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Effect of varying the size of flatbar stiffeners on the buckling behaviour of thin cylinders on local supports

Wesley Vanlaere†, Rudy Van Impe‡, Guy Lagae‡ and Thomas Maes‡†

Laboratory for Research on Structural Models, Ghent University, Technologiepark-Zwijnaarde 904, B-9052 Zwijnaarde, Belgium

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Abstract. A steel silo traditionally consists of a cylindrical and a conical shell. In order to facilitate emptying operations, the cylinder is placed on local supports. This may lead to dangerous stress concentrations and eventually to local instability of the cylindrical wall. In this contribution, the locally supported cylinder is strengthened by means of ring stiffeners and longitudinal stiffeners and the effect of their dimensions on the buckling stress is investigated. This study leads to a number of diagrams, each of them representing the effect of one of the dimensions on the buckling stress. In each diagram, the failure pattern corresponding to the buckling stress is indicated.

Key words: buckling; cylinders; local supports; stiffeners.

1. Introduction

In the field of structural engineering, a number of complex structures still need extensive research before their structural behaviour can be fully understood and the necessary design rules can be developed. With the advent of finite element programs and powerful computers with increasing capacity, some of these more complex structures can be investigated nowadays.

An important class of structures which are the object of ongoing research are thin-walled steel shells. Shell structures like water towers, storage tanks, silos, etc. are frequently used in the field of engineering because of their efficient shape. They appear to be simple structures, but quite often their thin wall is prone to local instability phenomena.

The behaviour of cylindrical shell structures has been investigated extensively in the past. Timoshenko (1936), Koiter (1945) and Flügge (1973) are but a few of the many excellent explorers of the vast area of structural stability who studied the buckling behaviour of circular cylinders subjected to uniform axial compression. These investigations led to the well-known expression for the critical stress σ_{cr} in an elastic cylinder with perfect geometry:

$$\sigma_{cr} = 0{,}605E\frac{t}{r}$$

† Scientific co-worker, Corresponding author, E-mail: Wesley.Vanlaere@UGent.be
‡ Professor
‡† Graduate Student

Where E is Young's modulus of elasticity and t and r denote the wall thickness and the radius of the cylinder respectively.

Experiments showed however that the failure stress was usually but a fraction of the one obtained with this analytical expression. Further investigation showed that the discrepancy between theory and observation was due to the presence of unavoidable imperfections of the cylinder geometry.

The outcome of this research gave birth to design rules which can be found in codes and standards, e.g. EUROCODE 3, part 1.6 (2004). Despite the tremendous efforts, the range of application of these design regulations is often restricted to structures with rather simple geometry and loading, e.g. circular cylinders under uniform axial compression.

In many practical situations, the design is far more complex and is often assisted by finite element computations. An example is a steel silo. Usually, this compound shell structure consists of two basic units: an elevated, upper cylindrical shell containing the bulk material and a lower portion with a conical shape in order to facilitate emptying operations. A limited number of columns support the cylinder in the transition zone between cylinder and cone. When the silo is filled, axial loads introduced above the column terminations is one of the main issues of design (Guggenberger 1992), and the assumption of a uniformly supported cylinder is no longer valid.

The effect of these local supports on the buckling behaviour of unstiffened cylinders was investigated by Guggenberger, who drafted design rules for this kind of cylinders. A reduction of the necessary amount of steel can be obtained by stiffening the cylinder in the area where the stress concentrations occur, i.e. the area above the local supports. In a proposition made by Rathé and Greiner (1996), the wall thickness of the lower part of the cylinder was increased.

Another and possibly better material saving solution consists of strengthening the cylinder by means of flatbar longitudinal stiffeners and ring stiffeners. Above each column termination, two longitudinal stringers with limited length are welded to the shell wall and at the same time upper and lower flatbar ring stiffeners skirt the cylinder at the stringer terminations. In this way, the extra material is placed in the areas where the danger of local instability is most pronounced.

Appropriate design rules for this kind of stiffening configuration are not available to date in the structural codes. It is the aim of the present ongoing research to develop such design rules. The research is conducted by performing numerical simulations with the finite element program ABAQUS (2003). The numerical model is validated by means of experiments on scale models. In the first phase of the research, the influence of the dimensions of the stiffeners on the buckling stress and the failure pattern was investigated. In the second phase, the design rule will be developed. The first phase has now been completed and the results will be presented in this contribution.

2. Locally supported cylinder

2.1 General

The main components of a steel silo are the cylindrical and conical shell. At the joint between these two shells, the entire structure rests on a limited number of supports. The principal advantage of this elevated silo is the accessibility of the silo during emptying operations.

