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Investigation of the behavior of a crack between two half-planes of functionally graded materials by using the Schmidt method

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Abstract. In this paper, the behavior of a crack between two half-planes of functionally graded materials subjected to arbitrary tractions is resolved using a somewhat different approach, named the Schmidt method. To make the analysis tractable, it is assumed that the Poisson's ratios of the mediums are constants and the shear modulus vary exponentially with coordinate parallel to the crack. By use of the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. This process is quite different from those adopted in previous works. Numerical examples are provided to show the effect of the crack length and the parameters describing the functionally graded materials upon the stress intensity factor of the crack. It can be shown that the results of the present paper are the same as ones of the same problem that was solved by the singular integral equation method. As a special case, when the material properties are not continuous through the crack line, an approximate solution of the interface crack problem is also given under the assumption that the effect of the crack surface interference very near the crack tips is negligible. It is found that the stress singularities of the present interface crack solution are the same as ones of the ordinary crack in homogenous materials.

Key words: crack; functionally graded materials; Schmidt method; the dual integral equations.

1. Introduction

In recent years, functionally graded materials (FGMs) have been widely introduced and applied to the development of thermal and structural components due to its ability to not only reduce the residual and thermal stresses but to increase the bonding strength and toughness as well. To help the development of such materials, many analytical and theoretical studies in fracture mechanics have been widely done. Erdogan and Wu (1997) analyzed a FGM strip containing an imbedded or an edge crack perpendicular to the surfaces. In particular, the use of a graded material as interlayers in the bonded media is one of the highly effective and promising applications in eliminating various

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shortcoming resulting from stepwise property mismatch inherent in piecewise homogeneous composite media (Lee and Erodogan 1994, Suresh and Mortensen 1977, Choi 2001). From the fracture mechanics viewpoint, the presence of a graded interlayer would play an important role in determining the crack driving forces and fracture resistance parameters. In an attempt to address the issues pertaining to the fracture analysis of bonded media with such transitional interfacial properties, a series of solutions to certain crack problems was obtained by Erdogan and his associates (Delale and Erdogan 1988, Chen 1990, Ozturk and Erdogan 1996). Among them there are the solutions for a crack in the non-homogeneous interlayer bounded by dissimilar homogeneous media (Delale and Erdogan 1988); and for a crack at the interface between homogeneous and non-homogeneous materials (Chen 1990, Ozturk and Erdogan 1996). Similar problems of delamination or an interface crack between a functionally graded coating and a substrate were considered in (Jin and Batra 1996, Bao and Cai 1997, Shbeeb and Binienda 1999). The dynamic crack problem for non-homogeneous composite materials was considered in Wang *et al*. (2000) but they considered the FGM layer as a multi-layered homogeneous medium. The crack problem in FGM layers under thermal stresses was studied by Erdogan and Wu (1996). They considered an unconstrained elastic layer under statically self-equilibrating thermal or residual stresses.

In this paper, the same problem that was treated by Delale and Erdogan (1983) is reworked using a somewhat different approach, named the Schmidt method (Morse and Feshbach 1958, Itou 1978). To make the analysis tractable, it is assumed that the Poisson's ratios $\eta^{(j)}$ ($j = 1, 2$) of the medium are constants and the material modulus $\mu^{(j)}$ ($j = 1, 2$) varies exponentially with coordinate parallel to the crack. The Fourier transform is applied and a mixed boundary value problem is reduced to two pairs of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surface. To solve the dual integral equations, the jumps of the displacements across crack surfaces are expanded in a series of Jacobi polynomials. This process is quite different from those adopted in the (Erdogan and Wu 1997, Lee and Erdogan 1994, Suresh and Mortensen 1977, Choi 2001, Delale and Erdogan 1988, Chen 1990, Ozturk and Erdogan 1996, Jin and Batra 1996, Bao and Cai 1997, Shbeeb and Binienda 1999, Wang *et al*. 2000, Erdogan and Wu 1996, Delale and Erdogan 1983) as mentioned above. In the previous works (Erdogan and Wu 1997, Lee and Erdogan 1994, Suresh and Mortensen 1977, Choi 2001, Delale and Erdogan 1988, Chen 1990, Ozturk and Erdogan 1996, Jin and Batra 1996, Bao and Cai 1997, Shbeeb and Binienda 1999, Wang *et al*. 2000, Erdogan and Wu 1996, Delale and Erdogan 1983), the unknown variables of dual integral equations are the dislocation density functions. This is the major difference. The numerical results are same as in Delale and Erdogan (1983) when the material properties are continuous through the crack line. It is also proved that the Schmidt method is performed satisfactorily. On the other hand, as discussed in Itou (1986), an exact solution of the interface crack problem had been given in England (1965) in spite of the incomprehensibility in fracture mechanics. However, from an engineering viewpoint, it is more desirable to seek a solution that is physically acceptable. Hence, the solving process of the present paper is expanded to solve the special case problem when the material properties are not continuous through the crack line. In this case, an approximate solution of the interface crack problem is given under the assumption that the effect of the crack surface interference very near the crack tips is negligible as discussed in (Erdogan and Wu 1993, Zhang 1989, 1986). For this special case (From practical view points, researchers in the field of functionally graded materials will not pay their attention in this case), it is found that the stress singularities of the present interface crack solution are the same as ones of the ordinary crack in homogeneous materials, while much problems have to be considered when the material properties are not continuous through the crack line.

2. Formulation of the crack problem

It is assumed that there is an interface crack of length $2l$ along the x-axis between two dissimilar FGM half-planes $-\infty < x < \infty$, $0 \le y < \infty$ and $-\infty < x < \infty$, $-\infty < y \le 0$ as shown in Fig. 1. In the global x-y coordinates the shear modulus of the FGM is assumed to be as follows

$$\mu^{(j)} = \mu_0^{(j)} e^{\beta^{(j)} x} \quad (j = 1, 2) \tag{1}$$

where $\beta^{(j)}$ is a constant (The superscript $j = 1, 2$ correspond to the upper half plane and the lower half plane through in this paper.). If $\mu_0^{(1)} = \mu_0^{(2)}$, $\eta^{(1)} = \eta^{(2)}$ and $\beta^{(1)} = \beta^{(2)}$, the problem in this paper will return to the same problem as discussed in Delale and Erdogan (1983). $\eta^{(j)}$ $(j = 1, 2)$ is the Poisson's ratio.

