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Hydrodynamic pressures acting on the walls of rectangular fluid containers

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Abstract. The dynamic response characteristics of a rectangular fluid container are investigated by using finite element method. The fluid is assumed to be linear-elastic, inviscid and compressible. A displacement-based fluid finite element was employed to allow for the effects of the fluid. A typical rectangular fluid container, which is used in recent studies, is considered for the numerical analysis. The North-South component of El Centro Earthquake records is used as input ground acceleration. Rigid and flexible fluid containers solutions are obtained for the chosen sample tank. Hydrodynamic pressures and sloshing motions are determined using Lagrangian fluid finite element. The results obtained from this study are compared with the results obtained by boundary-finite element method (BEM-FEM) and requirements of Eurocode-8. Based on the numerical analysis, some conclusions and discussions on the design considerations for rectangular fluid containers are presented.

Key words: rectangular fluid containers; seismic design; finite element method; EC-8.

1. Introduction

It is known that, some of the fluid containers are damaged in many earthquakes. Damage or collapse of these containers causes some unwanted events such as shortage of drinking and utilizing water, uncontrolled fires and spillage of dangerous fluids. Even uncontrolled fires and spillage of dangerous fluids subsequent to a major earthquake may cause substantially more damage than the earthquake itself (Priestley *et al.* 1986). Due to this reason this type of structures which are special in construction and in function from engineering point of view must be constructed well to be resistant against earthquakes. There have been numerous studies done for dynamic behavior of fluid containers; most of them are concerned with cylindrical tanks. But very few studies on the dynamic response of rectangular containers exist, when compared to that of the cylindrical tanks (Rammerstorfer Scharf and Fischer 1990).

Hoskins and Jacobsen gave the first report on analytical and experimental observations of rigid rectangular tanks under a simulated horizontal earthquake excitation (Hoskins and Jacobsen 1934). Graham and Rodriguez used spring-mass analogy for the fluid in a rectangular container (Graham and Rodriguez 1952). Housner proposed a simple procedure for estimating the dynamic fluid effects

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of a rigid rectangular tank excited horizontally by an earthquake (Housner 1957, 1963). An extended application of Housner's concept in the sense of a practical design rule is given by Epstein (1976). After these important studies some researches are made about rectangular fluid containers. These researches may be summarized as below:

- Studies related to flexibility of walls (Minowa 1980, Priestley 1986, Doğangün 1995, Doğangün Durmuş and Ayvaz 1996, 1997, Kim Koh and Kwahk 1996, Koh Kim and Park 1998)
- Studies related to seismically induced bending moments in walls (Haroun 1984)
- Studies related to sloshing (Bauer and Eidel 1987, Lepettier and Raichlen 1988, Haroun and Chen 1989)
- Studies related to soil-structure interaction (Kim Park and Jin 1998)
- Studies related to seismic isolation (Park Koh and Kim 2000)
- Experimental studies (Minowa 1984, Koh Kim and Park 1998)

First author could not compare the results of his dissertation, which is related to the dynamic behaviour of rectangular fluid containers, with the results obtained from numerical methods due to lack of information in 1995. However, very important papers related to this subject from Koh, Kim and their study group had begun to be published after the end of 1996. European Committee for Standardization prepared a new code named Eurocode-8 (1998). Part 4 of this code is related to tanks, silos and pipelines. But, requirements for rectangular tanks are very limited according to cylindrical tanks in this code. There is a statement related to rectangular storage tanks as "studies on the behavior of flexible rectangular tanks are not numerous, and the solutions are not amenable to a form suitable for direct use in design" in this code. Therefore, it is explained that the method suggested by New Zealand Code (Priestley *et al.* 1986) may be used as an approximation for design. In the current study, seismic analysis of a selected rectangular tank is made using Lagrangian fluid finite element and the results are compared with the result of boundary-finite element method developed by Koh, Kim and Park (1998) and the results obtained by using requirements of Eurocode-8.

