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On the numerical assessment of the separation zones in semirigid column base plate connections

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Abstract. The present paper concerns the mathematical study and the numerical treatment of the problem of semirigid connections in bolted steel column base plates by taking into account the possibility of appearance of separation phenomena on the contact surface under certain loading conditions. In order to obtain a convenient discrete form to simulate the structural behaviour of a steel column base plate, the continuous contact problem is first formulated as a variational inequality problem or, equivalently, as a quadratic programming problem. By applying an appropriate finite element scheme, the discrete problem is formulated as a quadratic optimization problem which expresses, from the standpoint of Mechanics, the principle of minimum potential energy of the semirigid connection at the state of equilibrium. For the numerical treatment of this problem, two effective and easy-to-use solution strategies based on quadratic optimization algorithms are proposed. This technique is illustrated by means of a numerical application.

Key words: structural steelwork, semirigid connections, column base plate, optimization

1. Introduction

Bolted steel column base plates are used in structural steelwork as connections to transfer internal forces of a steel column to its concrete pedestal. Although a rather large variety of cross section types is used, the base plate almost invariably sustains the orthogonal shape fixed to the concrete by symmetrically located bolts. This type of connection can indeed transfer a fair portion of the moment of the column to the concrete foundation. It is therefore, obvious that any improvement of the analysis methods for this type of bolted steel connections would lead to better design principles and more rational safety criteria as modern structural steelwork standards claim to offer. In this sense, the present study is a contribution to the research for the analysis and design of steel column base plate connections.

It is obvious that a column base plate under pure compression remains in perfect contact to the underlying concrete foundation. In this case, the compression forces can be assumed to follow a linear distribution and are perfectly transfered to the base through the contact interface. Should axial compression of the column not be the case, any arising tension is catered for by the fixing bolts. This rather simple assumption of linear interface forces distribution can not cover all possible loading conditions of a base plate. The general case is that end-plates are actually loaded by a combination of moment, shear and tension; these

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loads often cause separation on the contact surface between the deformable column base plate and the rigid concrete foundation. This way, the connection looses part of its bending rigidity behaving as a semirigid connection. Noteworthy is that this behaviour essentially depends on the moment-to-normal force proportion which is the dominant factor affecting the separation phenomena on the plate-concrete contact interface (Wald 1993). Extensive laboratory testing in combination to the study of a plethora of cases of collapse of steel structures certifies that bolted base plates may be partially separated from the foundation under certain loading conditions due to the unilateral nature of the contact. Such a semirigid behaviour of the connection often arises during seismic excitation of steel frames and, also, the moment-to-normal force ratio causing separation is highly dependent on the rigidity of the base plate (Astaneh, et al. 1992). Due to the complexity of the problem of accurately defining this reduction of the column base rigidity, this factor has been included only in a few standards, as are e.g. the Canadian standards and Eurocode 3, which dictate reduction of the critical column length according to the effective column base rigidity.

In the classical column base plate design, the plate is considered rigid and the bearing pressure distribution on the contact surface is assumed to be linear. Although such a simplifying assumption is considerably different from the actual base plate response, this method is widely used because it generally leads to results on the safe side. The column base plate analysis and design problem has already attracted the interest of numerous researchers who applied analytical, experimental and numerical approaches to investigate it. As an example we mention the analytical works where various normal reaction distributions on the column base plate contact surface in the presence of concentric (Fling 1970) and/or light loading (Stockwell 1975, Murray 1983) have been considered. Extensive experimental investigations aiming to the accurate description and evaluation of the response of column base connections have been conducted for both the cases of concentric (Dewolfe 1978) and eccentric loads (Dewolfe and Sansley 1983, Thambiratnam and Paramasivam 1986, Cook and Klinger 1992). On the other side, the numerical approaches based on the application of the finite element method have been recently applied to the analysis of column base plate connections (cf. e.g. Krishnamurthy 1978). Parametric three-dimensional finite element analysis of base plates (with respect to plate thickness and eccentricity of load) under axial loading and moment has been recently carried out (Thambiratnam and Krishnamurthy 1989), whereas nonlinear finite element analysis has also been performed (Rothert, et al. 1992). We can note here an extensive review on the results of all previous studies recently presented (Wald 1993). Results of considerable importance on the behaviour of semirigid connections in steel structures have been obtained by means of an appropriate modification of the classical finite element models in order to take into account the possibility of appearance of separation zones between the plate and the base. This phenomenon being of highly nonlinear nature, has to be numerically treated with utmost care when classical Structural Analysis methods are used, because it possesses certain traits of its own. The appropriate application of the methods of an actively developing branch of Mechanics called Nonsmooth Mechanics (cf. e.g. Panagiotopoulos 1985, Moreau, et al. 1988, Moreau and Panagiotopoulos 1988, 1993) appears very promising for the numerical simulation of the semirigid joint behaviour because it permits both the accurate theoretical and the exact numerical treatment of the local separation process on a steel base plate. Within such a theoretical framework, the separation problem of bolted end-plates has been already studied (Thomopoulos 1985, Abdalla and Stavroulakis 1989, Abdalla and Baniotopoulos 1991, 1992, Baniotopoulos, et al. 1992, Baniotopoulos and Abdalla 1993). Following these techniques, local noncontact zones between the base plate and the foundation are computed, whereas the loss of the rigidity of the arising semirigid connection is accurately evaluated. In particular, by introducing the classical unilateral contact law of Signorini (Fichera 1972) to simulate the contact conditions on the plate, the separation process is mathematically described in the continuous problem giving rise to a boundary value problem with respect to the displacements in variational inequality form. This mathematical technique expresses the principle of virtual work of the body at the state of equilibrium when the unilateral contact is included in the formulation. This formulation subsequently leads to the formation of a discrete quadratic programming problem (coupled by inequality constraints with respect to the displacements) that expresses the principle of minimum potential energy of the body at the state of equilibrium(Panagiotopoulos 1976, 1985). The transformation of the problem of guessing the contact zone into a constrained quadratic programming problem makes things much more manageable and allows us to use a wealth of theoretical work and software that is available from the mathematicians community. Therefore, the separation problem is numerically treated by means of any available quadratic optimization algorithm. It worths noting that a dual approach can also be employed giving rise to a similar variational inequality problem with respect to the stresses expressing in mathematical form, the principle of complementary virtual work of the connection at the state of equilibrium. Equivalently, this formulation leads to a quadratic programming formulation of the problem with respect to the stresses expressing the principle of complementary energy of the semirigid connection at the state of equilibrium (Panagiotopoulos 1976, 1985, Baniotopoulos, et al. 1992, Baniotopoulos and Abdalla 1993).

