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Dynamics of thick hygrothermal viscoelastic composite laminates through finite element method

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Abstract. An uncoupled computational model for analyzing the hygrothermal dynamic response of composite laminates has been developed. The constitutive equations, expressed in an integral form, and involving relaxation moduli are adopted, to describe the non-aging hygrothermorheologically simple materials. A Prony series represents the relaxation moduli is exploited in order to derive a recursive relationship, and thereby eliminate the storage problem that arises when dealing with material possessing memory. The problem is formulated in a descritized variational form. Mindlin and higher order finite elements are employed for spatial descretization, while the Newmark average acceleration scheme is exploited for temporal descritization. The adopted recursive formula uses only the details of the previous event to compute the details of the current one. Numerical results of the displacement fields of both thin and thick viscoelastic laminates problems are discussed to show up the effectiveness of Mindlin and higher-order shear theories.

Key words: composite laminates; viscoelastic; finite element; hygrothermal.

1. Introduction

Viscoelastic dynamic analysis can be carried out in either the time or complex frequency domain (Hashin 1970a, b, Cederbsum and Aboudi 1989). The dynamic analysis of such structures requires a more accurate model for material damping rather than that of the velocity proportional damping schemes (Saravanos and Chamis 1990). Chen and Zhu (1990) set up the equation of motion of the elastic-viscoelastic composite structure in the time domain instead of the Laplace frequency domain. They transformed the integro-differential equation of motion into a first-order differential equation, where they discussed response calculations in the time domain and modal analysis. In addition to the methods proposed in Chen and Zhu (1990), Chen and Chan (2000) developed an integral element to carry out a full scale visco-elastic dynamic analysis of structures.

For general load histories, including hygrothermal loads, and viscoelastic material, direct time integration schemes appear to be the most robust discretized tool. These schemes require large amounts of storage to retain all of the previous solutions needed to evaluate the current state. This deficiency can be remedied by the development of some sort of recursion relationships (Taylor *et al*.

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1970). Yi and Hilton (1994) used a recursive formulation to determine the dynamic response of hygrothermorheologically simple viscoelastic composite plates. The formulation accounted for in-plane and transverse mechanical loads and the developed recursive formula allows the current event is to be calculated in terms of results of the previous two events. A direct integration scheme similar to that of Taylor *et al.* (1970) was employed by Hammerand and Kapania (1999) to evaluate the hereditary integrals describing the linear viscoelastic behavior of hygrothermorheologically simple laminated plates and shells, where the resulting recursion relationship requires only the results of the previous two events to compute the current one.

In the present study, a discretized computational model is developed for analyzing the linear uncoupled dynamic response of hygrothermal viscoelastic composite laminates in the time domain and exploiting the integral form of the constitutive law. Both Mindlin-Reissner and higher-order shear theories are introduced and the consistent mass concept is adopted in the framework of finite element discretization procedure. The formulation accounts for in-plane and transverse mechanical loads as well as hygrothermal loads. The developed recurrence formula permits the new event to be determined in terms of the only previous one. To illustrate the versatility of the developed model, the dynamic analysis of composite laminate, subject to bending loads, is carried out.

2. Displacement fields and deformations

The viscoelastic composite laminate is assumed to be in a plane stress state with transverse shear deformations. In this study, small deformations, and consequently Caushy stresses, are considered. The displacement fields are expressed in either one of the following forms:

(i) Mindlin-Reissner Displacement Field (Transverse Shear Element 1)

$$U(X,t) = u_o + z\theta_x, \quad V(X,t) = v_o + z\theta_y, \quad W(X,t) = w_o \tag{1}$$

(ii) Higher-Order Displacement Field (Transverse Shear Element 2)

$$U(X,t) = u_o + z\theta_x + z^2 u_o^* + z^3 \theta_x^*, \quad V(X,t) = v_o + z\theta_y + z^2 v_o^* + z^3 \theta_y^*, \quad W(X,t) = w_o \tag{2}$$

Where $X(x, y, z)$ are global laminate co-ordinates, u_o, v_o, w_o are displacements at the midplane ($z = 0$) of the laminate, θ_x and θ_y are the angles of rotation about the y and x axes, u_o^*, v_o^* and θ_x^*, θ_y^* are higher-order terms of inplane displacements and rotational angles, respectively.

