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Motions of rigid unsymmetric bodies and coefficient of friction by earthquake excitations

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Abstract. Motions of an unsymmetric rigid body on a rigid floor subjected to earthquake excitations with special attention to coefficient of friction are investigated.

Motions of a body in a plane are classified (Ishiyama 1980) into six types, i.e. (1) rest, (2) slide, (3) rotation, (4) slide rotation, (5) translation jump, (6) rotation jump. Based upon the theoretical and experimental research work special attention is paid to the sliding of a body. The equations of motions and the behaviour of coefficient of friction in the time of floor excitation are studied. One of the features of this investigation is the introduction and estimation of the "time dependent" coefficient of friction. It has been established that the constant kinetic coefficient of friction $\mu(\text{kin}) \approx 0.8\mu(\text{stat})$ does not give the appropriate results. The method for the estimation of the friction coefficient variation during the time is given.

Key words: rigid unsymmetric body; earthquake; motion; slide; time-dependent coefficient of friction.

1. Introduction

The behaviour of a rigid body in the case of a strong ground motion has been the matter of study of a number of seismologists and earthquake engineers for over a hundred years. The first researchers who studied this problem were Milne and Perry in the year 1881. Ishiyama (1980, 1982) gave a complete overview of the methods describing the motion of a symmetric rigid body on rigid floor. He introduced the normal and tangent restitution coefficient in the analysis. These coefficients are dependent on different impacts between the body and the floor. These impacts appear only at the motions related to rotation and jumps.

In this paper, motions of a rigid unsymmetric body in response to earthquake excitations are theoretically and experimentally studied, extending and modifying the above-mentioned methods. The motions are classified into six types, i.e. rest, slide, slide rotation, translation jump and rotation jump, so that any motion of a body in a plane can be described. The non-linear equations of different types of motions have been solved numerically. All numerical results have been tested by experimental work (Zadnik 1991). The research is focused on the slide and slide rotation considering the effects of coefficient of friction between the body and the floor. One of the features of this investigation is the introduction of the time dependent coefficient of friction enabling us to estimate the magnitude of the horizontal body displacement.

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2. Motions of an unsymmetric body

The methodology and the differential equations describing a motion of a **symmetric body** were studied by other researchers (Ishiyama 1980). In this section the generalization of the methodology and the equations of the motion of an **unsymmetric body** exposed to the earthquake excitation are introduced originally.

2.1. Classification of motions of a body

When a body on a floor is subjected to earthquake excitations, it may be at rest while the excitations are not strong. When the excitations become strong enough, the body may rock, slide, jump or a combination of these motions may start. In this paper, the motions are classified into six types, i.e. (1) rest, (2) slide, (3) rotation, (4) slide rotation, (5) translation jump, (6) rotation jump; so that any motion of a body in a plane can be presented(see Fig.1).

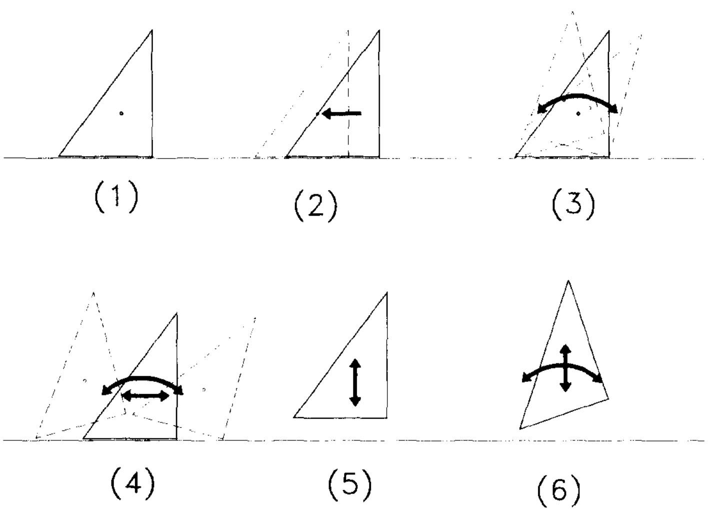

Fig. 1 Classifications of body motions: (1) rest; (2) slide; (3) rotation; (4) slide rotation; (5) translation jump; (6) rotation jump.

