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Buckling lengths of unbraced multi-storey frame columns

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Abstract. In several design codes and specifications, simplified formulae and diagrams are given for determining the buckling lengths of frame columns. It is shown that these formulae may yield rather erroneous results in certain cases. This is due to the fact that, the code formulae utilise only local stiffness distributions. In this paper, a simplified procedure for determining approximate values for the buckling loads of multi-storey frames is developed. The procedure utilises lateral load analysis of frames and yields errors in the order of 10%, which may be considered suitable for design purposes. The proposed procedure is applied to several numerical examples and it is shown that all the errors are in the acceptable range and on the safe side.

Key words: buckling load; buckling length; effective length; sway mode; unbraced frames; isolated subassembly; multi-storey frames; design codes.

1. Introduction

Determining the buckling (effective) lengths of frame columns is one of the significant phases of frame design. Theoretically, buckling length of an individual column is determined by calculating the system-buckling load of the frame. Since a full system instability analysis, may be quite involved for frames met in practical applications, simplified formulae and diagrams are given for determining the buckling lengths of frame columns in most of the design codes and specifications, (AISC 1988, ACI 1989). The so-called "Isolated subassembly approach" of specifications has been originally developed by Galambos (1968). Similar formulae and diagrams exist in other widely applied specifications such as and DIN 18800 (1990) and Eurocode 3 (2002).

A major limitation of the methods based on isolated subassembly approach is that they do not properly recognise the interaction effects of adjacent elements other than the ones at immediate neighbourhood of the joints. Hellesland and Bjorhovde (1996) have showed that this approach may result in significant errors in certain cases. Efforts to improve the applicability of subassembly approach include modifications proposed by Duan and Chen (1988, 1989) and an iterative procedure developed by Bridge and Fraser (1987). Another method of improvement for unbraced frames is the so-called "Storey buckling approach" which accounts for the horizontal interaction between columns

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in a storey (Yura 1971, LeMessurier 1977). White and Hajjar (1997) have showed that this approach may result in significant errors in unsymmetrical cases. Storey buckling approach has been the subject of several papers, among which Lui (1992), Aristizabal-Ochoa (1997) and Cheong-Siat-Moy (1999) may be stated. The works of Aristizabal-Ochoa and Cheong-Siat-Moy provide solutions for both braced and unbraced frames as well as "Partially braced frames". Aristizabal-Ochoa has further extended his studies to cover three-dimensional structures (Aristizabal-Ochoa 2002, 2003). Another interesting improvement approach is proposed by Hellesland and Bjorhovde (1997), which involves a postprocessing procedure using weighted mean values of buckling lengths. It has been stated that, it is necessary to consider a wider range of unbraced frames in order to confirm the practical applicability of the proposed method. Recently, in AISC (1999), the isolated subassembly approach has been abandoned and it has been stated that "…the effective length factor K of compression members shall be determined by structural analysis." However in several widely used codes (such as Eurocode 3) the subassembly approach and related charts and formulae are still being used.

In this study, a practical method is developed for determining the buckling lengths of columns in unbraced frames. The method is based on computing an approximate value for system buckling load by using the results of a fictitious lateral loading.

2. System buckling load of unbraced multi-storey frames

A multi-storey frame which is composed of beams and columns made of linear elastic material is under the effect of vertical loads as shown in Fig. 1(a).

Each axial load may be expressed as

$$N_{ij} = n_{ij}P \tag{1}$$

where n_{ij} is a dimensionless coefficient and P is an arbitrarily chosen load parameter. The frame is

