# Structural analysis based on multiresolution blind system identification algorithm

## Gee-Pinn James Too<sup>†</sup>, Chih-Chung Kenny Wang<sup>‡</sup> and Rumin Chao<sup>†</sup>

Department of System and Naval Mechatronic Engineering, National Cheng Kung University, Tainan, Taiwan, R.O.C

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**Abstract.** A new process for estimating the natural frequency and the corresponding damping ratio in large structures is discussed. In a practical situation, it is very difficult to analyze large structures precisely because they are too complex to model using the finite element method and too heavy to excite using the exciting force method; in particular, the measured signals are seriously influenced by ambient noise. In order to identify the structural impulse response associated with the information of natural frequency and the corresponding damping ratio in large structures, the analysis process, a so-called "multiresolution blind system identification algorithm" which combines Mallat algorithm and the bicepstrum method. High time-frequency concentration is attained and the phase information is kept. The experimental result has demonstrated that the new analysis process exploiting the natural frequency and the corresponding damping ratio of structural response are useful tools in structural analysis application.

Key words: structural analysis; the multiresolution blind deconvolution algorithm; Howell-Bunger gate.

#### 1. Introduction

There are two processes for the structural analysis. One is the numerical method, which sets up the analyzed model and then analyzes it using the finite element method. The model is complex and the boundary condition of structures is needed. The other is the exciting force method, which offers a force to excite the structure and uses Fourier Transform to obtain frequency response. Unfortunately, for large structures, like buildings, bridges, hoisting machines and the Howell-Bunger gate, which is a large control floating valves at a dam, it is difficult to determine the natural frequency and damping ratio of structures by using the finite element method because the analyzed model is so complex. Furthermore, the exciting force method cannot offer enough forces to excite the structure because the weight of structure is too great. Even though the structures are excited by enough force, the measured signals from large structures include serious ambient noise. For those reasons, it is worth paying greater attention to the problem of how to obtain the natural frequency and the corresponding damping ratio in large structures.

Blind system identification algorithm is a fundamental signal processing technology aimed at

<sup>†</sup> Professor

<sup>‡</sup> Graduate Research Assistant

reconstructing an unknown system. Most blind system identification algorithms are basically adaptive filtering algorithms (Haykin 1991, Doroslovacki 1996, Qu *et al.* 1996, Petraglia and Torres 2002), designed in such a way that they do not need the external supply of desired response. On the other hand, they are blind to the desired response.

In this paper, a so-called "multiresolution blind system identification algorithm" (MBSI) for estimating the natural frequency and the corresponding damping ratio in large structures is presented. The measured signal is to decompose a multicomponent signal into a number of single component signals by using Mallat algorithm (Mallat 1989). The MBSI algorithm with biceptrum method is used to reconstruct the single component signal. Since Gaussian noise has zero mean value after high order statistics analysis, the additive Gaussian noise of each single component is diminished by using the biceptrum method (Petropulu and Nikias 1993). Then, the enhanced signal is obtained after reconstructing and combining each single component. The new process is characterized by its strong robustness against additive Gaussian noise and the low computing cost.

The structure of the paper is as follows. The MBSI algorithm is demonstrated in Section 2. Section 3 introduces the relationship between WT and structural analysis. The experimental results by analyzing the natural frequency and its damping ratio of the Howell-Bunger Valve located at the bottom of a dam are discussed in Section 4. Finally, a brief summary is offered in Section 5.

#### 2. Multiresolution blind system identification algorithm

During recent years, there has been a lot of interest in using higher order statistics (HOS) (Boumahdi 1996, Martin and Nandi 1996) in blind system identification (BSI). The word "blind" simply means that the system's input is not available to (cannot not be seen by) the signal processor. The task of BSI is to identify the input and the system function from the output. There are several reasons behind the interest. First, higher order cumulants are blind to all kinds of Gaussian processes, hence cumulants suppress additive colored Gaussian noise. Therefore if the signal to be analyzed is contaminated by additive Gaussian noise, the noise will vanish in the cumulant domain. Thus, a greater degree of noise immunity is possible. Second, cumulants are useful for identifying non-minimum phase systems or for reconstructing non-minimum phase signals if the signal. Third, cumulants are useful for detecting and characterizing the properties of nonlinear systems.

The multiresolution blind system identification algorithm (MBSI) combines Mallat algorithm and the bicepstrum method. The multiresolution analysis of  $L^2(R)$  is an important concept for Mallat algorithm. Decomposition coefficients in a wavelet orthogonal basis are computed with a Mallat algorithm. The decomposition and construction algorithm is as shown in Fig. 1.

