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Thermo-elastic stability behavior of laminated cross-ply elliptical shells

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Abstract. In this work, thermo-elastic stability behavior of laminated cross-ply elliptical cylindrical shells subjected to uniform temperature rise is studied employing the finite element approach based on higher-order theory that accounts for the transverse shear and transverse normal deformations, and nonlinear in-plane displacement approximations through the thickness with slope discontinuity at the layer interfaces. The combined influence of higher-order shear deformation, shell geometry and non-circularity on the prebuckling thermal stress distribution and critical temperature parameter of laminated elliptical cylindrical shells is examined.

Key words: laminated shell; cross-ply; thermal buckling; critical temperature; non-circular; higher-order; finite element; elliptical cross-section.

1. Introduction

The cylinders made up of composite materials are increasingly being used by modern engineering industry such as aerospace, marine, nuclear and power, etc. The cross-section of such cylinders may become non-circular due to either the fabrication process or special design requirements like external shape/internal storage compartments in aerospace and submersible systems. These cylindrical shells subjected to extreme thermal environments during their service life may encounter thermo-elastic instability problems arising due to the compressive thermal stresses induced due to the boundary constraints. Further, the presence of out of roundness in the shell geometry will not only increase the level of geometrical complexity but may also adversely affect the stability behavior, in addition

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to the presence of bending-stretching coupling in the laminated cylindrical shells. Hence, the thermo-elastic buckling study of non-circular cylinders assumes importance.

The thermo-elastic buckling analysis of composite laminated shells has received limited attention in the literature compared to that of isotropic shells (Thornton 1993). The available studies on laminated shells are mostly dealing with the stability characteristics of circular cylindrical shells (Thangaratnam *et al*. 1989, 1990, Birman and Bert 1993, Shen 1997a, 1997b, 1998, 2002, Ganesan and Kadoli 2003). The studies pertaining to mechanics of cylindrical shells with non-circular cross-section have been reviewed by Soldatos (1999) and it can be concluded that the available literature on buckling analysis of anisotropic laminated non-circular shells are concerned with the mechanical loading situations considering axial compression/external pressure. To the authors' knowledge, thermo-elastic stability analysis of laminated composite non-circular cylindrical shells appears to be scarce in the literature except the work of Patel *et al*. (2004) wherein the authors have studied the thermal buckling of oval cylindrical shells employing higher-order finite element.

In the present work, the thermo-elastic stability characteristics of cross-ply elliptical cylindrical shells subjected to uniform temperature rise are studied employing finite element based on the higher-order displacement model (HSDT) including zig-zag term for in-plane displacements (u^k, v^k) and the variable transverse displacement (w^k) as:

$$u^k(x, y, z) = u_0(x, y) + z\theta_x(x, y) + z^2\beta_x(x, y) + z^3\phi_x(x, y) + S^k\psi_x(x, y)$$
$$v^k(x, y, z) = v_0(x, y) + z\theta_y(x, y) + z^2\beta_y(x, y) + z^3\phi_y(x, y) + S^k\psi_y(x, y)$$
$$w^k(x, y, z) = w_0(x, y) + zw_1(x, y) + z^2\Gamma(x, y) \qquad (1)$$

Here, u_0, v_0, w_0 are the displacements of a generic point on the reference surface; θ_x, θ_y are the rotations of normal to the reference surface about the y and x axes, respectively; w_1, β_x, β_y, Γ, ϕ_x, ϕ_y are the higher order terms defined at the reference surface. ψ_x and ψ_y are generalized variables associated with the zig-zag function $S^k = 2(-1)^k z_k/t_k$ with z_k being the local transverse coordinate with its origin at the centre of the kth layer and t_k is the corresponding layer thickness. The zig-zag function takes care of the inclusion of the slope discontinuity of u and v at the interfaces of the laminated shell.

The details of the finite element formulation are available in Patel *et al*. (2004) and for the sake brevity are not provided here. The study is carried out to highlight the combined influence of transverse shear deformations, non-circularity parameter, thickness and slenderness ratios on the prebuckling thermal stress distribution and critical temperatures of laminated elliptical cylindrical shells.