$u^{(j)}(x, y)$ and $v^{(j)}(x, y)$ represent the displacement components in the x- and y-directions, respectively, the constitutive relations for the non-homogeneous material are written as

$$\sigma_x^{(j)}(x, y) = \frac{\mu_0^{(j)} e^{\beta^{(j)} x}}{k^{(j)} - 1}\left[(1 + k^{(j)})\frac{\partial u^{(j)}}{\partial x} + (3 - k^{(j)})\frac{\partial v^{(j)}}{\partial y}\right] \quad (j = 1, 2) \tag{2}$$

$$\sigma_y^{(j)}(x, y) = \frac{\mu_0^{(j)} e^{\beta^{(j)} x}}{k^{(j)} - 1}\left[(1 + k^{(j)})\frac{\partial v^{(j)}}{\partial y} + (3 - k^{(j)})\frac{\partial u^{(j)}}{\partial x}\right] \quad (j = 1, 2) \tag{3}$$

$$\tau_{xy}^{(j)}(x, y) = \mu_0^{(j)} e^{\beta^{(j)} x}\left[\frac{\partial u^{(j)}}{\partial y} + \frac{\partial v^{(j)}}{\partial x}\right] \quad (j = 1, 2) \tag{4}$$

Where $k^{(j)} = 3 - 4\eta^{(j)}$ $(j = 1, 2)$ for the state of plane strain, $k^{(j)} = (3 - \eta^{(j)}) / (1+\eta^{(j)})$ $(j = 1, 2)$ for the state of generalized plane stress. The Poisson's ratio $\eta^{(j)}$ $(j = 1, 2)$ for FGMs, is taken to be a constant; owing to the fact its variation within a practical range has the rather insignificant influence on the value of the near-tip driving for fracture (Delale and Erdogan 1988, Chen 1990, Ozturk and Erdogan 1996). In the present paper, we just consider the plane strain problem.

In the absence of body forces, the elastic behavior of the medium with the variable shear modulus in (1) is governed by the following equations

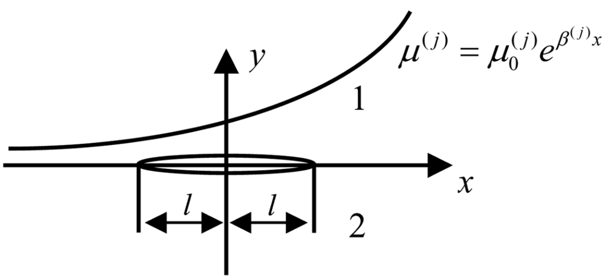

Fig. 1 Geometry of the interface crack between two dissimilar FGM half-planes and the variation of the shear modulus $\mu^{(j)} = \mu_0^{(j)} e^{\beta^{(j)} x}$ $(j = 1, 2)$

$$(1+k^{(j)})\frac{\partial^2 u^{(j)}}{\partial x^2}+(k^{(j)}-1)\frac{\partial^2 u^{(j)}}{\partial y^2}+2\frac{\partial^2 v^{(j)}}{\partial x\partial y}+\beta^{(j)}\left[(1+k^{(j)})\frac{\partial u^{(j)}}{\partial x}+(3-k^{(j)})\frac{\partial v^{(j)}}{\partial y}\right]=0 \quad (j=1,2) \tag{5}$$

$$(1+k^{(j)})\frac{\partial^2 v^{(j)}}{\partial y^2}+(k^{(j)}-1)\frac{\partial^2 v^{(j)}}{\partial x^2}+2\frac{\partial^2 u^{(j)}}{\partial x\partial y}+\beta^{(j)}(k^{(j)}-1)\left(\frac{\partial u^{(j)}}{\partial y}+\frac{\partial v^{(j)}}{\partial x}\right)=0 \quad (j=1,2) \tag{6}$$

3. Solution

The system of above governing equations is solved, using the Fourier integral transform technique to obtain the general expressions for the displacement components as

$$\begin{cases} u^{(1)}(x,y) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\sum_{j=1}^{2} A_j(s)e^{-\lambda_j y}e^{-isx}ds \\ v^{(1)}(x,y) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\sum_{j=1}^{2} m_j(s)A_j(s)e^{-\lambda_j y}e^{-isx}ds \end{cases} \tag{7}$$

$$\begin{cases} u^{(2)}(x,y) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\sum_{j=3}^{4} A_j(s)e^{-\lambda_j y}e^{-isx}ds \\ v^{(2)}(x,y) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\sum_{j=3}^{4} m_j(s)A_j(s)e^{-\lambda_j y}e^{-isx}ds \end{cases} \tag{8}$$

and from Eqs. (2)-(4), the stress components are obtained as

$$\begin{cases} \sigma_y^{(1)}(x,y) = \dfrac{\mu_0^{(1)}e^{\beta^{(1)}x}}{2\pi(k^{(1)}-1)}\displaystyle\int_{-\infty}^{\infty}\sum_{j=1}^{2}[-(k^{(1)}+1)m_j(s)\lambda_j - is(3-k^{(1)})]A_j(s)e^{-\lambda_j y}e^{-isx}ds \\ \tau_{xy}^{(1)}(x,y) = \dfrac{\mu_0^{(1)}e^{\beta^{(1)}x}}{2\pi}\displaystyle\int_{-\infty}^{\infty}\sum_{j=1}^{2}[-\lambda_j - im_j(s)s]A_j(s)e^{-\lambda_j y}e^{-isx}ds \end{cases} \tag{9}$$

$$\begin{cases} \sigma_y^{(2)}(x,y) = \dfrac{\mu_0^{(2)}e^{\beta^{(2)}x}}{2\pi(k^{(2)}-1)}\displaystyle\int_{-\infty}^{\infty}\sum_{j=3}^{4}[-(k^{(2)}+1)m_j(s)\lambda_j - is(3-k^{(2)})]A_j(s)e^{-\lambda_j y}e^{-isx}ds \\ \tau_{xy}^{(2)}(x,y) = \dfrac{\mu_0^{(2)}e^{\beta^{(2)}x}}{2\pi}\displaystyle\int_{-\infty}^{\infty}\sum_{j=3}^{4}[-\lambda_j - im_j(s)s]A_j(s)e^{-\lambda_j y}e^{-isx}ds \end{cases} \tag{10}$$

where s is the transform variable, A_j (j = 1, 2, 3, 4) are arbitrary unknowns.