Compared to traditional FIR and IIR mode blind system identification, the concept of the MBSI algorithm is that the raw signal is decomposed into several different resolution subspace signals with wavelet orthogonal basis, and then the bicepstrum method is used to estimate the inverse filter of each subspace. After the subspace signals are reconstructed with inverse filter, the enhanced signal is obtained by reconstructing each subspace signal and combining it.

The MBSI algorithm uses a time-frequency domain filter instead of traditional time domain filter. The multiresolution gives a controlled frequency resolution and the bicepstrum enhances the accuracy of the decomposition coefficients. Even though MBSI algorithm increases the operation of decomposition and construction for multiresolution and filtering operation in each subspace, the



Fig. 1 Multiresolution blind system identification algorithm

Table 1 The number of multiplication operations of the MBSI algorithm at 3 level and traditional FIR mode deconvolution

Deconvolution method	The number of multiplication operation
MBD at 3 level	(3MN/2) + (3NL/2) + (3NL/2) = ((3M/2) + 3L) N
Traditional FIR	2MN

where M is the order of inverse FIR, N is the length of input signal, and L is the length of scaling function

number of operations of MBSI algorithm is lower than traditional FIR mode deconvolution. Considering the Mallat algorithm, the operations are accompanied with a subsampling process, which allows the number operation to be gradually reduced. The multiplication operation of the MBSI algorithm and traditional FIR mode deconvolution are shown as Table 1.

#### 3. Wavelet transform

Wavelet transform (WT) of time-signal f(t) is defined by Daubechies (1990).

$$F(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \Psi\left(\frac{t-b}{a}\right) dt$$
(1)

The quantity  $\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right)$  given in the definition is referred to as the wavelet function.

The position variable *b* shifts the wavelet function along the time axis *t* of f(t) while the scale variable *a* expands or compresses the wavelet function  $\Psi_{a,b}(t)$ . Compared to Fourier transform, the scale variable *a* is equivalent to the inverse of the frequency.

In frequency domain, wavelet transform is rewritten as Eq. (2).

$$F_f(a,b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) \overline{\Psi}(a\omega) \exp(i\omega b) d\omega$$
(2)

where the  $f(\omega)$  and  $\overline{\Psi}$  are the frequency domain f(t) of and  $\Psi$ .

In order to demonstrate the relationship between WT and structure analysis, the Morlet wavelet (Daubechies 1992) is introduced because it is a modulated Gaussian signal from which the information of amplitude and phase can be kept. Its frequency domain is expressed as Eq. (3).

$$\overline{\Psi}(\omega) = \sigma \sqrt{2a\pi} e^{-\frac{\sigma^2(a\omega - \omega_0)^2}{2}} e^{-i\omega b}$$
(3)

Where  $\sigma$  is Gaussian parameter,  $\omega_0$  is modulated frequency, the scale is *a*.

If a signal is as Eq. (4), for example, A is amplitude,  $\omega_s$  is the frequency and  $\phi$  is the initial phase.

$$y(t) = A\cos(\omega_s t + \phi) \tag{4}$$

WT of the signal is as Eq. (5).

$$S(a,b) = A_{\sqrt{a}} \overline{\frac{\pi}{2}} e^{-\frac{(a\omega_s - \omega)^2}{2}} e^{i(\omega_s b + \varphi)}$$
(5)

According to Eq. (5), the coefficient of WT is proportional to the amplitude of the signal. The phase  $\varphi[S_f(a, b)] = \omega_n b$  of WT is consistent with the phase of the signal.

For structural analysis, the impulse response of a single degree of freedom system with a viscous damper is shown as Eq. (6).

$$x(t) = A \exp(-\xi \omega_n t) \cos(\omega_d t + \varphi_0)$$
(6)

$$\omega_d = \omega_n \sqrt{1 - \xi^2} \tag{7}$$

where A is amplitude,  $\omega_n$  is the frequency without damping,  $\omega_d$  is the frequency with damping,  $\xi$  is the damping ratio, and  $\varphi_0$  is the initial phase.

The WT of Eq. (6) is expressed by Eq. (8).

$$S(a,b) = A_{\sqrt{a}} \frac{\pi}{2} e^{-\xi \omega_n b} e^{-\frac{(a\omega_d - \omega)^2}{2}} e^{i(\omega_d b + \phi_0)}$$
(8)

If |S(a, b)| and  $\omega_d$  are known and then  $\xi \omega_n$  can be determined by Eq. (9).

$$\xi \omega_n = -\frac{d}{db} (\ln |S(a, b)|) \tag{9}$$

Finally,  $\omega_n$  and  $\xi$  are calculated by Eqs. (7) and (9).