In this paper two effective and easy-to-use solution strategies based respectively on two quadratic optimization algorithms are employed. The first method is based on the Theil-van de Panne algorithm which equivalently transforms the initial quadratic optimization problem into a sequence of classical Structural Analysis problems which in turn are solved by means of any available finite element code. This method is by no means a trial-and-error method since it provides two criteria for the systematic search of the optimal (actual) solution of the problem, as well as a criterion for the final control of a seemingly correct solution (Künzi and Krelle 1962). This method has been recently applied to the solution of numerous Structural Analysis problems formulated as quadratic optimization problems (Panagiotopoulos and Talaslidis 1980, Talaslidis and Panagiotopoulos 1982, Panagiotopoulos, et al. 1984). The second method is the Hildreth-d'Esopo algorithm that constitutes a typical iterative procedure being more easily programmable and computationally efficient, thus permitting the effective numerical treatment of problems with a large number of unknowns (fine finite element me shes) (Abdalla and Stavroulakis 1989, Abdalla and Baniotopoulos 1992).

2. A variational inequality and a quadratic programming formulation

2.1. The continuous problem

Let us consider an orthogonal cartesian system $0x_1x_2x_3$ and an elastic body Ω with boundary Γ made up of three nonoverlapping parts Γ_U, Γ_F and Γ_S. On Γ_U (respectively Γ_F) the displacements (respectively the surface forces) assume prescribed values U_i (respectively F_i), whereas on the part Γ_S, unilateral contact boundary conditions hold (Fig.1). Assuming that the unilateral contact is frictionless and defining as positive normal direction, the one directed

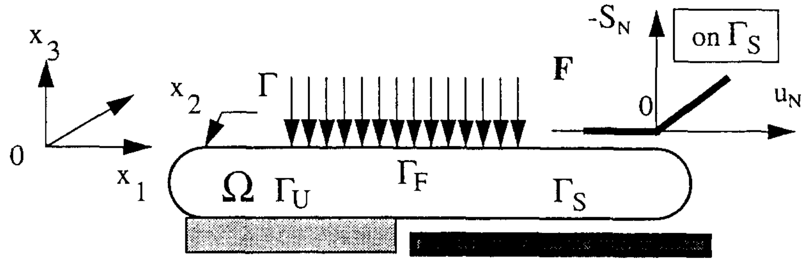

Fig. 1 On the continuous contact problem.

outwards of the boundary, the unilateral contact boundary conditions with respect to an elastic support are expressed in the following form

$$\text{if } u_N < 0 \quad \text{then} \quad S_N = 0 \tag{1}$$

$$\text{if } u_N \geq 0 \quad \text{then} \quad S_N + k(u_N) = 0 \tag{2}$$

where u_N (respectively S_N) denotes the part of the displacements normal to the boundary (respectively reaction forces) on Γ_S and $k(u_N)$ is a nondecreasing function. Relation (1) expresses that if the body looses contact from the support, then normal reaction is equal to zero, whereas if the body remains in contact with the support, then, a normal reaction does exist. The previous relations are schematically represented by means of the graph (u_N, $-S_N$) (Fig. 1).