It should be noted that, throughout this paper, only plane motions are considered and the following assumptions are used:

- the motion of the body is always in the x-y plane,
- the body and the floor are rigid,
- the body is unsymmetric,
- the floor is always horizontal,
- floor excitations are in the horizontal (X'') and/or vertical (Y'') direction,
- the time interval of the observation is very short. The body occupies the same position during that time interval.

2.2. Equations of motion for each type of motion

The equations of motion are derived for each type of motion.

The equations for **slide:**

$$x'' = -X'' - S\mu(g + Y'') \tag{1}$$

where x describes the horizontal displacement of the centre of gravity of a body relative to the floor; X'' and Y'' are the absolute horizontal and vertical accelerations of the floor, respectively; $S = sgn(x^{\circ\prime})$ if $x^{\circ\prime} \neq 0$ or $S = sgn(-X'')$ if $x^{\circ\prime} = 0$; g is the gravitational acceleration; μ is the surface kinetic coefficient of friction between the base of the body and the floor; the superscript $^\circ$ denotes the values of variables at the contact between the body and the floor; and $'$ and $''$ represent the first and the second derivative with respect to time, respectively.

$$y'' = y' = y = 0 \tag{2}$$

$$\theta'' = \theta' = \theta = 0 \tag{3}$$

where y denotes the vertical displacement of the centre of the body relative to the floor, and θ the rotation angle of the body.

The horizontal and vertical reaction forces from the floor per unit of mass are:

$$Rx = X'' + x'' \tag{4}$$

$$Ry = g + Y'' \tag{5}$$

It has to be stated that the governing equation of sliding (1) is not influenced by the variables describing the geometry of the body.

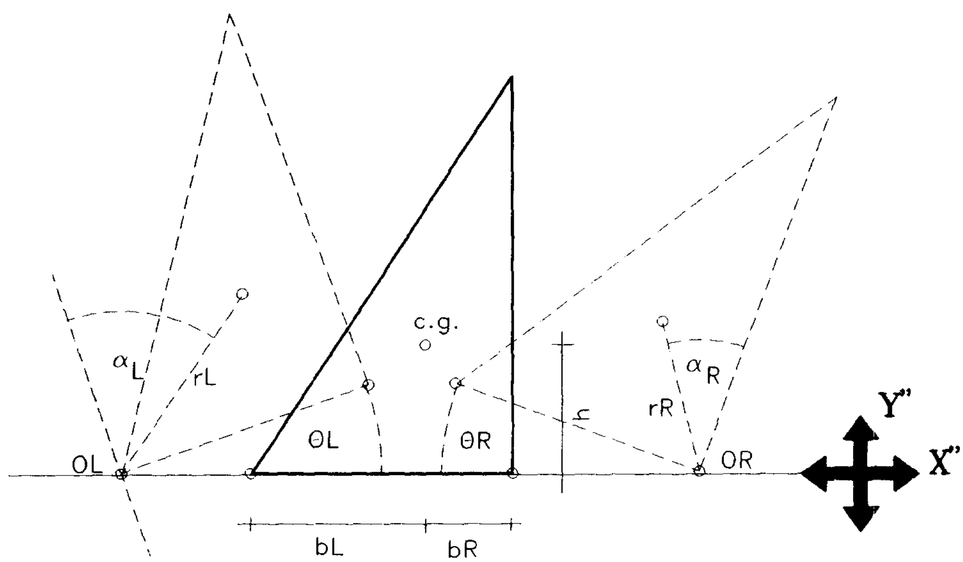

Fig. 2 An unsymmetric body in motion

The equations for **rotation**:

The situation at rotation is different. The values of variables which define the motion additionally depend on the geometry of the body, i.e. on the edge of the body which is at the moment the centre of rotation of the body (Left or Right edge, $j = L$ or R).

