Structural Engineering and Mechanics, Vol. 2, No. 4 (1994) 359-367
DOI: http://dx.doi.org/10.12989/sem.1994.2.4.359

Numerical and experimental study of large deflection of symmetrically laminated composite plates in compression

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Abstract. The stability behaviour of symmetrically laminated rectangular composite plates with loaded ends clamped and unloaded edges simply-supported, and subjected to uniform in-plane compression is investigated. A numerical and experimental investigation is presented in this contribution. The stacking sequence of the laminated glass/epoxy composite plates is symmetric about the middle surface and consists of 8-ply [0, 90, +45, −45]s lamination.

Numerical predictions were obtained through the use of the finite element method. The above plates were modelled with 8-noded isoparametric layered shell elements. The effect of the input parameters such as the degree and forms of prescribed initial imperfection and the incremental step size required for incremental loading, on the convergence of the solution is thoroughly examined.

Experimental results are presented for 10 test panels. All test panels were made from glass/epoxy unidirectional prepregs and have aspect ratio of 5.088. The laminate thicknesses were found to vary from 1.054 mm to 1.066 mm. Comparison of experimental data with predicted results show good correlation and give confidence in the finite element model.

Key words: finite element method; large deflection; laminated composite plate; compression; buckling; post-buckling.

1. Introduction

Thin plates when compressed in-plane suffer a detrimental phenomenon known as buckling. If these in-plane forces are sufficiently small, the equilibrium is stable and deformation is purely in-plane. As the magnitude of these in-plane forces increases, at a certain load level there is simultaneous occurrence of in-plane deformation with out-of-plane deflection. In this case, the stable equilibrium becomes unstable and the plate is said to have buckled. The importance of the buckling load is the initiation of a deflection pattern which will rapidly lead to very large out-of-plane deflections and eventually to complete failure if the load is increased further.

The buckling and post-buckling behaviour of rectangular isotropic plates in compression has been well researched and documented in Coan (1951), Yamaki (1959), Walker (1969) and Rhodes and Harvey (1971). The introduction of composite materials as the advanced light-weight materials of the future in the aircraft and aerospace industry, provided the required impetus for intense research into composite plate stability behaviour. There are already quite a number of notable

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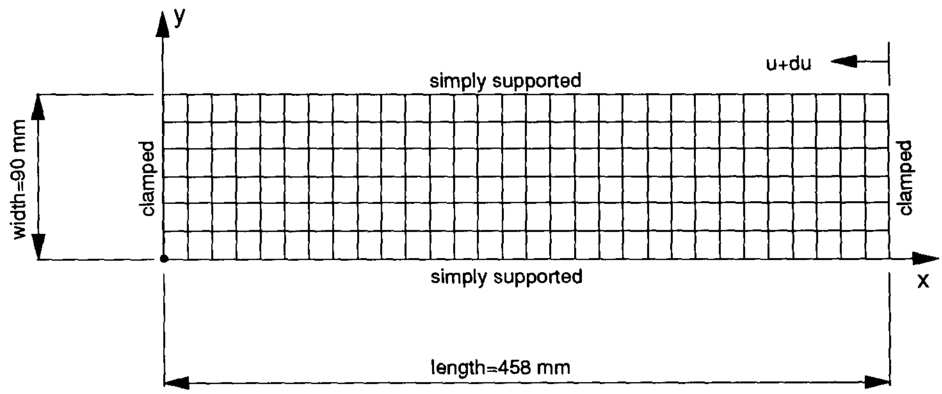

Fig. 1 Description of the geometry and support conditions on the finite element model.

publications on the stability problem of composite plates (Yusuff 1952, Chia 1972, Harris 1973, Banks 1977, Stein 1983). A recent monograph published in 1987 by Leissa presented a comprehensive review and indication on the vast amount of literature available related to this subject. Recent contributions to this field of study are found in Chai (1991) and Chai, *et al.* (1991a, 1991b).

