Structural Engineering and Mechanics, Vol. 16, No. 1 (2003) 83-99
DOI: http://dx.doi.org/10.12989/sem.2003.16.1.083

Prediction of concrete strength using serial functional network model

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(Received January 23, 2003, Accepted May 7, 2003)

Abstract. The aim of this paper is to develop the ISCOSTFUN (Intelligent System for Prediction of Concrete Strength by Functional Networks) in order to provide in-place strength information of the concrete to facilitate concrete from removal and scheduling for construction. For this purpose, the system is developed using Functional Network (FN) by learning functions instead of weights as in Artificial Neural Networks (ANN). In serial functional network, the functions are trained from enough input-output data and the input for one functional network is the output of the other functional network. Using ISCOSTFUN it is possible to predict early strength as well as 7-day and 28-day strength of concrete. Altogether seven functional networks are used for prediction of strength development. This study shows that ISCOSTFUN using functional network is very efficient for predicting the compressive strength development of concrete and it takes less computer time as compared to well known Back Propagation Neural Network (BPN).

Key words: functional network; prediction; concrete strength; error function - minimization - Lagrangian.

1. Introduction

Concrete is a mixture of paste and aggregates. The paste is basically a mixture of cement and water, binds the aggregates into rocklike mass as the paste hardens because of chemical reaction of the cement and water according to Kosmatka *et al.* (2002). There is an increase in strength of concrete while concrete hardens.

Even though in reliability studies, the concrete strength is treated as random variable, in this paper, the mean value of concrete strength is considered as nominal strength. However, the variability of concrete strength can be predicted using functional and neural networks by considering appropriate models and this is beyond the scope of the paper. The mix proportions, curing conditions and methods of mixing, transporting, placing and testing the concrete influence its compressive strength of concrete. Based on the strength of concrete at 7, 14 and 28 days, in

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modernized concrete construction and engineering judgement one will be in a position to remove the concrete forms, re-shore the slab, schedule the project and control the quality. In case of pre-stressed concrete construction, the prediction of early concrete strength enables the structural engineer to decide on post-tensioning.

Concrete strength, according to Snell *et al*. (1989) and Popovics (1998) has been predicted for many years by many investigators based on maturity concept of concrete, Chengju (1989) and Oluokun *et al*. (1990) which is defined as the integral of time and temperature of concrete above a datum temperature. In the previous investigations, only two parameters water cement ratio and curing temperature have been used in the regression equation to develop a fixed equation based on limited data. When the data is different from original, either it is not possible to predict the strength or the model should be updated not only its coefficients but also its equation form.

Functional Network (FN) or Artificial Neural Network (ANN) does not need such a specific equation form and instead it needs enough input-output data. Given any new data within the original range, it is possible to predict the output. Earlier, Artificial Neural Network has been used for the prediction of concrete strength by Kasperkiewicz *et al.* (1995) and Lee (2003). Functional networks have been originally proposed by Castillo *et al.* (1992, 1998a, 1998b, 2000a, 2000b).

The process of selecting the number of hidden layer and neurons in the hidden layer is a trial and error until a good fit to the data is obtained. But Functional Network (FN) does not suffer from this drawback. Functional networks were introduced by Castillo (1998a), Castillo *et al.* (2000a), and Castillo *et al*. (1998b), Castillo *et al.* (2000b) as a powerful alternative to ANN. Unlike Neural Networks (Kasperkiewicz *et al.* 1995), Functional Networks use domain knowledge in addition to data knowledge. The network initial topology is derived based on the modeling of the properties of the real world. Once this topology is available, functional equations allow one to obtain a much simpler equivalent topology. Although functional networks also can deal with data only, the class of problems where functional networks are most convenient is the classes where the two sources of knowledge about domain and data are available whereas Neural networks do not consider domain knowledge.

In this paper, Serial Functional Networks (SFN) has been used to predict the strength of concrete at 16 hours, 20 hours, 24 hours, 2 Days, 3 Days, 7 Days and 28 Days based on seven independent Functional Network architecture in which the output of one will be input to the other. It is also shown that functional networks are more efficient and powerful and take much less computer time compared to the prediction by Neural Network such as Back Propagation network as seen in the later discussion for 3 day strength of concrete.

The functional networks are introduced in Section 2 with general methodology, including its selection of initial topology and learning methods. In Section 3, the Associativity Functional Network is introduced and the methodology is explained. Single and serial Functional Network architecture is discussed in Section 4. In Section 5 the method is applied to predict the early and later strength of concrete using seven architectures. Conclusions are drawn in Section 6.

2. Functional networks

The main property of the Neural Network is its ability to learn from data by using structural and parametric learning methods. In Neural network, learning process is achieved by estimating the connection weights by minimizing the error (Kasperkiewicz *et al.* 1995). Functional Networks

(Castillo *et al.* 2000a) is a more generalization of Neural Network bringing together domain knowledge and data. There is no restriction of neural function in functional neurons and arbitrary functions are allowed. Another important property of functional network is the possibility of dealing with functional constraints of the model. The functional network uses two types of learning a) structural learning and b) parametric learning. In structural learning, the initial topology of the network, based on some properties available to the designer is arrived at and finally a simplification is made using functional equation to a simpler architecture. In parametric learning, neuron functions are estimated by considering the combination of shape functions.

