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Preliminary design of cable-stayed bridges for vertical static loads

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Abstract. This paper proposes a new method for the preliminary design of cable-stayed bridges that belong to the radial system subjected to static loads (self weight, traffic loads, concentrated loads, etc). The method is based on the determination of the each time existing relation between the tension forces of the cables and the corresponding bridge-deck deformations, and can be extended on any type of cable layout (fan, parallel, or mixed system). Galerkin's method is used for the final determination of the cable stresses and the bridge deformation. The determination of the equation, which gives the forces of the cables in relation to the deck's configurations, permits us to convert the problem to the solving of a continuous beam without cables.

Key words: cable-stayed bridges; design of bridges; preliminary analysis.

1. Introduction

Cable-stayed bridges have been known since the beginning of the 18^{th} century Leonard (1972), Chang and Cohen (1981), but they have been of great interest only in the last fifty years, particularly due to their special shape and also because they are an alternative solution to suspension bridges for long spans O'Connor (1971), Podonly and Scalzi (1976), Troitsky (1988), Gimsing (1997). The main reasons for this delay were the difficulties in their static and dynamic analysis, the various non-linearities, the absence of computational capabilities, the lack of high strength materials and the lack of construction techniques. There is a great number of studies concerning the behaviour (static, dynamic and stability) of cable-stayed bridges, some of which are referred to in this study here Tang (1971), Lazar (1972), Fleming (1979), Fleming and Egeseli (1980), Bruno and Grimaldi (1985), Nazmy and Abdel-Ghaffar (1990), Ermopoulos *et al.* (1992), Chatterje *et al*. (1994), Bruno and Golotti (1994), Khalil (1996), Bosdogianni and Olivari (1997), Virlogeux (1999).

In this paper a quick and efficient method of preliminary analysis of cable-stayed bridges that belong to the radial system is described. The parallel system will be covered in the future in another publication, since the mathematical treatment needs a different approach compared to the one used here. The shape functions of the corresponding continuous beam of the bridge deck without cables and Galerkin's method are used here. The determination of the equation, which gives the forces of

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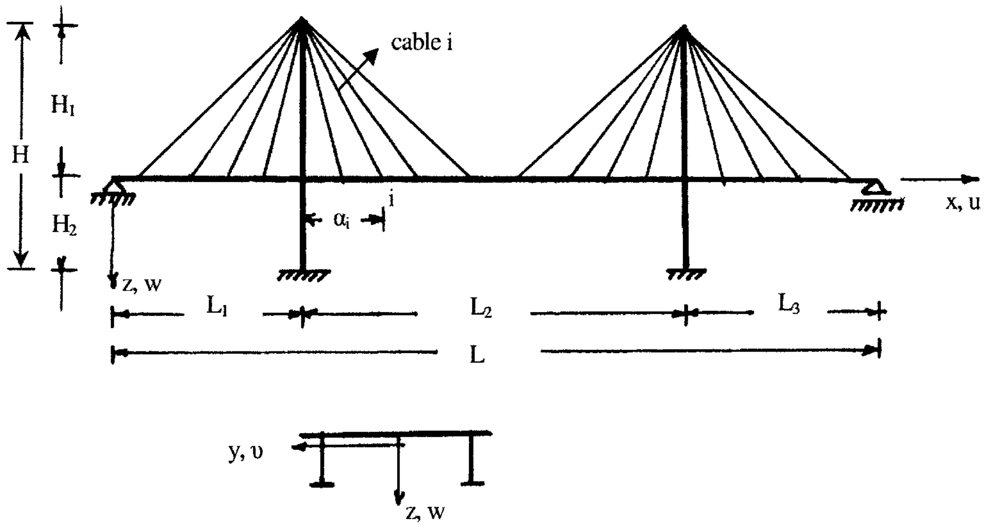

Fig. 1 Typical cable-stayed bridge of a radial system

the cables in relation to the deck's configurations, permits us to convert the problem to the solving of a continuous beam without cables.

2. Analysis

In the following analysis, a 2D model of a cable-stayed bridge shown in Fig. 1 is used.

The following assumptions are made:

- The pylon provides only vertical support to the deck of the bridge, so the deck can be characterized as a three- span continuous beam.
- The tangent modulus of elasticity E_s for the cables is used.
- The influence of axial forces either of the pylon or of the deck is neglected.

2.1 Deformation of the system deck-pylon

The relative horizontal deformation δu of the top of the pylon, in regard to the point of the vertical support of the deck for a load P_{pylon} that acts at its top and is vertical to its axes, is given by the formula:

$$\delta u = u_p - u_d = \frac{P_p \cdot H^3}{6E_pI_p}\left[2 - 3\left[\frac{H_2}{H}\right]^2 + \left(\frac{H_2}{H}\right)^3\right] \tag{1}$$

where:

P_p the horizontal force that acts at the top of the pylon

E_p the modulus of elasticity of the material of the pylon, and

I_p the moment of inertia of the cross-section of the pylon.

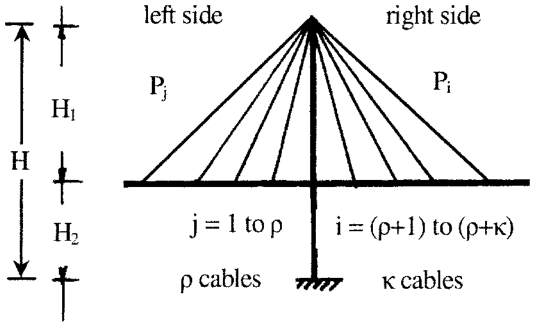

Fig. 2 The right and left-side cables of a radial system

2.2 Relation between axial loads P_i and vertical displacements w_i

The horizontal force that acts at the top of the pylon and causes the deformation u, is given by the following sum: $\sum_i P_i \cdot \sin\varphi_i - \sum_j P_j \cdot \sin\varphi_j$, where j and i are the cables on the left and right side connected at the top of the pylon as shown in Fig. 2.