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## 4. The natural frequency and damping resolution of Howell-Bunger gate

The Howell-Bunger gate (shown in Fig. 2) located at the bottom of a dam is used to release excessive water to control the flowing rate of water for emergent discharging. However, the structure of the Howell-Bunger gate is frustrated by hydroelastic instability and resonant vibration, caused by the interaction between the valve and rapid water (Lucey and Carpenter 1993, Bhattacharyya *et al.* 2000). There are two methods to find the natural modes of the system. For numerical FEM solution of natural modes, the model is so complex to set up and the boundary condition of the model is needed. The traditional exciting force method to get impulse response is hard to apply to this valve because the hammer cannot offer enough force to make this valve resonant in a practical situation. Consequently, the impact force is provided by rapid water when the gate is opened. Unfortunately, the impact force is unknown, moreover; ambient noise is generated and the signal-noise ratio (SNR) is reduced. To avoid the complex computation in numerical analysis and to enhance the impulse response signal of the Howell-Bunger gate in a practical situation, the MBSI algorithm based on Mallat algorithm and the bicepstrum method are used for extracting the natural frequency and damping ratio from the measured signal from the Howell-Bunger gate, especially in the case when impact force is unknown.

The measured signal and its wavelet transform (WT) are shown in Fig. 3 and Fig. 4, respectively. There are two main impulse responses at scale 86 and 36 in Fig. 4 and the amplitude and phase frequency responses of the corresponding coefficients are shown in Fig. 5 and Fig. 6, respectively. With the results of frequency responses, the identification of modes is very difficult because of the effect of ambient noise, especially where there is a low SNR.

In order to increase SNR and to reduce the effect of ambient noise, the MBSI algorithm is used to reconstruct the noise-free signal. The reconstructing signal and its WT are shown in Fig. 7 and Fig. 8. At scale 86 and 36, the amplitude and phase frequency response of the corresponding coefficient are shown in Fig. 9 and Fig. 10. The phase information of the signal is kept because the MBSI



Fig. 2 Howell-Bunger gate



Fig. 3 Measured signal from Howell-Bunger gate when it is opened



Fig. 4 The WT of measured signal

algorithm is used and the coefficient of WT is proportional to the amplitude of the signal. It helps to identify the exact natural frequency from the impulse response, since for real natural frequency, its phase should be  $180^{\circ}$  out of phase in its phase frequency response. This ensures that the scale 86 and 36 are really natural frequencies. In addition, SNR is increased from -9 dB to 23 dB by using MBSI algorithm.



Fig. 5 Frequency response and phase at scale 86 of WT of measured signal



Fig. 6 Frequency response and phase at scale 36 of WT of measured signal



Fig. 7 Measured signal after MBSI algorithm



Fig. 8 The WT of measured signal after MBSI algorithm



Fig. 9 Frequency response and phase at scale 86 of WT of measured signal after MBSI algorithm

Comparing Fig. 5 with Fig. 9, the results indicate that the MBSI algorithm can predict the natural frequency much more precisely than a regular WT analysis does. Fig. 6 and Fig. 10 also give the same indication. The damped frequency  $\omega_d$  and the natural frequency  $\omega_n$ , and its corresponding damping ratio of the Howell-Bunger gate can be evaluated from Eq. (7) and Eq. (8) and are shown in Table 2.

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Fig. 10 Frequency response and phase at scale 36 of WT of measured signal after MBSI algorithm

Table 2 The corresponding frequencies with damping and without damping, and the corresponding damping ratio of the Howell-Bunger gate

I Scale of WT	ltem	Natural frequency with damping, $(\omega_d/2\pi)$	Natural frequency without damping, $(\omega_n/2\pi)$	Damping ratio, $\xi$
36		95.2148 Hz	95.3313 Hz	0.0494
86		40.4358 Hz	40.4466 Hz	0.0231

### 5. Conclusions

This paper has demonstrated that the impulse response using the MBSI algorithm and WT can provide a reliable method to identify natural frequency and corresponding damping ratio for structural analysis. The advantages of the proposed approach are: First, use of the MBSI algorithm precludes the requirement of using the boundary condition for the finite element method to solve the natural frequency and its corresponding damping ratio. Second, in a practical situation, use of the MBSI algorithm based on Mallat algorithm and the bicepstrum method reduces the ambient noise and reconstructs the measured signal. It is helpful to identify structures from the impulse response, especially for structures which cannot be easily excited. In addition, MBSI can enhance SNR significantly, therefore, it can better predict natural frequency and damping ratio than a regular WT analysis does. Finally, the application of WT on structural analysis can be used to calculate the natural frequency and the damping ratio, because the coefficient of WT is proportional to the amplitude of the signal and phase information of the impulse response is kept by using MBSI algorithm.

Although this paper demonstrates reports for only two natural frequencies, the simultaneous occurrence of multiple natural frequencies can also be explored using this MBSI algorithm.

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