Functional networks consists of the following elements (see Fig. 1)

1. Storing units
   a) One layer of input storing units: This layer contains input data x_1, x_2, x_3 etc
   b) Intermediate layer units storing intermediate information f_4, f_5. These units evaluates a set of input values, coming from the previous layer and delivers a set of output values to the next layer.
   c) A layer of output units f_6
2. Layer of Computing Units f_1, f_2, f_3

A neuron in the computing unit evaluates a set of input values coming from a previous layer

3. A set of Directed Links. The functions are not arbitrary but they are determined by the structure of the network, like $x_7 = f_4(x_4, x_5, x_6)$, as explained in Fig. 1.

In addition to data, information about other properties of the function, such as associativity, commutativity and invariance, are used in selecting the final network. In a given functional network, neural functions are arbitrary but in neural networks they are sigmoidal, linear or radial basis and other functions. In functional networks, functions in which weights are incorporated are learned, and in neural networks, weights are learned. In some functional networks, the learning method leads to global minimum in a single step. Neural networks work well if the input and output data are normalized in the range of 0 to 1 but in Functional networks there is no such restriction. It can be pointed out that neural networks are special cases of functional networks.

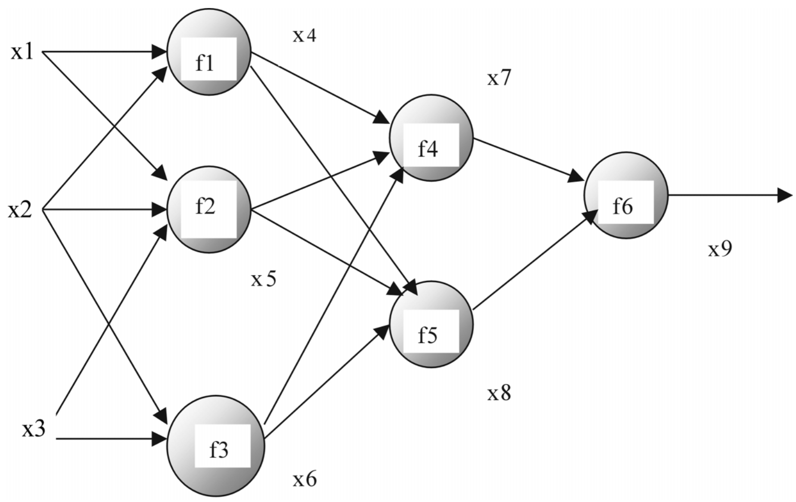


Fig. l Functional network

The following *eight-step* procedure is used for working with Functional Networks, FN.

Step. 1. Statement of the problem
Step. 2. Initial topology
Step. 3. Simplification of initial topology using functional equations
Step. 4. Arrive at conditions to hold for uniqueness
Step. 5. Data collection
Step. 6. Parametric learning by considering the linear combination of shape functions
Step. 7. Model Validation
Step. 8. if step 7 is satisfactory the model is ready to be used.

The learning method of functional network consists of obtaining the neural functions based on a set of data $D = (I_i, O_i)$ $(i = 1, 2\ldots\ldots n)$. The learning process is based on minimizing the *Euclidean Norm* of error function given by

$$E = \frac{1}{2}\sum_{i=1}^{n}(O_i - F(i))^2 \tag{1}$$

The approximate neural function '$f_i(x)$' may be arranged as

$$f_i(x) = \sum_{j=1}^{m} a_{ij}\phi_{ij}(X) \tag{2}$$

Where ϕ are '*shape functions*' with algebraic expressions $(1, x, x^2, \ldots, x^n)$ or Trigonometric functions such as (1, sin(x), cos(x), sin(2x), cos(2x), sin(3x), cos(3x)) or exponential functions. The associative optimisation function may lead to a system of linear or nonlinear algebraic equations.

3. The associativity functional network

Assume that for two inputs x_1, x_2 the output x_3 is given. We can construct functional network as shown in Fig. 2 using the functions f_1, f_2, and f_3 as

$$f_s(x_s) = \sum_{i=1}^{m_s} a_{si}\phi_{si} \quad \text{for } s = 1, 2 \text{ and } m_s \text{ can be any order} \tag{3}$$

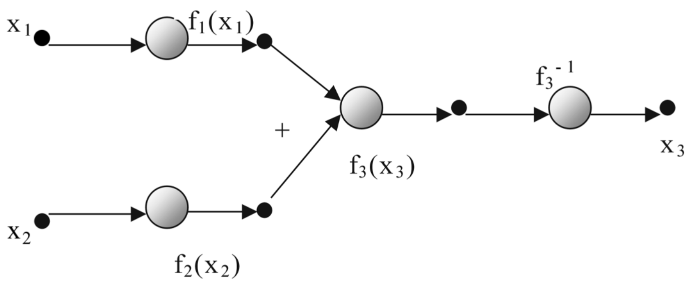


Fig. 2 Associativity functional network

ϕ_{si} can be polynomial, trigonometric or exponential or any admissible functions and herein we call them the shape functions. In this example we use only polynomial expressions as $<1, x, x^2, x^3 \ldots>$. The function f3 can be expressed as