The total relative deformation δu is:

$$\delta u = u_p - u_d = \frac{H^3}{6E_pI_p}\left[2 - 3\left[\frac{H_2}{H}\right]^2 + \left(\frac{H_2}{H}\right)^3\right]\left\{\sum_i P_i \cdot \sin\varphi_i - \sum_j P_j \cdot \sin\varphi_j\right\} \tag{2}$$

The elongation Δs_i of the cable i due to its axial force P_i is:

$$\Delta s_i = \frac{s_i \cdot P_i}{E_s \cdot A_i} \tag{3}$$

where s_i is the length of cable i in the undeformed state, E_s is its tangent modulus of elasticity and A_i is the area of its cross-section.

According to the geometry of the bridge shown in Fig. 3(a), and projecting the displacements on axis a-a the following equation is derived for the cables on the right side of the pylon:

$$u_p \cdot \sin\varphi_i + w_p \cdot \cos\varphi_i + (s_i + \Delta s_i)\cos\Delta\varphi_i = s_i + u_d \cdot \sin\varphi_i + w_i \cdot \cos\varphi_i \tag{4}$$

Neglecting w_p which is a very small quantity and replacing $\cos\Delta\varphi_i = 1$, we get:
$(u_p - u_d)\sin\varphi_i + \Delta s_i = w_i\cos\varphi_i$ or

$$\left\{\frac{H^3}{6E_pI_p}\left[2 - 3\left[\frac{H_2}{H}\right]^2 + \left(\frac{H_2}{H}\right)^3\right] \cdot \left(\sum_i P_i \cdot \sin\varphi_i - \sum_j P_j \cdot \sin\varphi_j\right)\right\} \cdot \sin\varphi_i + \frac{s_iP_i}{E_sA_i} = w_i\cos\varphi_i$$

and finally:

$$A \cdot \left(\sum_i P_i \cdot \sin\varphi_i - \sum_j P_j \cdot \sin\varphi_j\right) \cdot \sin\varphi_i + \frac{s_iP_i}{E_sA_i} = w_i\cos\varphi_i \tag{5}$$

where $i = (\rho + 1)$ to $(\rho + \kappa)$, $j = 1$ to ρ, and $A = \dfrac{H^3}{6E_pI_p} \cdot \left[2 - 3\left[\dfrac{H_2}{H}\right]^2 + \left(\dfrac{H_2}{H}\right)^3\right]$

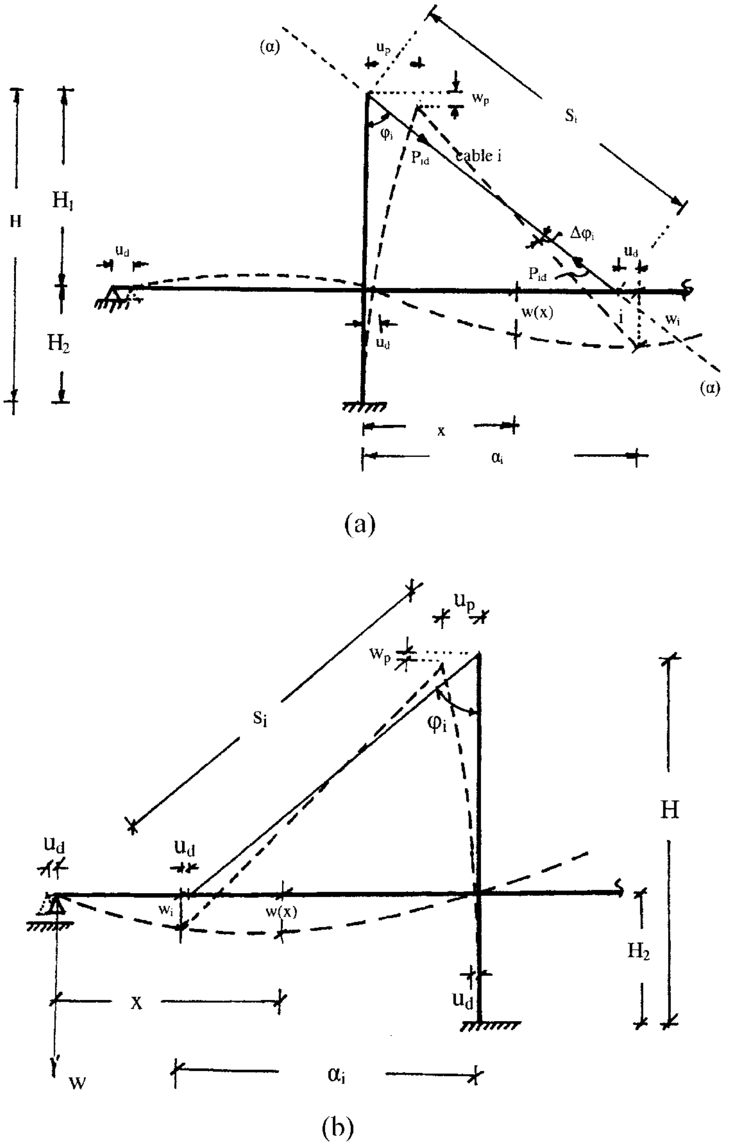

Fig. 3 Deformation of the system in the case of cables (a) right and (b) left to the pylon