When the silo is filled, the cylindrical shell is subjected to internal pressure in combination with vertical friction along the wall. In a numerical model, this vertical friction forces can be replaced by axial compression. The internal pressure can be included in the numerical simulation, but studies

(Guggenberger 2004) showed that this pressure usually is favourable with respect to the buckling strength. Therefore, this internal pressure is frequently omitted in an analysis of a steel silo (Herzig and Thiele 1997) and so for this study, the pressure will be neglected.

The loading of the silo causes axial compressive stresses to appear in the locally supported cylinder and stress concentrations are built up in the cylindrical shell near the supports. These stresses can cause local instability of the slender wall, which can lead to the collapse of the entire structure.

A possible solution to increase the buckling resistance of these locally supported cylinders involves the use of twin stringers above each support. Since the stress concentrations are situated near the supports, the height of the stringers is usually limited to a fraction of the cylinder height. The stringers are placed between two ring stiffeners. An example of such a stiffened cylinder can be seen in Fig. 1. If the cylinder is unstiffened, the buckles appear just above the supports. In Fig. 2, an eighth of such an unstiffened cylinder is modelled and symmetry boundary conditions are imposed to include the effect of the omitted part of the cylinder. The buckle appears above the support as mentioned.

If the cylinder is stiffened, the presence of these stiffeners increases the buckling resistance. However, if the buckling resistance of the stiffened area is too low, buckling can still occur in the

Fig. 1 A thin-walled cylinder with longitudinal stiffeners and ring stiffeners

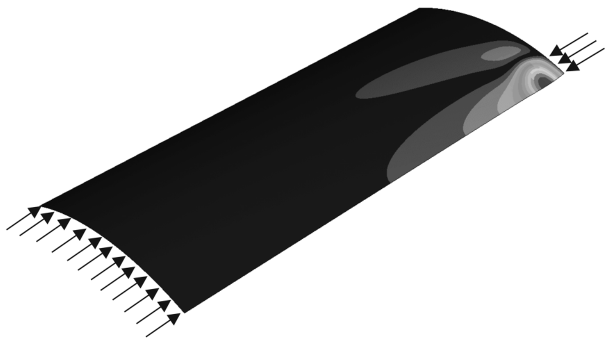
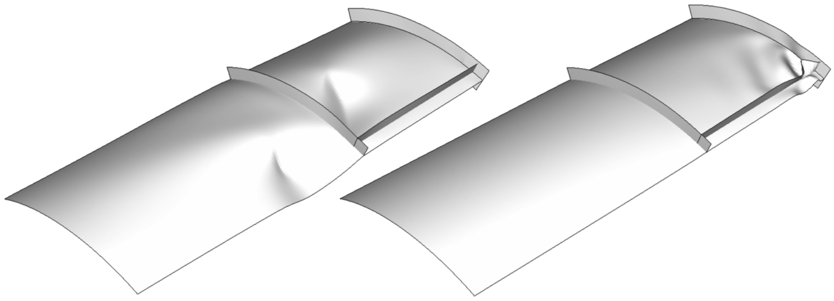


Fig. 2 Deformation of an unstiffened cylinder



(a) (b)

Fig. 3 (a) A buckle appears above the upper ring, followed by a buckle below this ring, (b) Buckles above the lower ring and in the longitudinal stiffener

stiffened zone, but at a higher load than in the unstiffened case. When the buckling resistance is sufficient, buckling occurs above the upper ring (Herzig and Thiele 1997). These two distinct failure patterns are shown in Fig. 3. The buckling resistance and failure pattern depend on the dimensions of the stiffeners. The influence of these dimensions on the buckling resistance is the subject of this investigation.

2.2 Dimensions of cylinder and stiffeners

Consider a cylinder with the dimensions shown in Table 1. The slenderness of the cylinder r/t equals 500. We use the dimensionless support width ratio (Guggenberger 2004) $\mu = n_{sup} \cdot w_{sup}/(2\pi r)$ which equals 0.146 for this cylinder. In this equation, n_{sup} is the number of supports and is equal to 4. The influence of n_{sup} on the structural behaviour can be assumed to be negligible, since Guggenberger showed that the buckling stress at a support of an unstiffened cylinder is independent of the number of supports.

The cylinder is stiffened with stringers and ring stiffeners whose dimensions are indicated in Tables 2, 3. The choice of these values is explained in Van Impe *et al.* (2001).