With the advancement of computational capability and availability of computers in the engineering industries, there is a wide range of finite element computer software commercially available for solving general or specific engineering problems. However details such as the effect of the incremental load step size and the degree and physical form of prescribed initial plate imperfection on the large deflection behaviour of plates are not available in literature. The objective of this contribution is thus twofold, the first is to redress this deficiency in the literature and the second is to enhance the present understanding of the large deflection behaviour of laminated composite plates in uniaxial compression.

2. Numerical approach

A commercially available finite element software computer code (ANSYS 1989) was chosen for the study. The element type used is an 8-noded isoparametric layered shell element suitable for analysing laminated composite plate problems and is capable of linear and non-linear analysis. The loading conditions imposed on the finite element model is uniform in-plane end-shortening. A typical finite element model describing the geometry of the plate and the support conditions is shown in Fig. 1. In this figure, the letter u represents the end shortening and du represents the change in the end shortening.

It is well known that the effects of initial imperfections are most detrimental if they are similar in form to the out-of-plane deflections due to buckling. The degree of initial imperfection of the plate is the maximum amplitude of the imperfection curve and is usually prescribed as a percentage of the plate's thickness. The initial imperfection of the plate model was prescribed in the form of a single term trigonometric function as illustrated in Fig. 2. Two trigonometric functions were used for the study:

$$\text{Function } F_1 \qquad w_o = A_m \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{1}$$

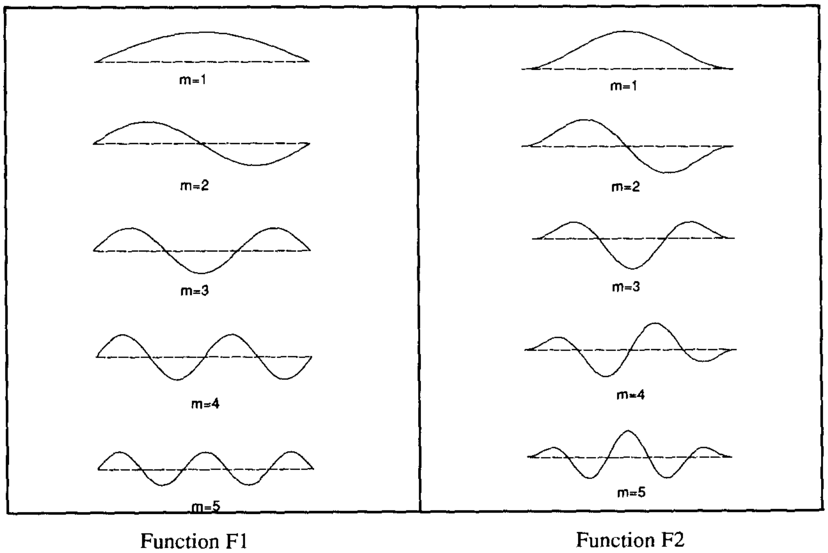

Fig. 2 Number of m-waves prescribed along the length of the plate.

$$\text{Function } F_2 \qquad w_o = A_m \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{2}$$

where x, y and z are the co-ordinates system of the plate, A_m is the maximum amplitude of the trigonometric function, a and b is the length and width of the plate respectively, w_o is the initial out-of-plane deflection of the plate, and m is the number of half wavelengths along the length of the plate which depends on the initial buckled mode shape. Function F_1 is a commonly used function for a plate simply supported along all edges and also applicable for a long plate ($a/b>5$) with unloaded long edges simply supported and loaded short ends other than simple supports. Function F_2 though not commonly used has been shown to give satisfactory stability results for a plate clamped on the loaded short ends and simply supported on the unloaded long edges (Chai and Khong 1991). In order to simulate the actual test loading condition, a uniformly end displacement was applied along the loaded ends at incremental steps. The incremental step size must be small enough such that the plate can numerically buckled at the bifurcation point without sacrificing computing time and relative accuracy. Guidelines given in some of the finite element software manuals are neither clear nor mentioned regarding the input of this parameter. For example, MARC (1991) states that generally a step size du of 1/5 to 1/15 of the buckling load (or the displacement at the buckling load Ucr) would be recommended. Thus step sizes of $Ucr/10$ and $Ucr/20$ were chosen for comparison purposes.