In a similar way according to the geometry of the bridge shown in Fig. 3(b), and projecting the displacements on axis a-a the following equation is derived for the cables on the left side of the pylon:

$$A \cdot \left(\sum_j P_j \cdot \sin\varphi_j - \sum_i P_i \cdot \sin\varphi_i \right) \cdot \sin\varphi_j + \frac{s_j P_j}{E_s A_j} = w_j \cos\varphi_j \tag{6}$$

with $i = (\rho + 1)$ to $(\rho + \kappa)$, $j = 1$ to ρ

From Eqs. (5) and (6) and by setting: $b_i = s_i / E_s A_i, \quad b_j = s_j / E_s A_j$
the following system is obtained:

$$\left.\begin{aligned}
&\text{left:} \quad A \cdot \frac{\sin^2\varphi_j}{b_j} \cdot \left(\sum_j P_j \cdot \sin\varphi_j - \sum_i P_i \cdot \sin\varphi_i\right) + \sin\varphi_j P_j = \frac{w_j \cdot \sin\varphi_j \cdot \cos\varphi_j}{b_j} \\
&\text{right:} \quad A \cdot \frac{\sin^2\varphi_i}{b_i} \cdot \left(\sum_i P_i \cdot \sin\varphi_i - \sum_j P_j \cdot \sin\varphi_j\right) + \sin\varphi_i P_i = \frac{w_i \cdot \sin\varphi_i \cdot \cos\varphi_i}{b_i}
\end{aligned}\right\} \tag{7}$$

with $i = (\rho + 1)$ to $(\rho + \kappa)$, $j = 1$ to ρ or

$$\left.\begin{aligned}
&A \cdot \left(\sum_j \frac{\sin^2\varphi_j}{b_j}\right) \cdot \left(\sum_j P_j \cdot \sin\varphi_j - \sum_i P_i \cdot \sin\varphi_i\right) + \sum_j \sin\varphi_j P_j = \sum_j \frac{w_j \cdot \sin\varphi_j \cdot \cos\varphi_j}{b_j} \\
&A \cdot \left(\sum_i \frac{\sin^2\varphi_i}{b_i}\right) \cdot \left(\sum_i P_i \cdot \sin\varphi_i - \sum_j P_j \cdot \sin\varphi_j\right) + \sum_i \sin\varphi_i P_i = \sum_i \frac{w_i \cdot \sin\varphi_i \cdot \cos\varphi_i}{b_i}
\end{aligned}\right\} \tag{8}$$

Adding Eqs. (8a) and (8b) finally the following equation is derived:

$$\left(\sum_j \sin\varphi_j P_j - \sum_i \sin\varphi_i P_i\right) = \frac{1}{2} \frac{\left(\sum_j \frac{\sin 2\varphi_j \cdot w_j}{b_j} - \sum_i \frac{\sin 2\varphi_i \cdot w_i}{b_i}\right)}{A \cdot \left(\sum_j \frac{\sin^2\varphi_j}{b_j} + \sum_i \frac{\sin^2\varphi_i}{bi}\right) + 1} \tag{9}$$

From Eqs. (5) and (6) the expressions of the cable forces P_i and P_j can be determined as follows:

$$P_j = \frac{\cos\varphi_j}{b_j} \cdot w_j - \frac{\sin\varphi_j}{b_j} \cdot \frac{1}{2} \cdot \frac{\left(\sum_j \frac{\sin 2\varphi_j \cdot w_j}{b_j} - \sum_i \frac{\sin 2\varphi_i \cdot w_i}{b_i}\right)}{\sum_j \frac{\sin^2\varphi_j}{b_j} + \sum_i \frac{\sin^2\varphi_i}{bi} + \frac{1}{A}} \tag{10a}$$

$$P_i = \frac{\cos\varphi_i}{b_i} \cdot w_i - \frac{\sin\varphi_i}{b_i} \cdot \frac{1}{2} \cdot \frac{\left(\sum_j \frac{\sin 2\varphi_j \cdot w_j}{b_j} - \sum_i \frac{\sin 2\varphi_i \cdot w_i}{b_i}\right)}{\sum_j \frac{\sin^2\varphi_j}{b_j} + \sum_i \frac{\sin^2\varphi_i}{bi} + \frac{1}{A}} \tag{10b}$$

with $i = (\rho + 1)$ to $(\rho + \kappa)$ and $j = 1$ to ρ

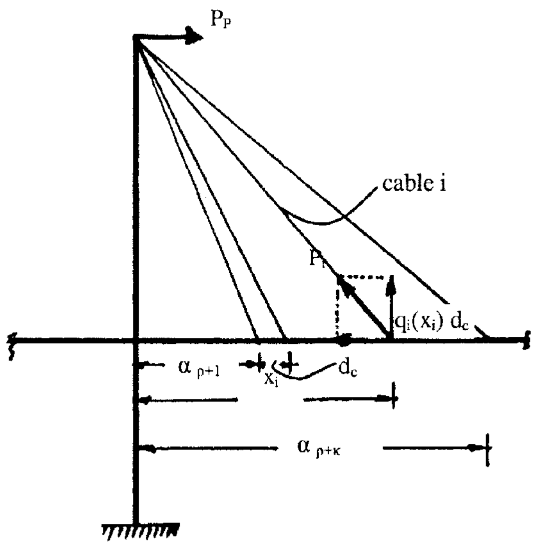

Fig. 4 The load $q(x)$, which expresses the effect of the cables on the bridge deck

2.3 Relation between distributed load q(x) of the cables and vertical displacement w of the deck

Let us consider that the cables are placed very densely at a distance d_c (Fig. 4). Then we can consider a distributed vertical load $q(x)$, extended from α_1 to α_ρ and from $\alpha_{\rho+1}$ to $\alpha_{\rho+\kappa}$, which at x_i will be equal to:

$$q(x_i) = \frac{1}{d_c} P_i \cdot \cos\varphi_i \tag{11}$$

It is evident that for a radial system the total horizontal force that acts at the top of the pylon and causes deformation u is:

$$P_p = \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i(x_i) \cdot \tan\varphi_i \cdot dx_i - \int_{\alpha_1}^{\alpha_\rho} q_j(x_j) \cdot \tan\varphi_j \cdot dx_j \tag{12}$$