Table 1 The global dimensions of the cylinder

Cylinder	Dimensions [mm]
Radius r	350
Length L	700
Wall thickness t	0.7
Support width w_{sup}	80

Table 2 The dimensions of the longitudinal stiffeners

Stringers	Dimensions [mm]
Width w_s	20
Height h_s	280
Spacing d_s	37
Thickness t_s	1.5

Table 3 The dimensions of the ring stiffeners

Ring stiffeners	Dimensions [mm]
Width lower ring w_{lr}	45
Width upper ring w_{ur}	20
Thickness lower ring t_{lr}	0.7
Thickness upper ring t_{ur}	1.5

3. Experiments on scale models

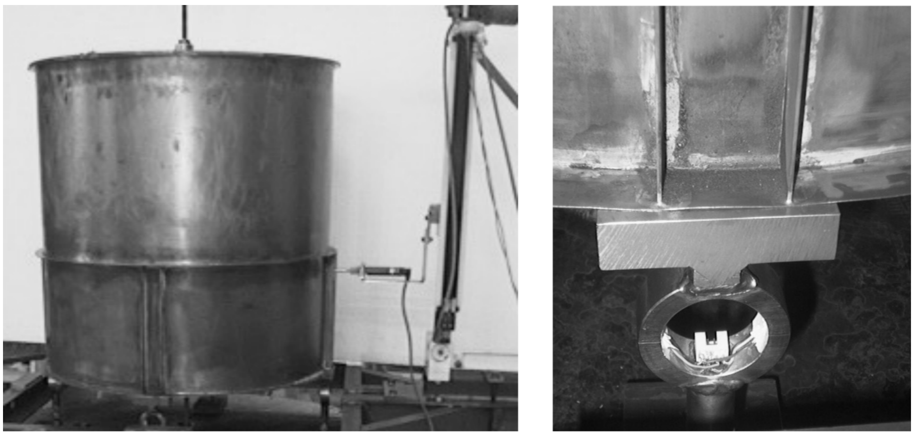
3.1 Test set-up

An important aspect of this research is the experimental part. Scale models of locally supported cylinders are tested, and the results are used to gain insight into the structural behaviour as well as to validate the numerical model.

The scale models rest on 4 support plates and a cover plate is placed on top of the cylinder as can be seen in Fig. 4. The axial compression in the cylinder is introduced by means of a hydraulic jack and a tie rod. The force in each support is measured by means of a dynamometer.

Since imperfections can play a major role in the behaviour of shells, the geometry of every scale model is measured before testing. The device that was used for this purpose can be seen in Fig. 4(a).

More information concerning the test set-up can be found in the work of Van Impe *et al.* (2001).

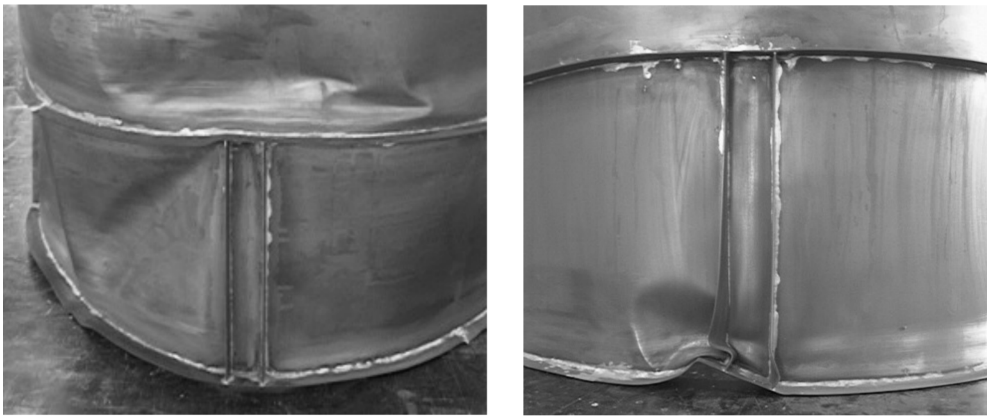


(a) (b)

Fig. 4 (a) The test set-up with the device to measure the imperfections, (b) A detail of a support plate and a dynamometer

3.2 Results

Up to now, a limited number of tests on similar scale models have been performed. The results of these tests show that there are two distinct failure mechanisms possible. The occurrence of a certain



(a) (b)

Fig. 5 (a) The failure pattern *cylinder* and (b) The failure pattern *stiffener*

failure mechanism is governed by the geometrical imperfections, the imperfections in the test set-up and the characteristics of the stiffening.

The first failure mechanism is characterised by the appearance of buckles in the cylindrical wall above the upper ring. As a consequence of this instability, secondary buckles can be formed just below this upper ring. In this contribution, this failure pattern will be referred to as *cylinder.* The pattern can be seen in Fig. 5(a).