Accelerations of the centre of gravity of the body in horizontal and vertical directions are expressed:

$$x''_j = -S' r_j \theta_j'' \cos(\alpha_j - |\theta_j|) - r_j \theta_j'^2 \sin(\alpha_j - |\theta_j|) \quad (6)$$

$$y''_j = -S' r_j \theta_j'' \sin(\alpha_j - |\theta_j|) - r_j \theta_j'^2 \cos(\alpha_j - |\theta_j|) \quad (7)$$

Angular acceleration at rotation:

$$\theta''_j = \frac{r_j}{i^2 + r_j^2} (X'' \cos(\alpha_j - |\theta_j|) - (g + Y'') \sin(\alpha_j - |\theta_j|)) \quad (8)$$

The equations for **slide rotation** (Fig.2) are a linear combination of equations for sliding and rotation:

$$x'' = -X'' - S\mu(g + Y'' + y'') \quad (9)$$

$$x'' = -X'' - S\mu\,(g + Y'' - \theta_j'^2 r_j \cos(\alpha_j - |\theta_j|) + S'\theta''_j r_j \sin(\alpha_j - |\theta_j|)) \quad (10)$$

Angular acceleration of the body is given for $j = L, R$ (Left or Right edge) as:

$$\theta_j'' = \frac{r_j}{i^2} ((g + Y'' - \theta_j'^2 r_j \cos(\alpha_j - |\theta_j|) + S'\theta_j'' r_j \sin(\alpha_j - |\theta_j|))^* \\ {}^*(S\mu \cos(\alpha_j - |\theta_j|) + S' \sin(\alpha_j - |\theta_j|)) \quad (11)$$

where r_j is the distance between the pole of rotation and the centre of gravity of the body, i is the radius of gyration of the body about the centre of gravity, α_j is the angle between the vertical line and the line connecting the base edge and the centre of gravity of the body at rest. $S = sgn(x^{\circ\prime})$ if $x^{\circ\prime} = 0$, $S = sgn(-X'')$ if $x^{\circ\prime} = 0$ and $S' = sgn(\theta_j)$ if $\theta_j \neq 0$, $S' = sgn(\theta_j')$ if $\theta_j = 0$ and $\theta_j' \neq 0$ or $S' = sgn(X'')$ if $\theta_j = \theta_j' = 0$.

The above equations describe some types of motion of unsymmetric body relative to the floor. All parameters, i.e. accelerations, velocities and displacements of the centre of gravity of the body in horizontal and vertical directions and rotation angles are obtained through numerical solution of these differential equations.

2.3. Transition of motion among different types of motions

In the previous section only three types of motions through differential equations were presented. In general, all six types of motions are possible when the floor is forced to move by earthquake floor excitation. A body starts to move from rest (1) and then undergoes one of the six types of motion, depending on some fulfilled conditions. Due to the earthquake excitations which vary with the time, the body is also exposed to changes in motion. The appearance of these changes and the transition among different types of motions are investigated. The conditions for transition of motion from slide (2) to other types of motion are shown in Table 1.

3. Coefficient of friction

With every sliding of rigid body on rigid base the friction force, acting in the opposite direction of motion, will appear in the contact surface. The appearance of this force depends on the external load acting on the body and, of course, on the micro relief and material characteristics of the contact surface. Humidity, temperature and adhesion forces, acting in the contact of two surfaces, also have an impact. All these influences can be indirectly de-

Table 1 Transition of motion from slide

Conditions for next motion				Next motion
$x'=0$	$h \mid Rx \mid \leq b_j \mid Ry \mid$	$Ry > 0$		(1) rest
$x' \neq 0$	$h \mid Rx \mid \leq b_j \mid Ry \mid$	$Ry > 0$	$\mu \leq \frac{\mid Rx \mid}{\mid Ry \mid}$	(2) slide
$x'=0$	$h \mid Rx \mid > b_j \mid Ry \mid$	$Ry > 0$	$\mu > \frac{\mid Rx \mid}{\mid Ry \mid}$	(3) rotation
$x' \neq 0$	$h \mid Rx \mid > b_j \mid Ry \mid$	$Ry > 0$	$\mu > \frac{\mid Rx \mid}{\mid Ry \mid}$	(4) slide rotation
		$Ry \leq 0$		(5) translation jump
		$Ry \leq 0$		(6) rotation jump

scribed by the coefficient of friction. The static coefficient of friction μ_S and the kinetic coefficient of friction μ_k are determined by conventional observation of stability conditions in the contact surface. Their interdependence is the following: $0 < \mu_k < \mu_S$.

