the theory of stochastic process whilst the other is the extrapolation method (Nowak 1994). This paper focuses on the former approach.

3.7.1 Application of stochastic theory

After identifying the stochastic processes for each bridge loading related parameters, the distribution parameters of every domain can be obtained. The maximum value of vehicle gross weight and axle weight for a design return period of bridge life can then be determined. For predicting the maximum value of vehicle gross weight and axle weight during the bridge service period, the probability distribution functions of the maximum value stochastic processes of vehicle gross weight stochastic processes and the maximum value of axle weight stochastic processes should be obtained. These processes can provide some information on the gross weight limitation and the axle weight in future formulated bridge design codes. These distribution functions are considered under inattentive traffic status and dense traffic status respectively. The inattentive traffic status will be used for formulating the codes for standard trucks and the dense traffic status for lane loadings.

1) Loose Status

(1) Gross weight

It can be seen from Table 7 that the distribution of gross weight is known to obey the Inverse Gaussian Distribution. Its parameters are:

Expected value = 20.04
Standard deviation = 28.30

From Table 7, it can also be seen that the time interval distributions of vehicles obey the Gamma Distribution whose parameters are:

Expected value α = 1.01
Standard deviation λ = 0.12

As α = 1.01, it is very close to 1.0. To simplify the calculation, the time intervals can be approximately described as an exponential distribution with α equal to 1.0. The subsequent density function can then be expressed as:

$$f(x) = \lambda \exp(-\lambda x) \quad x > 0$$

where λ is the distribution parameter and its estimated value is 0.12.

As the loading time of a vehicle on a bridge is very short, the gross weight stochastic processes on a bridge can be described by a filtered poisson process (Miao 2001).

Taking a week as the observation unit to analyse the maximum distribution of vehicle gross weight, the calculated time area of the maximum distribution of vehicle gross weight should be calculated as follows:

$$T = 7 \times 24 \times 3600 = 604800 \text{ seconds}$$

Since the parameter λ = 0.044 (calculated by MLE), the distribution function can be expressed as:

$$F(X) = \Phi[(\ln X - \mu)/\alpha] = \Phi[(\ln X - 20.04)/28.03] \tag{57}$$

The design life of highway bridges in Hong Kong is 120 years. The weekly observation data can be drawn randomly as a sample, and the maximum value of this weekly data can be taken to describe approximately the maximum value of a year. Then the maximum value of design life can be obtained by the following formula (Lin 1990):

$$F_T(X) = \exp\{-\lambda T[1 - F_M(X)]\} \tag{58}$$

Where $\lambda = 0.044$; $T = 120$ years; $F_M(X)$ is a maximum distribution of a year.

It can be seen that Eq. (58) is very difficult to solve. The calculation method used here is based on the Monte-Carlo approach. According to the theory of probability, if a function $F(X)$ is the distribution function of a stochastic variable x, $F(X)$ obeys an uniform distribution on area (0, 1). According to this principle, many counterfeit functions can be produced by means of the Monte-Carlo approach. These counterfeit random functions will be checked by the Kolmogorov-Smirnov approach.

The detail is described as follows:

a) Generate 100 stochastic processes on area (0,1) by the Monte-Carlo approach;

b) Line them from lowest to highest as $W(1)$, $W(2)$,…, $W(100)$;

c) Obtain $F_T(X)$ according to Eq. (58),

$$\exp\{-\lambda T[1 - F_M(X)]\} = W(i), \qquad i = 1, 2, \ldots, 100 \tag{59}$$

where $\lambda = 0.044$, $T = 120$ years

d) Obtain the random counterfeit functions $B(i)$, $i = 1, 2, \ldots, 100$ from the following:

According to Eq. (59),

$$F_M(X_i) = [1 + \ln W(i)]/\lambda T, \qquad i = 1, 2, \ldots, 100 \tag{60}$$

Letting,

$$F_M(X_i) = \exp\{-\lambda T[1 - F(Xi)]\} = \mu(i), \qquad i = 1, 2, \ldots, 100 \tag{61}$$

