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Optimization of active vibration control for random intelligent truss structures under non-stationary random excitation

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Abstract. The optimization of active bars' placement and feedback gains of closed loop control system for random intelligent truss structures under non-stationary random excitation is presented. Firstly, the optimal mathematical model with the reliability constraints on the mean square value of structural dynamic displacement and stress response are built based on the maximization of dissipation energy due to control action. In which not only the randomness of the physics parameters of structural materials, geometric dimensions and structural damping are considered simultaneously, but also the applied force are considered as non-stationary random excitation. Then, the numerical characteristics of the stationary random responses of random intelligent structure are developed. Finally, the rationality and validity of the presented model are demonstrated by an engineering example and some useful conclusions are obtained.

Key words: optimization; non-stationary random excitation; random intelligent truss structures; reliability constraints; active bars' placement; feedback gains.

1. Introduction

Intelligent truss structure is a self-adaptive structure that is utilized in some important fields. In this kind of truss structures, piezoelectric active bar is the structural active member to suppress mechanical vibrations, which are not only sensor but also actuator. Optimal placement of piezoelectric active bar is an important segment in the process of intelligent structural vibration control. The locations of active bars in intelligent truss structures affect the validity of active vibration control directly. Such as Rao *et al.* (1991) investigated the discrete optimal actuator location selection problem. They considered this problem in active controlled structures as cast in

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the framework of a zero-one optimization problem and a genetic algorithmic approach is developed to solve this zero-one optimization problem. Xu *et al.* (1994) presented an optimal design method for placement and gains of actuators and sensors in output feedback control systems. In this paper, a quadratic performance function was minimized using nonlinear programming. Their contribution is the derivation of analytical expressions for the gradients of the performance function. Peng *et al.* (1998) studied the active position control and vibration control of composite beams with distributed piezoelectric sensors and actuators with a finite element modal based on third order laminate theory. Wang *et al.* (2001) proposed a controllability index to quantify the controllability factor based on the state-coupled equation of beam structures with piezoelectric actuators and the index was utilized as an objective function to determine the optimal locations of piezoelectric actuators for vibration control of beam structures. Suk *et al.* (2001) introduced the Lyapunov control law for the slew maneuver of flexible space structure by using a time-domain finite element analysis. To optimize the gain set of the control system, an energy-based performance index was adopted and the gradients of the performance index with respect to each gain were derived. So far, however, almost of modeling on optimal placement of active bars in intelligent structures basically belongs to the determinate models, that is, all structural parameters, applied loads and control forces are regarded as determinate ones. Apparently, this kind of model can not reflect the influence of the randomness of intelligent structural parameters, loads and control forces on the optimal placement of active bars in intelligent structures. In recently years, great deals of research results of random structures have been published (such as Kaminski 2001, Singh and Yadav 2001, Gao and Chen 2003). As a matter of fact, in some situations the randomness of them must be considered. Such as, for one kind numerous or batch producing structures, their values of physical parameter of material and the geometric dimensions have randomness. Therefore, studying the optimization the active vibration control for random intelligent structures is of much realistic engineering background and important theoretic signification.

The results of the dynamic response analysis are the important base of the determination of the active bars' location. Because the random dynamic response analysis of stochastic structure is very complicated and difficult, it is only in the recent years that the stochastic finite element method based on perturbation technique has begun to be used for solving the dynamic response of structure with random parameters under stationary random excitation. Wall *et al.* (1987) researched the dynamic effects of uncertainty in structural properties when the excitation is random by use of perturbation stochastic finite element method (PSFEM). Liu *et al.* (1988) discussed the secular terms resulted from PSFEM in transient analysis of such a random dynamic system. Jensen *et al.* (1992) studied the response of systems with uncertain parameters to random excitation by extended the orthogonal expansion method. Zhao *et al.* (2000) studied the vibration for structures with stochastic parameters to random excitation by using dynamic Neumann stochastic finite element method, in which the random equation of motion for structure is transformed into a quasi-static equilibrium equation for the solution of displacement in time domain.

In this paper, intelligent truss structures are taken as researching objects. The problems of the optimization of active bar's placement and closed loop control system's gains are studied, in which not only the randomness of the physics parameters of structural materials, geometric dimensions and structural damping are considered simultaneously, but also the applied force are taken as non-stationary random excitation. Based on the maximization of dissipation energy due to control action, the performance function is developed[3]. Then, the optimal mathematical model with the reliability constraints on the mean square value of structural dynamic displacement and stress response is built.

The numerical characteristics of the non-stationary random dynamic responses of intelligent structures are developed. Through engineering example, the research on the optimal placement of active bar and the optimization of gains is developed.