2. Lagrangian approach

Basic formulation for fluid-structure interaction using finite element method with Lagrangian approach is summarized below:

Three assumptions made for this study are given below:

1) Fluid is compressible and linear elastic. The used finite element is based on a formulation in which the fluid strains are calculated from the linear strain-displacement equations. The only strain energy considered is associated with the compressibility of the fluid (Wilson and Khalvati 1983). The pressure volume relationship for a linear fluid is given by;

$$p = E_v \varepsilon_v \tag{1}$$

where the pressure p is equal to the magnitude of the mean stress, E_v is the bulk modulus of fluid, and ε_v is the volumetric strain.

2) Viscosity effects are negligible: This assumption is not contrary to the fact since the effect of viscosity for the dynamic behavior of fluid storage tanks is negligible and this effect decreases when dimensions of tanks increase (Priestley 1986).

3) Displacement field is constrained to be irrotational by introduction of a rotational stiffness. If the fluid is assumed to have no shear strength, and the elasticity matrix for the fluid is with the shear modulus set to zero. This results in a singular stress-strain matrix which in turn leads to spurious, zero-energy deformation modes for fluid elements and fluid meshes. A possible method of overcoming this problem is to assume a small value for shear modulus of the fluid. A second approach is to admit the inviscid behaviour and to use the implication that the fluid must be irrotational in nature. This behaviour can be enforced by the use of a penalty function as used in this study. Rotations (ε_{xr}, ε_{yr}, ε_{zr}) and constraint parameter (E_{11}, E_{22}, E_{33}, E_{44}) for the x, y and z directions which are necessary to satisfy the rotation constraints in the this assumption are as follows:

$$\varepsilon_{xr} = \frac{1}{2}\left[\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right] \qquad \varepsilon_{yr} = \frac{1}{2}\left[\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right] \qquad \varepsilon_{zr} = \frac{1}{2}\left[\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right] \tag{2}$$

$$E_{11} = E_v \qquad E_{22} = \psi_x E_v \qquad E_{33} = \psi_y E_v \qquad E_{44} = \psi_z E_v \tag{3}$$

where ψ_x, ψ_y and ψ_z are constraints parameter coefficients. From here, rotation pressures (p_{xr}, p_{yr}, p_{zr}) are as below;

$$p_{xr} = E_{22}\varepsilon_{xr} \qquad p_{yr} = E_{33}\varepsilon_{yr} \qquad p_{zr} = E_{44}\varepsilon_{zr} \tag{4}$$

The total potential energy (U) of the fluid system consist of the sum of the strain energy (Π_ε) and the increase in potential energy (Π_s) by taking into account the free surface oscillations of the fluid. The expression for this energy is as follows:

$$U = \Pi_\varepsilon + \Pi_s \rightarrow U = \frac{1}{2}\int \varepsilon^T E \varepsilon dV + \frac{1}{2}\int u_s \rho g (H + u_s) dv \tag{5}$$

where $\boldsymbol{E}$ is elasticity matrix, u_s is the vertical displacement of the fluid, H is the fluid height, g is the acceleration due to gravity and ρ is the mass density of fluid. The kinetic energy (T) of the fluid is;

$$T = \frac{1}{2}\int \rho v^T v dv \tag{6}$$

where v ($v^T = [\, v_x \;\; v_y \;\; v_z]$) is the velocity vector in the Cartesian coordinates.

Three-dimensional isoperimetric fluid element with eight nodes is considered in Lagrangian approach. Global (x, y, z) and local axes (r, s, t) are given in Fig. 1 for this element.

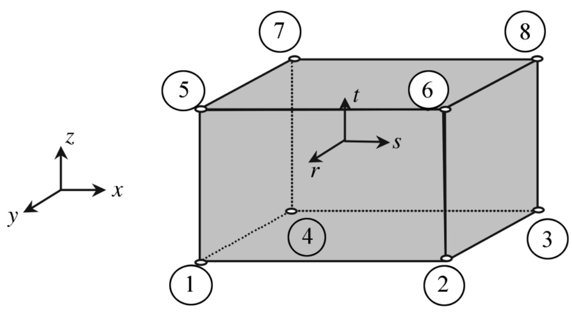


Fig. 1 Fluid finite element considered

Expressions for mass and rigidity matrices are given below;