$\lambda_j(s)(j=1,2)$ are the roots of the characteristic equation

$$\lambda^4-\left(2s^2+2is\beta^{(1)}+\beta^{(1)2}\frac{3-k^{(1)}}{k^{(1)}+1}\right)\lambda^2+s^2(s+i\beta^{(1)})^2=0 \tag{11}$$

and $m_j(s)$ $(j=1,2)$ are expressed for each root $\lambda_j(s)$ $(j=1,2)$ as

$$m_j(s)=\frac{-(k^{(1)}+1)s^2+(k^{(1)}-1)\lambda_j^2-is\beta^{(1)}(1+k^{(1)})}{\lambda_j[-2is+\beta^{(1)}(3-k^{(1)})]}\quad (j=1,2) \tag{12}$$

λ_j $(j=3,4)$ are the roots of the characteristic equation

$$\lambda^4-\left(2s^2+2is\beta^{(2)}+\beta^{(2)2}\frac{3-k^{(2)}}{k^{(2)}+1}\right)\lambda^2+s^2(s+i\beta^{(2)})^2=0 \tag{13}$$

and $m_j(s)$ $(j=3,4)$ are expressed for each root $\lambda_j(s)(j=3,4)$ as

$$m_j(s)=\frac{-(k^{(2)}+1)s^2+(k^{(2)}-1)\lambda_j^2-is\beta^{(2)}(1+k^{(2)})}{\lambda_j[-2is+\beta^{(2)}(3-k^{(2)})]}\quad (j=3,4) \tag{14}$$

The roots may be obtained as

$$\lambda_1=\sqrt{\frac{b^{(1)}+\sqrt{b^{(1)2}-4c^{(1)}}}{2}},\quad \lambda_2=\sqrt{\frac{b^{(1)}-\sqrt{b^{(1)2}-4c^{(1)}}}{2}} \tag{15}$$

$$\lambda_3=-\sqrt{\frac{b^{(2)}+\sqrt{b^{(2)2}-4c^{(2)}}}{2}},\quad \lambda_4=-\sqrt{\frac{b^{(2)}-\sqrt{b^{(2)2}-4c^{(2)}}}{2}} \tag{16}$$

where $b^{(1)}=2s^2+2is\beta^{(1)}+\beta^{(1)2}\dfrac{3-k^{(1)}}{k^{(1)}+1},\quad c^{(1)}=s^2(s+i\beta^{(1)})^2$

$$b^{(2)}=2s^2+2is\beta^{(2)}+\beta^{(2)2}\frac{3-k^{(2)}}{k^{(2)}+1},\quad c^{(2)}=s^2(s+i\beta^{(2)})^2$$

From Eqs. (7)-(10), it can be seen that there are four unknown constants (in Fourier space they are functions of s), i.e., A_j, $j=1,2,3,4$, which can be obtained from the following conditions:

$$\sigma_y^{(1)}(x,0)=\sigma_y^{(2)}(x,0)=-\sigma_0(x),\ \tau_{xy}^{(1)}(x,0)=\tau_{xy}^{(2)}(x,0)=-\tau_0(x),\ |x|\le l \tag{17}$$

$$\sigma_y^{(1)}(x,0)=\sigma_y^{(2)}(x,0),\quad \tau_{xy}^{(1)}(x,0)=\tau_{xy}^{(2)}(x,0),\quad |x|>l \tag{18}$$

$$u^{(1)}(x,0)=u^{(2)}(x,0),\quad v^{(1)}(x,0)=v^{(2)}(x,0),\quad |x|>l \tag{19}$$

where $\sigma_0(x)$ and $\tau_0(x)$ are known functions.

To solve the problem, the jumps of the displacements across the crack surfaces can be defined as follows:

$$f_1(x)=u^{(1)}(x,0)-u^{(2)}(x,0) \tag{20}$$

$$f_2(x)=v^{(1)}(x,0)-v^{(2)}(x,0) \tag{21}$$

In the solution of such problem, it is obviously that some unsurmountable mathematical difficulties will be encountered and have to be resort to a succinct procedure. In the present paper, it is decided to assume $\beta^{(1)} = \beta^{(2)} = \beta$. Applying the Fourier transforms and the boundary conditions (17)-(19), it can be obtained

$$[X_1]\begin{bmatrix} A_1(s) \\ A_2(s) \end{bmatrix} = [X_2]\begin{bmatrix} A_3(s) \\ A_4(s) \end{bmatrix} \tag{22}$$

$$[X_3]\begin{bmatrix} A_1(s) \\ A_2(s) \end{bmatrix} - [X_4]\begin{bmatrix} A_3(s) \\ A_4(s) \end{bmatrix} = \begin{bmatrix} \bar{f}_1(s) \\ \bar{f}_2(s) \end{bmatrix} \tag{23}$$

where

$$[X_1] = \begin{bmatrix} \dfrac{\mu_0^{(1)}[-(k^{(1)}+1)m_1(s)\lambda_1 - is(3-k^{(1)})]}{(k^{(1)}-1)} & \dfrac{\mu_0^{(1)}[-(k^{(1)}+1)m_2(s)\lambda_2 - is(3-k^{(1)})]}{(k^{(1)}-1)} \\ \mu_0^{(1)}[-\lambda_1 - im_1(s)s] & \mu_0^{(1)}[-\lambda_2 - im_2(s)s] \end{bmatrix}$$

$$[X_2] = \begin{bmatrix} \dfrac{\mu_0^{(2)}[-(k^{(2)}+1)m_3(s)\lambda_3 - is(3-k^{(2)})]}{(k^{(2)}-1)} & \dfrac{\mu_0^{(2)}[-(k^{(2)}+1)m_4(s)\lambda_4 - is(3-k^{(2)})]}{(k^{(2)}-1)} \\ \mu_0^{(2)}[-\lambda_3 - im_3(s)s] & \mu_0^{(2)}[-\lambda_4 - im_4(s)s] \end{bmatrix}$$

$$[X_3] = \begin{bmatrix} 1 & 1 \\ m_1(s) & m_2(s) \end{bmatrix}, \quad [X_4] = \begin{bmatrix} 1 & 1 \\ m_3(s) & m_4(s) \end{bmatrix}$$

A superposed bar indicates the Fourier transform. The Fourier transform is defined as follows:

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x)e^{isx}dx, \quad f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(s)e^{-isx}ds \tag{24}$$

