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Reliability of TLP tethers under extreme tensions

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Abstract. The tension leg platform (TLP) is a moored floating offshore structure whose buoyancy is more than its weight. The mooring system, known as tethers, is vulnerable to failure due to extreme (maximum and minimum) tensions. In the present study the reliability of these tethers under maximum and minimum tension (*ultimate limit state*) has been studied. Von-Mises failure criteria has been adopted to define the failure of a tether against maximum tension. The minimum tension failure criteria has been assumed to meet when the tethers slack due to loss of tension. *First Order Reliability method* (FORM) has been adopted for reliability assessment. The reliability, in terms of reliability index, and probability of failure has been obtained for twelve sea states. The probabilities of failure so obtained for different sea states have been adopted for the calculation of annual and life time probabilities of failure.

Key words: structural reliability; TLP; offshore; ultimate limit state; tethers.

1. Introduction

With the brilliant technological innovation of *Mars*, a gigantic Tension Leg Platform (TLP) in Gulf of Mexico, superiority of TLP in deep water offshore oil fields has been established. However, it has opened new challenges to the safety and reliability of these structures. The Tension Leg Platform is subjected to the severe environment due to combined wind, current, tide and waves. The mooring system of these platforms, known as tethers, is vulnerable to failure due to extreme (maximum and minimum) tensions. The present study has been devoted to study the reliability of these TLP tethers under maximum and minimum tensions. Faulkner *et al*. (1983), Lotsberg (1991), Banon, Cornell and Harding (1991), Sengupta and Ahmad (1996) and Siddiqui and Ahmad (2000) have studied various aspects of the reliability analysis against maximum and minimum tension in the tethers. Oran (1992) studied the highly localised bending stress in the TLP tethers near the supports. The closed form asymptotic formulas are developed to show the effect of bending stress on dynamic response. Siddiqui and Ahmad (2001) also studied the fatigue and fracture reliability of tethers under random loads.

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The wide review of the reliability analyses of TLP tethers shows that most of the past investigators in their reliability study assumed simple linear limit state functions based on classical theories of failure. Advanced modern theories of failure leading to nonlinear limit state functions were ignored. A comparative study of tether reliability for regular and random seas has also not been widely reported in the literature.

TLP is well suited for deep-sea conditions as the platform floats on the sea surface in compliant fashion under wind and wave forces. Massive sub structures as required for the fixed offshore platforms are replaced by light tubular tendons, strong enough to sustain the axial buoyancy forces. A TLP tether under study consists of a group of tubular members extending from platform bottom to the sea bed. Von-Mises failure criteria has been adopted to define its failure against maximum tension. The minimum tension failure has been assumed to occur when the tethers slack due to loss of tension (Banon and Harding 1989, Lotsberg 1991). The random variables, considered for the reliability analysis are: *pretension, dynamic uncertainty factor, yield strength, thickness, set down, translational* and *rotational mis-positionings of foundation.* An appropriate distribution for each random variable has been chosen and their statistical parameters are obtained. Apart from the axial stresses the limit state function also considers hoop and buckling stresses of tether members. A computationally efficient *first order reliability method* (FORM) has been adopted for reliability estimation. The reliability, in terms of reliability index, and probability of failure has been obtained for twelve sea states. The probabilities of failure so obtained for different sea states have been adopted for the calculation of annual and life time probabilities of failure.

2. Selection of sea states

A typical sea state has been defined in terms of significant wave height H_s, zero crossing period T_z and wind velocity u. A three parameter Weibull distribution, as recommended by Karadeniz, Vrouwenvelder and Bouma(1983) has been adopted for the long term probability distribution of the significant wave height as

$$f(H_s) = \frac{C}{B}\left(\frac{H_s - A}{B}\right)^{C-1} e^{-\left(\frac{H_s - A}{B}\right)^C} \tag{1}$$

where, A = lower limit of H_s; B = scale parameter and C = shape parameter.

The parameters A, B and C have been obtained from a scatter diagram of north sea location. These values are found to be as A = 0.594; B = 2.290; and C = 1.385 (Mathinsen and Bitner-Gregersen 1990).