3. Numerical results

For the preliminary study, a linear elastic buckling analysis was performed and a mesh size of 180 elements was found to give a satisfactory converged critical buckling stress of 12.2 MPa. The number of half buckle wavelengths (m-waves) predicted for the plate model was five and this was then used for the prescribed initial imperfection in the non-linear large deflection study.

Numerical results for the finite element model prescribed with the function F_2 are shown

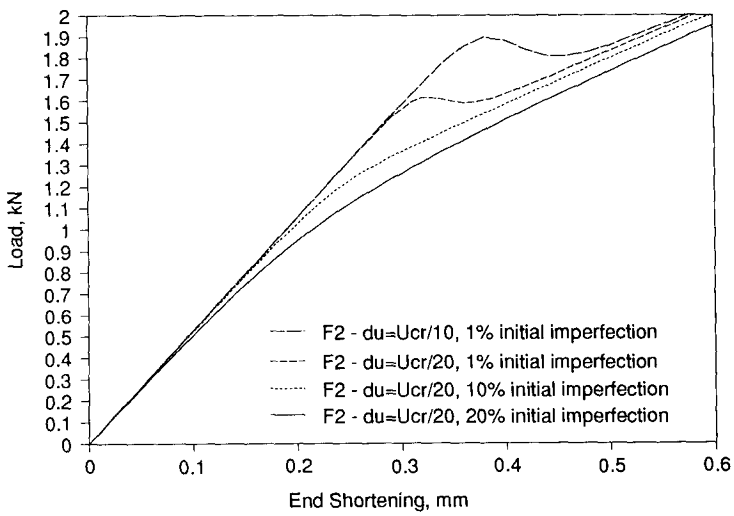

Fig. 3 Load versus end shortening behaviour (prediction using function F_2).

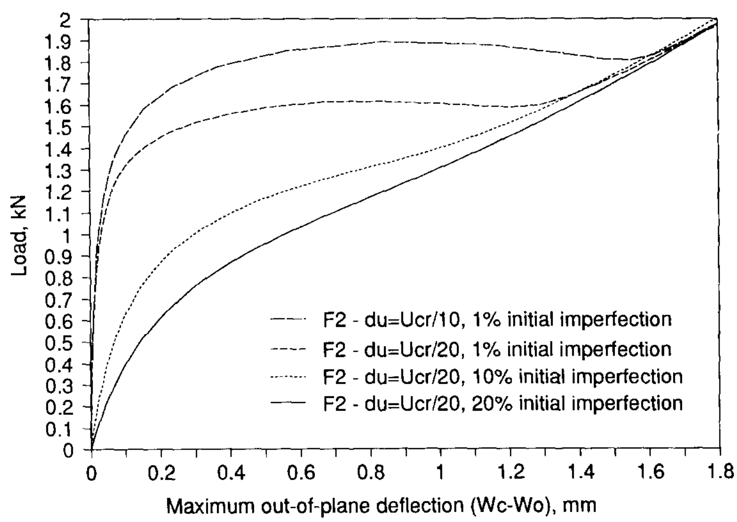

Fig. 4 Load versus out-of-plane deflection behaviour (prediction using function F_2).

in Figs. 3 and 4. It can be seen from these figures that with a 1% initial imperfection and $du = Ucr/10$, the result is a distinguishable hump. Keeping the degree of initial imperfection the same and refining $du = Ucr/20$, the hump becomes less obvious. The same effect can also be achieved by keeping the $du = Ucr/20$ and increasing the degree of initial imperfection to 10% and 20%, as shown in the figure. The notation (Wc-Wo) in Fig. 4 is the total deflection of the plate at the center minus the initial out-of-plane deflection at the center. Comparison of the predicted results using the two different forms of initial imperfections are shown in Figs. 5 and 6. Keeping the degree of initial imperfection at a constant of 10%, the figures show results for different values of du. The behavioural curves of the plate using function F_1 are always higher than those obtained using function F_2. This is to be expected as function F_2 seems to give a better representation of out-of-plane deflection form of the plate which is clamped on