$$f_3(x_3) = \sum_{i=1}^{2} a_{3i}\phi_{3i} \tag{4}$$

From the input functions we can construct

$$\hat{f}_3(x_3) = f_1(x_1) + f_2(x_2) \tag{5}$$

Then the error in the *j*th data is given by

$$e_j = f_1(x_{1j}) + f_2(x_{2j}) - f_3(x_{3j}) \tag{6}$$

The error can be written in matrix form

$$\text{as } e_j = <1, \quad x_{1j}, \quad x_{1j}^2, \quad \ldots, \quad 1, \quad x_{2j}, \quad x_{2j}^2, \quad \ldots, \quad -1, \quad -x_{3j}> \begin{Bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ \ldots \\ a_{21} \\ a_{22} \\ a_{23} \\ \ldots \\ a_{31} \\ a_{32} \end{Bmatrix} \tag{7}$$

or

$$e_j = <b_j> \{\underline{a}\} \tag{8}$$

The sum of the squares of the error for all the data is given by

$$E = \sum_{j=1}^{ndata} e_j^T \; e_j = <\underline{a}> \left(\sum_{j=1}^{ndata} \{b_j\}<b_j>\right)\{\underline{a}\} = <\underline{a}>[A]\{\underline{a}\} \tag{9}$$

To have uniqueness of solution assuming initial values $\{x_{k0}\}$ and the function α_k, we must have

$$f_k(x_{k0}) = \sum_{i=1}^{m_k} a_{ki}\phi_{ki}(x_{k0}) = \alpha_k \tag{10}$$

and writing it in matrix form we get

$$<\underline{a}>\begin{bmatrix} \{\phi_{10}\} & \{0\} & \{0\} \\ \{0\} & \{\phi_{20}\} & \{0\} \\ \{0\} & \{0\} & \{\phi_{30}\} \end{bmatrix} - <\alpha_1 \quad \alpha_2 \quad \alpha_3> \;=\; 0 \tag{11}$$

$$\text{or} \quad <\underline{a}>[\Phi_0] - <\alpha> = 0 \tag{12}$$

Using the Lagrangian multiplier technique, we define an augmented function as

$$R \;=\; E + <\underline{a}>[\Phi_0]\{\lambda\} - <\lambda>\{\alpha\} \tag{13}$$

$$\text{or} \quad R = <\underline{a}>[A]\{\underline{a}\} + <\underline{a}>[\Phi_0]\{\lambda\} - <\lambda>\{\alpha\} \tag{14}$$

We want to minimize R and thus

$$\frac{\partial R}{\partial\{\underline{a}\}} \;=\; 2[A]\{\underline{a}\} + [\Phi_0]\{\lambda\} \;=\; 0$$

$$\frac{\partial R}{\partial \lambda} \;=\; [\Phi_0]^T\{\underline{a}\} \;=\; \{\alpha\} \tag{15}$$

or

$$\begin{bmatrix} 2[A] & [\Phi_0] \\ [\Phi_0]^T & [0] \end{bmatrix} \begin{Bmatrix} \{\underline{a}\} \\ \{\lambda\} \end{Bmatrix} \;=\; \begin{Bmatrix} \{0\} \\ \{\alpha\} \end{Bmatrix} \tag{16}$$

$$\text{or} \quad [G]\{u\} = \{v\} \tag{17}$$

Note that $[G]$ matrix is symmetric and is called Vandermonde type matrix and it is nonsingular and is very poorly conditioned for large values of the degree of the equation. To get over this problem, one can either use the orthogonal polynomials or inputs and outputs are nomalized to lie between 0 –1 and computations are carried out in double precision. In this paper the second approach is used. Once we solve for unknowns $\{a\}$, for any given x_{1i}, x_{2i} one can write

$$\hat{f}_3(x_{3i}) \;=\; f_1(x_{1i}) + f_2(x_{2i}) \;=\; a_{31} + a_{32}x_{3i} \tag{18}$$

or

$$x_{3i} \;=\; (\hat{f}_3(x_{3i}) - a_{31})/a_{32} \tag{19}$$

If we assume higher order functions for $f_3(x_{3i})$ non-linear equations have to be solved for x_{3i} using the bisection or Newton-Raphson method. This is time consuming and hence for all the problems considered in this paper only a first order function has been assumed for $f_3(x_{3i})$.

4. Single and serial architecture

The simple way to determine the architecture of FN is to select the single one. The architecture should consist of all neurons of input consisting of basic information, material proportions, measurements and temperature and humidity of all days and output layer consisting of the strengths of concrete at 16 hours, 20 hours, 1 day, 2 days, 3 days, 7 days and 28 days. The data is shown in Table 1. Even though this architecture can give good results but it cannot appropriately predict the concrete strength when the curing temperature of a specific curing day is changed. This is because it uses the single architecture, which all nodes are fully connected, and thus all output neurons are influenced by all input neurons. For example, the temperature and humidity of 7th day to 28th days after pouring cannot actually influence the concrete strength development at first, second or third day after pouring. But the single architecture involves that input values of 7th day to 28th day can influence the output values of 1, 2 or 3rd day. Thus, the single architecture conceptually cannot be used for predicting the concrete strength development.