If $\lambda = 0.044$, $T = 120$ years, then the result:

$$\begin{aligned} F(X_i) &= \{1 + \ln[\mu(X_i)]\}/\lambda T \\ &= \Phi[(\ln X_i - \mu)/\sigma], \qquad i = 1, 2, \ldots, 100 \end{aligned} \tag{62}$$

Making,

$$F(X_i) = Z(i), \qquad i = 1, 2, \ldots, 100 \tag{63}$$

Functions $Z(1)$, $Z(2)$, …, $Z(100)$ can be obtained respectively through Eq. (59) and Eq. (63). $A(i)$ can then be obtained through checking the Gaussian distribution form:

$$A(i) = \varphi^{-1}[Z(i)], \qquad i = 1, 2, \ldots, 100 \tag{64}$$

With $(\ln X - \mu)/\sigma = A(i)$, then

$$X_i = \exp[\mu + \sigma A(i)] = B(i), \qquad I = 1, 2, \ldots, 100 \tag{65}$$

Where $\mu = 1.67$ and $\sigma = 0.82$.

Now the 100 random counterfeit functions $B(i)$, $i = 1, 2, \ldots, 100$, have been obtained. The Kolmogorov-Smirnov approach is used again to check whether Normal, Lognormal, Weibull, Gamma, Extreme Value Type-I and the Inverse Gauss Distribution describe the maximum value distribution of the gross weight. According to the simulated results, the Extreme-Value Type-I distribution describes the data best. The results show that the maximum value distribution of the gross weight stochastic process follows the Extreme-Value Type-I Distribution. That is:

$$F_T(X) = \exp\{-\exp[-(X - \alpha)/\beta]\} \tag{66}$$

Where α and β are distribution parameters, that can be obtained by MLE.

$$\alpha = 137.682 \qquad \beta = 41.784$$

(2) Axle weight

According to statistical analysis (see Table 7), the random process of axle weight can be described by the Inverse Gaussian Distribution (IGD). Its estimated statistical parameters are:

$$\mu = 4.25 \qquad \alpha = 7.05$$

Axle weight random processes on a bridge can be described as a filtered Poisson process and the maximum value stochastic process distribution is found to be described as an Extreme Value Type-I process using the same approach mentioned above,

$$F_T(X) = \exp\{-\exp[-(X - \alpha)/\beta]\} \tag{67}$$

where α and β are the value of MLE:

$$\alpha = 75.003 \qquad \beta = 19.359$$

2) Dense traffic status

(1) Gross weight

According to WIM data and statistical analysis, when the loading times of vehicles on a bridge are very short, the maximum value stochastic process of vehicle gross weight stochastic processes can be described as a Filtered Weibull Distribution (Miao 2001). The distribution of gross weight cross-section data obeys the Inverse Gaussian Distribution and its statistical parameters are:

$$\mu = 20.04 \qquad \sigma = 28.30$$

The maximum value distribution in design life of a bridge can also be determined by the Monte-Carlo approach. The steps are the same as that for the inattentive traffic status. The formula is:

$$F_M(X) = \exp\{-\lambda T^{\beta}[1 - F(X)]\} \tag{68}$$

where λ and β are the parameters, T is the design life of bridges.

According to the results of simulation, it can be seen that the extreme value distribution of gross weight under dense traffic status is described as a Weibull Distribution. Its distribution function is:

$$F_T(X) = 1 - \exp[-X/\alpha^{\beta}], \qquad X > 0 \tag{69}$$

where α and β are values of the maximum likelihood estimation:

$$\alpha = 358.275 \qquad \beta = 22.229$$

(2) Axle weight

The statistical parameters of the distribution of axle weight can be obtained by the same method as that for the gross weight. The maximum value distribution of axle weight under dense traffic status is also described as a Weibull distribution. Its statistical parameters are:

$$\alpha = 225.356 \qquad \beta = 12.340$$

3) Maximum value of gross vehicle weight and axle weight

The statistical analysis results of the maximum distribution of axle weight and gross weight are

Table 8 The maximum value distribution during bridge design life

Traffic status	Loading type (t)	The maximum value distribution in 120 years	Distribution	Parameters
Inattentive status	Gross weight	Extreme value Type-I distribution	α β	137.612 41.784
	Axle Weight		α β	75.003 19.359
Dense status	Gross weight	Weibull distribution	μ σ	358.274 24.692
	Axle Weight		μ σ	225.356 12.340

shown in Table 8. The maximum value of vehicle gross weight during bridge design life obeys the Extreme Value Type-I distribution or the Weibull Distribution respectively according to its traffic status.