Under the assumption of infinitesimal strains and displacements, the boundary value problem at hand consists of the equation of equilibrium, the compatibility relations, the constitutive relations relating stresses to strains and the boundary conditions holding on the boundary Γ_S. A field X^* of strains and displacements (ε_{ij}^* and u_i^* respectively) is kinematically admissible, if it satisfies the compatibility relations, the kinematical boundary conditions on Γ_U and the kinematical conditions imposed by the boundary conditions on Γ_S. Volume forces are denoted by p_i, whereas actual strains and displacements at the position of equilibrium by ε_{ij} and u_i respectively. Differences ($\varepsilon_{ij}^* - \varepsilon_{ij}$) and ($u_i^* - u_i$) represent kinematically admissible variations of the respective variables. Moreover, σ_{ij}^* denotes the stress field obtained from ε_{ij}^* by means of the elasticity law. By splitting now u_N^* into its positive and negative parts (u_{N+}^* and u_{N-}^* respectively) defined by the formulas

$$u_{N+}^* = \frac{u_N^* + |u_N^*|}{2} \tag{3}$$

and

$$u_{N-}^* = \frac{-u_N^* + |u_N^*|}{2} \tag{4}$$

which are nonnegative quantities, the principle of virtual work for the free body Ω (requirement that virtual work of internal forces be equal to the virtual work of external forces)

$$\int_\Omega \sigma_{ij}^* (\varepsilon_{ij}^* - \varepsilon_{ij}) \, d\Omega = \int_\Omega p_i (u_i^* - u_i) \, d\Omega + \int_{\Gamma S} S_{N_i} (u_{N_i}^* - u_{N_i}) \, d\Gamma + \int_{\Gamma_F} F_i (u_i^* - u_i) \, d\Gamma \;\; \forall u_i^* \in X^* \tag{5}$$

combined with the inequality describing the virtual work of the unilateral contact reactions

$$\int_{\Gamma_S} (S_{N_i}(u^*_{N_i} - u_{N_i}) + k(u_{N+})(u^*_{N+} - u_{N+}))\, d\Gamma \geq 0 \quad \forall u^*_{N_i} \in X^* \tag{6}$$

holding on Γ_S, the principle of virtual work for the body at the state of equilibrium is written in the following variational inequality form (for details, see Panagiotopoulos 1976)

$$\int_{\Omega} \sigma^*_{ij}(\varepsilon^*_{ij} - \varepsilon_{ij})\, d\Omega - \int_{\Omega} p_i(u^*_i - u_i)\, d\Omega + \int_{\Gamma_S} k(u_{N+})(u^*_{N+} - u_{N+}) d\Gamma - \int_{\Gamma_F} F_i(u^*_i - u_i)\, d\Gamma \geq 0 \quad \forall u^*_i \in X^* \tag{7}$$

Applying the method of special variations, it is proven that variational inequality (7) yields the equation of equilibrium and the boundary conditions on Γ_S and Γ_F and thus, it completely characterizes the position of equilibrium of the body Ω. From the standpoint of Mechanics, variational inequality (7) expresses the principle of virtual work in an inequality form for the body under consideration. It has been also proven that any solution of the variational inequality problem (7) minimizes the potential energy of the body over the set X^* of possible displacement configurations at the position of equilibrium

$$\Pi = \frac{1}{2}\int_{\Omega} \sigma_{ij}\varepsilon_{ij} d\Omega - \int_{\Omega} p_i u_i d\Omega + \int_{\Gamma_S} K(u_{N+}) d\Gamma - \int_{\Gamma_F} F_i u_i d\Gamma \tag{8}$$

where $K(.)$ is a convex function (due to the monotonicity of $k(.)$) defined by the following integral

$$K(\xi) = \int_{0}^{\xi} k(\xi) d\xi \tag{9}$$

Conversely, it has been proven that any solution of the problem of the minimization of (8) satisfies the variational inequality problem (7). It is also noteworthy that a dual approach with respect to stresses can also be equivalently applied. In this case, the formulated variational inequality problem expresses, from the standpoint of Mechanics, the principle of complementary virtual work. The latter gives rise to a minimization problem of the complementary energy of the body Ω (cf. e.g. Panagiotopoulos 1976, 1985).

2.2. The discrete problem

A typical bolted steel column base connection formed by a base plate fixed by bolts on the base is first considered (Fig. 2). In this connection, bolts are symmetrically located in lines parallel to the column sides. Due to the symmetry of this connection, bolts are equally stressed in each direction under pure eccentric normal loading, whereas under moment loading, additional forces develop on the plate tips that depend on the stiffness of the base plate. These prying forces developed on the plate reduce the contact pressure on certain zones between the plate and the base (Abdalla and Stavroulakis 1989). The development of the latter phenomenon greatly affects the overall structural response of the connection giving rise to a semirigid behaviour. It is therefore, obvious that in order to better estimate the column base connection behaviour, the separation zones have to be with accuracy determined by taking into account that on the contact zones, normal reactions do appear, whereas on the separation