The second failure mechanism has a failure pattern with buckles in the longitudinal stiffeners and in the cylindrical wall, just above the support plate. This failure pattern will be referred to as *stiffener.* An example of such a pattern is shown in Fig. 5(b).

4. The numerical model

4.1 General

This study deals with two main issues: experiments on scale models and numerical simulations. The experiments are described in the previous section. In this section, the focus is on the numerical modelling.

The numerical simulation of the stiffened shell is performed with the commercial FE package ABAQUS. In order to save valuable CPU-time, only a segment of 45° of the cylinder is modelled and symmetric boundary conditions are imposed. In the past, investigations were carried out to determine if this reduction of the model – which can restrict the deformation patterns – has a severe influence on the numerical results. The study showed that this is not the case and that the numerical simulations may be performed with the reduced model (Vanlaere *et al.* 2003b).

These considerations lead to the symmetric conditions that were chosen for the longitudinal edges of the model. The boundary conditions for the top and bottom part of the model have yet to be stated.

In the literature, two main possibilities are described for the modelling of the local force

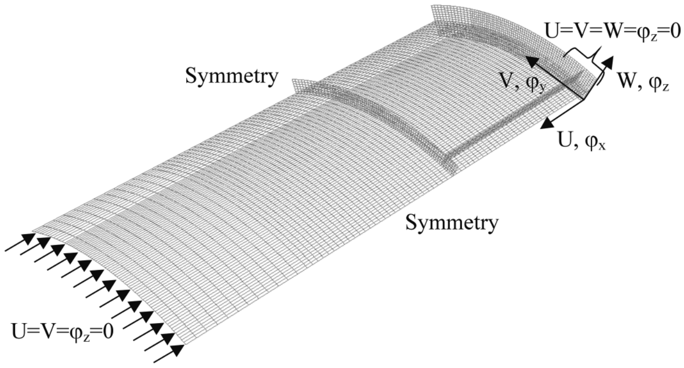


Fig. 6 The numerical model with the applied boundary conditions for the case “rigid supports”

introduction at the supports (Guggenberger 2004). A first possibility is modelling the supports as axially flexible. This means that the support causes uniform axial stress along the support width.

The second possibility is modelling the supports as axially rigid. This means that at the support no deformations in the axial direction are allowed and that the axial loads are applied uniformly at the top of the cylinder.

The assumption of flexible supports is the most conservative one, leading to the lowest strengths. However, the behaviour of actual supports will be somewhere in between the behaviour of flexible and rigid supports. Experiments show that the behaviour of actual supports corresponds best with rigid supports, and therefore rigid supports will be used for this first phase of the research.

For the second phase – the development of the design rule – it is important not to overestimate the strength of the cylinders and therefore flexible supports will be used in that phase. An overview of the boundary conditions for the model of the cylinder on local supports is given in Fig. 6.

Fig. 6 also shows the mesh of the model. This mesh is dense in the region near the supports and coarser elsewhere. In Abaqus, the material properties have to be inserted and typical values for steel are assumed, i.e. a Young’s modulus of 200 GPa and a Poisson’s ratio of 0.3. Tensile tests were performed on the steel plates that were used for the experiments. The obtained stress-strain curve for the plate of 0.7 mm had a 0.2% proof stress of 170 MPa. The stress-strain curve of a plate of 1.5 mm thickness corresponded to a 0.2% proof stress of 215 MPa. These non-linear stress-strain curves were used to define the material behaviour of the numerical model. For more information concerning the influence of these material properties, refer to Vanlaere *et al.* (2003a).

4.2 Comparison of results

For structural analyses of shells, imperfections usually play a major role. Therefore, the type of FE analysis that gives the best correspondence to the real behaviour of the shell is a geometrically and materially non-linear analysis with imperfections included (GMNIA) (EN 1993-1-6 2004). As imperfection shape, we take an eigenmode of the perfect shell. The failure patterns resulting from GMNIA analyses can be classified into two types. In the first, buckles appear above the upper ring as can be seen in Fig. 3(a). The second type of failure pattern is characterised by buckles just above the bottom ring. This pattern is depicted in Fig. 3(b). The same two failure patterns can be observed if the imperfections are omitted (geometrically and materially non-linear analysis – GMNA).

When comparing these numerical failure patterns to the failure patterns observed in the tests on the scale models, it is clear that the resemblance is good. In Vanlaere *et al.* (2003b) the failure loads and patterns of a number of GMNIA analyses are compared with the loads and patterns of corresponding experiments. One of the conclusions of this study is that the correspondence between the numerical model and experiments with scale models is very good. Furthermore, similar numerical calculations have been performed by Herzig and Thiele (1997) and in that study, the same failure patterns were observed.