The engineering strains expressed in the global laminate co-ordinate system are:

$$\bar{\varepsilon}_x(X,t) = \frac{\partial U}{\partial x}, \quad \bar{\varepsilon}_y(X,t) = \frac{\partial V}{\partial y}, \quad \bar{\varepsilon}_{xy}(X,t) = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}, \quad \bar{\varepsilon}_{yz}(X,t) = \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y},$$

$$\bar{\varepsilon}_{xz}(X,t) = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \tag{3}$$

For the two dimensional case, the engineering strains, $\bar{\varepsilon}$, are expressed in terms of the generalized strain vector, $\tilde{\varepsilon}$, which corresponds to the mid-plane at any point as:

$$\{\bar{\varepsilon}\} = [Z_c]\{\tilde{\varepsilon}\} \tag{4}$$

where the Z_c matrix and the $\tilde{\varepsilon}$ vector for each displacement field are given in Hassan *et al.* (1996).

3. Constitutive laws for orthotropic hygro-thermal visco-elastic lamina

The stress-strain relationships for orthotropic, non-aging, hygrothermorheologically simple, linear viscoelastic materials can be described in the principal material directions, $X_i(x_1, x_2, x_3)$, by the following hereditary integrals, (Yi and Hilton 1995):

$$\sigma_i(T, M, X_i, t) = \int_{-\infty}^{t} Q_{ij}(T, M, t-\tau)\frac{\partial}{\partial\tau}[\varepsilon_j(X_i, \tau) - \varepsilon_j^*(X_i, \tau)]d\tau \tag{5}$$

where σ_i is the stress tensor at temperature, T, moisture, M, and time, t; furthermore ε_j and ε_j^* are total and free hygrothermal strains which may be stated as:

$$\varepsilon_j^*(X_i, t) = \alpha_j\theta_T(X_i, t) + \beta_j\theta_M(X_i, t) \tag{6}$$

where α_j and β_j are thermal and hygroscopic expression coefficients which are independent of temperature, moisture and time; θ_T and θ_M are the changes in temperature and moisture from the stress-free state.

Based on the time-temperature-moisture superposition principles, the relaxation curves can be shifted, where master relaxation curves can be obtained at the reference temperature and moisture content. The relaxation moduli can be defined as:

$$Q_{ij}(T, M, t) = Q_{ij}(T_{rf}, M_{rf}, \zeta_{ij}) \tag{7}$$

where T_{rf} and M_{rf} are the reference temperature and moisture, and ζ_{ij} is the reduced time which is related to the temperature-moisture shift factor, ϕ_{ij}, as:

$$\zeta_{ij}(t) = \int_{o}^{t} \phi_{ij}[T(\tau), M(\tau)]d\tau \tag{8}$$

Assuming temperature and moisture are uniform throughout the laminate and timewise constant, Eq. (8) becomes as:

$$\zeta_{ij}(t) = \phi_{ij}(T, M)\cdot t \tag{9}$$

The relaxation moduli, Q_{ij}, and the reduced times, ζ_{ij}, can be rewritten in contracted notations as:

$$Q_r = Q_{ij}, \quad \zeta_r = \zeta_{ij} \tag{10}$$

where $r = 1, \ldots\ldots, 6$ for orthotropic lamina in a plane stress state with transverse shear deformations.

The Prony series represents the relaxation moduli as:

$$Q_r(T_{rf}, M_{rf}, \zeta_r) = Q_r^{\infty} + \sum_{p=1}^{N_r} Q_{rp} e^{-\zeta_r/\lambda_{rp}} \tag{11}$$

where λ_{rp} are the relaxation times, N_r are the numbers of terms in the series expansions and Q_r^{∞} and Q_{rp} are the fully relaxed and instantaneous moduli.