Table 1 Training data

No.	Input value																	
	Basic information(BI)							Material proportions(MP)								Measurement(ME)		
	a	b	c	d*	e*	f*	g*	h	i	j	k	l	m	n	o	p	q	r	s
1	A	20.6	30	KS	9	13	16	1.79	N	2.88	FA	3.92	8.72	8.46	AE	.098	23	1.8	26
2	A	20.6	30	KS	9	25	15	1.72	N	2.76	FA	3.82	8.69	8.79	AE	.098	20	3.4	27
3	A	23.5	40	KS	9	25	17	1.78	N	3.09	FA	4.21	8.5	8.52	AE	.108	21	4.5	26
			~									~						~	
24	A	20.6	30	KS	1	9	16	1.79	N	2.94	FA	3.23	8.72	8.46	AE	.098	18	5.5	17

*: Excluded variables for avoiding duplication in actually training
a: Concrete producer, *b*: Nominal concrete strength(MPa), *c*: Delivery time(min.),
d: region of pouring, *e*: month of pouring, *f*: day of pouring, *g*: time of pouring,
h: Weight of water (kN/m^3 of concrete),
i: Cement type (*N* = Normal type), *j*: Weight of cement (kN/m^3 of concrete),
k: Type of supplementary cementitious materials (FA = Fly Ash),
l: Weight of supplementary cementitious materials (kN/m^3 of concrete),
m: Weight of fine aggregate (kN/m^3 of concrete), *n*: Weight of coarse aggregate (kN/m^3 of concrete)
o: Admixture type (AE = Air-Entraining), *p*: Weight of admixture (kN/m^3 of concrete),
q: slump (cm), *r*: air content (%), *s*: concrete temperature (°C).

No.	Input value					Desired output value						
	Temp. and humidity history (T/H)					Test results of compressive strength in air curing (MPa)						
	Pouring day		~	28 days after pouring								
	a	*b*	~	*a*	*b*	16 h	20 h	24 h	48 h	3 D	7 D	28D
1	26	62		14	73	3.165	4.41	5.684	11.17	14.308	21.88	24.265
2	23	68		14	70	2.616	2.72	3.724	6.87	8.869	12.269	16.258
3	23	68	~	14	70	2.303	2.9	3.528	6.94	9.114	13.593	16.000
	~			~					~			
24	10	50		3.3	76	0.9898	1.362	1.881	3.254	5.39	12.054	17.63

a: Averaged temperature at a curing day (°C), *b*: Averaged humidity at a day (%).

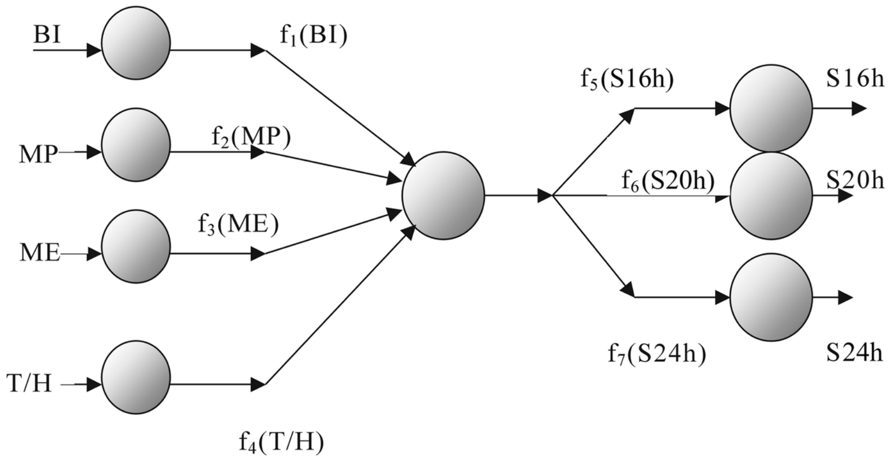
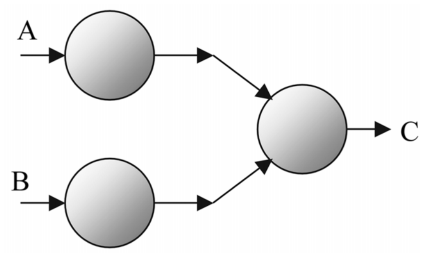


Fig. 3(a) FN I-III Architectures for prediction of early concrete strength



Architecture	A	B	C
FN IV	S(1D)	(T/H)- 1 day	S(2D)
FN V	S(2D)	(T/H)- 2 day	S(3D)
FN VI	S(3D)	(T/H)- 3-6 day (average)	S(7D)
FN VII	S(7D)	(T/H)-7-28 days(average)	S(28D)

Fig. 3(b) FN IV - VII Architectures for prediction of concrete strength at 2, 3, 7, 28 days

Serial functional network architecture can be introduced for solving problem rather than single architecture. It means all input and output neurons are divided into several serial architectures and it can produce results at intermediate stages as well as final stage. Fig. 3 shows proposed serial functional network which has multiple architectures composed of seven FN (I-VII).

The division of single FN into seven FN has accomplished by the relation of the temperature/ humidity and concrete strength in curing period as shown in Fig. 3 and also as given below.

4.1 Determination of input neurons

The number of input neurons in a functional network is determined from the variables that influence concrete strength. Because there are too many variables, it is unable to actually produce the input patters within a given time and also it is not practical in view of an engineering approach that allows a little error of about ±10%. All possible variables are considered in an initial development stage. Table 1 shows that 73 variables can be classified into four categories: basic information (BI, 3 variables), material proportions (MP, 9 variables), measurements (ME, 3 variables) and temperature and humidity history from pouring day to 28th day after pouring (T/H, 58 variables). These variables are reduced by taking the average temperature and humidity from 3-6 days and 7-28 days.