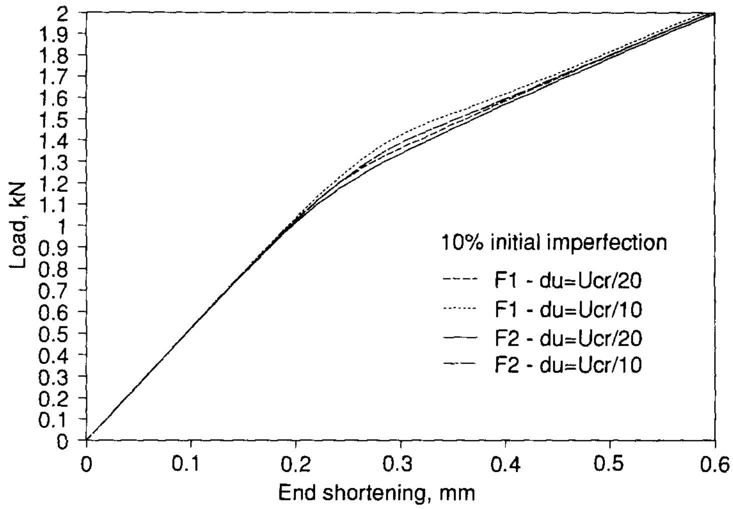

Fig. 5 Load versus end shortening behaviour (prediction using 10% initial imperfection).

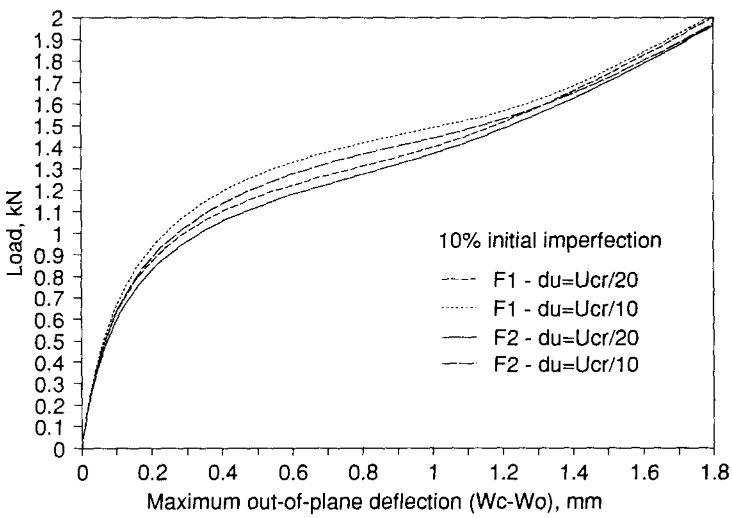

Fig. 6 Load versus out-of-plane deflection behaviour (prediction using 10% initial imperfection).

the loaded ends and simply supported on the unloaded edges, see Fig. 2. In addition, as can be seen from the figures the hump in the buckling region is less obvious even with $du = Ucr/10$ in comparison with the behaviour shown in Figs. 3 and 4.

Lastly for a plate prescribed with function F_2, 10% initial imperfection and $du = Ucr/20$, the effect of the number of m-waves on the large deflection behaviour of the 8-ply laminated composite plate is shown in Fig. 7. The figure shows that in the far post-buckling region (load>2 kN) the behavioural curves of the odd number of m-waves tend to group together. The same description applies also to the curves pertaining to even number of m-waves. From these comparisons, it would seem that the buckling load and the post-buckling path of the plate can suitably be changed by prescribing the required initial imperfection in the plate. It should also be noted that regardless of the number m-waves prescribed on the plate model, the plate will eventually