$$\boldsymbol{M} = \rho\int_v \boldsymbol{Q}^T\boldsymbol{Q}dV \rightarrow \boldsymbol{M} = \rho\sum_i\sum_j\sum_k \boldsymbol{\eta}_i\boldsymbol{\eta}_j\boldsymbol{\eta}_k\boldsymbol{Q}_{ijk}^T\boldsymbol{Q}_{ijk}\det\boldsymbol{J}_{ijk} \tag{7}$$

$$\boldsymbol{K} = \int_v \boldsymbol{B}^T\boldsymbol{E}\boldsymbol{B}dV \rightarrow \boldsymbol{K} = \sum_i\sum_j\sum_k \boldsymbol{\eta}_i\boldsymbol{\eta}_j\boldsymbol{\eta}_k\boldsymbol{B}_{ijk}^T\boldsymbol{E}\boldsymbol{B}_{ijk}\det\boldsymbol{J}_{ijk} \tag{8}$$

where $\boldsymbol{J}$ is the Jacobian matrix, $\boldsymbol{Q}_{ijk}$ is the interpolation function, η_i, η_j and η_k are weighting functions, $\boldsymbol{B}$ is the strain-displacement matrix which is obtained from $\boldsymbol{\varepsilon} = \boldsymbol{B}\,\boldsymbol{u}$ expression. Rigidity occurred from surface oscillations;

$$\boldsymbol{S} = \rho g\int_A \boldsymbol{Q}_s^T\boldsymbol{Q}_s dA \rightarrow \boldsymbol{S} = \sum_i\sum_j \boldsymbol{\eta}_i\boldsymbol{\eta}_j\boldsymbol{Q}_{sij}^T\boldsymbol{Q}_{sij}\det\boldsymbol{J}_{ij} \tag{9}$$

where $\boldsymbol{Q}_s$ is the interpolation function for two dimensional surface element. After the mass and rigidity matrices are obtained by Eqs. (7) and (8), total potential and kinetic energy expressions in the finite element can be written as;

$$U = \Pi_\varepsilon + \Pi_s \rightarrow U = \frac{1}{2}\boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u} + \frac{1}{2}\boldsymbol{u}_s^T\boldsymbol{S}\boldsymbol{u}_s \tag{10}$$

$$T = \frac{1}{2}\boldsymbol{v}^T\boldsymbol{M}\boldsymbol{v} \tag{11}$$

If the expressions for kinetic and potential energies are substituted into Lagrange equation, which is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_j}\right) - \frac{\partial T}{\partial u_j} + \frac{\partial u}{\partial u_j} = F_j \tag{12}$$

where u_j is the j^{th} displacement component and F_j is the applied external load, the governing equation can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} + \boldsymbol{S}\boldsymbol{u}_s = \boldsymbol{R} \tag{13}$$

where $\ddot{\boldsymbol{u}}$ is the acceleration and $\boldsymbol{R}$ is a general time varying load vector.

3. Eurocode-8 requirements

3.1 Assuming walls are rigid

The total pressure (p) is given by the sum of an impulsive (p_i) and a convective (p_c) contribution: for the tanks whose walls can be assumed as rigid:

$$p(z, t) = p_i(z, t) + p_c(z, t) \tag{14}$$

The impulsive (p_i) pressure is given by;

$$p_i(z, t) = q_0(z) \cdot \rho \cdot L \cdot A_g(t) \tag{15}$$

where L is the half-with of the tank in the direction of the seismic action, $q_0\,(z)$ is the function plotted in Fig. 2 and $A_g(t)$ is the ground acceleration.