The transformed relaxation moduli, $\bar{Q}_{ij}$, with respect to the global laminate coordinates (Yi and Hilton 1995) can be obtained by the following transformation:

$$\bar{Q}_{ij}(t) = \sum_{r=1}^{6} A_{ijr} Q_r(t) \tag{12}$$

where A_{ijr} are transformation coefficients (Assie 2001).

4. Finite element formulation and numerical algorithm

As regard to the dynamic orthotropic hygrothermal viscoelasticity, the Hamilton's variational functional is applied. Taking the first variation of the linear functional, the following equilibrium equations are obtained for each element:

$$M^e_{m'n'}(X)\ddot{u}_{n'}(t) + \sum_{r=1}^{6} \int_{-\infty}^{t} k^e_{m'n'}(\zeta_r - \zeta_r') \frac{\partial u_{n'}(\tau)}{\partial \tau} d\tau = f^e_{m'}(t) + f^{the}_{m'}(t) \tag{13}$$

where the individual element matrices and vectors are defined in Assie (2001).

The assembly procedure is carried out to obtain the global stiffness and mass matrices in addition to the load vectors. Therefore, the integro-differential equations are given as:

$$M_{mn}(X)\ddot{U}_n(t) + \sum_{r=1}^{6} \int_{-\infty}^{t} K_{mn}(\zeta_r - \zeta_r') \frac{\partial U_n(\tau)}{\partial \tau} d\tau = F_m(t) + F^{th}_m(t) \tag{14}$$

Where m', n' range from 1 to the total degrees of freedom per element, while m, n range from 1 to the total degree of freedom in the global system.

To avoid large storage requirements, a numerical approach suggested by Yi (Hammerand and Kapania 1999) and developed in Assie (2001) for the solution of Eq. (14) is employed. Assuming the system is free of load at $t < 0$, Eq. (14) becomes at t_ℓ as follows:

$$M_{mn}\ddot{U}_n(t_\ell) + \sum_{r=1}^{6} \left[\left[K_{mnr} + \sum_{p=1}^{N_r} K_{mnrp} \exp\left(-\frac{\zeta_r(t_\ell)}{\lambda_{rp}} \right) \right] U_n(0) + \right.$$
$$\left. \int_o^{t_\ell} K_{mn}[\zeta_r(t_\ell) - \zeta_r'(\tau)] \frac{\partial U_n(\tau)}{\partial \tau} d\tau \right] = F_m(t_\ell) + F^{th}_m(t_\ell) \tag{15}$$

The global hygrothermal force vector, $F^{th}_m(t_\ell)$, can be developed into a recursive formula as shown in Yi and Hilton (1995).

Applying Newmark average acceleration direct integral method, Eq. (15) would be reduced to the following recursive form:

$$\left[\left(\frac{2}{\Delta t_\ell}\right)^2 \tilde{H}_{mn}(t_\ell) + \left(\frac{2}{\Delta t_\ell}\right)\tilde{D}_{mn}(t_\ell) + \sum_{r=1}^{6} K_{mnr}\right] U_n(t_\ell) = \tilde{F}_m(t_\ell) +$$
$$\tilde{H}_{mn}(t_\ell)\left[\left(\frac{2}{\Delta t_\ell}\right)^2 U_n(t_{\ell-1}) + \frac{4}{\Delta t_\ell}\dot{U}_n(t_{\ell-1}) + \ddot{U}_n(t_{\ell-1})\right] +$$
$$\tilde{D}_{mn}(t_\ell)\left[\left(\frac{2}{\Delta t_\ell}\right) U_n(t_{\ell-1}) + \dot{U}_n(t_{\ell-1})\right] \tag{16}$$

where all terms of this equation are well defined in Assie (2001).

It is worthily noticed that Eq. (16) represents a simple recurrence formula for evaluating the displacement vector at time, t_ℓ, in terms of the event at time, $t_{\ell-1}$.