By solving four Eqs. (22)-(23) with four unknown functions, substituting the solutions into Eq. (17) and applying the boundary conditions, it can be obtained

$$\sigma_y^{(1)}(x,0) = \frac{e^{\beta x}}{2\pi}\int_{-\infty}^{\infty}[d_1(s)\bar{f}_1(s) + d_2(s)\bar{f}_2(s)]e^{-isx}ds = -\sigma_0(x), \quad 0 \le |x| \le l \tag{25}$$

$$\tau_{xy}^{(1)}(x,0) = \frac{e^{\beta x}}{2\pi}\int_{-\infty}^{\infty}[d_3(s)\bar{f}_1(s) + d_4(s)\bar{f}_2(s)]e^{-isx}ds = -\tau_0(x), \quad 0 \le |x| \le l \tag{26}$$

$$\int_{-\infty}^{\infty}\bar{f}_1(s)e^{-isx}ds = 0, \quad |x| > l \tag{27}$$

$$\int_{-\infty}^{\infty}\bar{f}_2(s)e^{-isx}ds = 0, \quad |x| > l \tag{28}$$

where $d_1(s), d_2(s), d_3(s)$ and $d_4(s)$ are known functions as follows:

$$[X_5] = [X_3] - [X_4][X_2]^{-1}[X_1], \quad \begin{bmatrix} d_1(s) & d_2(s) \\ d_3(s) & d_4(s) \end{bmatrix} = [X_1][X_5]^{-1}$$

To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$ the above two pairs of dual integral Eqs. (25)-(28) must be solved.

4. Solution of the dual integral equations

To solve the problem, the jumps of the displacements across the crack surfaces can be represented by the following series: (When the material properties are not continuous through the crack line, as mentioned above, the problem is solved under the assumptions that the effect of the crack surface overlapping very near the crack tips is negligible. These assumptions had been used in (Erdogan and Wu 1993, Zhang 1986, 1989). It can be obtained that the jumps of the displacements across the crack surface are finite, differentiable and continuous functions. Only in this case, the jumps of the displacements across the crack surfaces can be represented by the following series:)

$$f_1(x) = \sum_{n=0}^{\infty} a_n P_n^{(1/2,\,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{\frac{1}{2}}, \quad \text{for} \quad 0 \le |x| \le l \tag{29}$$

$$f_1(x) = 0, \quad \text{for} \quad |x| > l \tag{30}$$

$$f_2(x) = \sum_{n=0}^{\infty} b_n P_n^{(1/2,\,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{\frac{1}{2}}, \quad \text{for} \quad 0 \le |x| \le l \tag{31}$$

$$f_2(x) = 0, \quad \text{for} \quad |x| > l \tag{32}$$

where a_n and b_n are unknown coefficients, $P_n^{(1/2,\,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik 1980). The Fourier transform of Eqs. (29)-(32) are (Erdelyi 1954)

$$\bar{f}_1(s) = \sum_{n=0}^{\infty} a_n G_n \frac{1}{s} J_{n+1}(sl), \quad \bar{f}_2(s) = \sum_{n=0}^{\infty} b_n G_n \frac{1}{s} J_{n+1}(sl) \tag{33}$$

$$G_n = 2\sqrt{\pi}(-1)^n i^n \frac{\Gamma\left(n + 1 + \frac{1}{2}\right)}{n!} \tag{34}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq. (33) into Eqs. (25)-(28), it can be shown that Eqs. (27)-(28) are automatically satisfied. After integration with respect to x in $[-l, x]$, Eqs. (25)-(26) reduce to

$$\frac{1}{2\pi}\sum_{n=0}^{\infty} G_n \int_{-\infty}^{\infty} \frac{i}{s^2}[d_1(s)a_n + d_2(s)b_n] J_{n+1}(sl)[e^{-isx} - e^{isl}]ds = -\int_{-l}^{x} \sigma_0(s) e^{-\beta s} ds, \quad 0 \le |x| \le l \tag{35}$$

$$\frac{1}{2\pi}\sum_{n=0}^{\infty}G_n\int_{-\infty}^{\infty}\frac{i}{s^2}[d_3(s)a_n+d_4(s)b_n]J_{n+1}(sl)[e^{-isx}-e^{isl}]ds=-\int_{-l}^{x}\tau_0(s)e^{-\beta s}ds,\quad 0\le|x|\le l \tag{36}$$

From the relationships (Gradshteyn and Ryzhik 1980)

$$\int_0^{\infty}\frac{1}{s}J_n(sa)\sin(bs)ds = \begin{cases}\dfrac{\sin[n\sin^{-1}(b/a)]}{n}, & a\ge b\\ \dfrac{a^n\sin(n\pi/2)}{n[b+\sqrt{b^2-a^2}]^n}, & b\ge a\end{cases} \tag{37}$$

$$\int_0^{\infty}\frac{1}{s}J_n(sa)\cos(bs)ds = \begin{cases}\dfrac{\cos[n\sin^{-1}(b/a)]}{n}, & a\ge b\\ \dfrac{a^n\cos(n\pi/2)}{n[b+\sqrt{b^2-a^2}]^n}, & b\ge a\end{cases} \tag{38}$$

the semi-infinite integral in Eqs. (35)-(36) can be modified as:

$$\int_{-\infty}^{\infty}\frac{d_j(s)}{s^2}J_{n+1}(sl)[e^{-isx}-e^{isl}]ds = \begin{cases}\dfrac{2\delta_j}{n+1}\cos\left[(n+1)\sin^{-1}\left(\dfrac{x}{l}\right)\right], & n=0,2,4,6,\dots\\ \dfrac{-2i\delta_j}{n+1}\sin\left[(n+1)\sin^{-1}\left(\dfrac{x}{l}\right)\right], & n=1,3,5,7,\dots\end{cases}$$

$$+\int_{-\infty}^{\infty}\frac{1}{s}\left[\frac{d_j(s)}{s}-\delta_j\right]J_{n+1}(sl)[e^{-isx}-e^{isl}]ds\quad (j=1,4) \tag{39}$$

$$\int_{-\infty}^{\infty}\frac{d_j(s)}{s^2}J_{n+1}(sl)[e^{-isx}-e^{isl}]ds = \begin{cases}\dfrac{2\delta_j}{n+1}\left\{\cos\left[(n+1)\sin^{-1}\left(\dfrac{x}{l}\right)\right]-(-1)^{\frac{n+1}{2}}\right\}, & n=1,3,5,7,\dots\\ \dfrac{-2i\delta_j}{n+1}\left\{\sin\left[(n+1)\sin^{-1}\left(\dfrac{x}{l}\right)\right]+(-1)^{\frac{n}{2}}\right\}, & n=0,2,4,6,\dots\end{cases}$$

$$+\int_0^{\infty}\frac{1}{s}\left[\frac{d_j(s)}{s}-\delta_j\right]J_{n+1}(sl)[e^{-isx}-e^{isl}]ds+\int_{-\infty}^{0}\frac{1}{s}\left[\frac{d_j(s)}{s}+\delta_j\right]J_{n+1}(sl)[e^{-isx}-e^{isl}]ds\quad (j=2,3) \tag{40}$$

where $\lim_{s\to\pm\infty}d_1(s)/s=\delta_1$, $\lim_{s\to+\infty}d_2(s)/s=-\lim_{s\to-\infty}d_2(s)/s=\delta_2$, $\lim_{s\to+\infty}d_3(s)/s=-\lim_{s\to-\infty}d_3(s)/s=\delta_3=\delta_2$,