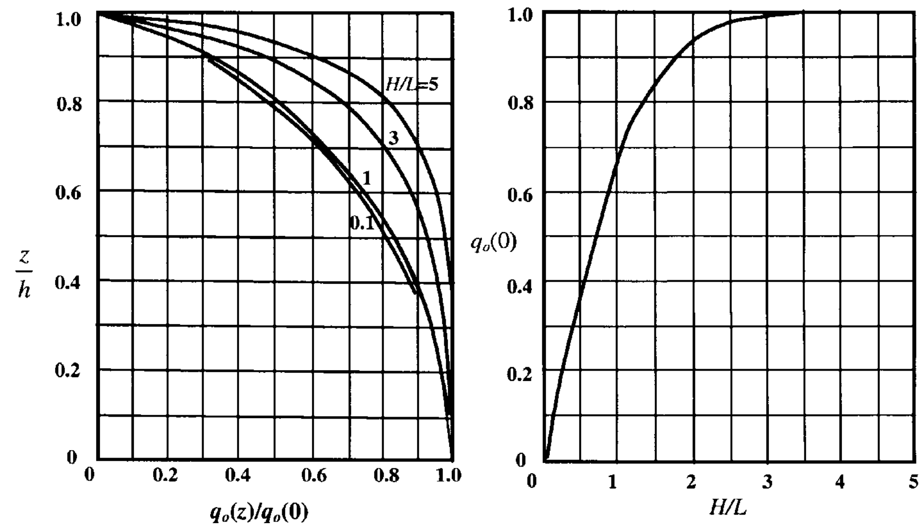
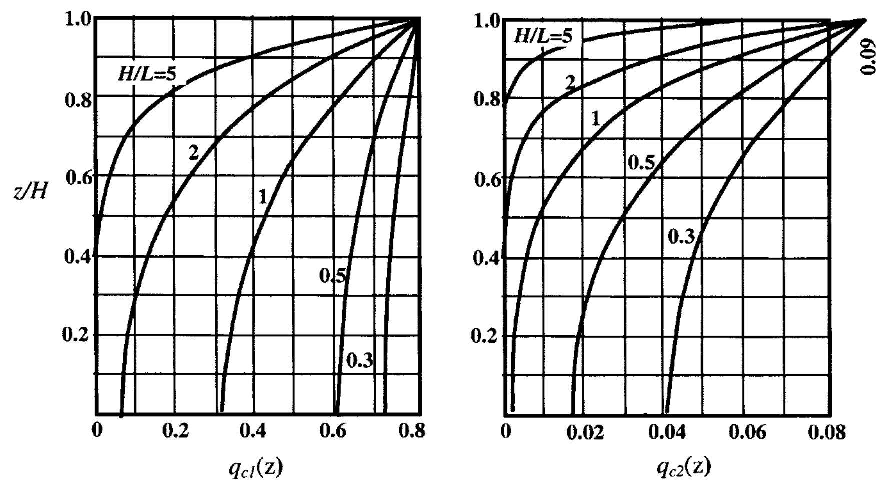


Fig. 2 Dimensionless impulsive pressure on rectangular fluid container wall

The convective pressure component is given by a summation of sloshing modes. The pressure is:

$$p_{cn}(z,t) = q_{cn}(z)\rho \cdot L \cdot A_n(t) \tag{16}$$

where $q_{cn}(z)$ is shown Fig. 3 for first and second sloshing modes and $A_n(t)$ is the acceleration response function of a simple oscillator having frequency of the n. mode



(a) first mode (b) second mode

Fig. 3 Dimensionless convective pressure on rectangular fluid container wall

The period of oscillation of the first sloshing mode is:

$$T_1 = \sqrt{\frac{L/g}{\frac{\pi}{2}\tanh\left(\frac{\pi H}{2 L}\right)}} \tag{17}$$

3.2 Assuming walls are flexible

It is concluded in Eurocode-8 that wall flexibility produces generally a significant increase of the impulsive pressures while leaving the convective pressures practically unchanged. So, for flexible rectangular storage tanks, an approximation which is suggested in Priestley (1986) is to use the same pressure distribution valid for rigid walls. But ground accelerations $A_g(t)$ in Eq. (15) replaced with the response acceleration of a simple oscillator having the frequency and the damping factor of the first impulsive tank-fluid mode. The period of vibration of the first impulsive storage tank-fluid horizontal mode is given approximately by:

$$T_f = 2\pi\sqrt{d_f/g} \tag{18}$$

Where d_f is the deflection of the wall on the vertical centre-line and at the height of the impulsive mass, when wall is loaded by a load uniform in the direction of the ground motion and of magnitude $m_i g/(4BH)$. Where B is the half with perpendicular to the direction of loading (earthquake direction) and m_i is the impulsive mass. This mass can be obtained from the equivalent cylindrical tank results and should include the wall mass (Eurocode-8 1998). For tanks without roofs the deflection d_f may be calculated assuming the wall to be free at the top and fixed on the other three sides.