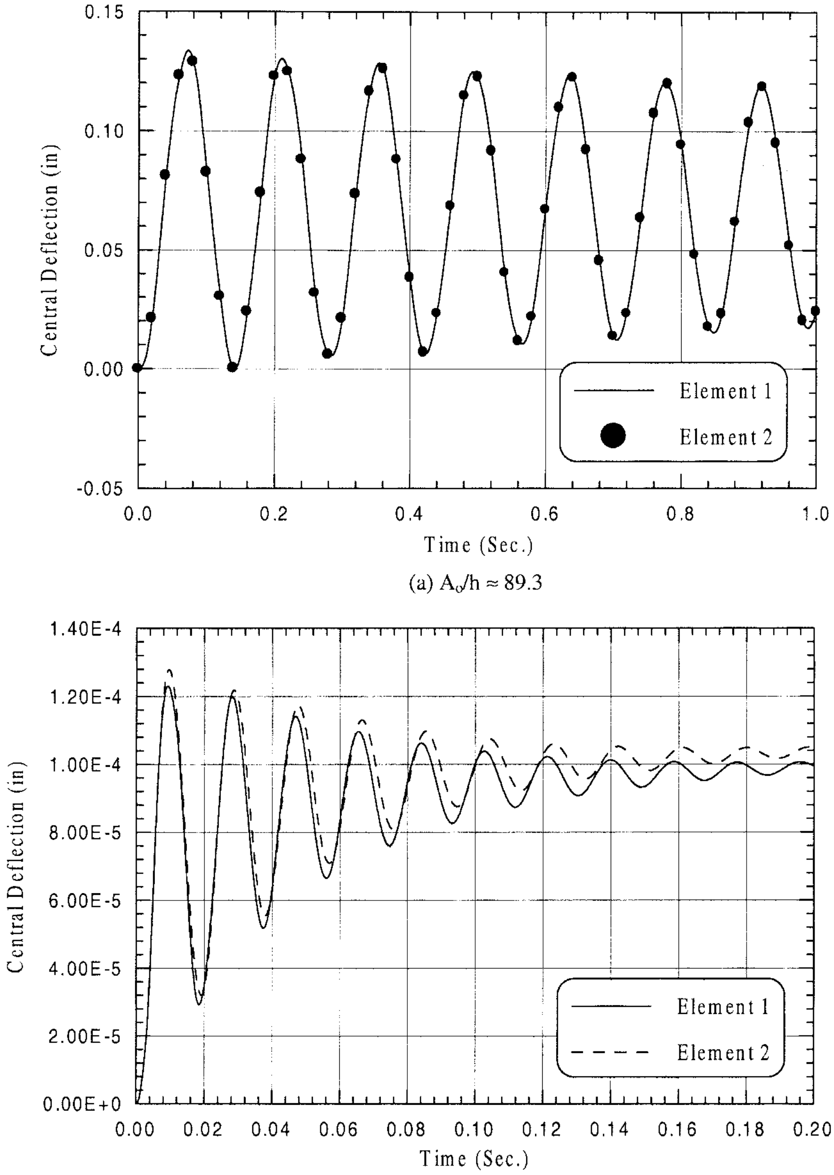
5. Numerical results

The suggested numerical approach, addressed in the previous section is incorporated into the framework of a traditional finite element model. The displacement models discussed in section (2) are exploited and heterosis element is adopted for discretization (Hassan *et al.* 1996).

As to the following examples, the dynamic central viscoelastic responses of simply supported composite laminates are evaluated. The laminates are subjected to uniformly distributed transverse unit step loads with zero initial conditions. All of the laminates have 20 in × 20 in square plan form, the thicknesses of all layers are equal and a time step size of 0.0001 sec. is used. Due to the biaxial symmetry of rectangular cross-ply laminates, one quadrant of the plate is discretized into four heterosis elements. The elastic properties of GY70/339 composites, their master relaxation curves and the shift factors are found in Yi (1987). It is assumed that $G_{12} = G_{13} = G_{23}$ (shear rigidities) for each lamina and Poisson's ratio ν_{23} and ν_{13} are assumed to be the same as ν_{12}. Since E_1 is controlled by fiber properties, it is assumed that the stiffness Q_{11} is time independent while other relaxation moduli such as Q_{12}, Q_{22}, Q_{33}, Q_{44}, and Q_{55} have the same time dependent function and the same shift factor. Also, all of the laminates are analyzed at 2.2% constant moisture content under isothermal conditions (T = 129°F).