$\lim_{s\to\pm\infty}d_4(s)/s=\delta_4=-\delta_1$, $\delta_1=-\dfrac{\mu_0^{(1)}\mu_0^{(2)}[(-1+k^{(2)})\mu_0^{(1)}+\mu_0^{(2)}(1-k^{(1)})]}{(\mu_0^{(1)}+\mu_0^{(2)}k^{(1)})(\mu_0^{(2)}+\mu_0^{(1)}k^{(2)})}$,

$\delta_2=-\dfrac{\mu_0^{(1)}\mu_0^{(2)}(\mu_0^{(1)}+k^{(2)}\mu_0^{(1)}+\mu_0^{(2)}+\mu_0^{(2)}k^{(1)})}{(\mu_0^{(1)}+\mu_0^{(2)}k^{(1)})(\mu_0^{(2)}+\mu_0^{(1)}k^{(2)})}$

For $\mu_0^{(1)} = \mu_0^{(2)} = \mu$ and $k^{(1)} = k^{(2)} = k$, $\delta_1 = 0$ and $\delta_2 = -\dfrac{2\mu}{k+1}$. When $\mu_0^{(1)} = \mu_0^{(2)} = \mu$ and $k^{(1)} = k^{(2)} = k$, this is the same case as in Delale and Erdogan (1983). The semi-infinite integral in Eqs. (39)-(40) can be evaluated directly. Eqs. (35)-(36) can now be solved for the coefficients a_n and b_n by the Schmidt method (Morse and Feshbach 1958, Itou 1978). For briefly, Eqs. (35)-(36) can be rewritten as

$$\sum_{n=0}^{\infty} a_n E_n^*(x) + \sum_{n=0}^{\infty} b_n F_n^*(x) = U_0(x), \quad 0 \le |x| \le l \tag{41}$$

$$\sum_{n=0}^{\infty} a_n G_n^*(x) + \sum_{n=0}^{\infty} b_n H_n^*(x) = V_0(x), \quad 0 \le |x| \le l \tag{42}$$

where $E_n^*(x), F_n^*(x), G_n^*, H_n^*(x)$, $U_0(x)$ and $V_0(x)$ are known functions. a_n and b_n are unknown coefficients.

From Eq. (42), it can be obtained:

$$\sum_{n=0}^{\infty} b_n H_n^*(x) = -\sum_{n=0}^{\infty} a_n G_n^*(x) + V_0(x) \tag{43}$$

It can now be solved for the coefficients b_n by the Schmidt method (Morse and Feshbach 1958, Itou 1978, Zhou *et al.* 1999a, b, Itou 1999, 2001). Here the form $-\sum_{n=0}^{\infty} a_n G_n^*(x) + V_0(x)$ can be considered as a known function temporarily. A set of functions $P_n(x)$, which satisfy the orthogonality condition

$$\int_{-l}^{l} P_m(x) P_n(x) dx = N_n \delta_{mn}, \quad N_n = \int_{-l}^{l} P_n^2(x) dx \tag{44}$$

can be constructed from the function, $H_n^*(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} H_i^*(x) \tag{45}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \ldots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \ldots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \ldots, d_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ d_{n0}, d_{n1}, d_{n2}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_{-l}^{l} H_i^*(x) H_j^*(x) dx \tag{46}$$

Using Eqs. (43)-(46), it can be obtained that

$$b_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \quad \text{with} \quad q_j = -\sum_{i=0}^{\infty} a_i \frac{1}{N_j} \int_{-l}^{l} G_i^*(x) P_j(x) dx + \frac{1}{N_j} \int_{-l}^{l} V_0(x) P_j(x) dx \tag{47}$$

So it can be rewritten

$$b_n = \sum_{i=0}^{\infty} a_i K_{in}^* + \sum_{j=n}^{\infty} \frac{M_{nj}}{N_j M_{jj}} \int_{-l}^{l} V_0(x) P_j(x) dx, \quad K_{in}^* = -\sum_{j=n}^{\infty} \frac{M_{nj}}{N_j M_{jj}} \int_{-l}^{l} G_i^*(x) P_j(x) dx \tag{48}$$

Substituting Eq. (48) into Eq. (41), it can be obtained

$$\sum_{n=0}^{\infty} a_n Y_n^*(x) = U_0(x) - W(x)$$

$$Y_n^*(x) = E_n^*(x) + \sum_{i=0}^{\infty} K_{ni}^* F_i^*(x), \quad W(x) = \sum_{n=0}^{\infty} F_n(x) \sum_{j=n}^{\infty} \frac{M_{nj}}{N_j M_{jj}} \int_{-l}^{l} V_0(s) P_j(s) ds \tag{49}$$

So it can now be solved for the coefficients a_n by the Schmidt method again as above mentioned. With the aid of Eq. (48), the coefficients b_n can be obtained.

5. Stress intensity factors

The coefficients a_n and b_n are known, so that the entire stress field can be obtained. However, in fracture mechanics, it is important to determine stresses $\sigma_y^{(1)}$ and $\tau_{xy}^{(1)}$ in the vicinity of the crack tips. In the case of the present study, $\sigma_y^{(1)}$ and $\tau_{xy}^{(1)}$ along the crack line can be expressed as:

$$\begin{aligned} \sigma_y^{(1)}(x, 0) &= \frac{e^{\beta x}}{2\pi} \sum_{n=0}^{\infty} G_n \int_{-\infty}^{\infty} \frac{1}{s} [d_1(s) a_n + d_2(s) b_n] J_{n+1}(sl) e^{-isx} ds \\ &= \frac{e^{\beta x}}{2\pi} \sum_{n=0}^{\infty} G_n \left\{ a_n \int_{-\infty}^{\infty} \left(\frac{d_1(s)}{s} - \delta_1 \right) J_{n+1}(sl) e^{-isx} ds + b_n \int_0^{\infty} \left(\frac{d_2(s)}{s} - \delta_2 \right) J_{n+1}(sl) e^{-isx} ds \right. \\ &\quad + b_n \int_{-\infty}^{0} \left(\frac{d_2(s)}{s} + \delta_2 \right) J_{n+1}(sl) e^{-isx} ds + \delta_1 a_n \int_{-\infty}^{\infty} J_{n+1}(sl) e^{-isx} ds \\ &\quad \left. + \delta_2 b_n \int_0^{\infty} J_{n+1}(sl) e^{-isx} ds - \delta_2 b_n \int_{-\infty}^{0} J_{n+1}(sl) e^{-isx} ds \right\} \end{aligned} \tag{50}$$