4. Housner method

Housner divided the hydrodynamic pressure into two components. The impulsive part represents the portion of fluid which moves in unison with the tank, while the convective component represents the portion of the fluid sloshing in the tank.

In addition to the assumptions made by Hoskins and Jacobsen (1934) Housner assumed that the fluid is kept between vertical membranes and those displacements are small. He suggested the following equation to determine the impulsive pressure.

$$p_i(z, t) = a(t)\rho H\sqrt{3}\left[\frac{z}{H} - \frac{1}{2}\left(\frac{z}{H}\right)^2\right]\tanh\left[\sqrt{3}\frac{L}{H}\right] \tag{19}$$

A gap which is 2% of the tank height is sufficient between the fluid free surface and the bottom surface of the top plate of the tank for the fluid oscillation freely. The following equations is suggested by Housner for the convective pressure, after kinetic and potential energy expressions of the fluid are substituted into Hamilton's principle.

$$p_c(z, t) = 0.527\rho L^2 \omega_n^2 \Phi \frac{\cosh\left[1.581\frac{H-z}{L}\right]}{\sinh\left[1.581\frac{H}{L}\right]}\sin(\omega_n t) \tag{20}$$

where ω_n is the angular frequency of the fluid, Φ is equal to S_a/g, where S_a is the spectrum acceleration and g is the acceleration due to gravity.

Housner method is used widely in the dynamic analysis of the tanks and it is incorporated into relevant design codes. Epstein (1976) improved Housner's work and presented design curves for estimating the bending and overturning moment induced by the hydrodynamic pressure for rectangular rigid tanks.

5. Numerical example

In this study, a rectangular storage tank with two different wall thicknesses is considered as shown in Fig. 4. This tank is selected as the same tank considered by Koh, Kim and Park (1988) to compare the results. In the example, the bulk modulus and density of fluid are taken to be 207×10^7 N/mm^2 and 1000 kg/m^3, respectively.

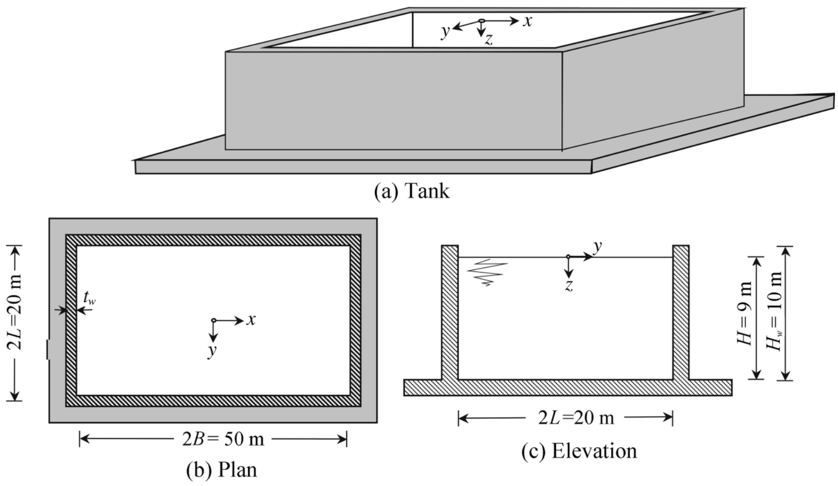


Fig. 4 Plan and elevation of the sample rectangular tank

It is assumed that the tank is subjected to N-S component of El Centro Earthquake in the direction parallel to the short side walls and it is fixed to the ground. Damping ratio for fluid is taken to be ξ = 0.5% as recommended in Eurocode-8.

In the analysis of the tank by finite element method using Lagrangian approach, the modified structural analysis program SAPIV (Bathe Wilson and Peterson 1974, Doğangün 1995) is used as the computational tool.