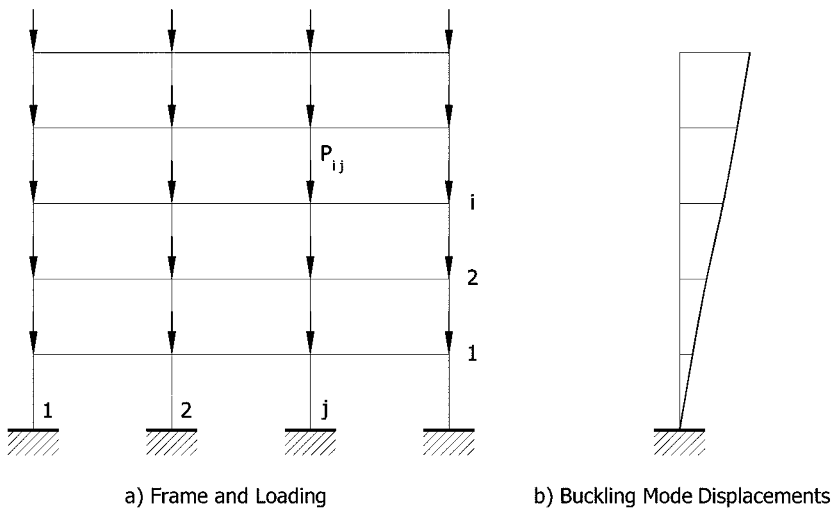

Fig. 1 Multi storey frame and buckling mode

in the state of “Stabile Equilibrium” and, if the axial deformations are neglected, all the displacements and deformations are zero. Internal forces of the frame columns consist of only axial forces $N_{i,j}$ while all the internal forces of beams are zero. However, when the load parameter reaches to a critical P_{cr} value, another state of “Unstable Equilibrium” may exist. The lateral displacement diagram corresponding to this new state, which is shown schematically in Fig. 1(b), is called the “Buckling Mode” of the structure (Horne and Merchant 1965). Once the buckling load parameter P_{cr} is determined, the buckling length s_{ij} of an individual column can be computed by

$$s_{ij} = \pi \sqrt{\frac{EI_{ij}}{n_{ij} P_{cr}}} \tag{2}$$

where EI_{ij} is the bending stiffness of the column.

In certain simple cases, buckling load parameter may be determined by using the so-called stability functions (Horne and Merchant 1965). For general cases however, it is necessary to utilise specially prepared software. In this paper, a practical method will be explained and applied to the numerical examples. The method, which is developed by using the procedure given by Çakiroglu (1977) is applied, by applying a simple quotient based on the results of lateral load analysis.

3. Buckling lengths according to design codes

In several design codes and specifications, simplified formulae and diagrams are given for calculating the buckling lengths of individual columns. These simple formulae have the advantage of enabling the designer to obtain the buckling lengths, without applying the tedious computations (or special software) which are necessary for the calculation of the overall-buckling load. In the following, the formulae of Eurocode 3 (2002) are presented as an example.

In Annex B1 of Eurocode 3 (2002), calculation of the “Buckling of components of building structures” is supplied as follows. First, the so-called distribution factors c_o and c_u for columns in a sway mode are computed by

$$c_o = \frac{1}{1 + \dfrac{\sum \alpha K_o}{K_s + K_{s,o}}} \tag{3}$$

$$c_u = \frac{1}{1 + \dfrac{\sum \alpha K_u}{K_s + K_{s,u}}} \tag{4}$$

Here

K_s, $K_{s,o}$, $K_{s,u}$ are the stiffness coefficients (*I/L* values) of columns,

K_o and K_u are the stiffness coefficients (*I/L* values) of beams,

α is a coefficient varying between 1 and 4 depending on the end conditions of beams.

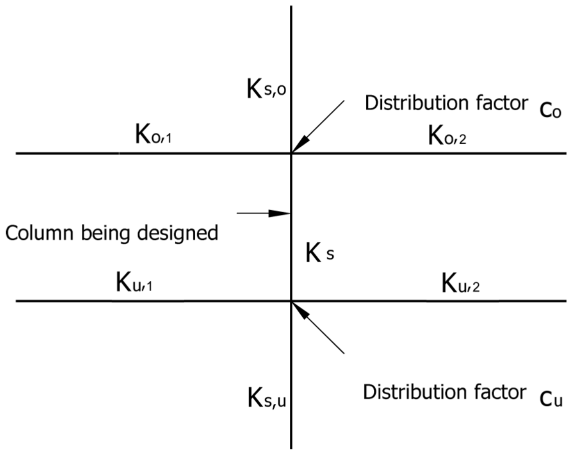

Fig. 2 Stiffness coefficients and distibution factors

Stiffness coefficients for various members are shown in Fig. 2.
Buckling (effective) length multiplier β is obtained first by solving the following equation for γ

$$\left[\frac{\gamma}{3(1/c_o-1)}-\frac{1}{\tan\gamma}\right]\left[\frac{\gamma}{3(1/c_u-1)}-\frac{1}{\tan\gamma}\right]-\frac{1}{\sin^2\gamma}=0 \tag{5}$$