Corresponding to a known significant wave height H_s, zero crossing period T_z may be obtained assuming the same probability of occurrence for T_z as for H_s using Eq. (2)

$$T_z = \sqrt{\left(\frac{32\pi H_s}{g}\right)} \tag{2}$$

The sea states have been simulated using Eqs. (1)-(2). For the present study twelve significant wave heights are selected such that $\sum f(H_s)\Delta H_s \approx 1$ i.e. the whole area under $f(H_s)$ and H_s have been divided into twelve rectangular strips of width ΔH_s. The area of each strip has been taken as $f(H_s)\Delta H_s$. ($f(H_s)\Delta H_s$) provides the magnitude of the corresponding probability of occurrence of

Table 1 Simulated sea states

Sea State	Significant wave height H_s (m)	Zero crossing period T_z (sec)	Probability of occurrence of the sea state
S1	17.15	13.26	0.00000037
S2	15.65	12.66	0.00000238
S3	14.15	12.04	0.00001437
S4	12.65	11.39	0.00007980
S5	11.15	10.69	0.00040572
S6	9.65	9.94	0.00187129
S7	8.15	9.14	0.00773824
S8	6.65	8.26	0.02822122
S9	5.15	7.26	0.08851105
S10	3.65	6.12	0.22831162
S11	2.15	4.69	0.43542358
S12	0.65	2.58	0.20942036

sea state defined by significant wave height H_s. T_z and u are then computed using Eqs. (2) and (3) respectively assuming that these are associated with the significant wave height (H_s) and have the same probability of occurrence as H_s. Table 1 shows the corresponding results of analysis.

3. Dynamic analysis

A deterministic time domain analysis has been carried out taking care of nonlinearities due to drag force, variable submergence, large deformations, randomly fluctuating tether tension. This nonlinear dynamic analysis considers the hydrodynamic loading due to random sea represented by Piersion Moskowitz (*PM*) spectrum idealized by Monte Carlo simulation (Ahmad 1996). Through this dynamic analysis, response time histories are generated and their statistics are used for the subsequent reliability analyses.

4. Reliability formulation

The present analysis assumes that the tethers of TLP are made up of a number of tubular tension elements. These elements are joined together through welding to make a complete tether length. Welded joints are designed to be stronger than the tether pipe and thus the failure would occur in the tether pipe and not at the weld. The failure of one of the tethers in one leg of TLP will cause increase in the stresses in other tethers which may add to the probability of catastrophic failure.

4.1 Limit state function

The reliability of a tether is governed by the calculation and prediction of the probability of limit state at any stage during their entire life. The limit state functions for the maximum and minimum tension cases (*Ultimate limit state*) have been formulated and established as given below

4.2 Maximum tension

The limit state function for the maximum tension has been derived according to the Von-Mises failure criteria expressed as:

$$g(\underline{z}) = -\log\left[\frac{\sigma_x^2 - \sigma_x\sigma_\theta + \sigma_\theta^2}{\sigma_y^2} \cdot \frac{1}{1-\left(\frac{\sigma_\theta}{\sigma_{E\theta}}\right)^2}\right] \tag{3}$$

where σ_x, σ_θ, $\sigma_{E\theta}$ are axial, hoop and elastic buckling hoop stresses respectively. These stresses are related to each other as under

$$\sigma_x = z_2 + z_3 + z_4 t_f + z_6 + z_8 + z_9 \tag{4}$$

$$\sigma_\theta = \frac{PD}{2t} \text{ and} \tag{5}$$

$$\sigma_{E\theta} = E\left(\frac{t}{D}\right)^2 \tag{6}$$

where P is the external hydrostatic pressure corresponding to the actual depth; D is the diameter; t_f is the stress in tether due to wave excitations obtained by the algebraic sum of all the tension components; z_2 is pretension stress; z_3 is stress in the tether due to tide and storm surge; z_4 is response uncertainty factor; z_6 is stress in tether due to set down; E is Young's modulus; z_8 is stress in tether due to translational mis-positioning of the foundation and z_9 is stress in tether due to rotational mis-positioning of the foundation.

4.3 Minimum tension

A TLP tether is said to be failed in minimum tension if due to sudden loss of tension tethers suddenly slack. Based on this criteria of *no slack* condition the following limit state function for minimum tension in tethers has been chosen

$$g(\underline{z}) = z_2 + z_3 - z_4 t_f + z_6 - z_8 - z_9 \tag{7}$$

Where t_f, z_2, z_3,........z_9 are same as defined in maximum tension.

4.4 Simplified limit state function

The above limit state function against maximum tension has been obtained adopting *Von-Mises theory of failure* in terms of nine random variables. The resulting limit state function, therefore is highly non-linear and complex. Limit state function for minimum tension case is though linear but still involves six random variables. It is sometimes imperative to use such detailed performance functions involving appropriate statistical distributions of various random variable. However, a simplified limit state model may be well suited for the preliminary study. It also helps in highlighting the significance of employing a complex limit state function. Keeping that in view the following simple and linear limit state functions have been presently employed involving only a few important random variables.

4.4.1 Maximum tension

The following simplified limit state function has been derived for maximum tension (tension exceeding yield) failure case

$$g(\underline{z}) = \sigma_y - z_2 - t_f z_4 \tag{8}$$

4.4.2 Minimum tension

The following simplified limit state function has been adopted for minimum tension (loss of tension) failure case:

$$g(\underline{z}) = z_2 - t_f z_4 \tag{9}$$

The random variable z_1, z_2 and z_4 have the same statistical parameters and probability distributions as for the original limit state functions (Table 3).