So Eq. (2) becomes:

$$\delta u = \frac{H^3}{6E_pI_p} \cdot \left[2 - 3\left[\frac{H_2}{H}\right]^2 + \left(\frac{H_2}{H}\right)^3\right] \cdot \left[\int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i(x_i)\tan\varphi_i dx_i - \int_{\alpha_1}^{\alpha_\rho} q_j(x_j)\tan\varphi_j dx_j\right] \tag{13}$$

Taking into account that $\tan\varphi_i = \frac{x_i}{H_1}$ and $\tan\varphi_j = \frac{L_j - x_j}{H_1}$ we can write:

$$\delta u = \frac{A}{H_1} \cdot \left[\int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i(x_i) \cdot x_i \cdot dx_i - \int_{\alpha_1}^{\alpha_\rho} q_j(x_j) \cdot (L_j - x_j) \cdot dx_j\right] \tag{14}$$

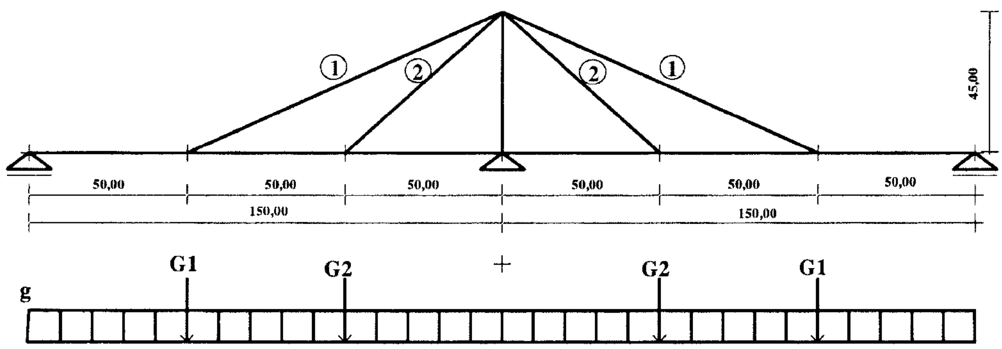

Fig. 5 Cable-stayed bridge studied by Kollbruner, Hajdin, Stipanic (1980) in their paper

Eq. (4), neglecting w_p as a very small quantity and replacing $\cos\Delta\varphi_i = 1$, leads to:

$$\left\{\frac{A}{H_1}\cdot\left(\int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i(x_i)\cdot x_i\cdot dx_i - \int_{\alpha_1}^{\alpha_\rho} q_j(x_j)\cdot(L_j - x_j)\cdot dx_j\right)\right\}\cdot\sin\varphi_i + \frac{s_i P_i}{E_s A_i} = w_i\cos\varphi_i \tag{15}$$

In a similar way according to the geometry of the bridge shown in Fig. 3(b) the following equation is derived for the cables on the left side of the pylon :

$$\left\{\frac{A}{H_1}\cdot\left(\int_{\alpha_1}^{\alpha_\rho} q_j(x_j)\cdot(L_j - x_i)\cdot dx_i - \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i(x_i)\cdot x_i\cdot dx_i\right)\right\}\cdot\sin\varphi_j + \frac{s_j P_j}{E_s A_j} = w_j\cos\varphi_j \tag{16}$$

From Eqs. (15) and (16) and by setting: $B_i = 1/E_s A_i$, $B_j = 1/E_s A_j$
we obtain the following:

$$\begin{aligned}
&\text{left:} \quad \left\{\frac{A}{H_1}\cdot\left(\int_{\alpha_1}^{\alpha_\rho} q_j\cdot(L_j - x_j)\cdot dx_j - \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i\cdot x_i\cdot dx_i\right)\right\}\cdot\sin\varphi_j + \frac{s_j B_j d_c q_j}{\cos\varphi_j} = w_j\cos\varphi_j \\
&\text{right:} \quad \left\{\frac{A}{H_1}\cdot\left(\int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i\cdot x_i\cdot dx_i - \int_{\alpha_1}^{\alpha_\rho} q_j\cdot(L_j - x_j)\cdot dx_j\right)\right\}\cdot\sin\varphi_i + \frac{s_i B_i d_c q_i}{\cos\varphi_i} = w_i\cos\varphi_i
\end{aligned} \tag{17}$$

with $i = (\rho + 1)$ to $(\rho + \kappa)$, $j = 1$ to ρ

Taking into account that:

$$s_i = \frac{x_i}{\sin\varphi_i}, \quad s_j = \frac{L_j - x_j}{\sin\varphi_j}, \quad \sin\varphi_i = \frac{x_i}{\sqrt{H_1^2 + x_i^2}}, \quad \sin\varphi_j = \frac{L_j - x_j}{\sqrt{H_1^2 + (L_j - x_j)^2}}$$

$$\cos\varphi_i = \frac{H_i}{\sqrt{H_1^2 + x_i^2}}, \quad \cos\varphi_j = \frac{H_1}{\sqrt{H_1^2 + (L_j - x_j)^2}} \tag{18}$$