and then by computing

$$\beta=\frac{\pi}{\gamma} \tag{6}$$

(Schneider 1996).
Alternatively, instead of using Eqs. (5) and (6), buckling length multiplier β may also be read from the diagrams given in the code. The buckling length s of an individual column is computed by

$$s=\beta L \tag{7}$$

where L is the length of the column.
Application of code formulae on several numerical examples have shown that erroneous results may be encountered for both sway and non-sway modes. This is mainly because, only local stiffness distributions are considered in these formulae, while the general behaviour of the frame is not taken into account. Discussion of buckling lengths of non-sway frames is left out of the scope of this study for the sake of brevity. The erroneous results encountered for sway mode will presently be demonstrated on several numerical examples.

3.1 Example 1

Dimensions and loading of a simple 5-storey frame is shown on the schematic elevation in Fig. 3.
Using special software prepared by Girgin (1996), which uses the system-buckling approach, the exact value of the buckling load for the frame is found to be

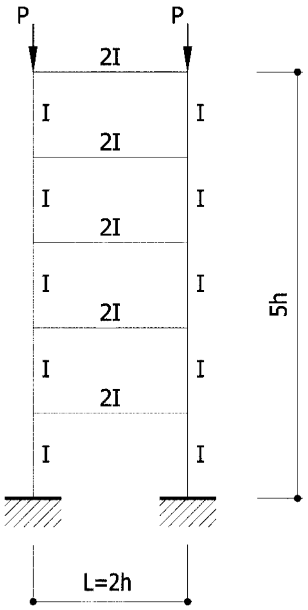

Fig. 3 Schematic elevation of Example 1

$$P_{cr} = 4.177\frac{EI}{h^2} \tag{8}$$

Substituting this value in Eq. (2) and taking $n_{ij} = 1$

$$s = 1.54\ h \tag{9}$$

is obtained for all the columns. Buckling length multipliers β calculated by using Eqs. (3), (4), (5) and (6) yield values of 1.24, 1.32 and 1.16 for topmost, intermediate and lowermost stories, respectively. Relative errors on β values vary between −24.7% and −11.7%, which may be considered rather high. Because the structure under consideration is chosen as being as regular as possible, hence satisfying all the assumptions in deriving the isolated subassembly equations.

3.2 Example 2

Dimensions and loading of another 5-storey frame is shown on the schematic elevation in Fig. 4. The characteristics of this frame are identical with the frame of Example 1, except that vertical loads *P*, exist at every joint.

The exact value of the buckling load for this frame is found to be

$$P_{cr} = 1.153\frac{EI}{h^2} \tag{10}$$

The exact values for buckling length multipliers are found as varying between 1.31 and 2.93. Relative errors on β values vary between −57.7% and −9.6%, which may be considered as excessive, i.e., not acceptable for design purposes. It is interesting to note that

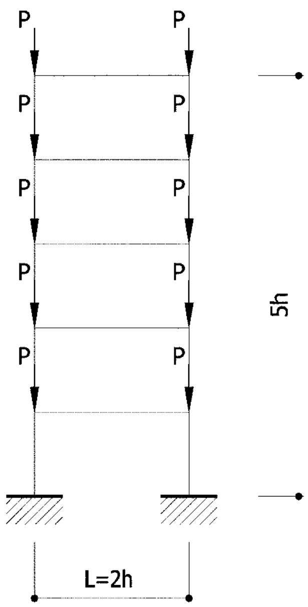

Fig. 4 Schematic elevation of Example 2

- This structure is again as regular as possible, thus satisfying all the assumptions for the isolated subassembly approach,
- Buckling length multipliers for Examples 1 and 2 are identical for Eurocode 3 formulae, because they do not take into account the axial force distribution.

3.3 Other design codes and evaluation

Similar formulae and diagrams for calculating buckling length multipliers are given in other design codes. The calculations for the above given examples have been carried out using AISC (1988) charts and ACI (1989) formulae and similar results are obtained. The ranges of errors for the codes under consideration are shown on Table 1.