5. Influence of the dimensions and position of the longitudinal stiffeners

With this numerical model of the stringer stiffened cylinder on local supports, the influence of some parameters can be examined. In this section, the influence of the dimensions and position of the longitudinal stiffeners is investigated. The parameters that are studied are:

- Height of the stringers h_s;
- Width of the stringers w_s;
- Thickness of the stringers t_s;
- Spacing of the stringers d_s.

When one of these parameters is investigated, the values of the other dimensions remain constant (see Tables 2 and 3).

For this study, a large number of analyses were executed. In the analyses, perfect shells were studied. If for instance an eigenmode is included as imperfection shape, then the influence of this imperfection would change with the variation of the stringer dimensions. Since only the influence of the stringer dimensions is investigated, the parametrical study is performed with GMNA analyses.

In addition, the material properties of the numerical model were altered. In order to validate the model, the material properties of the steel plates used to manufacture the scale models had to be included in the simulations. Tensile tests on these plates showed that these plates had a relatively low 0.2% proof stress. For the parametrical study, we will assume that the cylinder and the stiffeners are made out of steel with an ideal elastic-plastic material behaviour and a yield stress of 235 MPa. For the results of a similar parametrical study with the material properties of the steel plates of the scale models, refer to Vanlaere *et al.* (2003c).

The results of these GMNA analyses will be presented in a graph with the dimension of the varied parameter on the horizontal axis and the dimensionless mean axial buckling stress at a support on the vertical axis. This buckling stress equals the ratio σ_u/σ_{cr}, with σ_u the mean axial stress at a support at the time of buckling:

$$\sigma_u = \frac{F_u/4}{w_{\text{sup}} \cdot t + 2 \cdot w_s \cdot t_s}$$

and with F_u the failure load. The critical stress σ_{cr} for a cylinder with radius r = 350 mm and wall thickness t = 0.7 mm is equal to 242 MPa.

5.1 Height of the stringers

In this first subsection, the influence of the height of the stringers on the buckling strength and the failure pattern is investigated. A set of 13 GMNA calculations with different h_s-values were

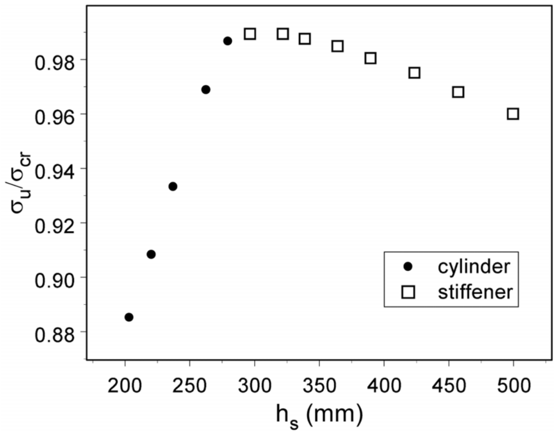


Fig. 7 The influence of the stringer height on the dimensionless buckling stress

performed. In Fig. 7, the results of these calculations are shown. The graph shows the influence of the stringer height on the dimensionless buckling stress. This influence can be approximated by two straight lines. The first line is formed from the results where buckles appear above the upper ring, i.e. the failure pattern *cylinder* of Fig. 3(a). The second line, which starts from a height of 300 mm, represents a failure pattern of the type *stiffener* as depicted in Fig. 3(b).

This bilinear diagram can be understood quite easily. Without stiffeners, the failure pattern of a cylinder on local supports exhibits buckles just above the supports (as can be seen in Fig. 2). When stringers are attached to the cylinder, the buckling strength of the stiffened region increases, leading to a higher failure load of the entire cylinder and therefore a higher buckling stress at the supports. When the height of the stringers is small, the buckling strength of the stiffened region is large enough to force the buckles to appear above the upper ring. This shift of the buckles causes an increasing buckling stress with increasing stringer height. However, the augmentation of the stringer height causes a decrease in the buckling strength of the stiffened region. For stringer heights above 300 mm, the buckling strength of the stiffened region is too low to prevent buckling in this region and the failure type *stiffener* appears. A further increase in the stringer height decreases the buckling stress.