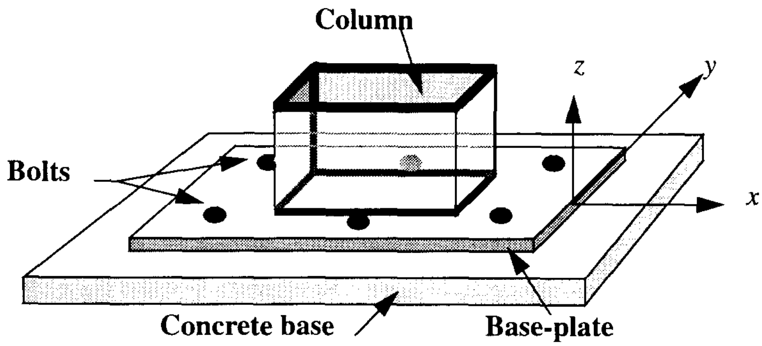

Fig. 2 A typical semirigid bolted column base plate connection.

zones, normal reactions are equal to zero. It must be also noted that separation and contact zones between the plate and the base are not *a priori* known.

In the present paper, a method for the numerical simulation of the semirigid behaviour of the connection under consideration is presented. This connection is discretized by means of an appropriately chosen finite element scheme. In particular, plate elements are used to simulate the behaviour of the base plate, whereas local separation conditions holding on the contact surface are realized by means of one-dimensional elastic couplers (fictitious springs -contact elements) of infinitesimal length which connect the adjacent nodes of the discretized contact fronts. The mechanical behaviour of these couplers simulating the possibility of partial separation of the plate and the base interface can be mathematically described for the i-th contact element by means of the Signorini-Fichera contact law (compare to relations (1), (2))

$$\text{if } u_z(i) > 0 \quad \text{then} \quad R_z(i) = 0 \tag{10}$$

$$\text{if } u_z(i) = 0 \quad \text{then} \quad R_z(i) \geq 0 \tag{11}$$

where $u_z(i)$ denotes the displacements along the normal (with respect to the plate) z-axis and $R_z(i)$ the respective reaction force. By means of relation (11) it is stated that if the region between the column plate and the base connected by the i-th coupler are in contact, then reaction force do exists, whereas relation (10) expresses that if separation takes place, reaction is equal to zero. It is also assumed that the response of the steel connection is not affected by any friction effects (frictionless type of contact).

Assembling relations (10),(11) in matrix terms for all the n imposed couplers, the following linear complementarity problem is formulated

$$\mathbf{u}_z \geq 0 \qquad \mathbf{R}_z \geq 0 \tag{12}$$

$$\mathbf{R}_z^T\mathbf{u}_z = 0 \tag{13}$$

where boldface letters denote vectors and matrices. Superscript ^{T} denotes transposed vectors or matrices. Linear complementarity problem (12),(13) completely describes mathematically the quasi-static local separation phenomenon along the steel column base plate.

Taking into consideration the experience of experiment data which certify that deformations are small on the base plate, the stiffness method is applied to the simulation of the structural response of the discretized model. In this way, the following matrix equation is obtained

$$Ku=P \tag{14}$$

where K is the stiffness matrix of the connection, u the displacement vector including vector u_z and P the load vector that includes both normal force and moments acting on the connection.

Relations (12)-(14) give rise to the following quadratic optimization problem

$$\Pi(u)=min\{ \frac{1}{2} u^T Ku - P^T u \mid Au \leq b \} \tag{15}$$

where A is an appropriately defined transformation matrix and b a vector which defines the inequality restrictions imposed by the inequalities(12), (13). Quadratic programming problem (15) expresses, from a mechanical point of view, the principle of minimum potential energy of the steel semirigid connection at the state of equilibrium. The actual displacements of the base plate, as well as the active contact and the separation zones between the base plate and the rigid foundation can be defined by solving problem (15). As has been previously noted for the continuous problem, a dual approach can also be employed for the formulation and the numerical treatment of the discretized problem. Such a dual approach equivalently gives rise to a quadratic programming problem of the same type where stresses are now the unknown variables appearing in the quadratic term (instead of the displacements) and the constraints consist of the equilibrium equation and the reaction forces appearing on the connection, i.e.

$$\Pi^C(s)=min \{ \frac{1}{2} s^T F_o s - s^T e_o \mid R_z \geq 0,\ Gs=P \} \tag{16}$$

where G (respectively F_o) is the equilibrium (respectively flexibility) matrix of the steel connection and s (respectively e_o) is the stress (respectively initial strain) vector. The constrained quadratic programming problem (16) expresses, from the standpoint of Mechanics, the principle of minimum complementary energy for the steel connection at hand at the state of equilibrium.

3. On the solution strategies

In the next paragraphs two methods from the theory of Optimization are applied for the numerical treatment of the discretized problems (15) and (16). The first method is the Theil-van de Panne method which is based on three theorems from the theory of Quadratic Optimization (Künzi and Krelle1962, Panagiotopoulos and Talaslidis 1980, Talaslidis and Panagiotopoulos 1982, Panagiotopoulos, et al. 1984). The method transforms the initial quadratic optimization problem into a sequence of classical Structural Analysis problems which can be solved by any available finite element code. The method provides two criteria to find the direction towards the actual solution of the problem and a final optimality test criterion for any seemingly correct solution.