2. Optimal mathematical model

2.1 Performance function

Following the finite element formulation, the equation of motion for an intelligent structure can be expressed as Chen *et al.* (1991)

$$[M]\{\ddot{u}(t)\} + [C]\{\ddot{u}(t)\} + [K]\{u(t)\} = v(t)\{F(t)\} + [B_1]\{F_P(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are the structural displacement vector, velocity vector and acceleration vector, respectively. $\{F(t)\}$ is the stationary random excitation vector and $v(t)$ is a time modulation function that denotes the non-stationary characteristic of the random force. $\{F_P(t)\}$ is the control force vector. The $[B_1]$ matrix consists of the active bars' direction cosines. In the following, the Wilson's damping hypothesis (Bathe 1995) will be adopted. With the modal expansion $\{u(t)\} = [\phi]\{z(t)\}$, the equation of motion takes the form

$$[I]\{\ddot{z}(t)\} + [D]\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^T v(t)\{F(t)\} + [\phi]^T[B_1]\{F_P(t)\} \tag{2}$$

where $[D] = diag[2\xi_j\omega_j]$, $[\Omega] = diag[\omega_j^2]$, $(j = 1, 2, ..., n)$, $[\phi] = [\phi_1 \cdot \cdot \cdot \phi_n]$, ω_j and ξ_j are jth order inherence frequency and modal damping of structure, respectively.

For active bars, a velocity feedback control law is considered. Since each active bar can be considered as a collocated actuator/sensor pair, the output matrix is the transpose of the input matrix. The output vector $Y(t)$ and the control force vector $\{F_P(t)\}$ can be respectively expressed as

$$Y(t) = [B_1]^T[\phi]\{\dot{z}(t)\} \tag{3}$$

$$\{F_P(t)\} = -[G]Y(t) = -[G][B_1]^T[\phi]\{\dot{z}(t)\} \tag{4}$$

where $[G]$ is the gain matrix, and let, $[G] = diag(g_j)$, g_j is jth element of the principal diagonal $(j = 1, 2, ..., n)$.

Substituting Eq. (4) into Eq. (2) yields the equation of the closed-loop system

$$[I]\{\ddot{z}(t)\} + ([D] + [\phi]^T[B_1][G][B_1]^T[\phi])\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^T v(t)\{F(t)\} \tag{5}$$

In the state-space representation, the equation of motion for the closed-loop system becomes

$$\{\dot{u}(t)\} = [A]\{u(t)\} \tag{6}$$

where $$\{u(t)\} = \{z(t) \quad \dot{z}(t)\}^T$$

$$[A] = \begin{bmatrix} 0 & [I] \\ -[\Omega] & -([D] + [\phi]^T[B_1][G][B_1]^T[\phi]) \end{bmatrix} \tag{7}$$

In order to find the optimal placement of the active bar and the optimal gain of closed-loop control system, based on the maximization of dissipation energy due to control action, the minimal energy stored in the structures is utilized as the performance and it can be expressed as

$$J = -\int_0^\infty \{\dot{z}(t)\}^T[\phi]^T[B_1][G][B_1]^T[\phi]\{\dot{z}(t)\}dt \tag{8}$$

Using the solution of Eq. (6), $\{u(t)\} = \exp([A]t)\{u(0)\}$, Eq. (8) can also be expressed as Abdullah (1998)

$$J = -\{u(0)\}^T \cdot \int_0^\infty e^{[A]T_t}[Q]e^{[A]t}dt \cdot \{u(0)\} \tag{9}$$

where $[Q] = \begin{bmatrix} [\Omega] & 0 \\ 0 & [I] \end{bmatrix}$. By using the method that described in Abdullah (1998), the performance function can be expressed as

$$J = -tr[K] \tag{10}$$

where the matrix [K] can be obtained by solving the Lyapunov equation.

$$[A]^T[K] + [K][A] = [Q] \tag{11}$$

2.2 Optimal mathematical model

For the intelligent truss structure with random parameters and the loads are non-stationary random excitations, the optimal mathematical model of active bar with the reliability constraints on the mean square value of the structural dynamic stress and displacement response can be built as

$$\text{find}: [B_1], [G]$$

$$\min: \quad J = -tr[K] \tag{12}$$

$$S.t.: \quad R^*_{\psi^2_\sigma} - P_r\{\psi^{2*}_\sigma - \psi^2_\sigma \geq \delta\} \leq 0 \tag{13}$$

$$R^*_{\psi^2_{uk}} - P_r\{\psi^{2*}_{uk} - \psi^2_{uk} \geq \delta\} \leq 0 \qquad (k = 1, 2, \ldots, n) \tag{14}$$

$$[B_1] \subset [B^*_1], [G] < [G^*] \tag{15}$$

In this model, [B_1] and [G] are design variables. $R^*_{\psi^2_\sigma}$ and $R^*_{\psi^2_{uk}}$ are the given values of reliability of the mean square value of stress and displacement response, respectively. $P_r\{\cdot\}$ is the reliability obtained from the actual calculation. ψ_σ^{2*} and $R_{\psi_{uk}}^{2*}$ are the given limit values of the mean square value of stress and displacement response, respectively. $[B_1]$, $[G]$, $R_{\psi_\sigma^2}^*$, $R_{\psi_{uk}^2}^*$, $P_r\{\cdot\}$, ψ_σ^{2*} and ψ_{uk}^{2*} can be random variables or determinate values. ψ_σ^2 is the mean square value of structural dynamic stress response and ψ_{uk}^2 is the mean square value of structural dynamic displacement response of the kth degree freedom, they are all random variables. δ is the given allowable deviation in order to avoid the destruction in the structure, which produced by the lack of the strength or the stiffness. $[B_1^*]$ are the bounds of $[B_1]$, $[G^*]$ are the upper bounds on the feedback gains.