5.1 Solution by assuming walls to be rigid

In this analysis, unit width tank model perpendicular to the direction of the motion is considered to compare the results of analytical and finite element methods. Finite element model of the fluid for rigid walls is given in Fig. 5. In this model, fluid is allowed to move freely in the vertical direction along the walls and in the horizontal direction at the bottom.

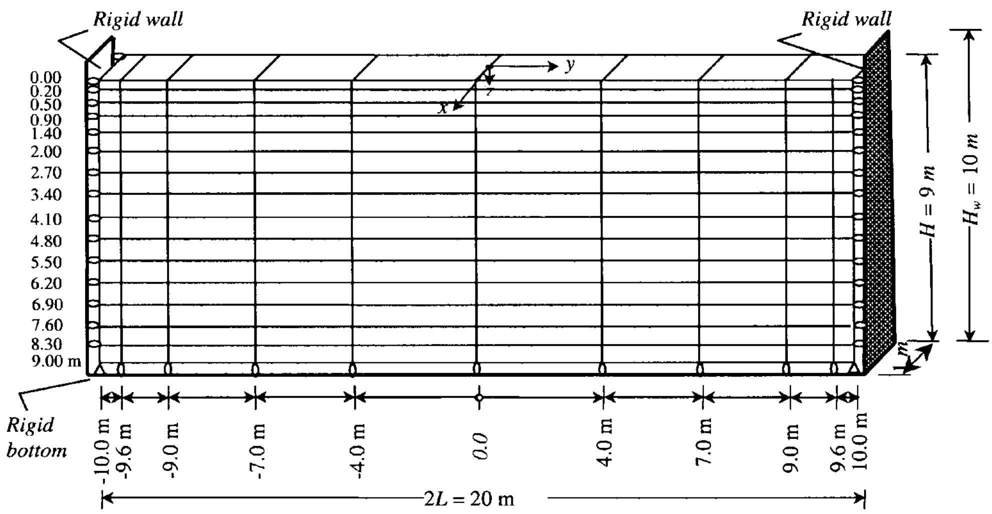


Fig. 5 Finite element mesh considered for unit width of tank in rigid solution

The distribution of hydrodynamic pressures obtained by Housner method, Eurocode-8 and this study are given in Fig. 6. As seen from this figure, the distribution of the hydrodynamic pressure obtained by these three different studies are generally good agreement. Hydrodynamic pressures obtained from this study are generally larger than that obtained by the other methods.

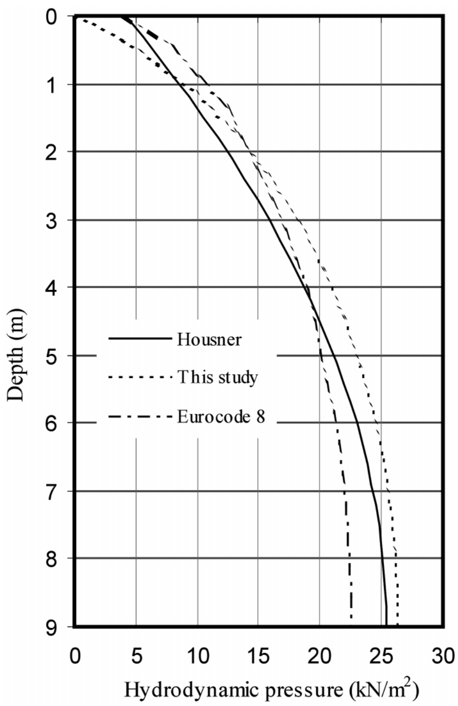


Fig. 6 Hydrodynamic pressure distributions obtained from assuming walls are rigid

The maximum wave heights (d_{max}) for the rigid tank are determined to be 0.49 m and 0.57 m by Housner and the finite element method, respectively. In order to obtain d_{max} by Housner method, the acceleration spectrum needed is taken to be 0.5 m/s, for the first mode T=5.3 sec. and 0.5% damping ratio are used.