2. Numerical results and discussion

Here, simply-supported layered elliptical cylinders with immovable edges are investigated as example of noncircular cylinders with doubly symmetric cross-section. The radius of curvature of the middle surface for elliptical cross-section is described as $R = (b^2/\kappa_o)\,(1 + \mu_o\cos 2\theta)^{-3/2}$ where κ_o $(=[(a^2 + b^2)/2]^{1/2})$, $\mu_o = (a^2 - b^2)/(a^2 + b^2)$ and θ is the angle between the tangent at the origin of y (circumferential co-ordinate) and the one at any point on the center-line (Fig. 1). The parameters a and b are the lengths of semi-major and -minor axes, respectively. The combined influence of higher-order shear deformation with thickness and length ratios (R_o/h and L/R_o, where R_o [= $C/2\pi$;

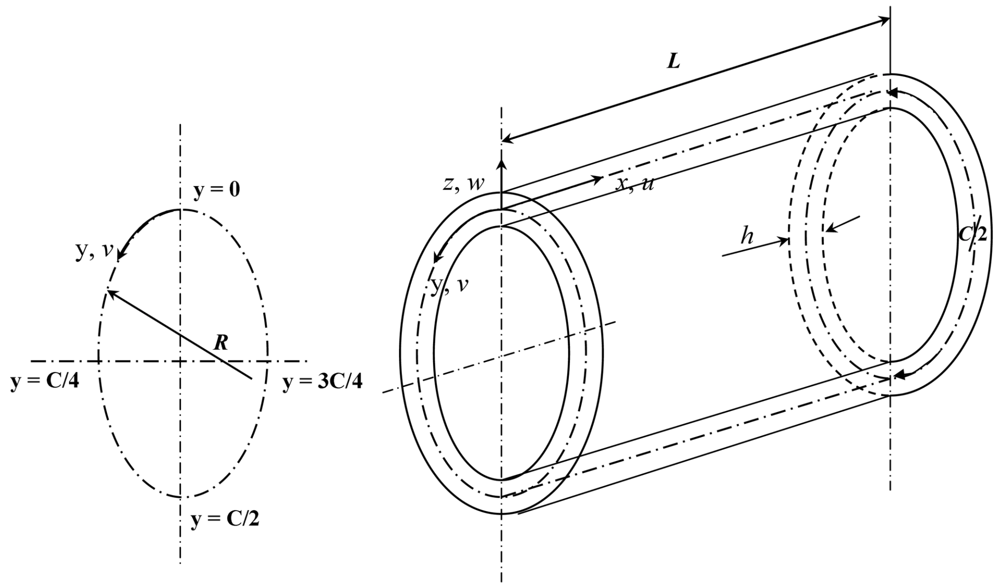

Fig. 1 Generalized coordinate system and cross-sectional details of the elliptical shell

C is the total circumferential length of the shell] is the average radius of curvature; L is length and h is thickness) and elliptical cross-sectional parameter (a/b) on the nondimensional critical temperature parameter T_{cr} ($= \Delta T\alpha_1 R_0/h$; ΔT is the rise in temperature) is examined. Furthermore, the critical temperature parameter variation with respect to various nominal circumferential wave numbers $\bar{n}$ defined based on the criterion suggested in the literature (Culberson and Boyd 1971) is studied considering four different possible classes of spatially fixed asymmetric modes (SS, SA, AS, AA), depending on whether they are symmetric (S) or antisymmetric (A) about the semi-major (y = 0 or C/2), and the semi-minor (y = C/4 or 3C/4) axes. The critical temperature values corresponding to axial half wave number m = 1 are presented here. Further, the results are also obtained using first-order shear deformation theory (FSDT) for comparison purpose. The material properties used here are: $E_1/E_2 = 40$, $G_{12}/E_2 = G_{13}/E_2 = 0.6$, $G_{23}/E_2 = 0.5$, $\nu_{12} = \nu_{23} = \nu_{13} = 0.25$, $\alpha_1 = 1 \times 10^{-6}/^0$C, $\alpha_2 = \alpha_3 = 10 \times 10^{-6}/^0$C where E, G, ν and α are Young's modulus, shear modulus, Poisson's ratio and coefficient of thermal expansion, respectively. The subscripts 1, 2 and 3 refer to the principal material directions. All the layers are of equal thickness and the ply-angle is measured with respect to the x-axis (meridional axis).