From the basic equations, describing individual body motions and from the equations, stating the conditions of transition among different kinds of motions (Table 1), it is evident that the parameter μ is an important independent variable which represents the coefficient of friction. Determination of this coefficient is always experimental and it is performed according to the usual and well-known procedures (Beer 1977, Hencher 1981). It is defined as the quotient of the friction force *Ft* and the gravitational force *Fg*, acting normally onto the base and which are experimentally determined:

$$\mu = \frac{Ft}{Fg} \tag{12}$$

Such definition and calculation of the coefficient of friction is sufficient for static approach. The coefficient is called the static coefficient of friction (μ_S). In the case of rapid sliding of body in the direction which is determined by the external load, there comes to a momentary fall of friction coefficient value, and a new value, called the kinetic (dynamic) friction coefficient (μ_k), is obtained. μ_k is also defined by the equation (10). Its order of magnitude is within the range of 75% to 80% of μ_S. The above-mentioned relations are well-known, and have been tested and confirmed many times (Beer 1977, Ishiyama 1980). It has to be emphasised that the coefficients of friction, determined in this way, were obtained by experiments where the body was moving in one direction only.

The behaviour of a free standing rigid body on rigid floor excitated by seismic load has been studied. With the sliding which appears in such cases it has been established that such kind of motion is much more complicated than the motion described in the previous paragraph. It will be explained in two parts. In the first part the behaviour of the body could be alternatively locally slipping forwards or backwards, i.e. in positive or negative direction around to the local equilibrium location. The second part of sliding could be explained as the result of the positive and negative differences in local displacements, which is reflected as the resulting global sliding of the body in the prevailing direction. More illustratively said, two steps forward and one step backward. Such method of sliding causes some kind of "rubbing" in the contact surfaces which is finally reflected in the modification of micro relief characteristics. Therefore the coefficient of friction is constantly changing. These changes depend also on the quality of the materials in the contact.

Such consideration has led us to a new definition of the coefficient of friction, reflecting the friction characteristics of the contact between the body and the floor more realistically. So the coefficient of friction is derived from the equation (9) in the form:

$$\mu(t)=\frac{-(x''+X'')}{S\ (g+Y''+y'')} \tag{13}$$

and named the time-dependent coefficient of friction. The variables X'' and Y'' in the equation (13) describe the floor excitation in horizontal and vertical directions, x'' and y'' express the corresponding accelerations of the centre of gravity of the body, S is the sign of the body motion and g is the gravitational acceleration. Time-dependency $\mu(t)$ means that the value of the coefficient of friction is determined for each time step within the observation period. In the equation (13) the variables S, x'' and y'' are experimentally determined, the others are known input values.

As it is evident from the equation (13), $\mu(t)$ can get positive or negative values. The calculated negative values are attributed to the tendencies of body motion in the negative direction. As the coefficient of friction can physically not be negative, only absolute values of $\mu(t)$ are assumed for further data processing.

With respect to the great number of experimentally obtained data which might deviate from the mean value, $|\ \mu(t)\ |$ is expressed in the form of a polynomial of n-th degree.

$$|\ \mu(t)\ | = a_0+a_1.t+a_2.t^2+....a_n.t^n \tag{14}$$

where $a_0, ..., a_n$ are coefficients of polynomial obtained on the basis of a linear regression. In our research the polynomials of the fourth degree acceptably represent the experimentally obtained data (Fig.3).

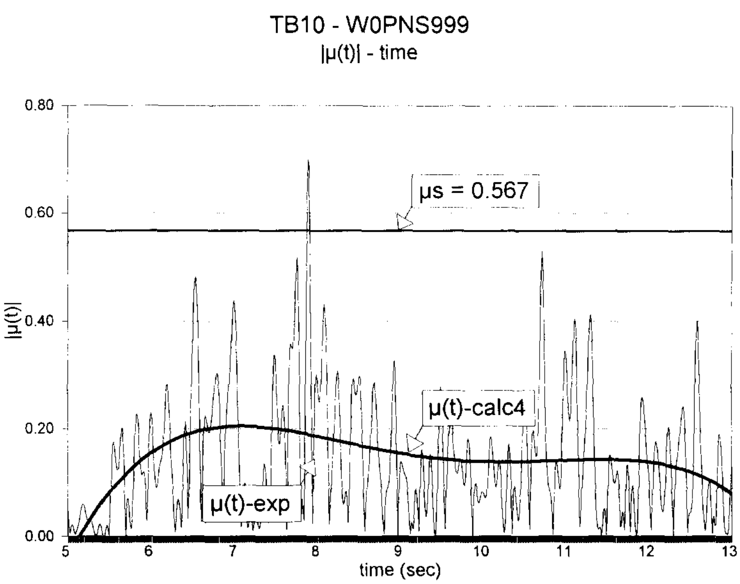

Fig. 3 $|\ \mu(t)\ |$ and polynomial of 4th degree

4. Experimental work

The basic goal of experimental work was to test and to confirm the suppositions in analytical approach when describing the motions of the model of the rigid body on the rigid floor. The work was divided into two different steps:

a) Testing under static loads where the behaviour of the contact between the body and the floor was observed by use of detachable mats.

b) Testing under dynamic loads where the horizontal component of harmonic or earthquake excitations was used under the same conditions as under a).