Eq. (17) can be written under the form of the following system of equations:

$$\begin{aligned} &\text{left:} \quad (1 + I_{1l}) \cdot \int_{\alpha_1}^{\alpha_\rho} q_j \cdot (L_j - x_j) \cdot dx_j - I_{1l} \cdot \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i \cdot x_i \cdot dx_i = I_{2l} \\ &\text{right:} \quad -I_{1R} \cdot \int_{\alpha_1}^{\alpha_\rho} q_j \cdot (L_j - x_j) \cdot dx_j + (1 + I_{1R}) \cdot \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i \cdot x_i \cdot dx_i = I_{2R} \end{aligned} \tag{19}$$

where:

$$I_{1l} = A \cdot E_s \cdot \int_{\alpha_1}^{\alpha_\rho} \frac{A_j(x_j) \cdot (L_j - x_j)^2}{[H_1^2 + (L_j - x_j)^2]^{3/2}} \cdot dx_j \qquad I_{1R} = A \cdot E_s \cdot \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} \frac{A_i(x_i) \cdot x_i^2}{[H_1^2 + x_i^2]^{3/2}} \cdot dx_i$$

$$I_{2l} = \int_{\alpha_1}^{\alpha_\rho} F_{1l} \cdot w_j(x_j) \cdot dx_j \qquad I_{2R} = \int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} F_{1R} \cdot w_i(x_i) \cdot dx_i$$

$$F_{1l} = \frac{E_s \cdot H_1^2 \cdot A_j(x_j) \cdot (L_j - x_j)}{[H_1^2 + (L_j - x_j)^2]^{3/2}} \qquad F_{1R} = \frac{E_s \cdot H_1^2 \cdot A_i(x_i) \cdot x_i}{[H_1^2 + x_i^2]^{3/2}}$$

$$A_j(x_j) = \frac{A_j}{d_c} \qquad A_i(x_i) = \frac{A_i}{d_c} \tag{20}$$

The solution of the above system of Eq. (19), gives the following:

$$\int_{\alpha_1}^{\alpha_\rho} q_j \cdot (L_j - x_j) \cdot dx_j = \frac{I_{2l} + I_{1R} \cdot I_{2l} + I_{1l} \cdot I_{2R}}{1 + I_{1l} + I_{1R}}$$

$$\int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i \cdot x_i \cdot dx_i = \frac{I_{2R} + I_{1R} \cdot I_{2l} + I_{1l} \cdot I_{2R}}{1 + I_{1l} + I_{1R}} \tag{21}$$

and finally:

$$\int_{\alpha_{\rho+1}}^{\alpha_{\rho+\kappa}} q_i \cdot x_i \cdot dx_i - \int_{\alpha_1}^{\alpha_\rho} q_j \cdot (L_j - x_j) \cdot dx_j = \frac{I_{2R} - I_{2l}}{1 + I_{1l} + I_{1R}} \tag{22}$$

The expression of the distributed vertical load $q(x)$ can be easily determined as follows:

$$q_j(x_j) = \frac{E_s \cdot A \cdot A_j(x_j) \cdot (L_j - x_j)}{[H_1^2 + (L_j - x_j)^2]^{3/2}} \cdot \frac{I_{2R} - I_{2l}}{1 + I_{1l} + I_{1R}} + \frac{E_s \cdot H_1^2 \cdot A_j(x_j)}{[H_1^2 + (L_j - x_j)^2]^{3/2}} \cdot w_j(x_i)$$

$$q_i(x_i) = \frac{E_s \cdot A \cdot A_i(x_i) \cdot x_i}{[H_1^2 + x_i^2]^{3/2}} \cdot \frac{I_{2l} - I_{2R}}{1 + I_{1l} + I_{1R}} + \frac{E_s \cdot H_1^2 \cdot A_i(x_i)}{[H_1^2 + x_i^2]^{3/2}} \cdot w_i(x_i) \tag{23}$$

2.4 Solution of the static problem applying Galerkin's method

2.4.1 Sparse distribution of cables

The equilibrium equation of the bridge deck is the following:

$$E_b I_b w''''(x) = p_{tot}(x) \tag{24}$$

where E_b is the modulus of elasticity of the bridge deck,
I_b the moment of inertia of the cross-section of the bridge deck,
$w(x)$ the total vertical displacement of the bridge deck, and

$$p_{tot}(x) = g(x) + p(x) + \sum_k P\delta(x - a_k) - \sum_j P_j \cos\varphi_j \delta(x - \alpha_j) - \sum_i P_i \cos\varphi_i \delta(x - \alpha_i) \tag{25}$$

in which $g(x)$ the dead load of the bridge deck
$p(x)$ the live load of the bridge
P_k are concentrated loads (dead or live load) at positions $x = a_k$
P_i the forces of the cables right to the pylon given by Eq. (10b)
P_j the forces of the cables left to the pylon given by Eq. (10a)

Eq. (24), taking into account Eq. (25) becomes:

$$E_b I_b w''''(x) = g(x) + p(x) + \sum_k P\delta(x - a_k) - \sum_j P_j \cos\varphi_j \delta(x - \alpha_j) - \sum_i P_i \cos\varphi_i \delta(x - \alpha_i) \tag{26}$$

Applying Galerkin's method a solution under the following form is investigated:

$$w(x) = \sum_{i=1}^{m} c_i \cdot \Psi_i(x) \tag{27}$$

where: c_i are unknown coefficients under determination and $\Psi_i(x)$ are arbitrarily chosen functions of x, which must satisfy the boundary conditions of the deck. In this case the shape functions of the corresponding continuous beam (which has the same characteristics with the bridge but without cables) are chosen (see Appendix). Introducing Eqs. (10) and (27) into (26), multiplying the outcome successively by Ψ_1, Ψ_2, …, Ψ_m and integrating from 0 to L the results, by taking into account the orthogonality conditions of the shape functions, a linear system of m equations is obtained, with unknowns the coefficients c_1, c_2, …, c_m which can be written under the following form:

$$A_{i1} \cdot c_1 + A_{i2} \cdot c_2 + \ldots + A_{im} \cdot c_m = B_i \qquad (i = 1, 2, \ldots, m) \tag{28}$$

Solving the above system the values of the unknowns c_1, c_2, …, c_m are obtained and thus the equation of the vertical deformation of the bridge is derived. Finally from Eq. (10) the values of the tensile forces of the cables are determined.