Firstly, the distribution of prebuckling in-plane stress resultants (N_{xx}, N_{yy}, N_{xy}) is studied by carrying out the static analysis of thick and thin shells subjected to uniform temperature rise, and their variation along length is highlighted in Fig. 2. It can be observed from this figure that the longitudinal stress resultant (N_{xx}) is uniform along length with mild variation whereas the circumferential/hoop component (N_{yy}) shows decreasing trend towards the center (x/L = 0.5) of the shell. Further, the hoop stress decays faster for thin shells and vanishes away from the supports. The presence of in-plane shear stress resultant (N_{xy}) with anti-symmetric variation about the center of the shell is also brought out. It is also observed from the circumferential variation of stresses (not shown here) that the stresses are uniform along circumferential direction for the circular case (a/b = 1) whereas non-circular geometry results in variation of stresses along circumference. The stress field

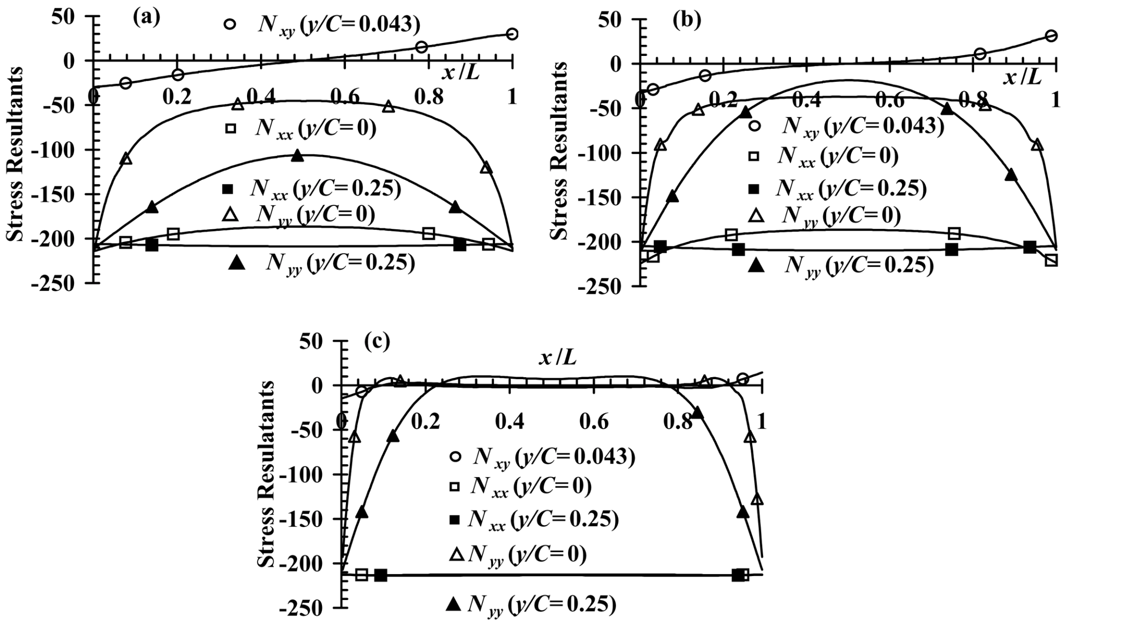

Fig. 2 Variation of in-plane stress resultants (N_{xx}, N_{yy} and N_{xy}) along the length (a/b = 2.6): (a) L/R_0 = 0.5, R_0/h = 5.0, (b) L/R_0 = 1.0, R_0/h = 5.0, (c) L/R_0 = 1.0, R_0/h = 100