5.2 Solution by assuming the walls to be flexible

In this solution, it is assumed that the walls have a prescribed flexibility depending on material and geometric characteristics. The flexible tank model for the analysis by the finite element method is given in Fig. 7. It is assumed that the bottoms of the walls are fixed.

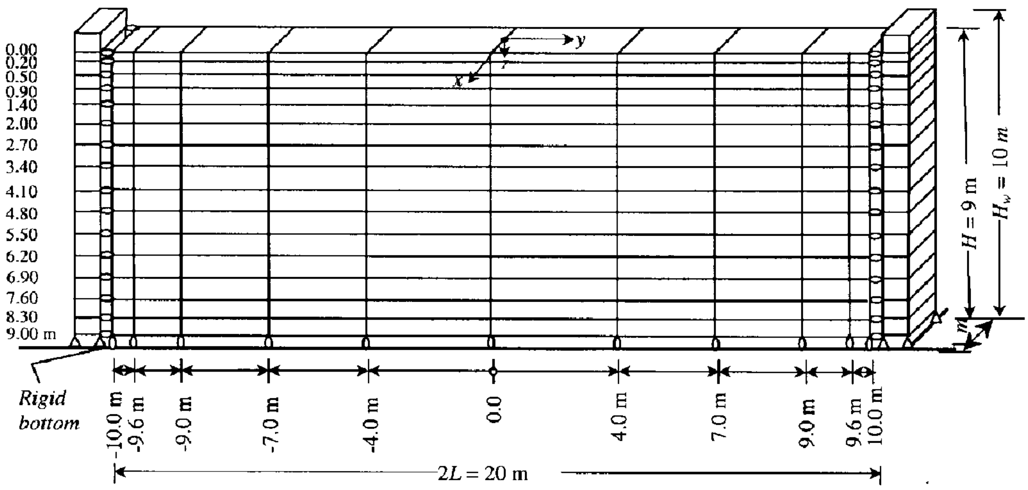


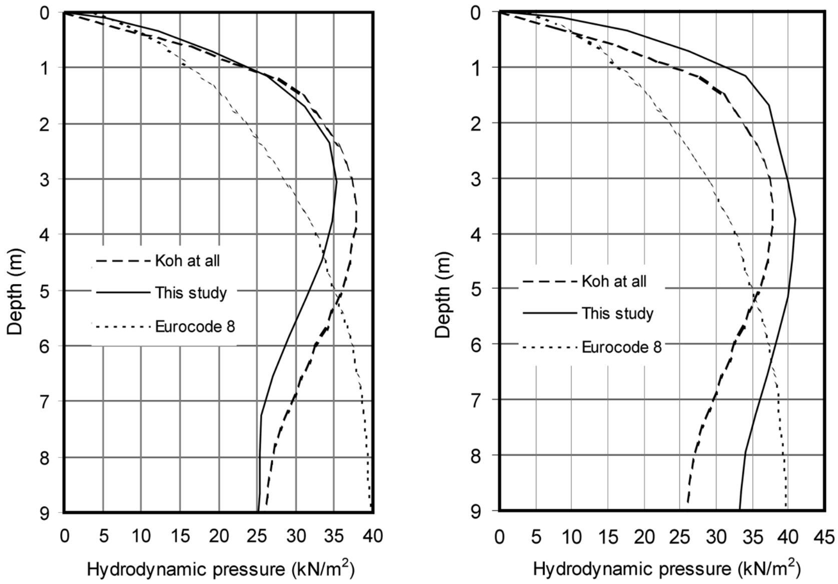
Fig. 7 Finite element mesh considered for unit width of flexible solution

In this solution; density, Young's modulus and Poisson's ratio of structural material are taken to be 2400 kg/m^3, 2.1×10^{10} N/m^2 and 0.17, respectively. Structural damping is selected as 3 per cent, which is the same as in reference (Koh Kim and Park 1988). Two different wall thicknesses are selected: t_w = 1.0 m and t_w = 0.5 m.

The hydrodynamic pressure distributions acting on tank wall are given Fig. 8 for wall thickness of 1.0 m and 0.5 m. As seen from these figures and from Fig. 4 the hydrodynamic pressure distribution obtained from the flexible solution is larger than that of the rigid solution.