4.2 Determination of output neurons

The concrete strength gradually develops with age. The compressive strength at 28 day generally represents an indication for design strength and quality control. The early concrete strength within 3 days after pouring is importantly considered whether the concrete is strong enough to handle removal of forms and reduction of shoring. The concrete strength for ISCOSTFUN is determined at seven different ages: 16 hours, 20 hours, 1 day, 2 days, 3 days, 7 days and 28 days. Therefore, the basic number of output neurons even though it is seven but in serial FN architecture there is only one output as shown as shown in Fig. 3.

4.3 Serial functional network architecture

FN-I predicts the early strength of 16-hour period based on two basic information as nominal concrete strength and delivery time in minutes and six material proportions as weight of water, weight of cement, weight of cementitious material, weight of fine aggregate, weight of coarse aggregate and weight of admixture (all weights/cu.m of concrete) and three measurements such as slump in cm, percentage of air content and concrete temperature and the average temperature and humidity on the pouring day. Altogether there are 13 inputs and one output as shown in Table 2.

FN-II predicts the early strength of 20-hour period based the above 13 inputs and hence this architecture consists of 13 inputs and one output.

FN-III predicts the early strength of 24-hour period based on the same 13 inputs and hence this architecture consists of 13 inputs and one output.

FN-IV predicts the two-day strength of concrete based on one-day strength and the average temperature and humidity on the first day after pouring. Hence this architecture consists of three inputs and one output.

FN-V predicts three-day strength of concrete based on two-day strength and the average temperature and humidity of the second day after pouring. Hence this architecture consists of three inputs and one output.

FN-VI predicts seven-day strength of concrete based on three-day strength and the average temperature and humidity of third to sixth day after pouring. Hence this architecture consists of three inputs and one output.

FN-VII predicts twenty-eighth day strength of concrete based on seven-day strength and the average temperature and humidity of seventh to twenty eighth day after pouring. Hence the architecture consists of three inputs and one output.

5. Strength of concrete

5.1 16 hour strength

First thirteen inputs and one output are normalized with respect to maximum and minimum values (see Table 2) such that minimum value of each variable maps into 0 of the normalized space and maximum maps into 1 of the normalized space. Even though this is not a requirement in Functional Networks, this method produces a well-conditioned matrix [G] in Eq. (17). Second order function is used for thirteen inputs and linear function is used for output as

Table 2 Normalized values of the input and output

S.No	Nom Con strength	Del time minutes	wt of water in 1cum of concrete	wt of cement in 1cum of concrete	wt of cem matl in 1cum of concrete	wt of sand in 1 cum of concrete	wt of coarse agg in 1 cum of concrete
1	0.1176471	0.2	0.66	0.176	0.1111111	0.725	0.32
2	0.1176471	0.2	0.52	0.128	0.1037037	0.7175	0.485
3	0.2058824	0.6	0.64	0.26	0.1333333	0.6675	0.35
4	0.1176471	0.6	0.8	0.248	0.0740741	0.73	0.165
5	0.2941176	0.6	0.76	0.428	0.1111111	0.62	0.205
6	0.0294118	0.2	0.56	0.052	0.0296296	0.7925	0.42
7	0.1176471	0.6	0.52	0.156	0.0518519	0.7175	0.485
8	0.6764706	0.6	0.44	0.864	0.1185185	0.4175	0.37
9	0.9705882	0.6	0.42	0.972	0.4592593	0.1025	0.655
10	0.1176471	0.6	0.34	0.096	0.0444444	0.7175	0.67
11	0.6764706	0.6	0.2	0.428	0.9481481	0.4075	0.32
12	0.1176471	0.6	0.52	0.156	0.0518519	0.7175	0.485
13	0.3823529	0.6	0.34	0.624	0.1481481	0.49	0.575
14	0.1176471	0.6	0.66	0.204	0.0592593	0.725	0.32
15	0.9705882	0.6	0.42	0.972	0.4592593	0.1575	0.7
16	0.2941176	0.2	0.76	0.428	0.1111111	0.62	0.205
17	0.1176471	0.2	0.52	0.156	0.0518519	0.7175	0.485
18	0.3823529	0.6	0.34	0.624	0.1481481	0.49	0.575
19	0.1176471	0.6	0.66	0.204	0.0592593	0.725	0.32
20	0.1176471	0.6	0.52	0.156	0.0518519	0.7175	0.485
21	0.1176471	0.2	0.52	0.156	0.0518519	0.7175	0.485
22	0.0294118	0.2	0.7	0.096	0.037037	0.8025	0.265
23	0.1176471	0.2	0.66	0.204	0.0592593	0.725	0.32
24	0.1176471	0.2	0.66	0.204	0.0592593	0.725	0.32
25	0.1176471	0.6	0.52	0.156	0.0518519	0.7175	0.485
26	0.2058824	0.6	0.64	0.288	0.0814815	0.6675	0.35
27	0.9705882	0.6	0.42	0.972	0.4592593	0.1025	0.655
28	0.5294118	0.6	0.3	0.744	0.0592593	0.48	0.64
Min	16.66 MPa	25	1.47 kN	2.45	0.245	5.88	7.84
Max	49.98 MPa	50	1.96 kN	4.9	1.568	9.8	9.8