The model, sized 72x50x50 *cm*, was made of concrete, grade 30(Fig. 4). The concept of the model was such as to enable an easy change of the contact conditions. At the bottom of the model of rigid body it was possible to change the mat made of different materials, as well as the material of the floor. There were no special requirements about the contact plane treatment. The recess to the body mass centre enabled the installation of accelerometers for measuring vertical and horizontal accelerations of the body centre. The concept of the model enabled combinations which were marked as M_{ij}, where:

M = model,

i = floor quality,

j = contact mat quality,

The values of "i" and "j" are represented as 1 for concrete, 2 for steel and 3 for timber.

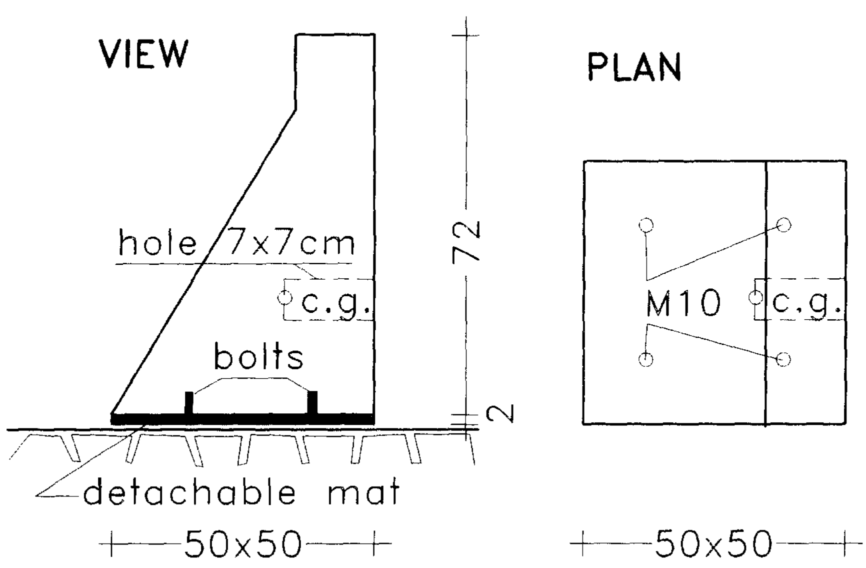

Fig. 4 Model of the rigid body

Based upon the above combinations, it was possible to find the critical conditions for slide, rotation or slide-rotation of the specimens and to follow the behaviour of the specimens for a few seconds and/or few centimetres of horizontal displacements. In the present paper our attention was focused only on the problem of the friction coefficient.

4.1. Testing on the vibroplatform

The model layout and the measuring equipment are shown in Fig.6. The motions of the model, relative to the vibroplatform, were observed. The effects of sliding and rotation were followed by measuring the horizontal and vertical displacements of the contact between the specimens and the floor. The acceleration of the centre of gravity of the body in both directions was measured as the most important data for later input in the analytical procedure. Checking the input, the acceleration of the vibroplatform and the acceleration at the top of the model were used for control. The following dynamic excitations were used:

- sinusoidal excitations of frequency 5.0 Hz and peak accelerations scaled within the range of $0.3g-0.5g$,
- earthquake excitations of original time history of the earthquake in Monte Negro in 1979, Petrovac $N-S$, having a peak acceleration of $0.23g$ and scaled within the range of up to $1.0g$.

During the experiments it was difficult to keep the body moving only in one vertical plane. The tendency of torsional rotation round the vertical axis was noticed several times. The irregularity of such motion is probably caused by the micro relief of the floor and eccentricity of the gravity centre of the body against the vertical plane.