In above optimal model, structural dynamic stress and displacement response constrains expressed by the probability form, which make the optimal problem difficult to solve. For this reason, the reliability constrains are transformed as the normal constrains by means of the second order moment theory on the reliability (Chen *et al.* 1994). Thus the reliability constrains Eqs. (13) and (14) can be respectively expressed as

$$\beta_{\psi_\sigma^2}^* - \frac{\mu_{\psi_\sigma^{2*}} - \mu_{\psi_\sigma^2} - \delta_{\psi_\sigma^2}}{(\sigma_{\psi_\sigma^{2*}}^2 + \sigma_{\psi_\sigma^2}^2)^{1/2}} \le 0 \tag{13a}$$

$$\beta_{\psi_{uk}^2}^* - \frac{\mu_{\psi_{uk*}^2} - \mu_{\psi_{uk}^2} - \delta_{\psi_{uk}^2}}{(\sigma_{\psi_{uk*}^2}^2 + \sigma_{\psi_{uk}^2}^2)^{1/2}} \le 0 \qquad (k = 1, 2, \ldots, n) \tag{14a}$$

where $\beta_{\psi_\sigma^2}^* = \Phi^{-1}(R_{\psi_\sigma^2}^*)$ and $\beta_{\psi_{uk}^2}^* = \Phi^{-1}(R_{\psi_{uk}^2}^*)$ are the given reliability of the mean square value of the structural dynamic stress response and the structural dynamic displacement response of the kth degree of freedom, respectively. $\Phi^{-1}(\cdot)$ denote the inverse functions of the distribution of random variables. $\mu_{\psi_\sigma^{2*}}$ and $\sigma_{\psi_\sigma^{2*}}^2$ are the mean value and variance of ψ_σ^{2*}, respectively. $\mu_{\psi_{uk}^{2*}}$ and $\sigma_{\psi_{uk}^{2*}}^2$ are the mean value and variance of the limit value of the mean square value of the structural dynamic displacement response of kth degree of freedom, respectively. $\mu_{\psi_\sigma^2}$ and $\sigma_{\psi_\sigma^2}^2$ are the mean value and variance of the mean square value structural dynamic stress response, respectively. $\mu_{\psi_{uk}^2}$ and $\sigma_{\psi_{uk}^2}^2$ are the mean value and variance of the mean square value of the structural dynamic displacement response of kth degree of freedom, respectively. $\delta_{\psi_\sigma^2}$ and $\delta_{\psi_{uk}^2}$ are the given allowable deviations of the mean square value of stress and displacement response, respectively. The numeral characteristics of these response random variables will be derived in the next chapters.

3. Structural non-stationary random dynamic response analysis of closed loop control system

Suppose that there are *ne* elements in an intelligent truss structure. In the structure, any element can be taken as passive bar or active bar. A piezoelectric bar is utilized as active bar. In order to utilize the unite form to express the structural stiffness and mass matrices, a kind of mixed element have been construct. A Boolean algebra value named θ is introduced in the mixed element, when $\theta = 0$, the mixed element is active element bar and when $\theta = 1$, the mixed element is passive element bar. In the following, expressions of the stiff matrix $[K]$ and mass matrix $[M]$ of intelligent truss structures in global coordinate will be developed by means of this kind of mixed element

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left\{\left[\theta\frac{E_m^{(e)}A_m^{(e)}}{l_m^{(e)}} + (1-\theta)\frac{c_{33}^{(e)} + (e_{33}^{(e)})^2/\varepsilon_{33}^{(e)}}{l_P^{(e)}}A_P^{(e)}\right][\bar{G}]\right\} \tag{16}$$

$$[M] = \sum_{e=1}^{ne}[M^{(e)}] = \sum_{e=1}^{ne}\left\{\frac{1}{2}(\theta\rho_m^{(e)}A_m^{(e)}l_m^{(e)} + (1-\theta)\rho_P^{(e)}A_P^{(e)}l_P^{(e)})[I]\right\} \tag{17}$$