Table 2 Continued

wt of admixture in 1 cum concrete	slump cm	air content %	concrete temp	Ave temp curing day	Ave humi curing day	test strength 16 hours MPa
0.0222222	0.1333333	0.08888889	0.44	0.86	0.55	0.364
0.0186667	0.2666667	0.26666667	0.48	0.78	0.7	0.289333333
0.0284444	0.2	0.38888889	0.44	0.78	0.7	0.246666667
0.0257778	0.3333333	0.43333333	0.44	0.7	0.7	0.354666667
0.1102222	0.2333333	0.44444444	0.44	0.7	0.7	0.457333333
0.0115556	0.3333333	0.26666667	0.44	0.453333333	0.85	0.150666667
0.0186667	0.2666667	0.44444444	0.52	0.453333333	0.85	0.230666667
0.2942222	0.4666667	0.22222222	0.52	0.543333333	0.6375	0.386666667
0.8097778	0.8666667	0.08888889	0.68	0.553333333	0.585	0.917333333
0.0151111	0.1333333	0.5	0.52	0.553333333	0.585	0.185333333
0.4773333	0.4666667	0.4	0.6	0.5	0.125	0.849333333
0.0186667	0.4	0.55555556	0.32	0.5	0.125	0.124
0.6151111	0.8666667	0.03333333	0.32	0.5	0.125	0.142666667
0.0222222	0.2666667	0.53333333	0.24	0.5	0.125	0.073333333
0.8577778	0.8666667	0.11111111	0.36	0.166666667	0.875	0.086666667
0.1102222	0.5333333	0.44444444	0.12	0.5	0.125	0.134666667
0.0186667	0.3333333	0.5	0.32	0.5	0.125	0.114666667
0.6151111	0.8666667	0.06666667	0.2	0.433333333	0.125	0.066666667
0.0222222	0.4	0.35555556	0.32	0.5	0.125	0.126666667
0.0186667	0.1333333	0.4	0.32	0.5	0.125	0.109333333
0.0186667	0.0666667	0.48888889	0.08	0.333333333	0.25	0.06
0.0142222	0.4333333	0.27777778	0.08	0.333333333	0.25	0.064
0.0222222	0.4	0.5	0.08	0.333333333	0.25	0.06
0.0222222	0.5333333	0.5	0.08	0.333333333	0.25	0.068
0.0186667	0.3333333	0.5	0.32	0.5	0.125	0.108
0.0284444	0.6	0.22222222	0.52	0.543333333	0.6375	0.2
0.8097778	0.8666667	0.04444444	0.44	0.5	0.125	0.373333333
0.4755556	0.5333333	0.38888889	0.36	0.066666667	0.25	0.036
0.00735	10	1	15	0	40	0.475
0.1176	25	10	40	30	80	7.84

output = test strength at 16 hours and all the others are inputs

$$f_i(x_i) = \sum_{j=0}^{2} a_{ij} x_i^j \qquad i = 1, 13 \tag{20}$$

$$f_{14}(x_{14}) = \sum_{j=0}^{1} a_{14,j} x_{14}^0 \tag{21}$$

and

$$f_{14}(x_{14}) = \sum_{i-1}^{13} f_i(x_i) = a_{14,0} + a_{14,1}x_{14} \tag{22}$$

or x_{14} is given by

$$x_{14} = \frac{f_{14}(x_{14}) - a_{14,0}}{a_{14,1}} \tag{23}$$

In Eq. (23), 1…13 denote inputs and 14 denotes the output either 16 hr, 20 hr or one day strength. FN-I architecture is used and to solve the equations, the initial X values are assumed to be 0.2 and the α values are assumed to be 0.8. The second order equation is used for the thirteen inputs and linear function is used for the output. The undetermined parameters in Eq. (21) for the input and in Eq. (22) for the output are given in Table 3. Fig. 4 shows the comparison of normalized values of

Table 3 Undetermined parameters for FN-I in Eqs. (21) and (22)

Input	a_{i0}	a_{i1}	a_{i2}
1	0.2738	1.2838	6.981
2	–0.8606	11.0614	–13.790
3	2.104	–6.932	2.0401
4	4	–15.892	–0.557
5	3.521	–13.908	1.509
6	1.153	2.19	–19.785
7	2.623	–8.656	–2.281
8	–0.028	3.7515	1.945
9	0.831	–0.247	0.4607
10	0.548	1.618	–1.776
11	0.849	–0.770	2.629
12	1.37	–3.665	4.069
13	.8843	–0.4636	0.2109
Output 14	–0.972	0.857	

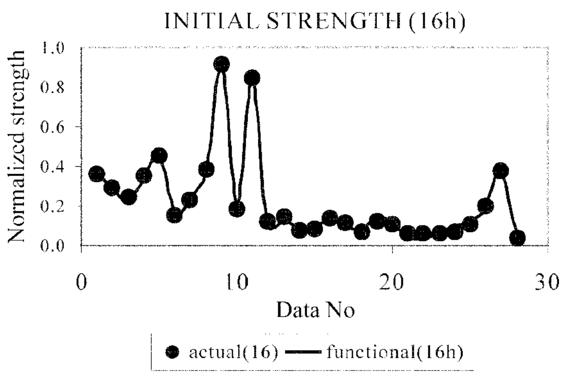


Fig. 4 Comparison of 16 h strength by FN and actual test values

strength at 16 h obtained from Functional Networks and the actual values. It is observed that a correlation coefficient of 0.9977 is obtained between functional network values with actual values.