Symmetric and unsymmetric stacking sequences, $(0^o{}_{10}/90^o{}_{10})_s$ and $(0^o{}_{20}/90^o{}_{20})_t$, are used to compare between Element type (1) and Element type (2) in analyzing thin/thick viscoelastic laminates. The thickness of each lamina is 0.0056 in and 0.1 in for the thin and thick plates, respectively.

From Figs. 1 and 2, it is shown that the viscoelastic deflections obtained by using both Element type (1) and Element type (2) are almost coincident for the thin plate limit where the plate side to thickness ratio, A/H, equals 89.3. Meanwhile, in the thick limit of Element type (2) the model exhibits higher viscoleastic deflection than that obtained from Element type (1), where a large discrepancy is observed in unsymmetric laminates. In general, it is noticed that the response of thick laminates rapidly decay than that of the thin ones. Also, it is noticed that the viscoelastic damping dominantly affects the frequency of the unsymmetric thick laminates. These obtained results are highly consistent with the concepts of higher-order shear and Mindlin-Reissner theories.



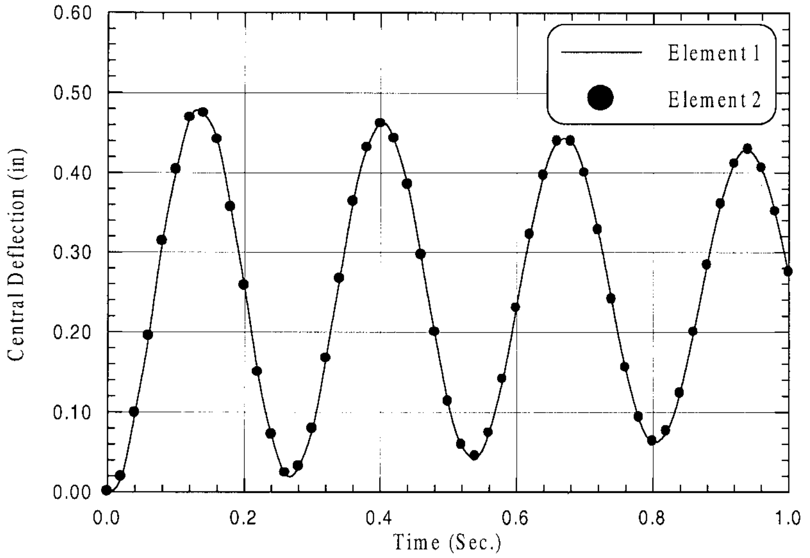
(a) $A_o/h \approx 89.3$

(b) $A_o/h = 5.0$

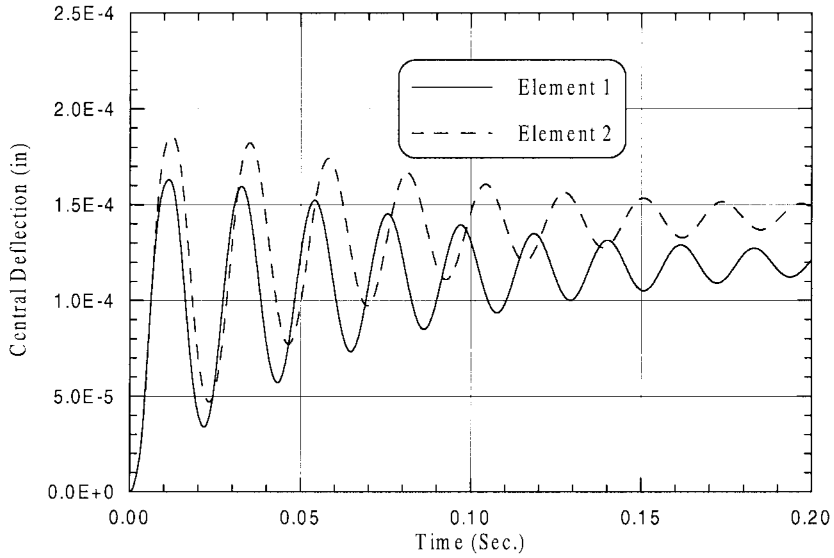
Fig. 1 The viscoelastic deflection of a simply supported plate with $(0°_{10}/90°_{10})_s$ -stacking sequence