It is clearly seen that all the considered codes yield errors, which are almost of the same order. This is due to the fact that all codes use similar formulae, which consider only the local (isolated) stiffness distributions. However, investigations carried on a number of numerical examples have shown that, buckling length multipliers are dependent on

- Axial force distribution,
- Number of stories,
- Position of the individual element

Table 1 Error ranges of buckling length parameters (%)

	Example 1	Example 2
Eurocode 3 DIN 18800	–24.7~–14.3	–57.7~–9.6
AISC (1988)	–16.9~3.2	–50.5~8.9
ACI (1989)	–13.0~1.3	–50.2~6.8

together with local stiffness distributions. It is concluded that, the buckling length multipliers should be determined by taking into account all these factors i.e., considering not only the local stiffness distributions, but also the overall characteristics of the structure.

4. A simplified procedure for determining the buckling load

In the following, a practical method will be explained and applied to the numerical examples. The method, which is developed by using the procedure given by Çakiroglu (1977), is applied by using a simple quotient based on the results obtained by standard frame analysis software.

Consider the fictitious lateral loading shown in Fig. 5 applied to the frame shown in Fig. 1. It is assumed that this loading provides displacements identical to (or proportional with) those corresponding to the buckling mode.

The buckling load parameter can be determined by using Betti's Reciprocal Theorem applied to the states shown in Figs. 1 and 5. According to this theorem, it may be written that

$$W_1 = W_2 \tag{11}$$

where W_1 is the virtual work of the force system in Fig. 1(a) in conjunction with the displacements in Fig. 5(b), and W_2 is the virtual work of the force system in Fig. 5(a) in conjunction with the displacements in Fig. 1(b), (Neal 1964). Since the displacements of Figs. 1(b) and 5(b) are assumed as being the same, the displacements and deformations corresponding to the lateral fictitious loading will be used in the following.

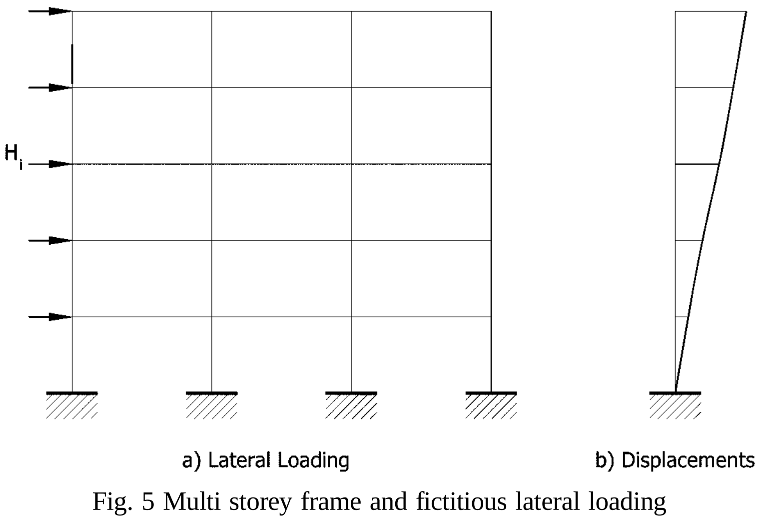

Fig. 5 Multi storey frame and fictitious lateral loading

4.1 Determination of W_1

According to the Principle of Virtual Works, W_1 can be computed as the work done by the internal forces of the loading shown in Fig. 1, in conjunction with the deformations induced by the fictitious lateral loading. The displacement diagram of an infinitely small portion of one of the

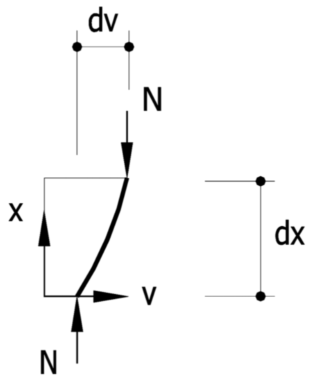

Fig. 6 Displacement diagram of a column portion

columns together with the internal forces is shown in Fig. 6.

If the axial deformations are neglected, the virtual work in this small portion can be computed by the product of the couple Ndv and the rotation dv/dx. Hence, the virtual work on any column can be obtained by

$$w = \int_{x=0}^{h} Ndv\frac{dv}{dx} \tag{12}$$

or

$$w = nP\int_{x=0}^{h}\left(\frac{dv}{dx}\right)^2 dx \tag{13}$$

where h denotes the height of the individual column. The total virtual work can be expressed as

$$W_1 = \sum w = P\sum n\int_{x=0}^{h}\left(\frac{dv}{dx}\right)^2 dx \tag{14}$$

Here the summation will be carried out for all the columns. It must be noted that, the indices are omitted for the sake of simplicity. In the following, the calculations related to the integral expression at the right hand side of Eq. (14) will be carried out. Since the integrand contains the derivative of the lateral displacements, it suffices to consider relative displacements.