The second method for the numerical treatment of the problem is the Hildreth-d' Esopo algorithm, a classical iterative procedure with the great advantage of being easily programmable and computationally efficient .

3.1. The Theil-van de Panne method

A solution strategy to the search for the solution of problem (15) will be sketched in the sequel. By means of the term "active contact zone" or "active contact region" of the base plate, we mean a set of couplers where displacements along the z-axis are equal to zero. The term "nonactive contact zone" or "separation zone" is referred to those couplers where displacements are greater than zero. The method is based on three theorems proved in the theory of Optimization which, translated into the language of Mechanics, give rise to the following three propositions:

Proposition 1. If the solution $\boldsymbol{U}^o$ of the unrestricted quadratic programming problem violates some of the inequality constraints imposed by the unilateral contact boundary conditions, then the actual (optimal) solution of the problem $\hat{\boldsymbol{U}}$ has to satisfy at least one of the violated constraints in equality form.

Proposition 2. If a solution $\boldsymbol{U}^V$ of an intermediate problem violates certain of the inequality constraints imposed by the unilateral contact boundary conditions, then, at least one of the violated constraints must be satisfied in equality form in order to obtain the actual (optimal) solution of the problem $\hat{\boldsymbol{U}}$.

Proposition 3. A solution U^V which satisfies all the inequality restrictions of the problem imposed by the unilateral contact boundary conditions, coincides with the actual (optimal) solution of the initial quadratic optimization problem $\hat{\boldsymbol{U}}$, if and only if, omitting one by one the restrictions of the problem holding as equalities and solving in turn each one of the resulting problems, the respective solution is found to violate the unilateral contact restriction at the same contact element.

The solution method is based on the application of the previous Propositions. The procedure begins by calculating the solution of the unrestricted quadratic programming problem. If this solution $\boldsymbol{U}^o$ violates certain inequality restrictions imposed by the unilateral contact boundary conditions, then at least one of these inequality restrictions must be satisfied in equality form in the actual (optimal) solution $\hat{\boldsymbol{U}}$ of the problem (Proposition 1). In the sequel, if the solution of problem(15) violates certain inequality constraints, then, in order to approximate the actual solution, we must consider, in addition to the equality constraints already holding, at least one more of the constraints violated by the intermediate solution $\boldsymbol{U}^V$, to hold as equality (Proposition 2). An intermediate solution $\boldsymbol{U}^V$ which does not violate any unilateral contact restriction is the actual solution of the problem, if and only if, omitting one by one the previously imposed equalities and solving the problem, the respective solutions are found to violate the inequality restrictiction at the very coupler where the equality constraint was omitted (Proposition 3). The employment of these propositions leads to the solution of the initial quadratic problem by solving iteratively a sequence of classical (bilateral) Structural Analysis problems: combining the three propositions appropriately, the quadratic optimization problem is linearized in the sense that within each step a bilateral Structural Analysis problem is solved and this is done by means of any available finite element code. Iterative steps are not arbitrarily chosen, but they are controlled by the criterion of Proposition 2. The final optimality criterion defined by Proposition 3 checks whether a seemingly correct solution is the actual one or not. It is worth noting that since there exists systematic choice for the active constraints set and an optimality criterion, the method can by no means be characterized as a trial-and-error method, although it possesses the characteristic of repeated attempts with a varying constraint set.

3.2. The Hildreth-d'Esopo method

The application of the Hildreth-d'Esopo algorithm is in the sequel applied to the solution of the quadratic optimization problem (15). The well-known Kuhn-Tucker optimality conditions for problem (15) are written in the following form

$$\boldsymbol{Au}+\boldsymbol{y}=\boldsymbol{b} \tag{17}$$

$$\boldsymbol{Ku}+\boldsymbol{A}^T\boldsymbol{f}=\boldsymbol{P} \tag{18}$$

$$\boldsymbol{y}\geq \boldsymbol{0},\quad \boldsymbol{f}\geq \boldsymbol{0},\quad \boldsymbol{y}^T\boldsymbol{f}=0, \tag{19}$$

where $\boldsymbol{y}$ is a vector corresponding to the unilateral constraints of the problem and $\boldsymbol{f}$ the vector of reactions on the same constraints. Solving equation (18) with respect to $\boldsymbol{u}$, the following relation is obtained

$$\boldsymbol{u}=-\boldsymbol{K}^{-1}(\boldsymbol{A}^T\boldsymbol{f}-\boldsymbol{P}), \tag{20}$$

and then, putting

$$\boldsymbol{h}=-\boldsymbol{AK}^{-1}\boldsymbol{P}+\boldsymbol{b} \tag{21}$$

and

$$\boldsymbol{F}=\frac{1}{2}\boldsymbol{AK}^{-1}\boldsymbol{A}^T, \tag{22}$$

relations (17)-(19) are written as follows

$$2\boldsymbol{Ff}-y=-h \tag{23}$$

$$\boldsymbol{y}\geq \boldsymbol{0},\quad \boldsymbol{f}\geq \boldsymbol{0},\quad \boldsymbol{y}^T\boldsymbol{f}=0. \tag{24}$$