2.4.2 Dense distribution of cables

The equilibrium equation of the bridge deck is given by Eq. (24), where:

$$p_{tot}(x) = g(x) + p(x) + \sum_k P\delta(x - a_k) - q(x) \tag{29}$$

in which $q(x)$ is the load of the cables given by Eq. (23).

Eq. (24), taking into account Eq. (29) becomes:

$$E_b I_b w''''(x) = g(x) + p(x) + \sum_k P\delta(x - a_k) - q(x) \tag{30}$$

Applying Galerkin's method a solution under the form of Eq. (27) is investigated. Following the same procedure as in paragraph 2.4.1 a linear system of m equations similar to that of Eq. (28) is obtained. The solution of this system gives the values of the coefficients c_i, while Eq. (27) leads to the displacement of the bridge deck.

Using the equations of paragraphs 2.4.1 or 2.4.2 one can find, for the loading g (self weight), the needed negative constructional deformation of the deck (see Figs. 7 and 8) and also the corresponding cable tensions (prestressing loads and predeformation).

3. Numerical results and discussion

3.1 Sparse distribution of cables

In order to check the accuracy of the above-described method, the bridge which is studied by C. F. Kollbruner, N. Hajdin, B. Stipanic (1980) is considered. It is a cable-stayed bridge with two equal spans of 150 m. The bridge is loaded by the uniformly distributed dead load g = 120 kN/m and by the concentrated loads G_1 = 400 kN and G_2 = 300 kN, due to the weights of the cables, anchor heads and anchor girders (Fig. 6). The bridge has the following characteristics:

L_1 = 150 m, L_2 = 150 m, $E_b = E_p = 2.1 \cdot 10^8$ kN/m^2, $E_c = 2.05 \cdot 10^8$ kN/m^2, I_b = 1.2 m^4,
I_p = 0.6 m^4, H = 45 m, H_1 = 45 m, H_2 = 0, A_1 = 0.0449 m^2, A_2 = 0.0296 m^2

The subscripts b, p and c refer to the deck, the pylon and the cables correspondingly.

Applying the analysis presented here the configuration of the deck shown in Fig. 7 is determined and the tensile forces of the cables are found as shown in column 2 of Table 1. In addition the same bridge has been solved using the Computer Program SOFISTIK of Sofistik Gmbh, column 3. The cable forces given by the above mentioned paper are also presented in column 4.

It can be seen that the accuracy of the proposed method of analysis is acceptable for a preliminary design of a cable-stayed bridge.

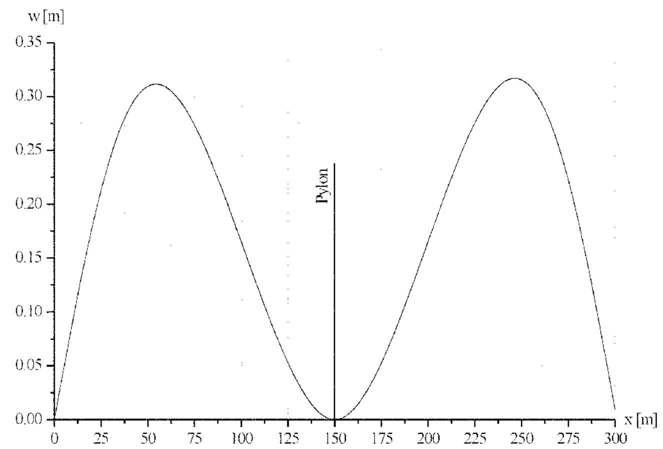

Fig. 6 Configuration of the deck of the bridge shown in Fig. 6 calculated by the presented analysis

Table 1 Comparison of cable forces after solution of the bridge by 3 different methods

Cables	Authors' method (kN) (1)	SOFISTIK (kN) (2)	Results from paper (kN) (3)	[(1)-(2)]%	[(1)-(3)]%
T_1	12850	13450	13860	–4.46%	–7.28%
T_2	9560	9320	9027	+2.57%	+5.9%

3.2 Dense distribution of cables

In the case of a bridge with a dense distribution of cables the bridge of Fig. 1 with the following characteristics is considered:

$L_1 = 80$ m, $L_2 = 200$ m, $L_3 = 80$ m, $E_b = 2.1 \cdot 10^5$ N/mm^2, $E_p = 2.1 \cdot 10^5$ N/mm^2, $I_b = 1.2$ m^4, $I_p = 30 \cdot I_b$, $E_c = 2.1 \cdot 10^5$ N/mm^2, $H = 45$ m, $H_1 = 40$ m, $H_2 = 5$ m, $\alpha_1 = 0$, $\alpha_2 = 80$ m, $\alpha_3 = 0$, $\alpha_4 = 100$ m, $\alpha_5 = 100$ m, $\alpha_6 = 200$ m, $\alpha_7 = 0$, $\alpha_8 = 80$ m, $m = 1200$ kg, $g = 120$ kN/m, $p = 120$ kN/m

The cross-section of the cables is supposed to change according to the following law (see Bruno and Golotti 1994):

$$A(x) = \frac{g}{\sigma_g \cdot \cos\varphi} \tag{31}$$

where: g the uniform distributed self weight of the deck
σ_g the initial tensile stress of the stay's curtain, due to g: $\sigma_g = \sigma_\alpha \cdot \frac{g}{g+p}$
σ_a the allowable stress of cables, and
p the design live load.

The application of the proposed method led to the following results:

1. For load case 1 (only dead load $g = 120$ kN/m on the bridge deck) the plot of Fig. 7 for the

equivalent vertical distributed load, which replaces the action of the cables on the deck, and the plot of Fig. 8 for the deck deformation were obtained.