The bending moment and relative displacement diagrams of an individual column are shown in Fig. 7. α is a dimensionless coefficient designating the location of the point of contraflexure and δ denotes the relative storey displacement.

The deformation expression of the column is

$$\frac{d^2v}{dx^2} = -\frac{M(x)}{EI} \tag{15}$$

where the bending moment function $M(x)$ may be expressed as

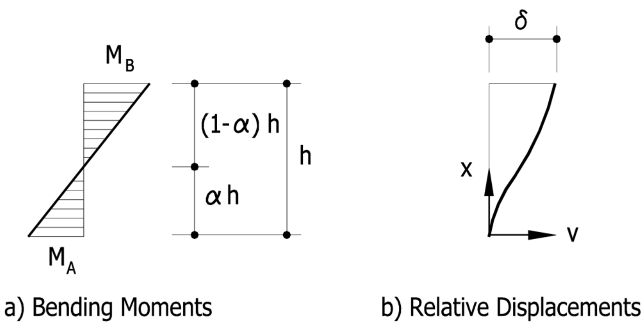

Fig. 7 Column bending moment and relative displacement diagrams

$$M(x) = M_A\left(1 - \frac{x}{\alpha h}\right) \tag{16}$$

Substituting $M(x)$ into Eq. (15) and integrating twice with the boundary conditions

$$v = 0 \quad \text{for} \quad x = 0 \quad \text{and}$$
$$v = \delta \quad \text{for} \quad x = h$$

yields

$$v(x) = \left(\frac{\delta}{h} + \frac{M_A h(3\alpha - 1)}{6EI\alpha}\right)x + \frac{M_A}{12EI\alpha h}x^2(x - 3\alpha h) \tag{17}$$

After substituting the derivative of $v(x)$ into Eq. (14) and carrying out the integral

$$W_1 = P\sum n\frac{\delta^2}{h}\chi \tag{18}$$

is obtained. Here χ denotes a dimensionless coefficient given by

$$\chi = 1 + \left(\frac{M_A h^2}{EI\delta}\right)^2\left(\frac{1}{12} - \frac{1}{12\alpha} + \frac{1}{45\alpha^2}\right) \tag{19}$$

The rather interesting variation of χ will be discussed presently.

4.2 Determination of W_2

The virtual work of the force system in Fig. 5(a) in conjunction with the displacements in Fig. 1(b) Fig. 5(b) can simply be written as

$$W_2 = \sum H_i d_i \tag{20}$$

where H_i and d_i represent the lateral storey loads and storey displacements, respectively. The summation will be carried out for all stories. Eq. (20) may more conveniently be expressed in terms of storey shears Q_i and relative storey displacements δ_i as

$$W_2 = \sum Q_i \delta_i \tag{21}$$

4.3 Simplified buckling load formula

Substituting the expressions for W_1 and W_2 given respectively by Eqs. (18) and (21) into Eq. (11) and solving for P (P_{cr}), the buckling load is obtained as

$$P_{cr} = \frac{\sum\limits_{stories} Q_i \delta_i}{\sum\limits_{Columns} n_{ij} \dfrac{\delta_i^2}{h_i} \chi_{ij}} \tag{22}$$

It must be noted that, this formula is approximate since the lateral loading corresponding to the buckling load displacements, are not known initially. However, application on several numerical examples has shown that, the value of P_{cr} is not strongly dependent to the initial choice of lateral loads. It may be recommended that, lateral load at each joint should be selected as proportional to the vertical load P_{ij} existing at the joint.

4.4 The χ coefficients

It is seen that when applying Eq. (22), it is necessary to compute χ_{ij} coefficients for each individual column. As can be seen in Eq. (19), these coefficients are dependent on the bending moments; hence, a tedious amount of computation is required. However, it can be shown that, χ values vary in a rather narrow range and can easily be simplified.