The latter relations constitute the Kuhn-Tucker optimality conditions for the following problem

$$\mathit{II}(\boldsymbol{l})=min\{\frac{1}{2}\boldsymbol{f}^T\boldsymbol{Ff}+\boldsymbol{h}^T\boldsymbol{f}\mid \boldsymbol{f}\geq \boldsymbol{0}\} \tag{25}$$

where matrix $\boldsymbol{F}$ is a flexibility matrix defined by equation (22) relating contact forces to the corresponding unilateral contact displacements. When the solution of problem (15) exists, then problem (25) does also have a solution and this is unique. Problem (25) can be numerically treated by means of the Gauss-Seidel method. During the iterative steps $p=0, 1, 2, \ldots$, the following iterative values are considered

$$l_i^{p-1}=max\,\{0,\ \omega_i^{p+i}\} \tag{26}$$

where

$$\omega_i^{p+i}=-\frac{1}{g_{ii}}\left(\sum_{j=1}^{i-1} g_{ij}l_j^{p+1}+\frac{h_i}{2}+\sum_{j=j+1}^{m} g_{ij}l_j^p\right)\quad for\ i=1,\ 2,\ \ldots\mathrm{m} \tag{27}$$

and m is the number of the constraints of the problem. Iterations stop when the computed contact reactions in two successive solution steps pass the imposed accuracy criteria, i.e. when

$$\|\, l_j^p-l_j^{p+1}\| \leq \varepsilon$$

where the symbol $|| \cdot ||$ denotes an appropriately defined norm.

4. Numerical application

The bolted column base plate connection of Fig.3 is considered. Due to the double symmetry of the connection, the discretized model can be limited to the one-fourth of the column base plate (Fig.4, shadedregion). For the numerical simulation of the column base plate response,

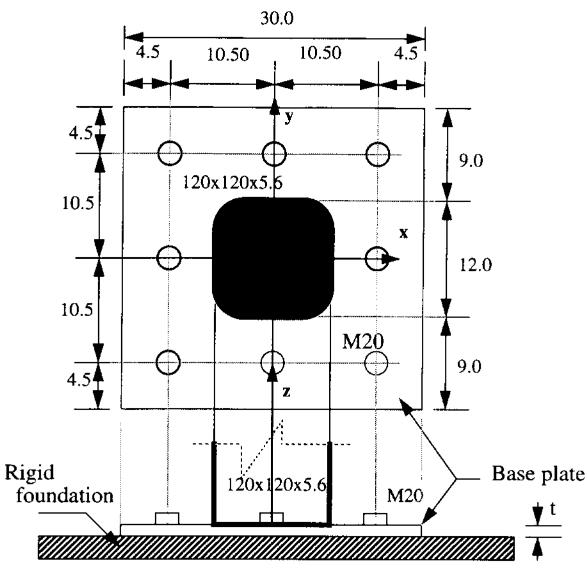

Fig. 3 Data of the numerical example.

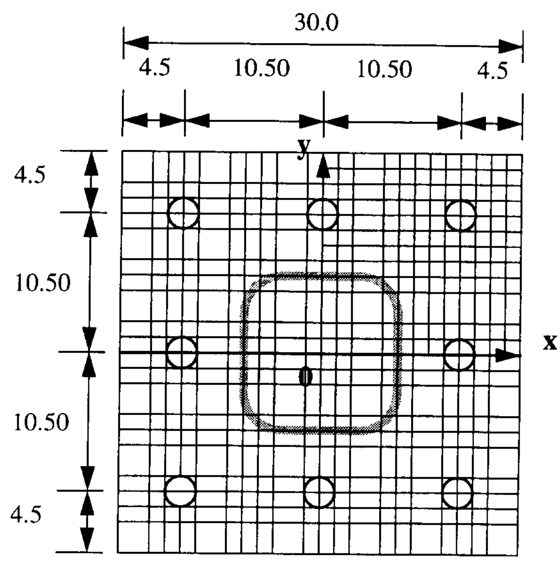

Fig. 4 The discretized model.

the plate element #6 from the library of the SAP IV-B program with six degrees of freedom has been employed. The degrees of freedom corresponding to the in-plane displacement and the rotation around the z-axis are taken fixed. This way, nodal displacements along the z-axis and rotations around x- and y-axis do appear. Contact elements obeying to the unilateral contact law (10), (11) are used to simulate the possibility of partial separation between the steel plate and the rigid support. In order to improve the accuracy of the numerical solution, the mesh has to be dense enough around the neighbourhood of the bolts and the column tips (Fig.4), whereas aspect ratio of the mesh must, in all cases, be between 0.5 and 2.