5.2 Width of the stringers

The influence of the stringer width was investigated by performing 10 GMNA calculations. The results of this study are shown in Fig. 8. In this figure, the results are categorised into three failure patterns. The first failure pattern exhibits buckles in the lower half of the cylinder. This failure pattern appears for very small stringer widths. The failure pattern *stiffener* appears when the stringer width is increased, but the buckling strength of the stiffened area is too small to prevent buckles in this region. When the width equals 20 mm, the buckling strength of this region is large enough to force the buckles to appear above the upper ring. This is the failure pattern *cylinder.*

Fig. 8 not only shows the effect of the stringer width on the failure pattern, but also on the buckling stress. The graph shows that an increase of the stringer width leads to a decrease of the buckling stress. This seems illogical, certainly for small stringer widths. The explanation for this

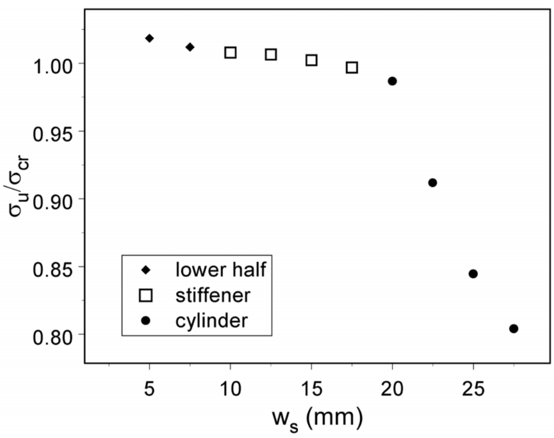


Fig. 8 The influence of the stringer width on the dimensionless buckling stress

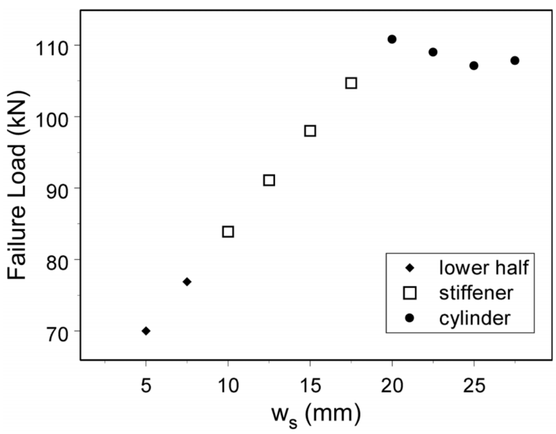


Fig. 9 The influence of the stringer width on the failure load

decrease lies in the fact that the stringer width has an influence on the numerator and the denominator of the expression for the buckling stress σ_u. Fig 9 shows the same results, but here the dimensionless buckling stress on the vertical axis is replaced by the failure load. In this graph, the failure load increases with increasing stringer width till the buckles are forced to appear above the upper ring. From then on, the increase of the width only has a small impact on the failure load.

5.3 Thickness of the stringers

In this subsection, the effect of the thickness is examined. The results of 9 GMNA calculations can be seen in Fig. 10. A thin stringer doesn't sufficiently reinforce the lower side of the cylinder and allows the appearance of buckles above the lower ring, i.e. failure pattern *stiffener*. If the

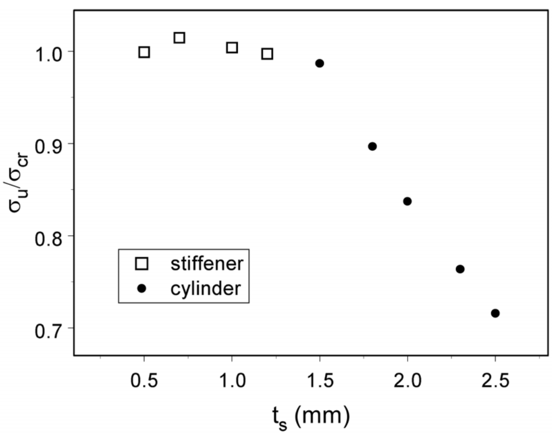


Fig. 10 The influence of the stringer thickness on the dimensionless buckling stress

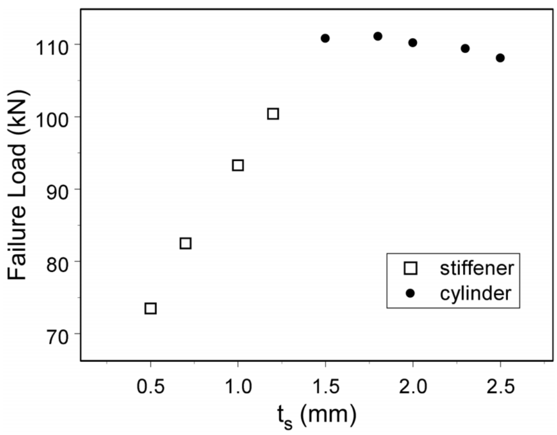


Fig. 11 The influence of the stringer thickness on the failure load

thickness of the stringers increases, the buckling strength of the stiffened area increases. For a thickness of 1.5 mm, this buckling strength is sufficient to cause the failure pattern *cylinder*.