2. For load case 2 (dead load g = 120 kN/m, live load p = 120 kN/m on the bridge deck) the plot of Fig. 9 for the equivalent vertical distributed load, which replaces the action of the cables on the deck, and the plot of Fig. 10 for the deck deformation were obtained.
3. For load case 3 (dead load g = 120 kN/m, live load p = 120 kN/m only on the first span of the bridge deck) the plot of Fig. 11 for the equivalent vertical distributed load, which replaces the action of the cables on the deck, and the plot of Fig. 12 for the deck deformation were obtained.
4. For load case 4 (dead load g = 120 kN, live load p = 120 kN only on the second span of the bridge deck) the plot of Fig. 13 for the equivalent vertical distributed load, which replaces the action of the cables on the deck, and the plot of Fig. 14 for the deck deformation were obtained.

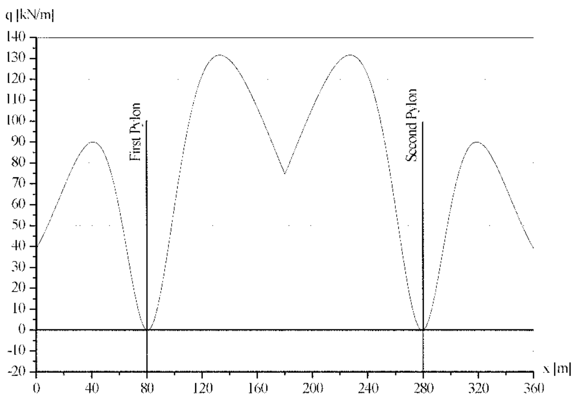

Fig. 7 Cable tension for load case 1 (only dead load g = 120 kN/m on the bridge deck)

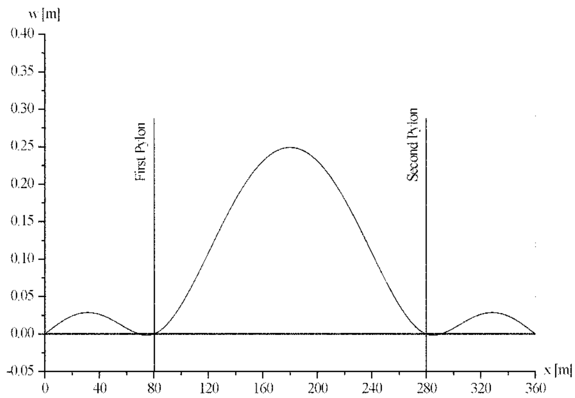

Fig. 8 Deck configuration for load case 1 (only dead load g = 120 kN/m on the bridge deck)

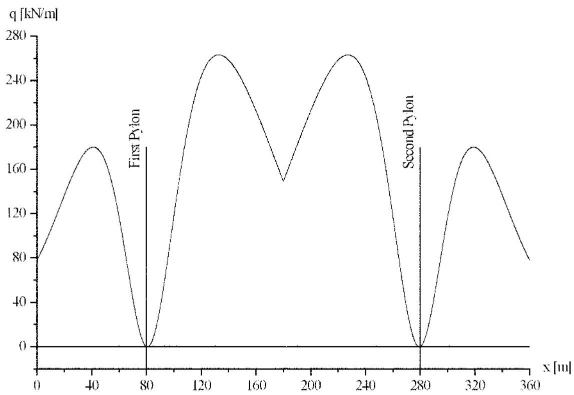

Fig. 9 Cable tension for load case 2 (dead load g = 120 kN/m, live load p = 120 kN/m on the bridge deck)

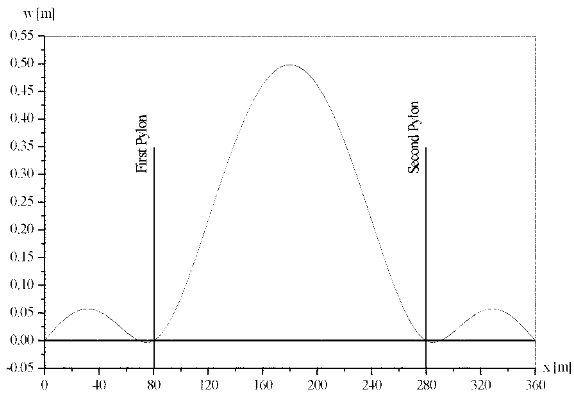

Fig. 10 Deck configuration for load case 2 (dead load g = 120 kN/m, live load p = 120 kN/m on the bridge deck)

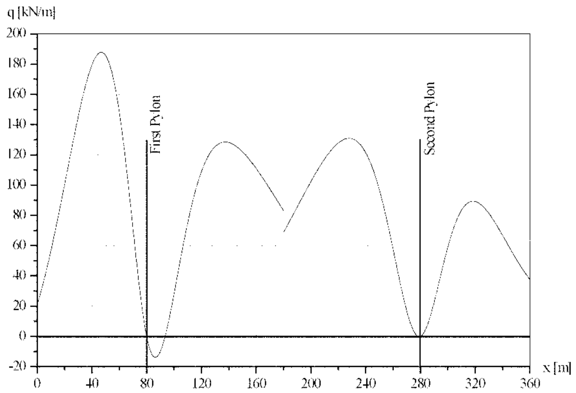

Fig. 11 Cable tension for load case 3 (dead load g = 120 kN/m, live load p = 120 kN/m only on the first span of the bridge deck)

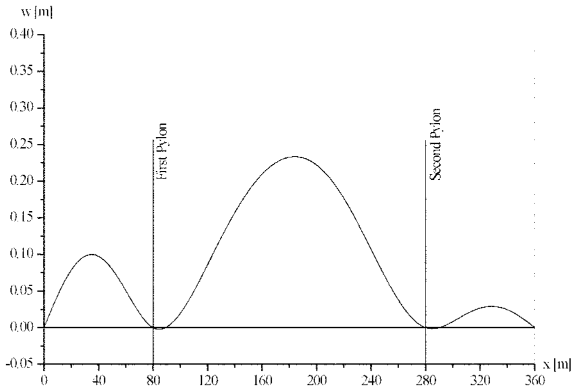

Fig. 12 Deck configuration for load case 3 (dead load g = 120 kN/m, live load p = 120 kN/m only on the first span of the bridge deck)