$$\begin{aligned} \tau_{xy}^{(1)}(x, 0) &= \frac{e^{\beta x}}{2\pi} \sum_{n=0}^{\infty} G_n \int_{-\infty}^{\infty} \frac{1}{s} [d_3(s) a_n + d_4(s) b_n] J_{n+1}(sl) e^{-isx} ds \\ &= \frac{e^{\beta x}}{2\pi} \sum_{n=0}^{\infty} G_n \left\{ a_n \int_0^{\infty} \left(\frac{d_3(s)}{s} - \delta_2 \right) J_{n+1}(sl) e^{-isx} ds + a_n \int_{-\infty}^{0} \left(\frac{d_3(s)}{s} + \delta_2 \right) J_{n+1}(sl) e^{-isx} ds \right. \\ &\quad + b_n \int_{-\infty}^{\infty} \left(\frac{d_4(s)}{s} + \delta_1 \right) J_{n+1}(sl) e^{-isx} ds + \delta_2 a_n \int_0^{\infty} J_{n+1}(sl) e^{-isx} ds \\ &\quad \left. - \delta_2 a_n \int_{-\infty}^{0} J_{n+1}(sl) e^{-isx} ds - \delta_1 b_n \int_{-\infty}^{\infty} J_{n+1}(sl) e^{-isx} ds \right\} \end{aligned} \tag{51}$$

By examination of Eqs. (50)-(51) shows that, the singular part of the stress field can be obtained from the relationships as follows (Gradshteyn and Ryzhik 1980):

$$\int_0^\infty J_n(sa)\cos(bs)ds = \begin{cases} \dfrac{\cos[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a > b \\ -\dfrac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b > a \end{cases}$$

$$\int_0^\infty J_n(sa)\sin(bs)ds = \begin{cases} \dfrac{\sin[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a > b \\ \dfrac{a^n\cos(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b > a \end{cases}$$

For $l < x$, the singular part of the stress field can be expressed respectively as follows:

$$\sigma = \frac{\delta_2 e^{\beta x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^\infty J_{n+1}(sl)e^{-isx}ds + (-1)^n\int_0^\infty J_{n+1}(sl)e^{isx}ds\right] = -\frac{\delta_2 e^{\beta x}}{\pi}\sum_{n=0}^{\infty} b_n G_n Q_n(x) \tag{52}$$

$$\tau = \frac{\delta_2 e^{\beta x}}{2\pi}\sum_{n=0}^{\infty} a_n G_n\left[\int_0^\infty J_{n+1}(sl)e^{-isx}ds + (-1)^n\int_0^\infty J_{n+1}(sl)e^{isx}ds\right] = -\frac{\delta_2 e^{\beta x}}{\pi}\sum_{n=0}^{\infty} a_n G_n Q_n(x) \tag{53}$$

where $Q_n(x) = \begin{cases} \dfrac{(-1)^{\frac{n}{2}} l^{n+1}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{n+1}}, & n = 0, 2, 4, 6, \dots \\ \dfrac{i(-1)^{\frac{n+1}{2}} l^{n+1}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{n+1}}, & n = 1, 3, 5, 7, \dots \end{cases}$

For $x < -1$, the singular part of the stress field can be expressed respectively as follows:

$$\sigma = \frac{\delta_2 e^{\beta x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^\infty J_{n+1}(sl)e^{-isx}ds + (-1)^n\int_0^\infty J_{n+1}(sl)e^{isx}ds\right] = -\frac{\delta_2 e^{\beta x}}{\pi}\sum_{n=0}^{\infty} b_n G_n Q_n^*(x) \tag{54}$$

$$\tau = \frac{\delta_2 e^{\beta x}}{2\pi}\sum_{n=0}^{\infty} a_n G_n\left[\int_0^\infty J_{n+1}(sl)e^{-isx}ds + (-1)^n\int_0^\infty J_{n+1}(sl)e^{isx}ds\right] = -\frac{\delta_2 e^{\beta x}}{\pi}\sum_{n=0}^{\infty} a_n G_n Q_n^*(x) \tag{55}$$

where $Q_n^*(x) = \begin{cases} \dfrac{(-1)^{\frac{n}{2}} l^{n+1}}{\sqrt{x^2-l^2}[|x|+\sqrt{x^2-l^2}]^{n+1}}, & n = 0, 2, 4, 6, \dots \\ \dfrac{-i(-1)^{\frac{n+1}{2}} l^{n+1}}{\sqrt{x^2-l^2}[|x|+\sqrt{x^2-l^2}]^{n+1}}, & n = 1, 3, 5, 7, \dots \end{cases}$

The values of the stress concentration at the right tip of the crack can be given as follows

$$K_{\mathrm{I}} = \lim_{x \to l^+} \sqrt{2(x-l)} \cdot \sigma = -\frac{2\delta_2 e^{\beta l}}{\sqrt{\pi l}} \sum_{n=0}^{\infty} (-1)^n b_n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \tag{56}$$

$$K_{\mathrm{II}} = \lim_{x \to l^+} \sqrt{2(x-l)} \cdot \tau = -\frac{2\delta_2 e^{\beta l}}{\sqrt{\pi l}} \sum_{n=0}^{\infty} (-1)^n a_n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \tag{57}$$

The values of the stress concentration at the left tip of the crack can be given as follows

$$K_{\mathrm{I}}^* = \lim_{x \to -l^-} \sqrt{2(|x|-l)} \cdot \sigma = -\frac{2\delta_2 e^{-\beta l}}{\sqrt{\pi l}} \sum_{n=0}^{\infty} b_n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \tag{58}$$

$$K_{\mathrm{II}}^* = \lim_{x \to -l^-} \sqrt{2(|x|-l)} \cdot \tau = -\frac{2\delta_2 e^{-\beta l}}{\sqrt{\pi l}} \sum_{n=0}^{\infty} a_n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \tag{59}$$