5. Results

Over hundred experiments with different contact conditions and excitations were made. These experiments were followed by analytical work. The results of investigations presented in Figures 3 and 5 are representative to illustrate our research work.

Figure 3 presents the results of measuring the static coefficient of friction μ_S in comparison with the measured dynamic coefficient of friction (curve $\mu(t)$-exp) and the coefficient of friction obtained by regression analysis of measured data (curve $\mu(t)$-calc4). In the concrete example TB10 the quality of the contact wood/steel was studied. The measured static coefficient of friction was $\mu_S=0.567$. The dynamic coefficient of friction was measured at earthquake excitation with peak acceleration of $a_{max}=0.77g$. So $\mu(t)$-exp(erimental) reaches different values within the range of 0 to 0.7 in the time period observed. It is obvious that these values largely deviate from the classical definition of dynamic (kinetic) coefficient of friction μ_k, which in the case TB10 amounts to $\mu_k \approx 0.8\ \mu_S \approx 0.448$. This statement is confirmed by the regression curve $\mu(t)$-calc4, which represents the equation (14), by means of which the new definition of the coefficient of friction has been described. With the accuracy which would assure the right order of magnitude of $\mu(t)$, the behaviour of this coefficient could be extrapolated also for a period of time exceeding the measured one.

The residual displacements of the body relative to the rigid floor were obtained by numerical integration of the differential equations of the body motion. The results, shown in Fig. 5, were acquired by introducing the newly defined coefficient of friction into the numerical analysis of body motion. Graph 5a. is explained analogically as Fig. 3, only that here the harmonic sinusoidal floor excitation with a frequency of $5Hz$ and acceleration $a_{max}=0.49g$ is used. Graph 5b. represents the curve of measured horizontal accelerations of the body gravity centre (x''-exp) as function of time. For the sake of comparison also the analytically obtained curve of horizontal accelerations of the body gravity centre, determined by the aforementioned differential

a.)
b.)
c.)