where $[K^{(e)}]$ is the eth element's stiffness matrix, $[M^{(e)}]$ is the eth element's mass matrix. $[I]$ is a 6-order identity matrix. $\rho_m^{(e)}, A_m^{(e)}$ and $l_m^{(e)}$ are the eth passive bar's mass density, cross-section area and length, respectively. $\rho_P^{(e)}, A_P^{(e)}$ and $l_P^{(e)}$ are the eth active bars' mass density, cross-section area and length, respectively. $E_m^{(e)}$ is the eth passive bar's elastic module. $c_{33}^{(e)}, e_{33}^{(e)}$ and $\varepsilon_{33}^{(e)}$ are the eth active bar's elastic module, piezoelectric force/electrical constant and dielectric constant, respectively. $[\bar{G}]$ is a 6×6 matrix, where $\bar{g}_{11} = \bar{g}_{44} = 1, \bar{g}_{14} = \bar{g}_{41} = -1$, other elements of $[\bar{G}]$ are all equal to zero.

Here introduce another expression as follow:

$$E_P^{(e)} = c_{33}^{(e)} + (e_{33}^{(e)})^2/\varepsilon_{33}^{(e)} \tag{18}$$

$E_P^{(e)}$ just is a generalized elastic module of piezoelectric active bars while considering the mechanic-electronic coupling effect.

Substituting Eq. (18) into Eq. (16) yields

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left\{\left[\theta\frac{E_m^{(e)}A_m^{(e)}}{l_m^{(e)}} + (1-\theta)\frac{E_P^{(e)}A_P^{(e)}}{l_P^{(e)}}\right][\bar{G}]\right\} \tag{19}$$

In the closed loop control system, because the production and response process of $\{F_P(t)\}$ is determined by the non-stationary random excitation, $\{F_P(t)\}$ is the non-stationary random force vector too, and these two variables are full positive correlation.

Let
$$g(t)\{P(t)\} = v(t)\{F(t)\} + [B_1]\{F_P(t)\} \tag{20}$$

where $g(t)$ is a time modulation function, which denotes the non-stationary characteristic of the random force; $\{P(t)\}$ is a stationary random force vector. Then, Eq. (1) can be expressed as

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = g(t)\{P(t)\} \tag{21}$$

Eq. (21) is a set of differential equations coupled to each other. Its formal solution can be obtained in terms of the decoupling transform and Duhamel integral, that is

$$\{u(t)\} = \int_0^t [\phi][h(t-\tau)][\phi]^T g(\tau)\{P(\tau)\}d\tau \tag{22}$$

where $[h(t)]$ is the impulse response function matrix of the structure and can be expressed as

$$[h(t)] = diag(h_j(t)) \tag{23}$$

where

$$h_j(t) = \begin{cases} \dfrac{1}{\overline{\omega}_j}\exp(-\xi_j\omega_j t)\sin\overline{\omega}_j t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (j = 1, 2, \ldots, n) \tag{24}$$

where $\overline{\omega}_j = \omega_j(1-\xi_j^2)^{1/2}$.

From Eq. (22), the correlation function matrix of the structural displacement response can be obtained

$$\begin{aligned}[][R_u(t_1, t_2)] &= E(\{u(t_1)\}\{u(t_2)\}^T) \\ &= \int_0^{t_1}\int_0^{t_2}[\phi][h(t-\tau_1)][\phi]^T g(\tau_1)[R_P(\tau_1, \tau_2)]g(\tau_2)[\phi][h(t-\tau_2)]^T[\phi]^T d\tau_1 d\tau_2\end{aligned} \tag{25}$$

where $[R_u(t_1, t_2)]$ is the correlation function matrix of the displacement response of the structure; $[R_P(\tau_1, \tau_2)]$ is the correlation function matrix of the $\{P(t)\}$.

By performing $[R_u(t_1, t_2)]$ a Fourier transformation, the power spectral density matrix of the structural displacement response can be obtained:

$$[S_u(\omega, t)] = [\phi][H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)][\phi]^T \tag{26}$$

where $[S_u(\omega, t)]$ is the power spectral density matrix of the displacement response; $[S_P(\omega)]$ is the power spectral density matrix of $\{P(t)\}$, $[H^*(\omega)]$ is the conjugate matrix of $[H(\omega)]$, $[H(\omega)]$, is the frequency response function matrix of the structure and can be expressed as

$$[H(\omega)] = diag[H_j(\omega)] \tag{27}$$

where

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + i \cdot 2\xi_j\omega_j\omega} \quad (i = \sqrt{-1}) \quad (j = 1, 2, \ldots, n) \tag{28}$$

Integrating $[S_u(\omega, t)]$ within the frequency domain, the mean square value matrix of the structural displacement response can be obtained

$$[\psi_u^2] = \int_0^\infty [S_u(\omega, t)]d\omega = \int_0^\infty [\phi][H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)][\phi]^T d\omega \tag{29}$$

where $[\psi_u^2]$ is the mean square value matrix of the structural displacement response.