6. Numerical calculations and discussion

As discussed in the works (Morse and Feshbach 1958, Itou 1978, Zhou *et al.* 1999a,b, Itou 1999, 2001), it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series in Eqs. (35)-(36) are retained. The behavior of the sum of the series keeps steady with the increasing number of terms in Eqs. (35)-(36). For the case in which the material constants of the materials are different, the material constants of the upper half plane functionally graded materials are assumed as $\mu^{(1)} = 77.0e^{\beta x}(\times 10^9 \mathrm{N/m^2})$, $\eta^{(1)} = 0.28$, and the material constants of the lower half plane functional graded materials are assumed as $\mu^{(2)} = 66.5e^{\beta x}(\times 10^9 \mathrm{N/m^2})$ and $\eta^{(2)} = 0.3$. The crack surface loading $-\sigma_0(x)$ and $-\tau_0(x)$ will simply be assumed to be a polynomial of the form as follows:

$$-\sigma_0(x) = -p_0 - p_1\left(\frac{x}{l}\right) - p_2\left(\frac{x}{l}\right)^2 - p_3\left(\frac{x}{l}\right)^3$$

$$-\tau_0(x) = -s_0 - s_1\left(\frac{x}{l}\right) - s_2\left(\frac{x}{l}\right)^2 - s_3\left(\frac{x}{l}\right)^3$$

Since the problem is linear, the results can be superimposed in any suitable manner. The results are obtained by taking only one or two of the eight input parameters p_0, p_1, p_2, p_3, s_0, s_1, s_2 and s_3 nonzero at a time. The values of the stress concentration K are calculated numerically. The results of the present paper are shown in Fig. 2 to Fig. 6 and the Table 1 to Table 2.

From the results, the following observations can be made:

(i) The aim of the present paper is to give a new approach to resolve the same problem as in Delale and Erdogan (1983). It can be seen that the results of the present paper is the same as

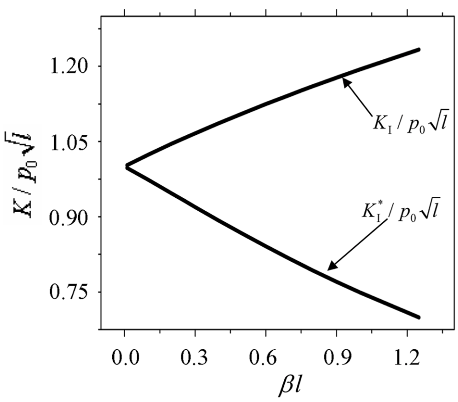

Fig. 2 Stress intensity factors versus βl for $\sigma_0(x) = p_0$, $\tau_0(x) = 0$ and $(\mu_0^{(1)}, \eta^{(1)}) = (\mu_0^{(2)}, \eta^{(2)}) = (66.5 \times 10^9\ \mathrm{N/m^2}, 0.3)$

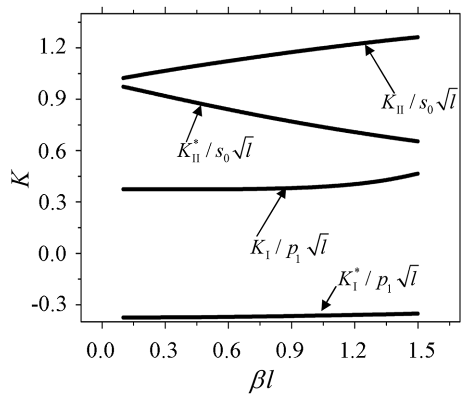

Fig. 3 Stress intensity factors versus βl for $\sigma_0(x) = p_1\left(\frac{x}{l}\right)$, $\tau_0(x) = s_0$ and $(\mu_0^{(1)}, \eta^{(1)}) = (\mu_0^{(2)}, \eta^{(2)}) = (66.5 \times 10^9\ \mathrm{N/m^2}, 0.3)$

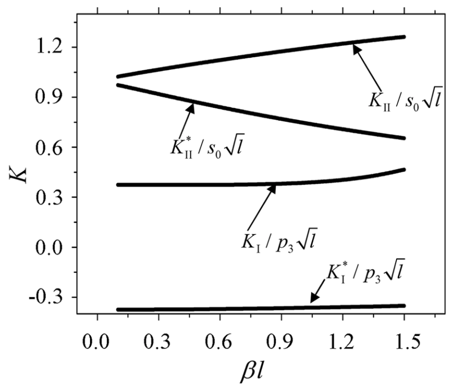

Fig. 4 Stress intensity factors versus βl for $\sigma_0(x) = p_3\left(\frac{x}{l}\right)^3$, $\tau_0(x) = s_0$ and $(\mu_0^{(1)}, \eta^{(1)}) = (\mu_0^{(2)}, \eta^{(2)}) = (66.5 \times 10^9\ \mathrm{N/m^2}, 0.3)$

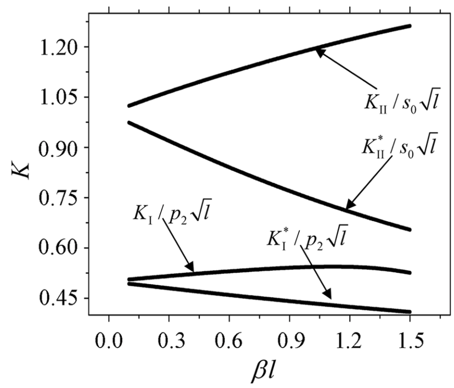

Fig. 5 Stress intensity factors versus βl for $\sigma_0(x) = p_2\left(\frac{x}{l}\right)^2$, $\tau_0(x) = s_0$ and $(\mu_0^{(1)}, \eta^{(1)}) = (\mu_0^{(2)}, \eta^{(2)}) = (66.5 \times 10^9\ \mathrm{N/m^2}, 0.3)$

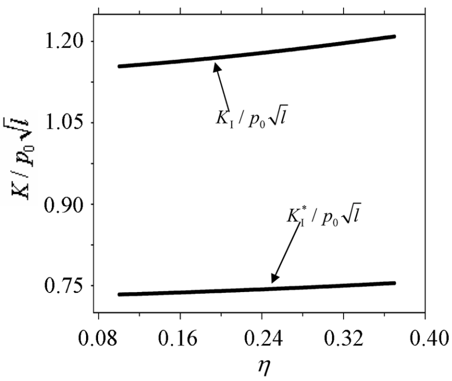

Fig. 6 Stress intensity factors versus η for $l = 1.0$, $\sigma_0(x) = p_0$, $\tau_0(x) = 0$ and $\mu_0^{(1)} = \mu_0^{(2)} = 66.5 \times 10^9\ \mathrm{N/m^2}$, $\eta^{(1)} = \eta^{(2)} = \eta$

ones in Delale and Erdogan (1983) when $(\mu_0^{(1)}, \eta^{(1)}) = (\mu_0^{(2)}, \eta^{(2)})$ as shown in Fig. 2. It is also proved that the Schmidt method is performed satisfactorily. In the paper (Delale and Erdogan 1983), the unknown variables of dual integral equations are the dislocation density functions. However, the case of $(\mu_0^{(1)}, \eta^{(1)}) \neq (\mu_0^{(2)}, \eta^{(2)})$ was not considered in Delale and Erdogan (1983).