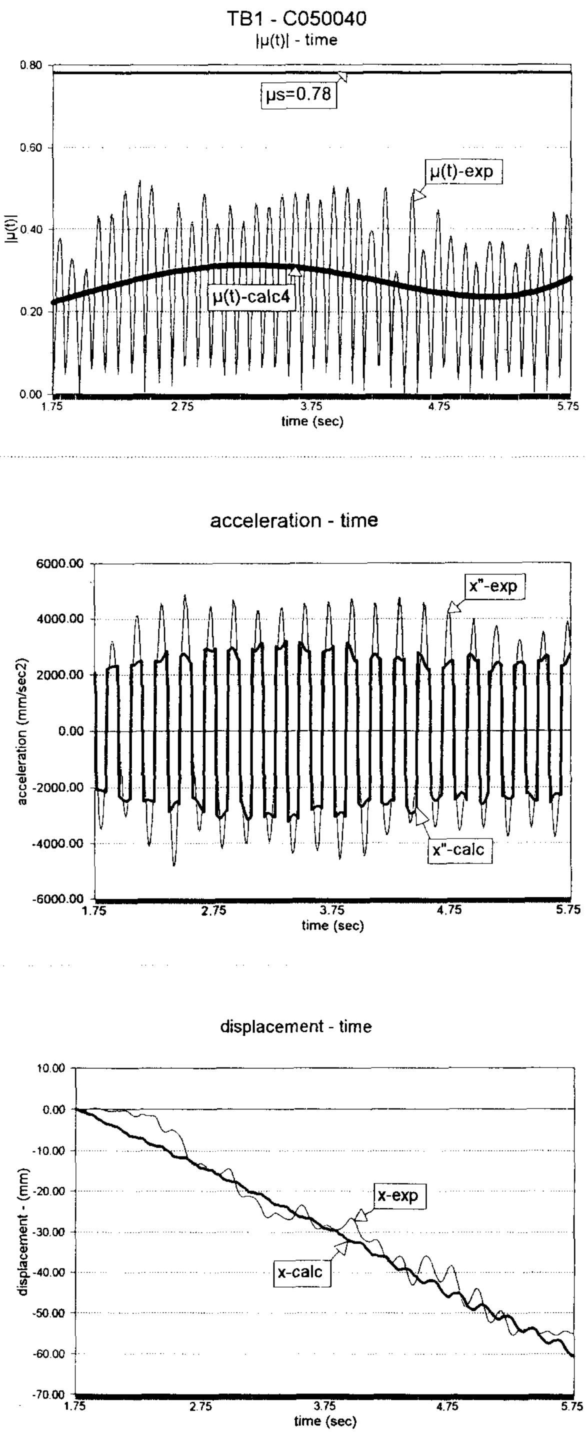

Fig. 5 Comparison of measured and analytically obtained results

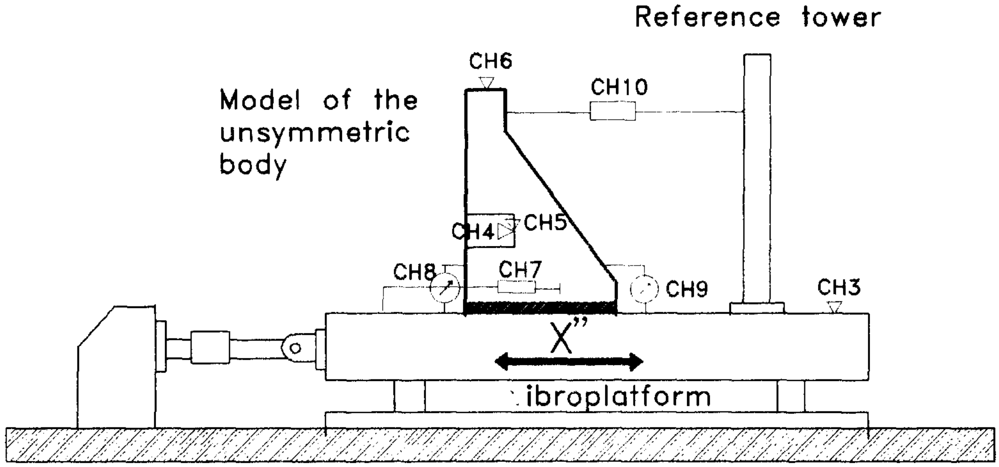

Fig. 6 Model on the vibroplatform

equations of motions and application of the new definition of the coefficient of friction ($\mu(t)$-calc4), are presented. The analytically obtained horizontal accelerations are smaller than the measured ones which is understandable due to the fact that the data $\mu(t)$ used in the equations of body motion had been obtained by the regression analysis. $\mu(t)$ is acting as a kind of filter cutting off the peaks of accelerations as compared to the measured ones. The results shown in Graph 5c are measured and the analytical displacements are obtained after numerical integration processes have been performed. In our opinion the order of magnitude of the differences between the measured and analytical results is acceptable, particularly taking into account the unpredictable character of an earthquake as the main reason for sliding of the body.

It could be generally stated that the analytical results, showing the displacements of the body, correspond to the experimental ones. Sliding is the predominant type of motion in all cases of our experiments. The obtained results are acceptable especially due to the stochastic character of the whole problem of body motion (the microrelief of the contact, earthquake), which is described by deterministic approach.

We think that such a method could be used in practice to predict residual displacements as the basis for structural solutions where such predictions are important.

6. Conclusions

From the theoretical and experimental analyses, the following conclusions can be drawn:

(1) The motions of an unsymmetric rigid body on a horizontal rigid floor in response to earthquake excitations can be described by differential equations for each of six different types of motion.

(2) All types of motion are influenced by static and/or kinetic coefficient of friction.

(3) The coefficient of friction is exposed to great variations during the period of observation and has a stochastic character actually, although being deterministically studied in our case. There are two main reasons for the stochastic character of $\mu(t)$: the unpredictable

form of microrelief of the contact area and stochastic character of the earthquake load.

(4) The slide is not influenced by the unsymmetricity of the shape of the rigid body.

(5) Conventional measurement of μ_k is not sufficient for the purposes of stability analysis in earthquake engineering. The loading velocity or mass acceleration of the body, which is in some kind of motion, is very important and has a great influence on the magnitude of the coefficient of friction. With the classical experiment for μ_k the body velocities are not as high as in the case of the body moving on the floor at an earthquake. On the other hand, the body moves on the same area many times in both directions(forwards and backwards). It comes to rubbing and the body moves on some kind of powder appearing in contact area, produced by body motion. So the average value of μ_k has to be lower than the value of μ_k obtained in a conventional way. According to our experiments, $\mu(t)$ should be within the range of 15%~30% of the μ_s, sometimes even lower.

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