1) Gross weight

To substitute the distribution parameters in Table 8 into Eq. (30), the distribution function of gross weight under inattentive traffic status can be expressed as:

$$F_T(X) = \exp\{-\exp[-137.682(X - 41.784)]\} \tag{70}$$

Similarly, the distribution function of gross weight under dense traffic status can be obtained by substituting the corresponding distribution parameters from Table 8 into Eq. (7):

$$F_T(X) = 1 - \exp[-(X/24.692)^{358.274}] \tag{71}$$

According to these two formulae, the maximum gross weight value in bridge design life under a given probability constant, say 0.95, can be obtained.

By solving Eq. (70), the maximum value of gross weight under inattentive traffic status is:

$$W_G = 41.80 \text{ (tons)}$$

By solving Eq. (71), the maximum value of gross weight under dense traffic status is:

$$W_G = 24.70 \text{ (tons)}$$

According to the above statistical results, it can be seen that heavy vehicles rarely pass the central areas of cities. The maximum loading of a single vehicle in these areas is far less than that of those in other areas. However, it can be seen that the traffic can be described as dense status. Distributed loading is the best form to describe this kind of traffic.

2) Axle weight

The axle weight maximum value distribution during bridge design life can be computed as follows:

Under the inattentive traffic status, substitute the corresponding distribution parameters from Table 8 into Eq. (29):

$$F_T(X) = \exp\{-\exp[-75.003(X-19.359)]\} \tag{72}$$

Under the dense traffic status, substitute the corresponding distribution parameters from Table 8 into Eq. (7):

$$F_T(X) = 1 - \exp[-(X/12.34)^{225.356}] \tag{73}$$

By solving Eq. (72), the maximum value of axle weight during bridge design life is:

$$W_L = 19.36 \text{ (tons)}$$

By solving Eq. (73), the maximum value of axle weight during bridge design life is:

$$W_L = 12.40 \text{ (tons)}$$

The results of maximum axle weight show that there are not many vehicles with axle-groups found in dense traffic status.

It is interesting to note that the maximum value of gross weight and axle weight during bridge design life, say 120 years, calculated according to the above approach are very close to their legal limitations of Hong Kong which are 42 tonnes for gross weight and 10 tonnes for axle weight.

3.7.2 Extrapolation

The approach of extrapolation is used to obtain the maximum value of bending moments and shears. For each surveyed truck, bending moments and shear forces are calculated for a wide range of simple spans. Then the cumulative distribution functions are plotted on normal probability paper for different spans. According to the trend of every curve, the needed maximum values for various time periods are obtained.