Let us consider the basic equation used in the approximate methods of lateral load analysis, which may be expressed as

$$\delta = \frac{Q}{k\dfrac{12EI}{h^3}} \tag{23}$$

Here Q denotes the shear force of the individual column and k is a dimensionless coefficient varying between 0 and 1, which depends on the stiffnesses of beams at each end of the column, (Muto 1964). Eq. (23) can alternatively be written as

$$\delta = \frac{\dfrac{M_A}{\alpha h}}{k\dfrac{12EI}{h^3}} = \frac{M_A h^2}{12k\alpha EI} \tag{24}$$

from which

$$\frac{M_A h^2}{EI\delta} = 12k\alpha \tag{25}$$

is obtained. Substituting this into Eq. (19)

$$\chi = 1 + 144k^2\alpha^2\left(\frac{1}{12} - \frac{1}{12\alpha} + \frac{1}{45\alpha^2}\right) \tag{26}$$

is found. It is seen that this new expression is dependent only to the two dimensionless variables, namely k and α. The variation of χ is shown on the diagrams in Fig. 8.

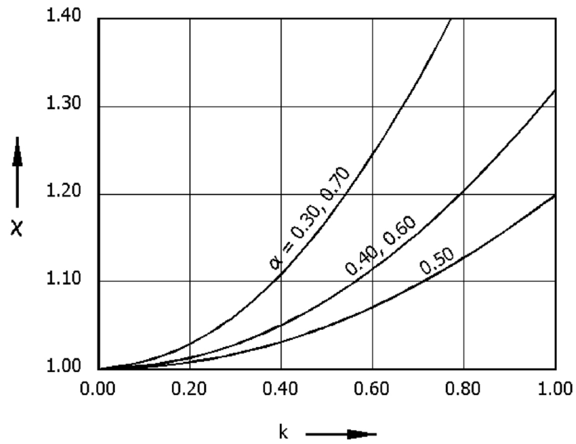

Fig. 8 Theoretical variation of χ values

It is well known that, when k approaches to unity, α assumes values near 0.50. Moreover, calculations carried out on the columns of several numerical examples, have yielded the results shown as dots on Fig. 9.

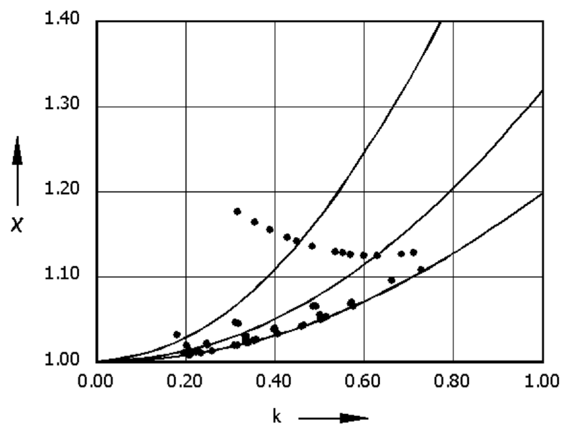

Fig. 9 Variation of χ values for numerical examples

It may be concluded that, approximately lower half of the figure is valid for practical purposes. Considering this narrow range for values of χ, it is reasonable to assume a constant and conservative value of

$$\chi = 1.20$$

for practical purposes. Thus, Eq. (22) takes the rather practical form of

$$P_{cr} = \frac{\sum\limits_{stories} Q_i \delta_i}{1.20 \sum\limits_{columns} n_{ij} \frac{\delta_i^2}{h_i}} \tag{27}$$

4.5 Analysis procedure

Buckling lengths of frame columns can be determined as follows:

- Apply lateral forces proportional to the vertical loads at each joint,
- Compute relative storey displacements using any existing software,
- Compute the critical load P_{cr} by using Eq. (27),
- Determine the buckling lengths of columns by using Eq. (2).

5. Numerical examples

In the following, the procedure outlined above will be applied to several numerical examples and the results will be discussed.

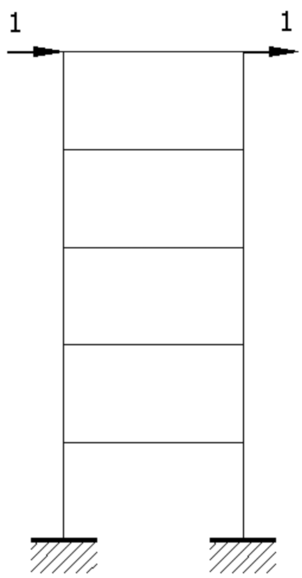

Fig. 10 Fictitious lateral loading for Example 1

Table 2 Buckling load calculations for Example 1

Storey	Q	$\frac{EI}{h^3}\delta$	$\frac{EI}{h^3}Q\delta$	n	$\frac{(EI)^2}{h^5}n\frac{\delta^2}{h}$
5	2.00	0.2090	0.4180	1.00	0.04368
4	2.00	0.2446	0.4892	1.00	0.05983
3	2.00	0.2478	0.4956	1.00	0.06140
2	2.00	0.2380	0.4760	1.00	0.05664
1	2.00	0.1561	0.3122	1.00	0.02437
Sum			2.1910		0.24592