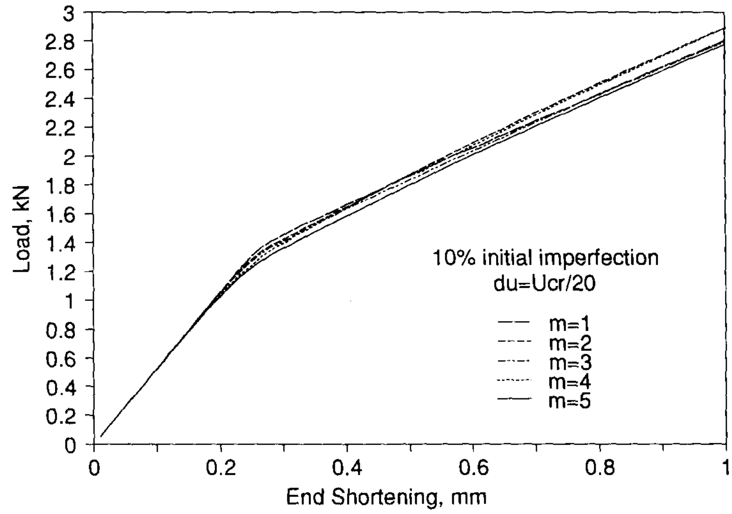

Fig. 7 Load versus end shortening behaviour (varying the number of m-waves initial imperfection).

buckled into five half wavelength mode shape.

4. Experimental investigation

The test plates were made in-house using Heatcon 9500 composite curing table. The materials used were Fibredux 913G-E-5-30 unidirectional *E*-glass fibre prepregs (30% epoxy resin) supplied by Ciba-Geigy. Heatcon 9500 is capable of controlling the temperature cycle recommended by Ciba-Geigy but it is however limited to only pressure applied through a vacuum pump. Nevertheless the material properties tests using ASTM D3039-76 and D3518-76 shows consistency in the results. The following mechanical properties were found to be: $E_{11}=46$ GN/m^2, $E_{22}=15$ GN/m^2, $\nu_{12}=0.276$ and $G_{12}=4.17$ GN/m^2. Details of the experimental test rigs, test set-up and testing procedures can be found in Chai, *et al.* (1991b). The description of the test plates made in-house is given in Table 1, the dimensions given in the table are measured between the supports. The thickness shown in the table for each test plate is the average value of ten different thicknesses measured randomly throughout the plate. The overhung strip in Table 1 refers to the amount of overhang of the plate's unloaded edges from the knife edge supports. All test plates were made as flat as possible without imposing any initial imperfections.

In addition, two strain gauges (Strain Gauge No. 1 and Strain Gauge No. 2) were bonded on the top and directly opposite bottom surface of each test plate, and located at the region where the maximum buckle crest was predicted. All together two sets of five test plates per set were made, i.e. GE1-1 to GE1-5 and GE2-1 to GE2-5. All test plates were of 8-ply [0, 90, +45, −45]s stacking sequence.

5. Numerical comparisons

In order to eliminate any possible influence caused by the variation in the plate's cross-sectional dimensions, applied stresses instead of loads were used for comparison. The applied stress was

Table 1 Summary of experimentally obtained critical buckling stresses

Test panels	Length (mm)	Width (mm)	Thickness (mm)	Overhung strip (mm)	Experimental critical buckling stress (MPa)
GE1-1			1.066	1	11.0
GE1-2			1.066	2	12.5
GE1-3	458	90	1.058	3	11.0
GE1-4			1.063	4	11.5
GE1-5			1.060	5	11.5
GE2-1			1.066	1	11.5
GE2-2			1.049	2	10.5
GE2-3	458	90	1.054	3	10.8
GE2-4			1.054	4	11.5
GE2-5			1.057	5	12.0

calculated by dividing the load by the overall cross-sectional area across the width. The results for the critical buckling stress are tabulated in Table 1. The experimental critical buckling load from which the stress is determined, is obtained using the membrane strain reversal method on the measured strain result. This method was first proposed by Coan (1951) for isotropic plate and has been applied to laminated composite plate (Chai, *et al.* 1991b). It can be seen from the table that the experimental results for all test plates correlated rather well with the numerical prediction of 12.2 MPa (note that this value is based on laminate thickness of 1.066 mm). All test panels were observed to have buckled with five half buckle wavelengths (i.e. $m=5$) during the test.