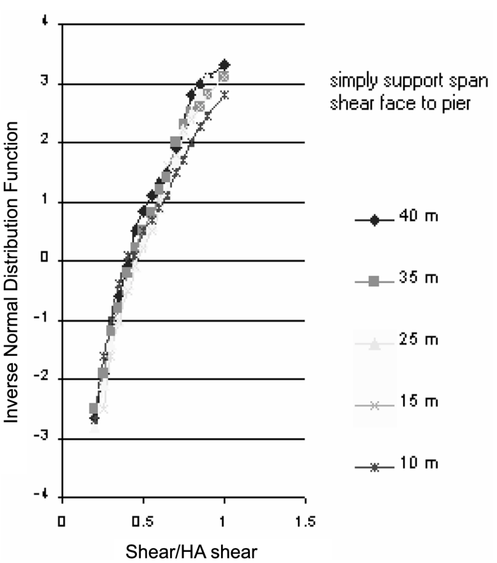

Fig. 7 CDF of truck shears from HK WIM data of general sample

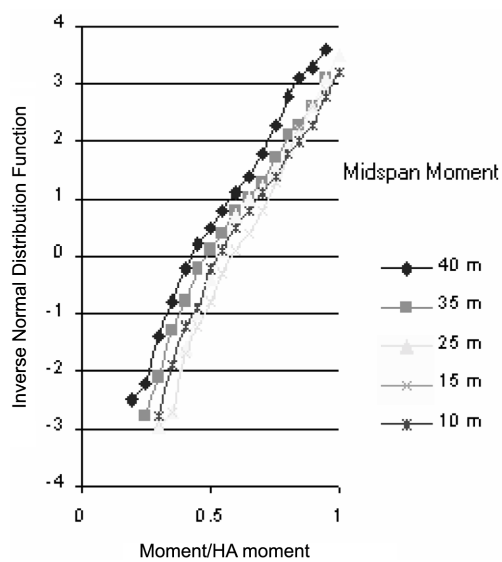

Fig. 8 CDF of truck moments from HK WIM data of general sample

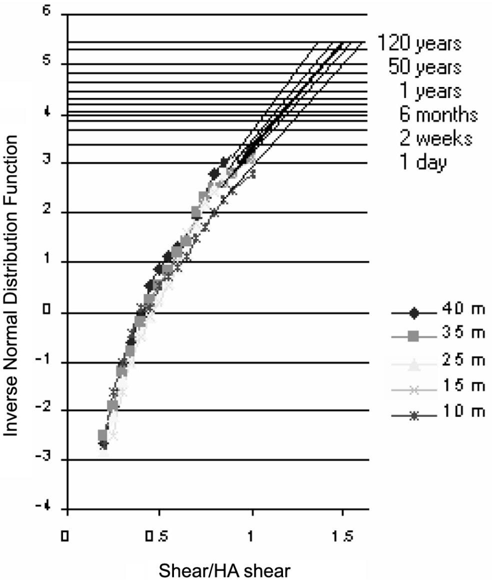

Fig. 9 Extrapolated CDF of truck shears

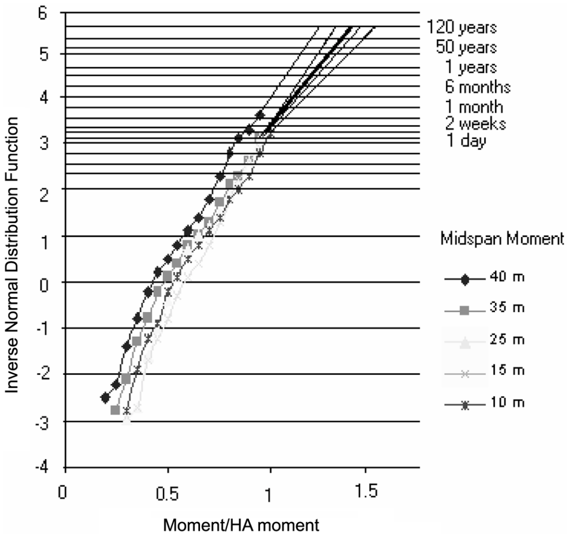

Fig. 10 Extrapolated CDF of truck moments

The moments and shears were calculated separately for the randomly assembled sub-sample from the surveyed WIM data between 1985 and 1995 and the Hong Kong bridge design HA loading. The bias of moments or shears from the WIM data divided by the moments or shears from the HA can be obtained. The resulting cumulative distribution functions of moments and shears are plotted in Figs. 7 and 8. The results do not indicate any considerable change in maximum moments and shears in all samples.

The maximum bending moments and shears for various time periods can be extrapolated from the cumulative distribution function in the above Figures. The extrapolation distribution functions are shown in Figs. 9 and 10.

Let N be the total number of trucks in time period T. According to the surveyed data, there were about 3,000 trucks passing a test site a day. Therefore, in T = 120 years, the number of trucks, will be about 1.3×10^8. The probability level corresponds to N is $1/N$; for $N = 1.3 \times 10^8$, the probability is 7.7×10^{-9}, which corresponds to z = 5.38 on the vertical scale, as shown in Fig. 9 and Fig. 10.

Table 9 Number of trucks vs. time period and probability

Time period T	Number of trucks N	Probability $1/N$	Inverse normal z
120 years	130 000 000	7.7×10^{-9}	5.38
50 years	54 750 000	1.8×10^{-8}	5.17
1 year	1 095 000	9.1×10^{-7}	4.45
6 months	547 500	1.8×10^{-6}	4.29
2 weeks	42 000	2.8×10^{-5}	3.75
1 day	3 000	3.3×10^{-4}	3.07

The number of trucks (N), the probability ($1/N$), and the inverse normal distribution value (z) corresponding to various time periods (T), from 1 day to 120 years, are shown in Table 9.