6. Conclusions

A numerical algorithm for analyzing dynamic responses of orthotropic hygrothermal viscoelastic composite laminates has been developed in the time domain. The integral form of the constitutive laws is used. Mindlin-Reissner and higher-order transverse shear deformation theories are utilized in the finite element formulations employing the consistent mass matrix. The proposed algorithm is incorporated in the framework of the finite element the displacement model. Two different finite



(a) $A_o/h \approx 89.3$



(b) $A_o/h = 5.0$

Fig. 2 The viscoelastic deflection of a simply supported unsymmetric Laminate with $(0^o{}_{20}/90^o{}_{20})_t$ -lamination scheme

element types are exploited. The developed recurrence formula permits the current displacement vector to be computed using only the previous one. Hygrothermal environment causes degradation of the stiffness, and consequently, decaying of the vibration amplitude. The effectiveness of each type of elements, in simulating the response of thin/thick viscoelastic laminates, is discussed.

References

Assie, A.E. (2001), "Transient vibration of viscoelastic composite plates", Ph.D. Thesis, Zagazig University, Zagazig, Egypt.

Cederbsum, G. and Aboudi, J. (1989), "Dynamic response of viscoelastic laminated plates", *J. Sound Vib.*, **133**(2), 225-238.

Chen, Q. and Chan, Y.W. (2000), "Integral finite element method for dynamical analysis of elastic-viscoelastic composite structures", *Comput. Struct.*, **74**(3), 51-64.

Chen, Q. and Zhu, D.M. (1990), "Vibrational analysis theory and application to elastic-viscoelastic composite structures", *Comput. Struct.*, **37**(4), 585-595.

Hammerand, D.C. and Kapania, R.K. (1999), "Thermoviscoelastic analysis of composite structures using a triangular flat shell element", *AIAA Journal*, **37**(2), 238-247.

Hashin, Z. (1970a), "Complex moduli of viscoelastic composites - I. General theory and applications to particulate composites", *Int. J. Solids Structures*, **6**, 539-552.

Hashin, Z. (1970b), "Complex moduli of viscoelastic composites - II. Fiber reinforced materials", *Int. J. Solids Structures*, **6**, 797-807.

Hassan, M.M., Assie, A.E. and Mahmoud, F.F. (1996), "Static analysis of symmetrical and unsymmetrical laminated plates using heterosis finite element models", *Journal of Engineering and Applied Science*, Faculty of Engineering, Cairo University, **43**(4), 733-748.

Saravanos, D.A. and Chamis, C.C. (1990), "Mechanics of damping for fiber composite laminates including hygrothermal effects", *AIAA Journal*, **28**(10), 1813-1919.

Taylor, R.L., Pister, K.S. and Goudreau, G.L. (1970), "Thermo-mechanical analysis of viscoelastic solids", *Int. J. Numer. Meth. Eng.*, **2**(1), 45-59.

Yi, S. (1987), "Time-dependent behavior of composite laminates in hygrothermal environments", M.Sc. Thesis, University of Washington, Seattle, WA.

Yi, S. and Hilton, H.H. (1994), "Dynamic finite element analysis of viscoelastic composite plates in the time domain", *Int. J. Numer. Meth. Eng.*, **37**, 4081-4096.

Yi, S. and Hilton, H. (1995), "Hygrothermal effects on viscoelastic responses of laminated composites", *Composite Engineering*, **5**(2), 183-193.