4.5 Annual and life time probability of failure

For annual and life time probabilities of failure, probability of failure corresponding to each sea state is determined (P_{f_i}). This probability of failure is then multiplied by annual probability of occurrence of corresponding sea state (say f_i) to obtain following expression for the estimation of annual probability of failure

$$P_f = \frac{\sum_{i=1}^{N} P_{f_i} f_i}{\sum_{i=1}^{N} f_i} = \sum_{i=1}^{N} (P_{f_i} f_i) \tag{10}$$

where N is the number of possible sea states and $\sum_{i=1}^{N} f_i = 1$

Life time (T_L) probability of failure has then been obtained by Eq. (11)

$$P_F(T_L) = 1 - \exp(-T_L P_f) \tag{11}$$

Reliability index (β) corresponding to above annual or life time probability of failure has been given as:

$$\beta = -\Phi^{-1}(P_f) \tag{12}$$

where $\Phi^{-1}(\,)$ is the inverse of the standardized normal distribution function.

5. Numerical study

For numerical study a TLP shown in Fig. 1 has been chosen. The principal dimensions of the TLP are as given in Table 2. The structural mass of the TLP is assumed to be lumped at the center of gravity of the platform. For twelve sea states (Table 1) a detailed dynamic analysis of TLP has been carried out for regular and long crested random wave. Participation of variable submergence and instantaneous buoyancy have been assumed to be active and significantly influencing the heave response of TLP. The twelve time histories, so obtained are statistically analyzed and response statistics have been obtained. Reliability analyses have then been carried out separately for Maximum and Minimum tension failures.

5.1 Random variables

In order to make the study more realistic *pretension, dynamic uncertainty factor, yield strength, thickness, set down, translational* and *rotational mis-positionings of foundation* have been considered random in nature and assumed to follow some appropriate probability distributions (Table 3).

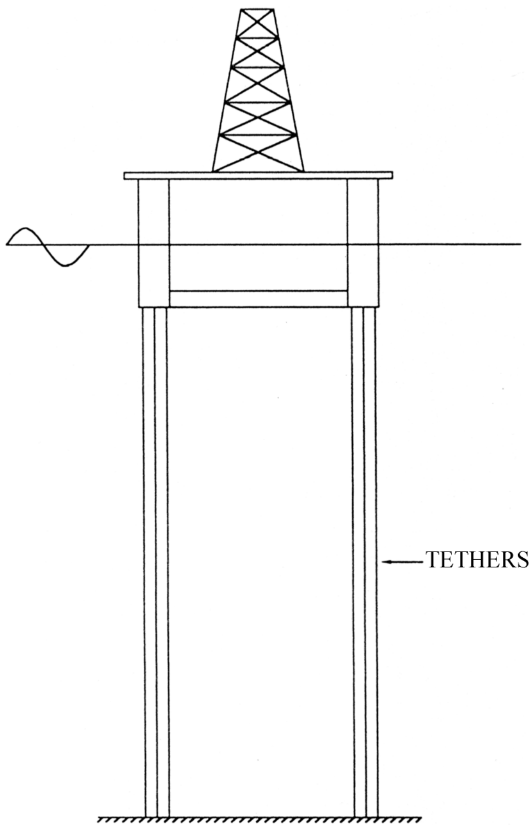

Fig. 1 Tension leg platform

Table 2 TLP description (Fig. 1)

Platform height	80.3 m
Platform weight	2.081×10^8 N
Total Pretension	1.220×10^8 N
Draft	26.60 m
Corner column diameter	14.20 m
Center to center column spacing	58.30 m
Pontoon length	58.30 m
Pontoon Diameter	11.00 m
Center of gravity above the base line	35.85 m
Tether length	473.4 m
Length of the each element of the tether	9.75 m
Thickness of tether pipe	3.34 cm
Young's modules of tether material	2.019×10^{11} N/m^2
Number of columns	4
Number of joints/tether	50
Service life	20 years

Table 3 Random variables

Random Variables	Distribution	Distribution parameters		
		Mean	COV	Other
Material yield strength (z_1)	Lognormal	350 MPa	0.05	-
Pretension (z_2)	Normal	12480 tonns	0.05	-
Tide and surge stress (z_3)	Uniform	4.5 MPa	0.05	a = 4.0 MPa b = 5.0 MPa
Response uncertainty (z_4)	Normal	1.00	0.08	-
Thickness of tether (z_5)	Normal	34.3 mm	0.01	-
Set down stress (z_6)	Uniform	0.70 MPa	0.05	a = 0.4 MPa b = 1.0 MPa
Young's modulus (z_7)	Normal	2.1×10^{05} MPa	0.05	-
Translational mispositioning (z_8)	Uniform	1.5 MPa	0.05	a = 1.0 MPa b = 2.0 MPa
Rotational mispositioning (z_9)	Uniform	3.0 MPa	0.05	a = 2.0 MPa b = 4.0 MPa