The mean maximum bending moments and shears corresponding to various time periods can be read directly from the Figure. For example, for a 20 m span and T = 50 years, the mean maximum bending moment is 1.26 times the HA moment. It is equal to the horizontal coordinate of intersection of the extrapolated distribution and z = 5.17 on vertical scale. For a 35 m span and T = 120 years, the mean maximum bending moment is 1.4 times the HA moment. Similar calculations can be carried out for other periods of time.

3.7.3 Comparison of the two methods

To meet the demands, the loading carried by a heavy vehicle will be getting heavier and heavier, a bridge design loading should meet the needs of this development. The maximum gross vehicle weight within a bridge design life, say 120 years, should be obtained and considered in the development of bridge design loading models. Two methods, statistical analysis and extrapolation, are introduced above to obtain the required maximum values. The comparison of these two methods is made below:

- The statistical analysis approach used in this paper is an effective method to obtain the CDF of related parameters such as gross vehicle weight, axle weight, axle spacing and so on. These obtained parameters are the base of formulating the bridge live load models. This approach can provide a distinct CDF and a required value of any parameter under a given probability level. The disadvantage of this approach is very complicated. The simulated samples should be large enough and the process of random assembling is quite tedious.
- The extrapolation approach proposed by Nowak (1994) provides an easy and effective way to obtain the maximum value of related parameters. It can avoid complicated simulating calculations. However, the approach may be subjective and the accuracy depends on the experience of the researchers.

4. Conclusions

The WIM data from Hong Kong has been statistically analyzed in this paper. The mathematical distributions of gross vehicle weights, axle weights and headway are obtained. The maximum values of axle weight and gross vehicle weight are compound stochastic processes. Their mathematical distributions are also determined. Then the maximum gross weights and axle weights within bridge design life under a probability level are calculated. The results are summarized in Table 10.

Table 10 The probability distribution types of Hong Kong traffic and their parameters

Vehicle loadings	Distribution types	Distribution parameters	
		Mean value	Standard deviation
Gross weight (t)	IGD	20.044	28.303
Axle weight (t)	IGD	4.250	7.054
Headway (m)	LD	4.975	1.137
Axle spacing (m)	LD	0.4.6	1.044

The maximum value of gross vehicle weight from Hong Kong WIM data in observed ten years obeys EV-1D and the statistical parameters are: expected value = 137.612, standard deviation = 41.784.

The maximum value of gross vehicle weight within bridge design life, say 120 years, is: 41.80 tonnes for inattentive status and 24.70 tonnes for dense status.

The maximum value of axle weight from Hong Kong WIM data in observed ten years obeys EV-1D, the statistical parameters are: expected value = 75.003; standard deviation = 19.36.

The maximum value of axle weight within bridge life, say 120 years, is: 19.36 tonnes inattentive status and 12.40 tonnes for dense status.

Acknowledgements

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Notation

$f(x)$	: probability distribution function
$F(x)$	: distribution function
$L(\theta)$	: the likelihood function of θ
σ	: the expected value
μ	: the mean value
$s(t)$	: the filtered Weibull process
$\gamma(\alpha)$	: gamma function
MLE	: Maximum Likelihood Estimation
$\hat{\mu}$ $\hat{\sigma}$	: the MLE of the distribution parameters
$\hat{\beta}$ $\hat{\lambda}$	: the MLE of the distribution parameters
$\bar{X}$	: mean value
G	: the gross vehicle weight vector
$\{G_i\}$	: the *i*th sample of the gross weight vehicle in the time section
P	: the axle weight vector
$\{P_i\}$	: the *i*th sample of the axle weight in the time section
S	: the axle spacing vector of a sample
$\{S_i\}$	: the *i*th sample of the axle spacing in the time section
H	: the headway vector of a sample
$\{H_i\}$	: the *i*th sample of the headway in the time section
T	: the time interval vector of a sample in a measuring time section
$\{T_i\}$	: the *i*th sample of time interval in the time section