Taking the contact of the plate with the foundation to be frictionless, the nodes of the lower surface of the plate elements are connected to the rigid base by means of contact elements of length tending to zero. Those elements that correspond to the regions where bolts are located, exhibit the elastic characteristics of steel, whereas the elements that simulate the unilateral contact have 30 times greater flexibility. The lower part of the steel column (120 x120x5.6 mm) is subjected to a moment loading in y-axis transferred into the discretized base plate model composed by 169 plate elements and 196 contact elements. Although it is redundant, it is noted that the formulation of the method permits taken into account axial and shear column forces in a straightforward way. The solution procedure starts by assembling

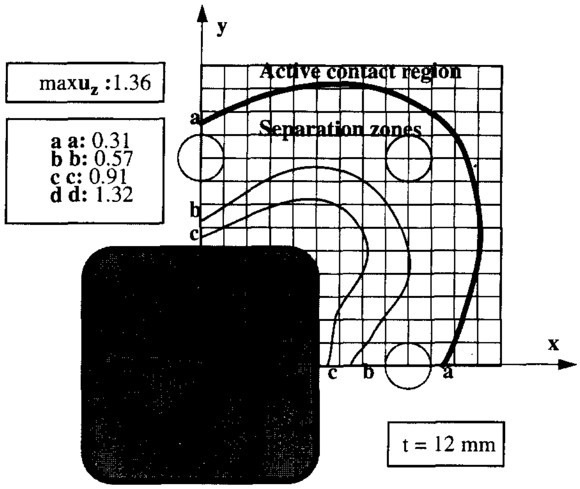

Fig. 5 Deformation isolines and output data.

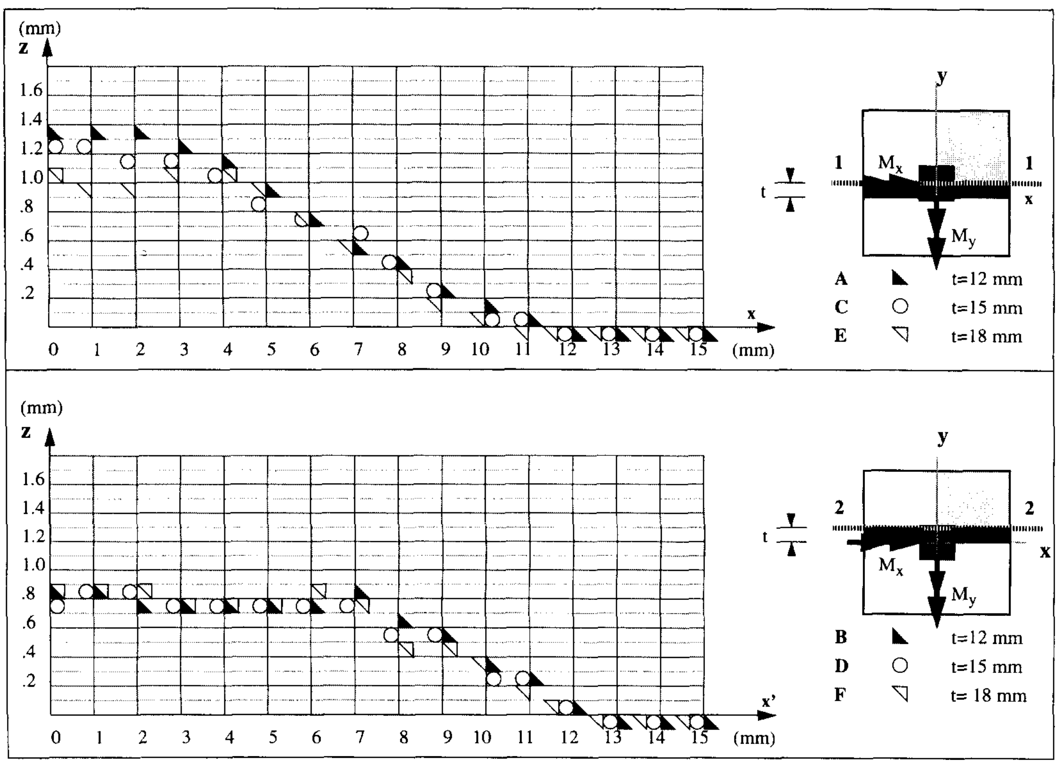

Fig. 6 Deformation graphs :
Plate thickness t=12 mm (A) Section 1-1 (B) Section 2-2.
Plate thickness t=15 mm (C) Section 1-1 (D) Section 2-2.
Plate thickness t=18 mm (E) Section 1-1 (F) Section 2-2.