The maximum wave heights (d_{max}) for the flexible solution for wall thicknesses of 1.0 m and 0.5 m are obtained 0.59 m and 0.65 m, respectively. Wave heights are larger than those obtained from rigid solution. But, increase in wave height due to flexibility is smaller than the results obtained BEM-FEM solution (Koh Kim and Park 1998).

As seen from Fig. 8, the results obtained by this study are generally in agreement with the results obtained by BEM-FEM. Hydrodynamic pressures determined by these two approaches are very close to each other at bottom and top of the wall with thickness of 1.0 m. For this thickness, at about mid-height of the wall the results a bit differ from each other. But, for 0.5 m wall thickness, hydrodynamic pressures determined by this study are larger than the pressures obtained by BEM-



(a) walls thickness of 1.0 m (b) walls thickness of 0.5 m

Fig. 8 Hydrodynamic pressure distribution obtained from assuming walls are flexible

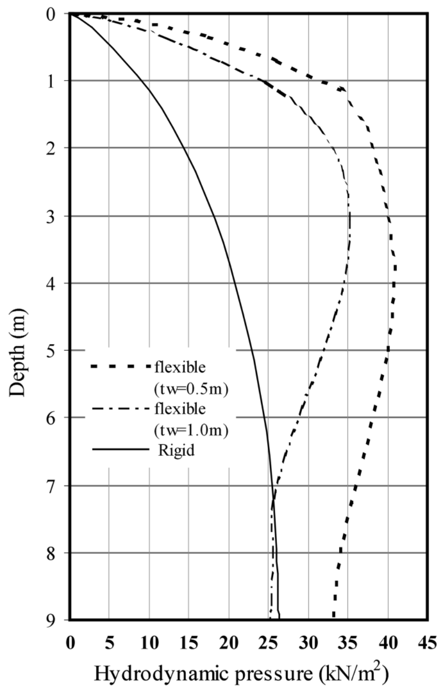


Fig. 9 Hydrodynamic pressure distributions obtained from this study

FEM along the fluid depth. Nevertheless, the shapes of the hydrodynamic pressure distributions are similar for the two approaches. Hydrodynamic pressures obtained by Eurocode-8 are smaller than the pressures obtained by this study and BEM-FEM over about mid-height of the wall and larger than the pressures obtained by this study and BEM-FEM near the bottom of the wall.

The hydrodynamic pressure distributions acting on the tank walls obtained from this study are compared in Fig. 9 for assuming the walls are rigid and flexible (wall thickness of 1.0 m and 0.5 m).

6. Conclusions

The dynamic behavior and seismic design consideration of rectangular storage tank were investigated. A Lagrangian fluid finite element was presented for the analysis of fluid-structure system taking into account the free surface sloshing motion. The results were compared to verify with the results obtained from the coupled BEM-FEM developed in recent years and the results obtained by using the Eurocode-8. Conclusion drawn from this study may be summarized as:

a) The hydrodynamic pressure distribution and magnitude obtained by using finite element method, Housner method and Eurocode-8 are generally in agreement with that for rigid solution. This conclusion shows that the method used in this study can be used as efficiently for rigid solution of rectangular fluid container.

b) The hydrodynamic pressure distributions for assuming rigid and flexible walls differ from each other in magnitude and in shape. The hydrodynamic pressures for flexible storage tanks are generally larger than that for rigid storage tanks. The difference between hydrodynamic pressures for rigid and flexible walls increases rapidly from the top of the wall to about mid-height, and then this difference decreases rapidly to the bottom of the wall. This conclusion agrees with recent studies using BEM-FEM which verified experimental results.

c) The maximum wave height obtained from this study for assuming rigid wall is 0.08 m larger than that obtained by Housner method. The wave heights are not changed as BEM-FEM solution with wall Thickness at 1.0 m and 0.5 m. So, further improvements are necessary for sloshing problem in rectangular fluid containers.

d) It is recommended that the subject on effect of wall flexibility on hydrodynamic pressure should be investigated and design rules should be presented in the earthquake code.

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