(ii) When the material properties are not continuous along the crack line, an approximate solution of the interface crack problem is given as shown in Table 1 to Table 2 under the assumption that the effect of the crack surface interference very near the crack tips is negligible (From practical view points, researchers in the field of functionally graded materials will not pay their attention in this case). The solving process is quite different from the other works such as in (Erdogan and Wu 1997, Lee and Erdogan 1994, Suresh and Mortensen 1977, Choi 2001, Delale and Erdogan 1988, Chen 1990, Ozturk and Erdogan 1996, Jin and Batra 1996, Bao and Cai 1997, Shbeeb and Binienda 1999, Wang *et al*. 2000, Erdogan and Wu 1996, Delale and Erdogan 1983). It can be obtained that the stress singularities of the present paper are the same as ones of the ordinary crack in homogeneous materials when the material parameters don't continue through the crack line. During the solving process for this case, the mathematical difficulties would not be met, i.e., the oscillatory stress singularity and the overlapping of the crack surfaces do not appeared near the interface crack tips, while much problems have to be considered.

(iii) It can be obtained that the shear stress field is independent of the tension loading when $(\mu_0^{(1)}, \eta^{(1)}) = (\mu_0^{(2)}, \eta^{(2)})$ from the results in Fig. 3 to Fig. 5. However, the shear stress field is dependent of the tension loading when $(\mu_0^{(1)}, \eta^{(1)}) \neq (\mu_0^{(2)}, \eta^{(2)})$ as shown in Table 1 to Table 2.

(iv) From the results as shown Fig. 2 to Fig. 5, it can be obtained that the values of the stress intensity factors vary approximately linearly with the variable βl for a uniformly loading. The values of the stress intensity factors K_{I} and K_{II} tend to increase with increasing of βl as shown

Table 1 The values of the stress concentration K versus βl for the case of $(\mu_0^{(1)}, \eta^{(1)}) \neq (\mu_0^{(2)}, \eta^{(2)})$ and $l = 1.0$ under crack surface loading $\sigma_0(x) = p_0$ and $\tau_0(x) = s_0$ ($\mu_0^{(1)} = 77 \times 10^9$ N/m^2, $\eta^{(1)} = 0.28$, $\mu_0^{(2)} = 66.5 \times 10^9$ N/m^2 and $\eta^{(2)} = 0.3$)

βl	$K_{\mathrm{I}}/p_0\sqrt{l}$ (the right tip of the crack)	$K_{\mathrm{II}}/s_0\sqrt{l}$ (the right tip of the crack)	$K_{\mathrm{I}}^*/p_0\sqrt{l}$ (the left tip of the crack)	$K_{\mathrm{II}}^*/s_0\sqrt{l}$ (the left tip of the crack)
0.1	1.02500	1.02608	0.975972	0.974991
0.2	1.04856	1.04902	0.949966	0.949902
0.3	1.07096	1.07212	0.923700	0.922808
0.4	1.09255	1.09375	0.897719	0.896868
0.5	1.11343	1.11469	0.872256	0.871447
0.6	1.13366	1.13497	0.847452	0.846682
0.7	1.15329	1.15465	0.823406	0.822673
0.8	1.17235	1.17376	0.800188	0.799492
0.9	1.19086	1.19233	0.777848	0.777186
1.0	1.20886	1.21040	0.756413	0.755783
1.1	1.22634	1.22795	0.735895	0.735295
1.2	1.24329	1.24498	0.716289	0.715719
1.3	1.25965	1.26143	0.697583	0.690740
1.4	1.27531	1.27720	0.679755	0.679237

Table 2 The values of the stress concentration K versus βl for the case of $(\mu_0^{(1)}, \eta^{(1)}) \neq (\mu_0^{(2)}, \eta^{(2)})$ and l = 1.0 under crack surface loading $\sigma_0(x) = p_3\left(\frac{x}{l}\right)^3$ and $\tau_0(x) = s_0$ ($\mu_0^{(1)} = 66.5 \times 10^9$ N/m^2, $\eta^{(1)}$ = 0.3, $\mu_0^{(2)} = 77 \times 10^9$ N/m^2 and $\eta^{(2)}$ = 0.28)

βl	$K_{\mathrm{I}}/p_3\sqrt{l}$ (the right tip of the crack)	$K_{\mathrm{II}}/s_0\sqrt{l}$ (the right tip of the crack)	$K_{\mathrm{I}}^*/p_3\sqrt{l}$ (the left tip of the crack)	$K_{\mathrm{II}}^*/s_0\sqrt{l}$ (the left tip of the crack)
0.1	0.383740	1.02321	–0.388456	0.971727
0.2	0.382271	1.04488	–0.389018	0.944447
0.3	0.380751	1.06535	–0.389139	0.917096
0.4	0.379314	1.08501	–0.388840	0.890211
0.5	0.378117	1.10400	–0.388149	0.864016
0.6	0.377358	1.12239	–0.387097	0.838637
0.7	0.377282	1.14027	–0.385721	0.814155
0.8	0.378201	1.15771	–0.384056	0.790624
0.9	0.380513	1.17476	–0.382138	0.768076
1.0	0.384724	1.19148	–0.380002	0.746522
1.1	0.391486	1.20792	–0.377679	0.725990
1.2	0.401632	1.22409	–0.375199	0.706373
1.3	0.416227	1.23999	–0.372586	0.687730
1.4	0.436632	1.25555	–0.369865	0.670027
1.5	0.464584	1.27064	–0.367056	0.653202

in Fig. 2 to Fig. 5. However, the values of the stress intensity factors K_{I}^* and K_{II}^* decrease with increase of βl.

(v) The effect of Possion's ratio on the stress intensity factors in plane strain is shown in Fig. 6. The result is shown that this effect is rather insignificant.

(vi) From the results, it can be obtained that the Schmidt method can be used to solve the mix boundary crack problem.

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