Then the mean square value of the kth degree of freedom of the structural dynamic displacement response can be expressed as

$$\psi_{uk}^2 = \vec{\phi}_k \cdot \int_0^\infty [H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)]d\omega \cdot \vec{\phi}_k^T \quad (k = 1, 2, \ldots, n) \tag{30}$$

where $\vec{\phi}_k$ is the kth line vector of the matrix $[\phi]$.

According to the relationship between node displacement and element stress, the stress response of the eth element in the truss structure can be expressed as

$$\{\sigma(t)^{(e)}\} = E^{(e)} \cdot [B] \cdot \{u(t)^{(e)}\} \quad (e = 1, 2, \ldots, n_e) \tag{31}$$

where $\{u(t)^{(e)}\}$ is the displacement response of the nodal point of the eth element, $\{\sigma(t)^{(e)}\}$ is the stress response of the eth element, $[B]$ is the geometric matrix of the eth element. $E^{(e)}$ is the elastic module of the eth element.

From Eq. (31), the correlation function matrix of the eth element stress response can be obtained

$$[R_\sigma^{(e)}(\tau)] = E(\{\sigma(t)^{(e)}\}\{\sigma(t+\tau)^{(e)}\}^T) = E^{(e)}[B][R_u^{(e)}(\tau)][B]^T E^{(e)} \tag{32}$$

where $[R_\sigma^{(e)}(\tau)]$ is the correlation function matrix of the eth element stress response.

Furthermore, the power spectral density matrix of the stress response of the eth element can be obtained

$$[S_\sigma^{(e)}(\omega)] = E^{(e)}[B][S_u^{(e)}(\omega)][B]^T E^{(e)} \tag{33}$$

where $[S_\sigma^{(e)}(\omega)]$ is the power spectral density matrix of the stress response of the eth element.

Then, the mean square value matrix of the eth element stress response can be expressed as

$$[\psi^2_{\sigma^{(e)}}] = E^{(e)}[B][\psi^2_{u^{(e)}}][B]^T E^{(e)} \tag{34}$$

where $[\psi^2_{\sigma^{(e)}}]$ is the mean square value matrix of the eth element stress response.

4. Numerical characteristics of the non-stationary random response of random intelligent structures

4.1 Numerical characteristics of natural frequency random variable

Here, the randomness of ξ_j, $\rho_m^{(e)}$, $\rho_P^{(e)}$, $A_m^{(e)}$, $A_P^{(e)}$, $l_m^{(e)}$, $l_P^{(e)}$, $E_m^{(e)}$ and $c_{33}^{(e)}$ are considered simultaneously. From Eq. (18), it can be obtained easily that $E_P^{(e)}$ is random variable. The randomness of physical parameters and geometric dimensions will lead the structural matrices $[K]$ and $[M]$ having randomness. It can be obtained that the randomness of the structural matrices $[K]$ and $[M]$ will lead the structural inherence frequency ω_j having randomness.

In the following, the computing expression of the numerical characteristics of jth order inherence can be deduced by means of the algebra synthesis method (Gao *et al*. 2003).

$$\begin{aligned}\mu_{\omega_j} = \omega_j^{\#}\left(\frac{\mu_E}{\mu_\rho\mu_Z}\right)^{\frac{1}{2}}\{[1 &+ \nu_A^2 + \nu_Z^2 + \nu_\rho^2 + \nu_A^2\nu_Z^2 + \nu_A^2\nu_\rho^2 + \nu_Z^2\nu_\rho^2 + \nu_A^2\nu_Z^2\nu_\rho^2 \\ &- c_{E_\rho}\cdot(\nu_A^2 + \nu_E^2 + \nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2 + \nu_Z^2 + \nu_\rho^2 + \nu_A^2\nu_Z^2 + \nu_A^2\nu_\rho^2 + \nu_Z^2\nu_\rho^2 + \nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]^2 \\ &- \frac{1}{2}[2\nu_A^2 + \nu_E^2 + \nu_Z^2 + \nu_\rho^2 + \nu_A^2\nu_Z^2 + \nu_A^2\nu_E^2 + \nu_A^2\nu_\rho^2 + \nu_Z^2\nu_\rho^2 + \nu_A^2\nu_Z^2\nu_\rho^2 \\ &- 2c_{E_\rho}\cdot(\nu_A^2 + \nu_E^2 + \nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2 + \nu_Z^2 + \nu_\rho^2 + \nu_A^2\nu_Z^2 + \nu_A^2\nu_\rho^2 + \nu_Z^2\nu_\rho^2 + \nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]\}^{1/4}\end{aligned} \tag{35}$$

where μ_{ω_j} is the mean value of the jth order inherence.