The large deflection results for the each set of five test plates are presented respectively in Figs. 8 and 9. From the figures, it can be seen that the behavioural trend is generally well predicted even in the immediate post-buckling region. One point to note of the scatter in the experimental data in post-buckling region, is that this is probably caused by the different value of the overhung strip used in the test panels. The discussion for this effect is outside the scope of this paper and thus will not be mentioned further as the research is still on-going.

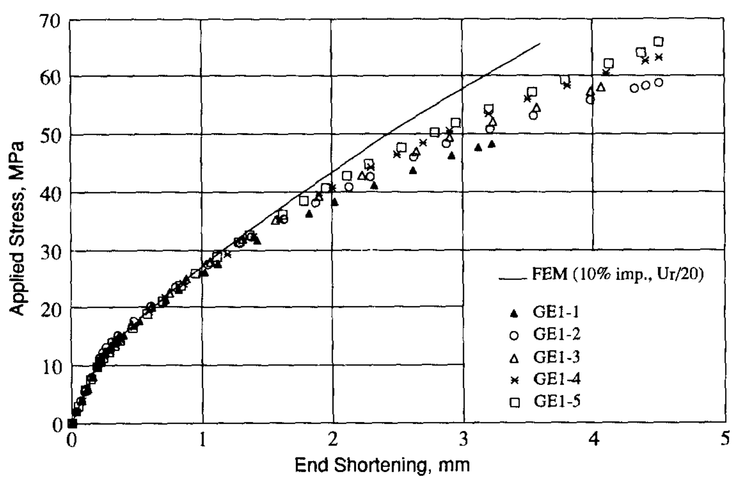

Fig. 8 Applied stress versus end shortening behaviour for group 1 test plates.

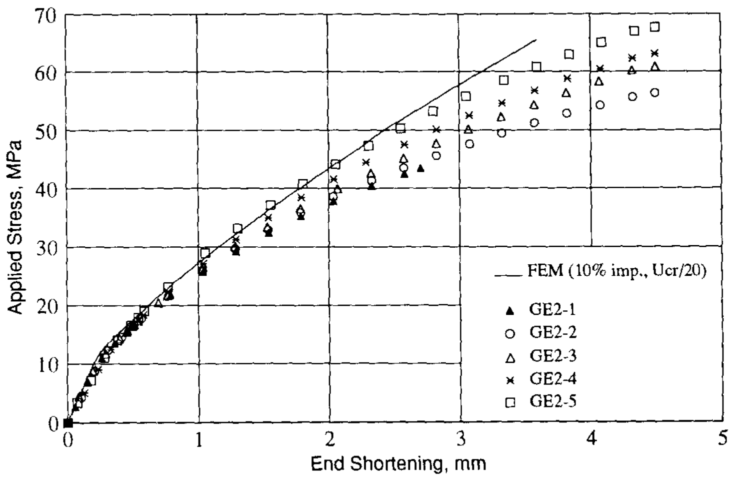

Fig. 9 Applied stress versus end shortening behaviour for group 2 test plates.

6. Conclusions

The form and degree of initial imperfection prescribed on the finite element plate model were shown to affect the structural behaviour of the laminated composite plates in particular in the buckling and post-buckling region. The step size of the incremental end shortening was shown to affect the finite element solution especially in the bifurcation region. It was also shown that the number of incremental loading steps can be kept to a minimum in conjunction with a higher degree of initial imperfection prescribed on the plate model. Hence this will help in reducing the computational time and speed up the rate of convergency in the solution.

The large deflection in-plane compression tests performed on ten symmetrically laminated composite plates confirmed the predicted structural behaviour obtained using finite element method. With the wide availability of commercial finite element computer software, this paper will provide some of the missing links to the solution of large deflection problems of structures using finite element method.

Acknowledgements

The authors would like to extend their appreciation to the Ministry of Finance, Singapore for funding this project under the research grant MOF R&D 5/89 and to Nanyang Technological University for the use of the laboratory facilities. The authors are also grateful to the undergraduate students who were involved in both the finite element modelling and experimental programmes.

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