Similar to the effect of the stringer width, an increase in the thickness leads to a decrease of the buckling stress. Again the explanation can be found in the appearance of the stringer thickness in the denominator of the expression of the buckling stress. In Fig. 11, the buckling stress on the vertical axis is replaced by the failure load of the cylinder. This figure shows that an increase in the stringer thickness leads to an increase in the failure load as long as the failure pattern is characterised by buckles in the stiffened region. When the buckles are forced to appear outside this region, a further increase in the thickness doesnt substantially influence the failure load.

5.4 Spacing of the stringers

In this final subsection, the effect of the spacing of the two stringers above a support is investigated. The results of 9 GMNA calculations are presented in Fig. 12. For closely spaced stringers, the buckles are forced to appear outside the stiffened region. The failure pattern can be classified as *cylinder*. When the distance between the stringers gets larger than 49.2 mm, the buckling of the cylinder and the stiffeners above the support can't be avoided and the failure pattern changes to a pattern of the type *stiffener*. A further increase of the spacing leads to a decreasing buckling stress. When the stringers are placed outside the supported area (d_s > 80 mm), the stringers lose a large part of their functionality and the buckling stress decreases rapidly.

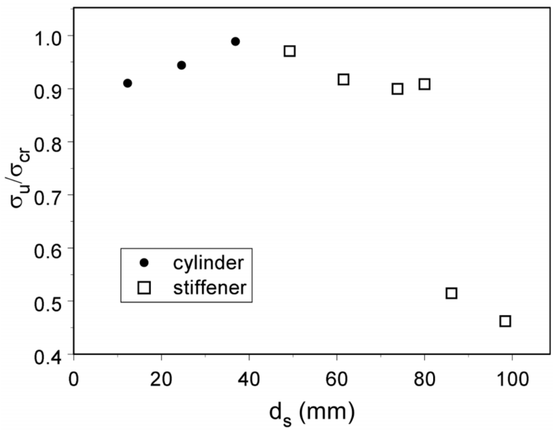


Fig. 12 The influence of the spacing on the dimensionless buckling stress

6. Influence of the dimensions of the ring stiffeners

In the previous section, the influence of the dimensions of the stringers were investigated. The cylinder, however, is not only stiffened by means of these stringers, but above and beneath the stringers, ring stiffeners are placed. These ring stiffeners also play an important role in the structural behaviour of the cylinder.

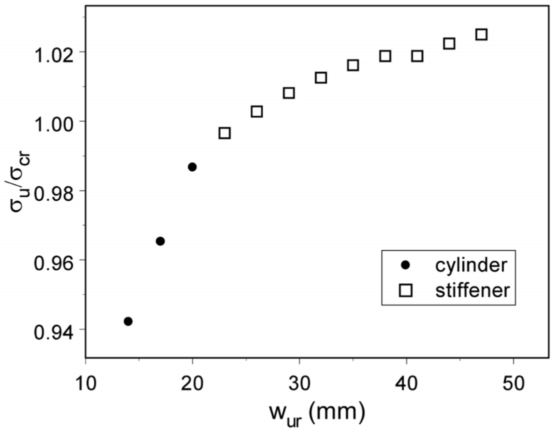


Fig. 13 The influence of the width of the upper ring on the dimensionless buckling stress

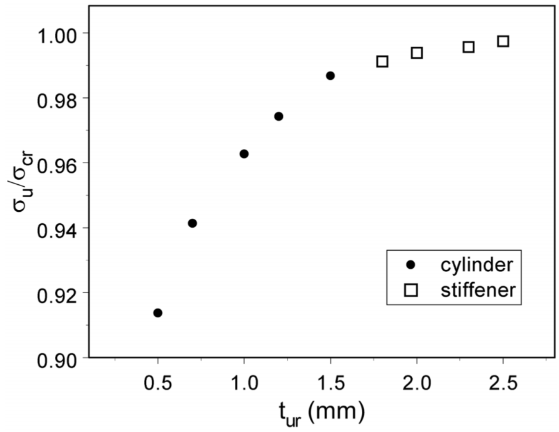


Fig. 14 The influence of the thickness of the upper ring on the dimensionless buckling stress

6.1 The dimensions of the upper ring

For the upper ring, the effect of the width and the thickness on the buckling stress and the failure pattern is studied. The results of the study on the width are shown in Fig. 13. In this graph, the buckling stresses of 12 GMNA calculations form a smooth curve. For narrow rings, the failure pattern of the cylinder is characterised by buckles above this upper ring. When the width of the ring is increased, the buckling resistance of the region near the upper ring increases, which leads to increasing buckling stresses. When the ring width surpasses 23 mm, the buckling resistance near the upper ring is sufficient to change the failure pattern. For wide rings, the buckles appear above the lower ring. However, an increase in the ring width still increases the buckling stress.