5.2 20 hour and 24 hour strength

Using the architectures FN-II and FN-III with 13 inputs the early strength of 20 hour and 24 hour are obtained. Figs. 5 and 6 show the comparison of normalized values of strength obtained from Functional Networks and the actual values. It is observed that correlation coefficients of 0.9993 and 0.9992 are obtained between functional network values and the actual value.

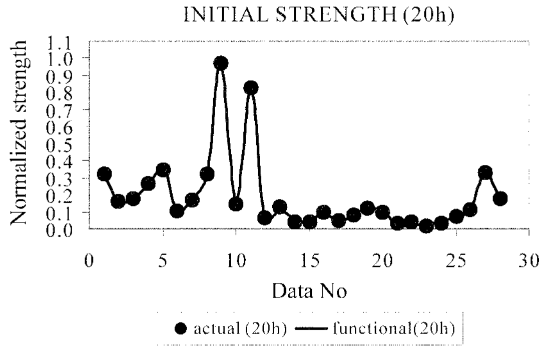


Fig. 5 Comparison of 20 h strength by FN and actual test values

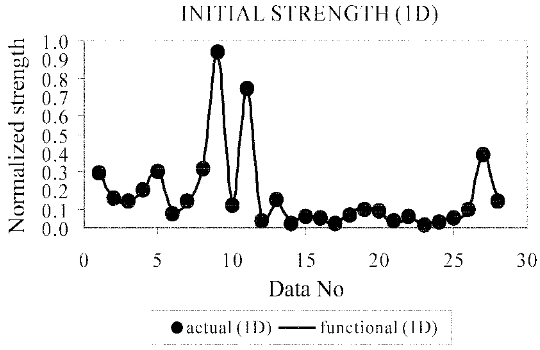


Fig. 6 Comparison of 1 D strength by FN and actual test values

5.3 2 day 3 day 7 day and 28 day strength

Similarly FN-IV, V, VI and VII are used to predict the normalized strengths of concrete at 2 day, 3 day, 7 day and 28 days. It is observed that except FN-IV the correlation coefficients obtained in other three architectures are greater than 0.95 and for FN-IV, the correlation coefficient is 0.90976. Figs. 7-10 show the comparison of normalized values of strength obtained from Functional

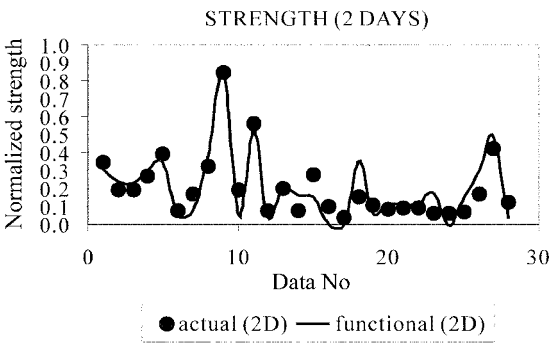


Fig. 7 Comparison of 2 D strength by FN and actual test values

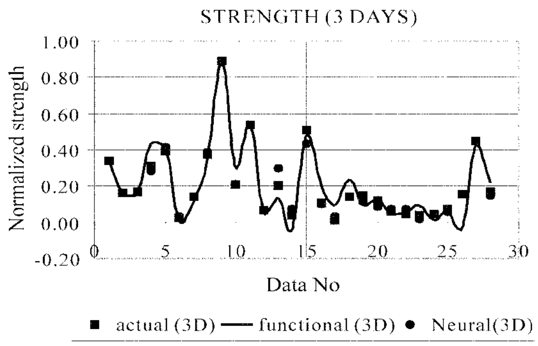


Fig. 8 Comparison of 3 D strength by FN, neural and actual test values

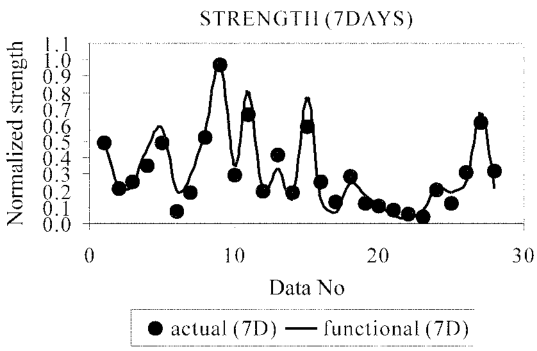


Fig. 9 Comparison of 7 D strength by FN and actual test values

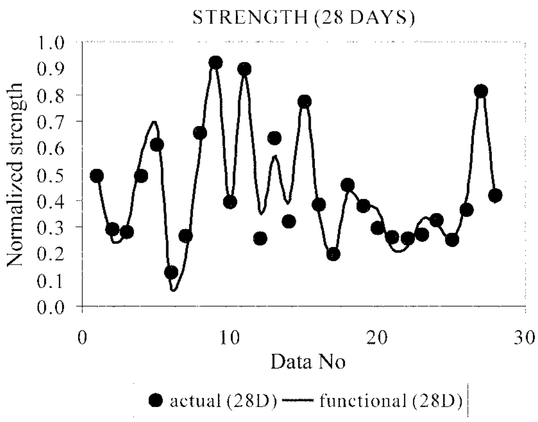


Fig. 10 Comparison of 28 D strength by FN and actual test values

Table 4 Initial values, order of the equation and correlation coefficient

FN Architecture	Initial values of X	Initial Values of α	Order of Equation	Correlation Coefficient
I	0.2	0.8	2	0.9977
II	0.2	0.8	2	0.9993
III	0.2	0.8	2	0.9992
IV	0.001	0.005	3	0.90976
V	0.001	0.005	7	0.9539
VI	0.4	0.8	3	0.9570
VII	0.1	0.1	8	0.9739