5.1 Example 1

Dimensions and loading of the first example is the same as shown on the schematic elevation in Fig. 3 of Section 3.1. The fictitious lateral loading is shown in Fig. 10.

After carrying out lateral load analysis for the fictitious loading, storey relative displacements δ are obtained. The terms used for the application of Eq. (27) is shown on Table 2.

Applying Eq. (27) yields

$$P_{cr} = \frac{2.1910}{1.20 \times 2 \times 0.24592}\frac{EI}{h^2} = 3.712\frac{EI}{h^2}$$

which has an error of –11.1%. Computing the buckling lengths of columns by using Eq. (2) gives

$$s = 1.63\ h$$

for all the columns. This value has an error of 6.1%, which is on the safe side. It is interesting to note that the buckling lengths (and errors) of all the columns are the same due to the fact that they are computed by using the same equation used for exact calculations, namely Eq. (2).

Buckling load calculations are repeated by using wind and earthquake loadings for the same frame, and the errors on buckling lengths are found as 1.5% and 4.5%, respectively. It can be deduced that, any lateral loading can be used in determining the approximate buckling load value, without largely effecting the results.

5.2 Example 2

Dimensions and loading of the second example is the same as shown on the schematic elevation in Fig. 4 of Section 3.2. The fictitious lateral loading is shown in Fig. 11, where loads are chosen as being proportional to vertical loads at the joints.

The terms used for the application of Eq. (27) are shown on Table 3.

Applying Eq. (27) yields

$$P_{cr} = \frac{21.7914}{1.20 \times 2 \times 8.4045}\frac{EI}{h^2} = 1.080\frac{EI}{h^2}$$

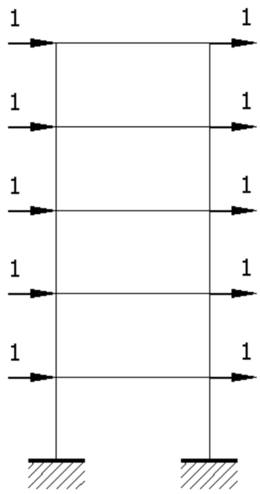

Fig. 11 Fictitious lateral loading for Example 2

Table 3 Buckling load calculations for Example 2

Storey	Q	$\frac{EI}{h^3}\delta$	$\frac{EI}{h^3}Q\delta$	n	$\frac{(EI)^2}{h^5}n\frac{\delta^2}{h}$
5	2.00	0.2635	0.5270	1.00	0.0694
4	4.00	0.5007	2.0028	2.00	0.5014
3	6.00	0.7419	4.4514	3.00	1.6512
2	8.00	0.9344	7.4752	4.00	3.4924
1	10.00	0.7335	7.3350	5.00	2.6901
Sum			21.7914		8.4045

Table 4 Buckling length multipliers for Example 2

Storey	β (Exact)	β (Prop. Method)	Relative error (%)
5	2.93	3.02	3.3
4	2.07	2.14	3.3
3	1.69	1.75	3.3
2	1.46	1.51	3.3
1	1.31	1.35	3.3

which has an error of –6.3%. Buckling length multipliers β, which are calculated by means of Eq. (2), are shown and compared with the exact values on Table 4.

Here again, all the buckling length parameters have the same error and are on the safe side.

Buckling load calculations are repeated by using wind and earthquake loadings for the same frame, and the errors on buckling lengths are found as 4.1% and 1.6%, respectively.