In Fig. 14, the effect of the ring thickness is displayed. The results of 9 simulations lead again to a fluent curve. For thin rings, the buckling resistance of the region near the upper ring is not sufficient to prevent the failure pattern *cylinder*. With increasing ring thickness, this buckling resistance increases which leads to increasing buckling stresses. Finally, for rings with a thickness of 1.8 mm or more, the failure pattern changes and the buckles are forced to the area above the lower ring.

6.2 The dimensions of the lower ring

Also for the lower ring, the width and the thickness are the parameters that will be varied. In Fig. 15, the results of 9 GMNA simulations with different ring widths are shown. These results demonstrate that a narrow lower ring leads to the failure mechanism *stiffener*. An increasing width leads to an increasing buckling stress. When the change in ring width has increased sufficiently the buckling resistance of the region above this ring, the buckles are forced to appear above the upper ring. When this failure pattern is dominant, a change in ring width has no significant influence on the buckling stress.

Fig. 16 shows the results for the study of the ring thickness. Very thin rings (i.e. a thickness of 0.5 mm) lead to the failure pattern *stiffener*. Thicker rings prevent this failure pattern and the buckles

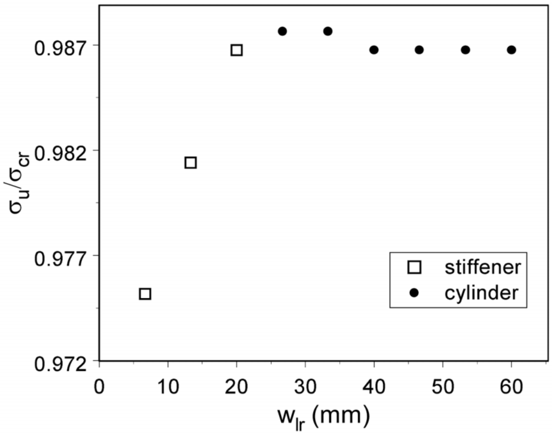


Fig. 15 The influence of the width of the lower ring on the dimensionless buckling stress

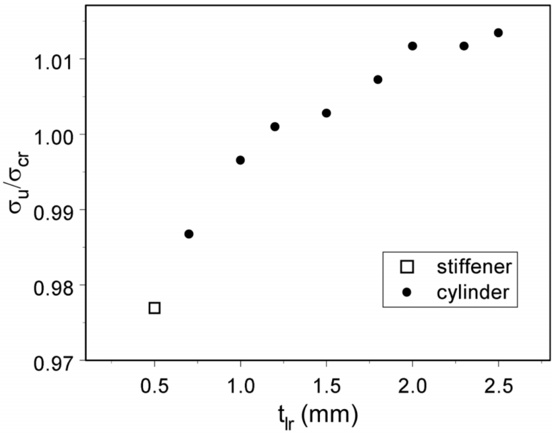


Fig. 16 The influence of the thickness of the lower ring on the dimensionless buckling stress

are shifted near the upper ring. However, an increase in the thickness still leads to increasing buckling stresses.

The two figures show that the effect of the dimensions of the lower ring on the buckling stress is less pronounced than for the other dimensions investigated. The presence of the lower ring is however important because this ring guarantees in practical situations the circularity of the lower rim of the cylinder.

7. Conclusions

With this contribution, the first phase of a study has been reported. The influence of the dimensions of the stiffeners on the buckling stress and failure mechanism of stringer stiffened cylinders has been investigated. This study showed that the dimensions have a significant effect on the structural behaviour of a locally supported cylinder. Not only is the influence on the buckling stress very pronounced (especially for the stringer dimensions), but they also determine whether local instability appears above the upper ring in the cylindrical wall (failure mechanism *cylinder*) or above the lower ring in the cylinder and the stringers (failure mechanism *stiffener*).

This investigation will continue with a second phase. In this phase, a large number of parametrical studies with the validated numerical model will be performed, so that a design rule for stringer stiffened cylinders on local supports can be developed.

Acknowledgements

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