$$\sigma_{\omega_j} = \omega_j^{\#}\left(\frac{\mu_E}{\mu_\rho \mu_Z}\right)^{\frac{1}{2}}\{[1 + v_A^2 + v_Z^2 + v_\rho^2 + v_A^2 v_Z^2 + v_A^2 v_\rho^2 + v_Z^2 v_\rho^2 + v_A^2 v_Z^2 v_\rho^2$$

$$-c_{E_\rho}\cdot(v_A^2 + v_E^2 + v_A^2 v_E^2)^{1/2}\cdot(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2 v_Z^2 + v_A^2 v_\rho^2 + v_Z^2 v_\rho^2 + v_A^2 v_Z^2 v_\rho^2)^{1/2}]$$

$$-\{[1 + v_A^2 + v_Z^2 + v_\rho^2 + v_A^2 v_Z^2 + v_A^2 v_\rho^2 + v_Z^2 v_\rho^2 + v_A^2 v_Z^2 v_\rho^2$$

$$-c_{E_\rho}\cdot(v_A^2 + v_E^2 + v_A^2 v_E^2)^{1/2}\cdot(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2 v_Z^2 + v_A^2 v_\rho^2 + v_Z^2 v_\rho^2 + v_A^2 v_Z^2 v_\rho^2)^{1/2}]^2$$

$$-\frac{1}{2}[2v_A^2 + v_E^2 + v_Z^2 + v_\rho^2 + v_A^2 v_Z^2 + v_A^2 v_E^2 + v_A^2 v_\rho^2 + v_Z^2 v_\rho^2 + v_A^2 v_Z^2 v_\rho^2$$

$$-2c_{E_\rho}\cdot(v_A^2 + v_E^2 + v_A^2 v_E^2)^{1/2}\cdot(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2 v_Z^2 + v_A^2 v_\rho^2 + v_Z^2 v_\rho^2 + v_A^2 v_Z^2 v_\rho^2)^{1/2}]\}^{1/2}\}^{1/2}$$

$$v_z = v_{l^2} = \frac{\sqrt{4v_l^2 + 2v_l^4}}{1 + v_l^2} \tag{36}$$

where σ_{ω_j} is the mean variance of the jth order inherence; the symbol v denotes variation coefficient; c_{E_ρ} is the correlation coefficient of variables E and ρ. $\omega_j^{\#}$ can be obtained from the structural conventional dynamic characteristic computation.

4.2 Numerical characteristics of the non-stationary random response of the closed loop system of the stochastic intelligent structure

The randomness of the structural damping, dynamic characteristics and the non-stationary stochastic excitation will lead the structural dynamic response (dynamic displacement and dynamic stress) of the closed loop control system having randomness. In the following, expressions of the numerical characteristics of the structural stationary response random variables will be derived.

From Eq. (30), the mean value and mean variance of the mean square value of the kth degree of freedom of the structural dynamic displacement response can be deduced by means of the random variable's functional moment method.

$$\mu_{\psi_{uk}^2} = \mu_{\vec{\phi}_k}\cdot\int_0^{\omega_c}\mu_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]}d\omega\cdot\mu_{\vec{\phi}_k^T} \tag{37}$$

$$\sigma_{\psi_{uk}^2} = \mu_{\vec{\phi}_k}\cdot\{\int_0^{\omega_c}\{\mu^2_{[H(\omega)]}(\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]})^2\sigma^2_{[H^*(\omega)]}$$

$$+\sigma^2_{[H(\omega)]}(\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]})^2\mu^2_{[H^*(\omega)]}$$

$$+\sigma_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\sigma_{[H^*(\omega)]}\}d\omega\}^{1/2}\cdot\mu_{\vec{\phi}_k^T} \qquad (k = 1, 2, \dots, n) \tag{38}$$

$$\sigma_{[H(\omega)]} = diag\left\{\frac{\{[(2\mu_{\omega_j} + i\cdot 2\mu_{\xi_j}\omega)\cdot\sigma_{\omega_j}]^2 + [(i\cdot 2\mu_{\omega_j}\omega)\cdot\sigma_{\xi_j}]^2\}^{1/2}}{(\mu^2_{\omega_j} - \omega^2 + i\cdot 2\mu_{\xi_j}\mu_{\omega_j}\omega)^2}\right\} \quad (j = 1, 2, \dots, n) \tag{39}$$

In Eqs. (37) and (38), $\mu_{\psi^2_{uk}}$ and $\sigma_{\psi^2_{uk}}$ are mean value and mean variance of ψ^2_{uk}, respectively.

From Eqs. (37) and (38), the variation coefficient of the mean square value of the *k*th degree of freedom of the structural dynamic displacement response can be obtained

$$\nu_{\psi^2_{uk}} = \sigma_{\psi^2_{uk}} / \mu_{\psi^2_{uk}} \tag{40}$$

where $\nu_{\psi^2_{uk}}$ is the variation coefficient of the random variable ψ^2_{uk}.