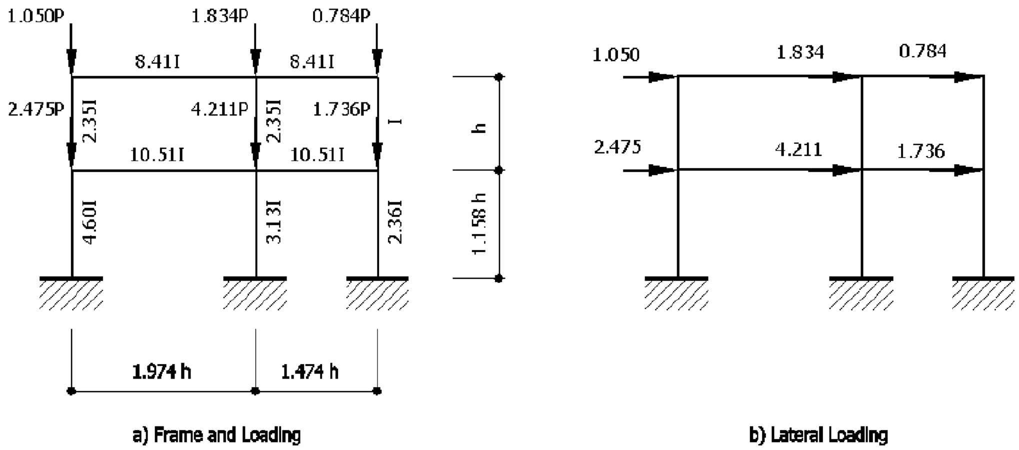

Fig. 12 Schematic elevation and loadings of Example 3

5.3 Example 3

Dimensions and loading of the last example, which is adopted from Lui (1992), is shown in Fig. 12(a). Fictitious lateral loading is chosen as shown in Fig. 12(b).

The exact value of the buckling load for the frame is found to be

$$P_{cr} = 5.191\frac{EI}{h^2}$$

according to Girgin (1996). The terms used for the application of Eq. (27) are shown on Table 5.

Applying Eq. (27) yields

$$P_{cr} = \frac{2.7215}{1.20 \times 0.4344}\frac{EI}{h^2} = 5.221\frac{EI}{h^2}$$

Table 5 Buckling load calculations for Example 3

Storey	Q	h_i	$\frac{EI}{h^3}\delta$	$\frac{EI}{h^3}Q\delta$	Column	n	$\frac{(EI)^2}{h^5}n\frac{\delta^2}{h_i}$
2	3.668	h	0.0966	0.3543	Left	1.050	0.0098
					Middle	1.834	0.0171
					Right	0.784	0.0073
1	12.090	$1.158h$	0.1958	2.3672	Left	3.525	0.1167
					Middle	6.045	0.2001
					Right	2.520	0.0834
Sum				2.7215			0.4344

Table 6 Buckling length multipliers for Example 3

Storey	Column	β (Exact)	β (Prop. Method)
2	Left	2.063	2.057
	Middle	1.561	1.556
	Right	1.557	1.553
1	Left	1.360	1.356
	Middle	0.857	0.854
	Right	1.152	1.149

which has an error of 0.58%. Buckling length multipliers β are shown and compared with the exact values on Table 6. As for the previous examples, all the buckling length parameters have the same error of –0.3%.

Buckling load calculations are repeated by using wind and earthquake loadings for the same frame, and the errors on buckling lengths are found as –0.4% and –2.3%, respectively.

The errors of the results obtained by using the proposed method have been found less than the errors in the original paper (Lui 1992). The same example has been solved by using the methods proposed by Aristizabal-Ochoa (1997) and Hellesland and Bjorhovde (1997) as well. It is found that the errors of the proposed method are less than the ones found by both.

6. Conclusions

In this paper, determination of buckling lengths of multi-storey frame columns is investigated. The main conclusions derived, may be summarised as follows:

1. It is shown that, simplified formulae and diagrams, which are given in several design codes and specifications, may yield rather erroneous results for buckling lengths of the columns. This is due to the fact that the code formulae refer only to local stiffness distributions, instead of the overall behaviour of the structures.
2. A simplified procedure for determining the approximate value for the system buckling load of multi-storey frames is developed. Buckling lengths of columns may then be calculated by means of a simple formula.
3. The procedure, which utilises lateral load analysis of the frames yields errors, which are less than 10% for all the examples. This order may be considered acceptable from the designer's point of view.
4. The buckling load value is not strongly dependent on the choice of lateral loading. Hence any existing lateral loading on the frame under consideration may be used without losing a significant amount of accuracy.
5. The proposed procedure is applied to several numerical examples and it is seen that all the errors are in the acceptable range and on the safe side.

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