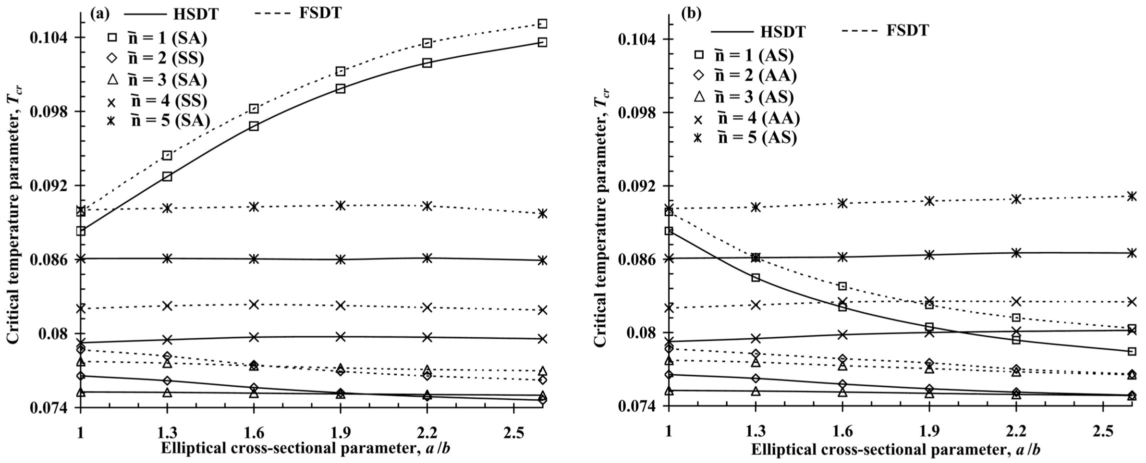

Fig. 3 Critical temperature parameter (T_{cr}) versus elliptical cross-sectional parameter (a/b) curves for eight-layered $(0^0/90^0)_4$ elliptical shells (R_0/h = 5, L/R_0 = 0.5): (a) SS and SA modes, (b) AA and AS modes

predicted in the prebuckling analysis is employed for evaluating the geometric matrix and in turn for the stability analysis.

Next, the study on the thermo-elastic stability of eight-layered thick elliptical cylindrical shells ($[0^0/90^0]_4$, L/R_o = 0.5; R_o/h = 5) for different elliptical cross-sectional parameter (a/b) is carried out and the results are depicted in Fig. 3. It can be observed from this figure that the first-order model over predicts the buckling temperature values corresponding to various nominal circumferential wave numbers compared to the higher-order model as expected. The percentage difference in the

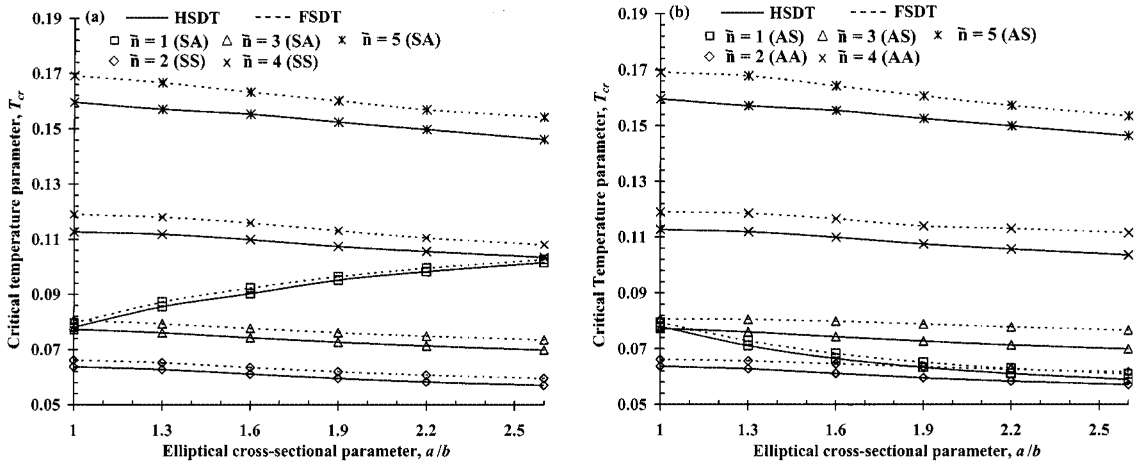

Fig. 4 Critical temperature parameter (T_{cr}) versus elliptical cross-sectional parameter (a/b) curves for eight-layered $(0^0/90^0)_4$ elliptical shells (R_0/h = 5, L/R_0 = 1.0): (a) SS and SA modes, (b) AA and AS modes

temperature parameter predicted using FSDT and HSDT increases with nominal circumferential wave number ($\bar{n}$). It can be further viewed from this figure that the critical temperature variation corresponding to wave number $\bar{n}$ = 1 case shows significant increasing trend for SA mode and decreasing one for AS mode with a/b ratio. It can also be noticed from Fig. 3 that the rate of change of critical temperature parameter with respect to non-circularity decreases for higher modes ($\bar{n}$ > 1). The nominal circumferential wave number corresponding to lowest buckling temperature is sensitive to cross-sectional parameter (a/b) and it may shift to higher/lower wave numbers.