COV : Coefficient of variation
a, b : Parameters of uniform probability distribution

There are a number of uncertainties involved in the estimation of both vertical and horizontal responses. These uncertainties involved in the estimation of responses are modeled by inclusion of a *response uncertainty factor.* All motion responses are multiplied by this factor in order to incorporate their uncertain variations. This response uncertainty factor has been considered as normally distributed with unit mean and COV of 8% (Lotsberg 1991).

6. Discussion of results

Tables 4 and 5 show that both in regular and long crested sea idealizations the annual and life time reliability indices are quite above the usual target reliability index which normally ranges from 3-5. This indicates that the structure under study is quite safe and reliable against maximum tension failure irrespective of idealization of the sea state. However, the probability of failure due to minimum tension (tension loss) is more than probability of failure due to maximum tension (tension above yield).

Table 4 Annual & life time failure P_f & β for maximum tension

	Regular sea		Long crested	
	P_f	β	P_f	β
Annual	1.36×10^{-09}	5.947	3.50×10^{-22}	7.139
Life Time	2.73×10^{-08}	5.436	6.99×10^{-21}	6.715

Table 5 Annual & life time P_f & β for minimum tension

	Regular sea		Long crested sea	
	P_f	β	P_f	β
Annual	7.05×10^{-08}	5.106	5.30×10^{-07}	4.880
Life Time	1.41×10^{-06}	4.507	1.06×10^{-05}	4.252

Tables 6 and 7 show the comparison of reliability results obtained using original and simplified limit state functions for maximum and minimum tension failure. The result of maximum tension case show that for all the sea idealizations the probability of failure is lesser in case of simplified limit state function.

Table 6 Effect of limit state function (maximum tension)

Sea idealization	Original limit state		Simplified limit state	
(Sea State S1)	P_f	β	P_f	β
Regular sea	3.66×10^{-03}	2.68	8.45×10^{-05}	3.76
Long crested sea	7.75×10^{-16}	7.97	4.11×10^{-29}	11.14

Table 7 Effect of limit state function (minimum tension)

Sea idealization	Original limit state		Simplified limit state	
(Sea State S1)	P_f	β	P_f	β
Regular sea	3.31×10^{-02}	1.84	4.12×10^{-02}	1.74
Long crested sea	2.34×10^{-01}	0.73	2.70×10^{-01}	0.61

The results of minimum tension using simplified limit state model show a close proximity with the original limit state model. It is because the original and simplified limit state functions both are linear in nature. This shows that influences of tide and surge, set down and foundation mispositionings which are neglected in simplified limit state function, are not very significant. Thus, for quick and preliminary design one may ignore these random variables and adopt the simplified limit state function of the type mentioned above.

7. Conclusions

Reliability analysis of TLP tubular tethers have been carried out for twelve sea states under regular wave and long crested random sea idealizations. Sea states which cause high probability of failure of TLP tethers have low probability of occurrence and vice versa. For the present TLP under regular and long crested random sea the annual and life time reliability indices are quite above the usual target reliability indices ranging from 3 to 5. The present structure is, therefore, quite safe and reliable against maximum tension failure mode irrespective of the sea state idealization. However, the probability of failure due to minimum tension (tension loss) is more than probability of failure due to maximum tension (tension above yield).

Reliability analysis has been extended employing a simplified limit state function. A simplified linear limit state function is easy to handle and computationally economical. For maximum tension, simplified limit state model underestimates the probability of failure compared to a realistic nonlinear limit state model. For minimum tension, however, simplified model shows close proximity with the original limit state model.

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Notation

The following symbols are used in this paper:

β	: Reliability index
σ_θ	: Hoop stress
$\Phi(\ )$	: Cumulative distribution function for a standard normal variable
$\Phi^{-1}(\ )$	: Inverse of standardized normal distribution function
$\sigma_{E\theta}$	: Elastic buckling hoop stress
σ_s^2	: Variance
ρ_w	: Mass density of water
σ_{x}	: Axial stress in tether
D	: Diameter of tether
$E[\ ]$	: Expected value
f_i	: Annual probability of occurrence of sea state
FORM	: First order reliability method
$g(\ )$	: Limit state function
H_s	: Significant wave height
P_f	: Probability of failure
T_L	: Life time
TLP	: Tension leg platform
$\underline{z}$	: Vector of basic random variables
$z_1, z_2,...z_n$	: Random variables