Networks and the actual values. The initial values and α values, the order of the equation and the correlation coefficients for the above seven FN architectures are given in Table 4. For 28 day strength the average error between actual values and the values obtained from Functional Networks is 11.7%.

For three day strength, back propagation neural network (BPN) also has been used for comparison with learning rate of 0.6 and momentum factor of 0.9 and sigmoidal gain of 1 and the network is trained for 10000 iterations. It took 160 seconds in Pentium III with 333.3 MHz speed with 64 MB RAM and the correlation coefficient of 0.99 is obtained between actual and BPN values. Whereas the time taken in functional network is only 5 seconds.

5.4 Traditional method

The strength of concrete is related to the maturity of concrete that can be expressed as a simple mathematical function of time and temperature. The maturity method has been widely accepted at civil engineering practice due to its simplicity. The prediction results by maturity method are compared to those of FN model and the tested values as shown in Fig. 11. For the 25th data, with the following parameter values: constant averaged temperature 5 deg centigrade, W/B = 55%,

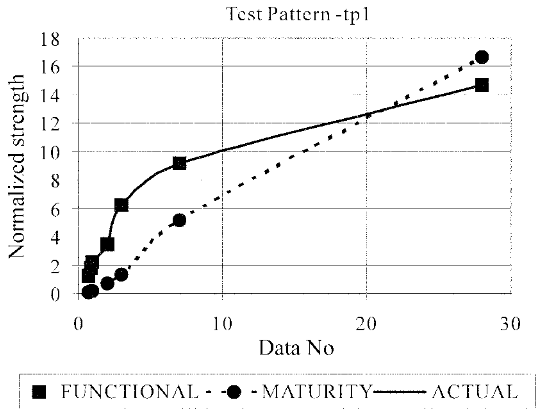


Fig. 11 Comparison of predicted compressive strength by FN with test values and logistics equation

$F\infty$ = 210, m = 4.66, k = 1.83, in the maturity method the following logistics equation is used (Han and Han 2001).

$$F_c = \frac{F_\infty}{1 + \exp(-k.\log M + m)} \tag{24}$$

where F_∞ is the ultimate strength of concrete, M is the maturity of concrete, k and m are experimental constants. Correlation coefficient of 0.9481 is obtained between functional network values and maturity strength equation. There is error between these two methods because maturity method requires only two variables as inputs: water to cement ratio and average temperature during curing periods. Therefore, the usefulness of maturity method by existing publications is not enough to predict the concrete strength development of test patterns used in this study.

6. Conclusions

The FN based model has been developed for predicting the concrete strength development. The following conclusions are obtained from this study.

1. In Artificial Neural Networks, weights are trained whereas in functional networks the functions are trained. In Functional networks, in addition to data knowledge domain knowledge also is required. Using this domain knowledge, serial functional network is applied to predict the early and later strength of concrete.
2. Serial functional networks are more suitable rather than single one FN for predicting the strength in both early and later periods. Even though this paper does not support this conclusion explicitly, this is obvious since the 2nd day strength is not influenced by the temperature and humidity from 3-28 days.
3. Functional Networks predict the strength quite accurately since the correlation coefficients obtained from FN values and the actual value are greater than 0.95 except for 2-day prediction in which case the correlation coefficient is 0.90976.
4. Functional Networks take much less computer time as compared with conventional BPN as seen for three-day strength of concrete.

5. FN based model predicts better than traditional maturity method within the cylinder test data used in this study. It could deal with enough factors to influence the concrete strength development.
6. It is not required to normalize the input and output data. Normalization is carried out so that resulting matrix is well conditioned.

Acknowledgements

The senior author thanks the management and Principal Dr. S. Vijayarangan for giving necessary facilities to carry out the research work reported in the paper. The second author thanks the support from Hyundai Development Company. The authors thank the anonymous reviewers for their suggestions to improve the standard of the paper.

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Notation

a : undetermined parameters
A : Matrix
b : undetermined parameters
BI : Basic inputs

c	: undetermined parameter
d	: undetermined parameter
D_i	: i^{th} set of input and output
E	: Euclidean Error norm
f	: function
$F(i)$	: Output obtained from Functional Network
g	: function
H	: function
I	: Input
k	: Experimental constant in Eq. (24)
m	: order of the equation
m	: Experimental constant in Eq. (24)
M	: Maturity of concrete
ME	: Measurement
MP	: Material properties
$ndata$	: number of data
O	: Target Output
T/H	: Temperature and Humidity
x	: Input vector
α	: Constant
ϕ	: Shape function