The influence of length ratio on the stability characteristics is highlighted in Fig. 4 considering eight layered shells with L/R_o = 1 and R_o/h = 5. In general, the critical temperature parameter variation with cross-sectional parameter (a/b) is qualitatively similar to that of short shells (L/R_0 = 0.5). However, the rate of change of temperature parameter with non-circularity is higher compared to short shells. One can also notice from Figs. 3 and 4 that the rate of change of critical temperature parameter for the $\bar{n}$ = 1 case decreases at higher elliptical cross-sectional parameter (a/b). Further, the comparison of Figs. 3 and 4 reveals that the short non-circular shells may buckle in anti-symmetric modes (AA and AS) whereas the shells with L/R_o = 1.0 may prone to buckling in symmetric modes (SS and SA) at lowest critical temperature parameter.

To investigate the effect of non-circularity on the stability characteristics of thin shells, the analysis is carried out for the eight-layered shells with geometrical parameters R_o/h = 100 and L/R_o = 1.0. Their stability behavior is depicted in Fig. 5 for symmetric (SS and SA) and anti-symmetric (AA and AS) modes. It may be seen from this figure that the FSDT and HSDT models yield almost the same results as expected. The nominal circumferential wave number corresponding to lowest buckling temperature shifts to higher values compared to thick cases considered here. The critical temperature parameter significantly changes with elliptical parameter (a/b) due to the strong coupling among the various modes corresponding to circular case and the lowest buckling temperature decreases with the non-circularity. The temperature parameter curves corresponding to circumferential wave numbers away from the critical wave number depict the increasing trend at lower non-circularity and decreasing trend for highly non-circular shells.

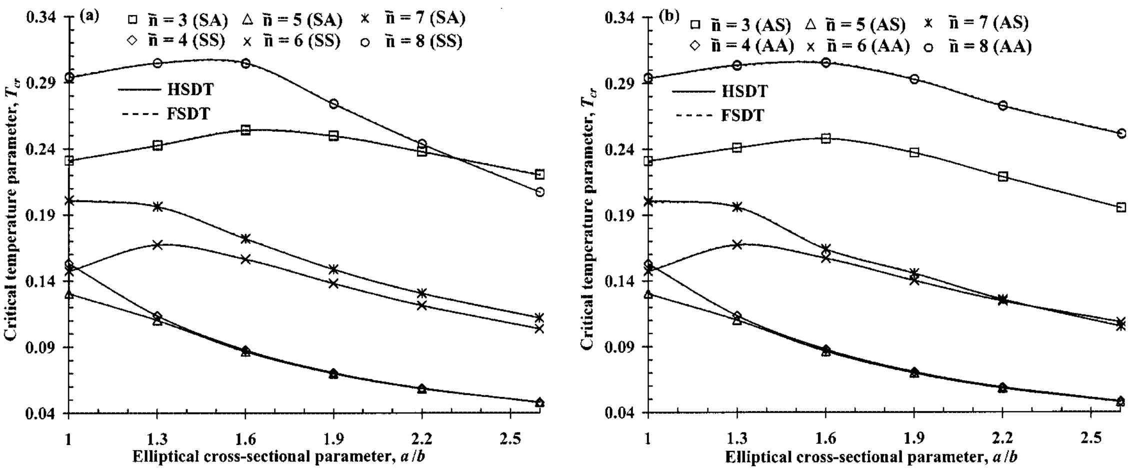

Fig. 5 Critical temperature parameter (T_{cr}) versus elliptical cross-sectional parameter (a/b) curves for eight-layered $(0^0/90^0)_4$ thin elliptical shells (R_0/h = 100, L/R_0 = 1.0): (a) SS and SA modes, (b) AA and AS modes

3. Conclusions

The influences of elliptical cross-sectional parameter on the onset of buckling temperature of laminated cross-ply elliptical cylindrical shells are demonstrated through a parametric study. The following observations can be made from the detailed analysis carried out here:

(i) The prebuckling stress distribution is uniform along circumferential direction for the circular shells whereas non-circular geometry yields variation of stresses along the circumferential direction.

(ii) The introduction of non-circularity can considerably change the buckling temperature values depending on thickness and length ratios, and nominal circumferential wave number.

(iii) Variation of the critical temperature parameter for wave number $\bar{n}$ = 1 case shows significant increasing trend for SA mode and decreasing one for AS mode.

(iv) The percentage difference between FSDT and HSDT models increases with the nominal wave number for thick shells.

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