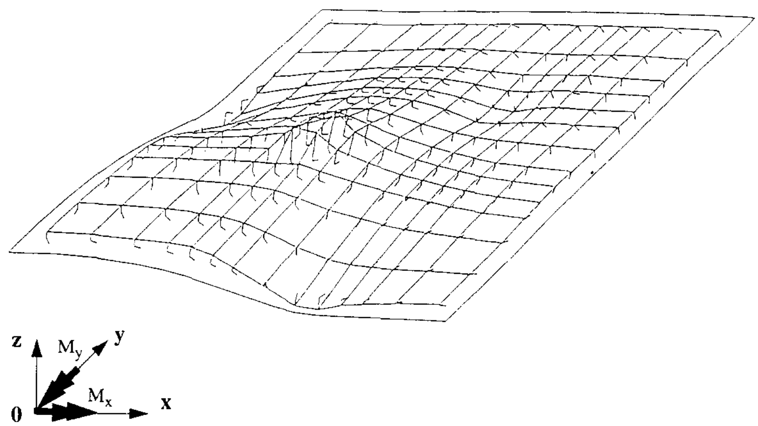

Fig. 7 Deformed shape of the column base plate for thickness t = 12 mm

the stiffness matrix of the discretized steel connection. Then, the quadratic programming problem is formulated having as subsidiary conditions the nonnegativity of the z-displacements of the base plate. Applying the Theil-van de Panne method, the contact boundary conditions are iteratively imposed as equality boundary conditions of the problem and the arising problems are successively solved. Each resulting solution is checked for possible violation of the inequality constraints. The procedure stops when no inequality restrictions are violated by the results of an intermediate problem. This seemingly correct solution is controlled by means of the optimality criterion of proposition 3: the imposed equality subsidiary conditions are relaxed one-by-one and the results of the respective solutions are checked for possible violation of the inequality constraint on the very contact element that has been relaxed. By means of this method, the solution of the problem is obtained within six steps. For a base plate thickness of 12 mm, the isolines of the z-displacements of the column base plate are depicted in Fig. 5, whereas the plate deformations along sections 1-1 and 2-2 are shown in Fig. 6 (cases A and B). Note that all these diagrams illustrate the vertical displacements of the base plate where the positive direction is upwards. The deformed base plate configuration is depicted in Fig. 7. Similar results have been obtained for the base plate for a thickness of 15 mm and 18 mm which are presented in the diagrams C, D and E, F in Fig.6 respectively.

The numerical results calculated by the application of the present method qualitatively conform quite well to those obtained by experiment. The analysed base plate exhibits a behaviour corresponding to the so-called flexible base plate behaviour which has recently been experimentally investigated. In particular, the numerical results of the present method show similar behaviour patterns to those obtained experimentally for flexible base plates: vertical displacements along the centerline of flexible base plates take in both cases their maximum values (of the same class of magnitude) on the center of symmetry, whereas active contact (i.e. compressive) zones appearing near the edge of the plate is circa 20% of the plate half length (Cook and Klinger 1992, p. 1656). However, more systematic research using exactly the same types of connections and loading has to be carried out in the future in order to accurately calibrate the proposed model. The inherently missing datum of the numerical prediction of contact zones is the fact that the concrete surface where the base plate rests, is never absolutely level. Small asperities or bumps (of the order of 10^{-4}-10^{-5} m) are of the order of magnitude of the z-displacements differential, thus producing significant alteration in both the internal forces of the plate and the contact zone. As this aspect seems entirely random in nature, a stochastic study of it might be futile for practical reasons but, it is going to constitute a permanent difficulty for experimentalists and a source for discrepancies between simulation and experiment.

5. Conclusions

The presented study aims to contribute to the investigation of the structural response of the semirigid column base plate connections by combining Finite Element and Nonsmooth Mechanics methods. Within this framework, the not *a priori* known zones of active contact and separation can be calculated with accuracy. The proposed solution strategies can effectively treat the resulting quadratic programming problem. Those are based on theTheil-van de Panne and the Hildreth-d'Esopo method. The first one has the advantage that it transforms the initial quadratic programming problem into a sequence of classical Structural Analysis problems solved by any available finite element code. The second one is easily programmable and makes the process fully automatic.

The proposed method determines the semirigid behaviour of the base connection easily and takes into account the highly nonlinear separation phenomena on the contact surface which can otherwise empirically be incorporated into the structural system. This way, a phenomenological constitutive law describing the semirigid behaviour of the base connection can be proposed in order to be used in computer programs that deal with structural analysis of steelwork. Therefore, the presented analysis method has to be considered as a contribution to the search for both the optimal design of semirigid column base plate connections and the development of more cost effective and safe computer programs for metal structural analysis.

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Notation

Ω : Elastic body
Γ_U : part of the boundary Γ where displacements have a prescribed value U_i
Γ_F : A part of the boundary Γ where forces have a prescribed value F_i
Γ_S : A part of the boundary Γ where unilateral contact conditions hold
ε_{ij} : Strain tensor
u_N : Displacements normal to the boundary Γ_S
u_{N+} : Positive part of u_N

u_{N-} : Negative part of u_N
S_N : Reaction normal to Γ_S
X^* : Set of kinematically admissible displacements and strains
$\boldsymbol{u}$: Displacement vector including vertical displacements
$\boldsymbol{K}$: Stiffness matrix
$\boldsymbol{P}$: Load vector including axial and shear forces and moment
$\boldsymbol{F}$: Flexibility matrix
^{T} : Superscript denoting transposed vector and matrices
U^o : Solution of the unrestricted problem
U^V : Intermediate solution of the problem
$\hat{U}$: Actual (optimal) solution of the problem

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