From Eq. (34), the expressions of numerical characteristics of the element stress response can be deduced by means of the algebra synthesis method

$$\mu_{\left[\psi^2_{\sigma^{(e)}}\right]} = (\mu_{E^2} + \sigma_{E^2}) \cdot [B] \cdot \mu_{\left[\psi^2_{u^{(e)}}\right]} \cdot [B]^T \quad (e = 1, 2, \ldots, n_e) \tag{41}$$

$$\begin{aligned}\sigma_{\left[\psi^2_{\sigma^{(e)}}\right]} = \Bigg\{ & (\mu_{E^2} + \sigma_{E^2})^2 \cdot \left([B] \cdot \sigma_{\left[\psi^2_{u^{(e)}}\right]} \cdot [B]^T\right)^2 \\ & + (4\mu_{E^2}\sigma_{E^2} + 2\sigma_{E^4}) \cdot \left([B] \cdot \mu_{\left[\psi^2_{u^{(e)}}\right]} \cdot [B]^T\right)^2 \\ & + (4\mu_{E^2}\sigma_{E^2} + 2\sigma_{E^4}) \cdot \left([B] \cdot \sigma_{\left[\psi^2_{u^{(e)}}\right]} \cdot [B]^T\right)^2 \Bigg\}^{1/2} \quad (e = 1, 2, \ldots, n_e)\end{aligned} \tag{42}$$

where $\mu_{\left[\psi^2_{\sigma^{(e)}}\right]}$ and $\sigma_{\left[\psi^2_{\sigma^{(e)}}\right]}$ are the mean value and mean variance of the mean square value of the *e*th element stress response, respectively.

From Eqs. (41) and (42), the variation coefficient of the mean square value of the *e*th element stress response $\nu_{\left[\psi^2_{\sigma^{(e)}}\right]}$ can be obtained

$$\nu_{\left[\psi^2_{\sigma^{(e)}}\right]} = \sigma_{\left[\psi^2_{\sigma^{(e)}}\right]} / \mu_{\left[\psi^2_{\sigma^{(e)}}\right]} \quad (e = 1, 2, \ldots, n_e) \tag{43}$$

5. Examples

A 20-bar planar intelligent truss structure is utilized as an example. Active bar's and passive bar's materials and their parameters' value are given in Table 1. In order to solve the optimal problem, two steps are adopted.

Table 1 Intelligent truss structure's physical parameters

	Active bar (PZT-4)	Passive bar (steel)
Mean value of mass density ρ	7600 kg/m^3	7800 kg/m^3
Mean value of elastic module c_{33}	8.807*10^{10} N/m^2	2.1*10^{11} N/m^2
Piezoelectric force/electric constant e_{33}	18.62 C/m^2	-
Dielectric constant ε_{33}	5.92*10^{-9} C/Vm	-
Cross section area A	3.0*10^{-4} m^2	3.0*10^{-4} m^2

In the first step, reliability constrains of dynamic stress and displacement are neglected and the feedback gains are considered as constant. Then, each element bar is taken as active bar in turns, the corresponding performance function's value is calculated. The optimal location of active bar can be determined based on the computational results.

In the second step, after the active bar's optimal placement is determined, considering the reliability constrains, the optimization of feedback gains, that is, minimization of feedback gains will be developed.

5.1 Optimal placement of active bar

Let the closed loop control system feedback gains g = g_j = 50, each element bar is taken as active bar in turns, the corresponding performance function's value is given in Table 2.

From Table 2, it can be seen that the 1st element or the 4th element is utilized as active bar, the effect of active vibration control for the intelligent truss structure is best. However, the 20th element is utilized as active bar, the effect of active vibration control for the intelligent truss structure is worst.

Table 2 The computational results of performance function (g = 50)

Element	1	2	3	4	5	6	7	8	9	10
Value of J	–123.06	–117.83	–117.83	–123.06	–96.45	–85.76	–78.49	78.49	–85.76	–68.02
Element	11	12	13	14	15	16	17	18	19	20
Value of J	–71.17	–60.02	–60.02	–71.17	–58.33	–44.29	–36.75	–36.75	–44.29	–31.21

5.2 Optimization of feedback gains

In order to compare, the 1st element and the 20th element are utilized as active bar respectively. Considering the reliability constrains, the optimization of feedback gain is developed. The elastic module E, mass density ρ, bars' length L, bars' cross-section area A and structural damping (ξ_j) are all random variables, and let $\mu_{\xi_j} = \mu_{\xi} = 0.01$. A ground level acceleration act on the structure, $F(t)$ is a Gauss stationary random process and its mean value is zero. Its self-power spectral density can be expressed as Li *et al*. (2002)

$$S_{FF}(\omega) = \frac{1 + 4(\xi_g \omega / \omega_g)^2}{(1 - \omega^2/\omega_g^2)^2 + 4(\xi_g \omega / \omega_g)^2} S_0 \tag{44}$$

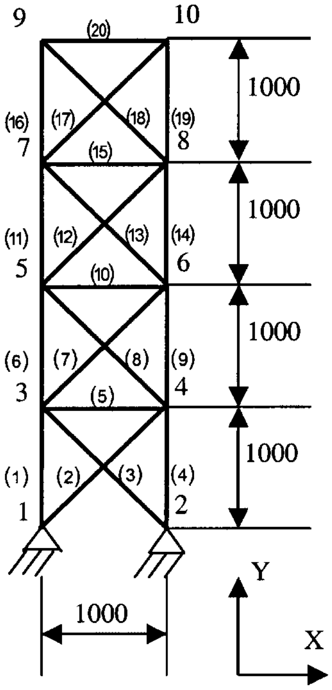
where $\omega_g = 16.5$, $\xi_g = 0.7$, $S_0 = 15.6\ \mathrm{cm}^2/\mathrm{s}^3$. $v(t)$ is the time modulation function and can be expressed as

$$v(t) = \begin{cases} (t/t_b)^2 & 0 \le t < t_b \\ 1.0 & t_b \le t < t_c \\ \exp[-\alpha(t - t_c)] & t \ge t_c \end{cases} \tag{45}$$

where t_b = 7.1s, t_c = 19.5s, α = 0.16.

$\mu_{\psi_\sigma^{2*}}$ and $\mu_{\psi_{uk}^{2*}}$ are all random variables. Their mean values are $\mu_{\psi_\sigma^{2*}} = \pm 2000(\text{Mpa}^2)$ and $\mu_{\psi_{uk}^{2*}} = \pm 3.0000(\text{mm}^2)$, respectively. In addition, $R^*_{\psi_\sigma^2} = R^*_{\psi_{uk}^2} = 0.95$.

To compare the optimal results of the determinate model and random model, they are all adopted in computational process. In the determinate model, the mean values of all random variables are regarded as determinate quantity, and their variation coefficients are taken as zero. In the random



(unit: mm)

Fig. 1 20-bar planar intelligent truss structure

Table 3 The computational results of feedback gains (*Dynamic analysis by Monte-Carlo simulation method)

	1st element utilized as active bar				16th element utilized as active bar			
Design variables	Original value	Determinate model	Random model (I)	Random model (II)	Original value	Determinate model	Random model (I)	Random model (II)
G	50	49.27	69.51	85.04	50	62.23	81.79	105.77
$^*G$			*69.53	*85.07			*81.82	*105.83
$\mu_{\psi_\sigma^2}$ (Mpa^2)	1737.6.	1999.7	1582.8	1253.4	2179.3	1999.1	1582.5	1252.9
$^*\mu_{\psi_\sigma^2}$ (Mpa^2)			*158.31	*1253.8			*1582.9	*1253.5
$\mu_{\psi_{uk}^2}$ (mm^2)	2.7493	2.8454	2.3741	1.9018	3.3079	2.8468	2.3739	1.9014
$^*\mu_{\psi_{uk}^2}$ (mm^2)			*2.3743	*1.9022			*2.3744	*1.9019
$R_{\psi_\sigma^2}$		0.47	0.98	0.98		0.47	0.98	0.98
$R_{\psi_{uk}^2}$		0.51	0.95	0.95		0.51	0.95	0.95

model, in order to investigate the effect of the dispersal degree of random variables E, ρ, l, A and ξ_j on the optimal results, the values of variation coefficients of parameters E, ρ, l, A, ξ_j, ψ_σ^{2*} and ψ_{uk}^{2*} are taken as two groups respectively. I: $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_{\xi_j} = \nu_{\psi_\sigma^{2*}} = \nu_{\psi_{uk}^{2*}} = 0.02$. II. $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_{\xi_j} = \nu_{\psi_\sigma^{2*}} = \nu_{\psi_{uk}^{2*}} = 0.2$ The corresponding optimal results are given in Table 3. In addition, in order to verify our method, the optimal results are given in Table 3, in which the random structural no-stationary random responses are obtained by Monte-Carlo simulation method.

From Table 3, it can be seen easily that the optimal results of the method proposed in this paper accord with that of the random structural no-stationary random responses are analyzed by Monte Carlo simulation method, by which the validity of our method is verified.

6. Conclusions

(1) The optimal results of normal model (that is determinate model) and random models are different. The optimal result of determinate model fulfill the normal constraints, but the result can not fulfill the reliability constrains. From the probability standpoint, in common instance, the former one is the infeasible solution of the later one.

(2) In order to attain the same effect of active vibration control for intelligent truss structure, different elements are utilized as active bar, the corresponding optimal results of feedback gain are remarkably different. The randomness of the physics parameters of structural materials, geometric dimensions and structural damping affect on the optimal results of feedback gains notably. The optimal value of feedback gains will increase remarkably along with the increase of the variation coefficients of these random variables.

(3) The results of the example show that the areas of the system where the most energy is stored are the optimal location of an active bar in order to maximize its damping effect. The example also shows that the model and solving method presented in this paper are rational and feasible.

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