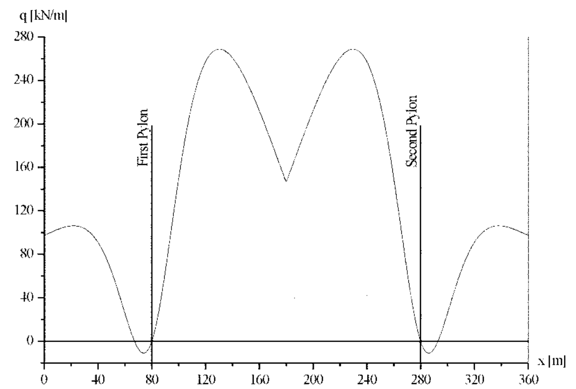

Fig. 13 Cable tension for load case 4 (dead load g = 120 kN/m, live load p = 120 kN/m only on the second span of the bridge deck)

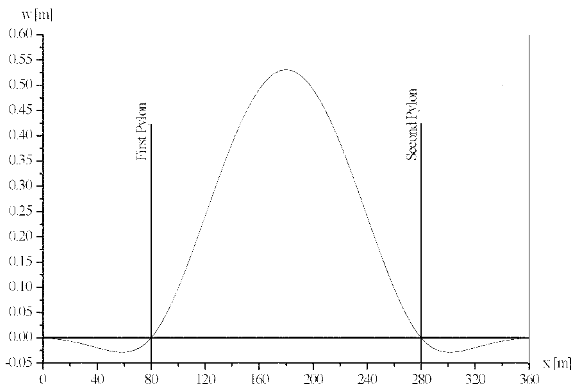

Fig. 14 Deck configuration for load case 4 (dead load g = 120 kN/m, live load p = 120 kN/m only on the second span of the bridge deck)

4. Conclusions

On the basis of the chosen model, the following conclusions may be drawn:

1. The cable tensile forces P_i and the distributed load $q(x)$ that expresses the effect of the cables can be determined with adequate accuracy, according to the results included in Table 1.
2. The proposed procedure of preliminary design based on Galerkin's method is adequately quick and efficient.
3. The obtained results compared to those of a usual analysis can be considered as satisfactory.

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Appendix

1. Two-span continuous beam

Eigenfrequencies equation:

$$\cosh\lambda l_2 \cdot \sin\lambda l_1 \cdot \sin\lambda l_2 \cdot \sinh\lambda l_1 + \cosh\lambda l_1 \cdot \sin\lambda l_1 \cdot \sin\lambda l_2 \cdot \sinh\lambda l_2$$
$$- \cos\lambda l_2 \cdot \sinh\lambda l_1 \cdot \sinh\lambda l_1 \cdot \sinh\lambda l_2 - \cos\lambda l_1 \cdot \sinh\lambda l_2 \cdot \sinh\lambda l_1 \cdot \sinh\lambda l_2 = 0$$

Shape-functions equation:

$$\Psi_n(x_1) = \frac{1}{\sin\lambda_n l_1}\sin\lambda_n x_1 - \frac{1}{\sinh\lambda_n l_1}\sinh\lambda_n x_1 \qquad \text{for} \quad 0 \le x_1 \le l_1$$
$$\Psi_n(x_2) = -\cot\lambda_n l_2 \sin\lambda_n x_2 + \cos\lambda_n x_2 + \coth\lambda_n l_2 \sinh\lambda_n x_2 - \cosh\lambda_n x_2 \qquad \text{for} \quad 0 \le x_2 \le l_2$$

where: $\lambda_n = \left(\frac{m\omega_n^2}{EI}\right)^{0.25}$

2. Three-span continuous beam

Eigenfrequencies equation:

$$
\begin{aligned}
&\cosh\lambda_n l_2 \cdot \cosh\lambda_n l_3 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 + \\
&\cosh\lambda_n l_1 \cdot \cosh\lambda_n l_3 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_2 - \\
&\cos\lambda_n l_2 \cdot \cosh\lambda_n l_3 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_2 - \\
&\cos\lambda_n l_1 \cdot \cosh\lambda_n l_3 \cdot \sin\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_2 + \\
&\cosh\lambda_n l_1 \cdot \cosh\lambda_n l_2 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_3 - \\
&\cos\lambda_n l_3 \cdot \cosh\lambda_n l_2 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_2 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_3 + \\
&\cos^2\lambda_n l_2 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_3 - \\
&2 \cdot \cos\lambda_n l_2 \cdot \cosh\lambda_n l_2 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_3 + \\
&\cosh^2\lambda_n l_2 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_3 - \\
&\cos\lambda_n l_1 \cdot \cosh\lambda_n l_2 \cdot \sin\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_3 + \\
&\sin\lambda_n l_1 \cdot \sin^2\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_3 - \\
&\cos\lambda_n l_3 \cdot \cosh\lambda_n l_1 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_2 \cdot \sinh\lambda_n l_2 \cdot \sinh\lambda_n l_3 - \\
&\cos\lambda_n l_2 \cdot \cosh\lambda_n l_1 \cdot \sin\lambda_n l_1 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_2 \cdot \sinh\lambda_n l_3 + \\
&\cos\lambda_n l_2 \cdot \cos\lambda_n l_3 \cdot \sin\lambda_n l_1 \cdot \sinh\lambda_n l_{12} \cdot \sinh\lambda_n l_2 \cdot \sinh\lambda_n l_3 + \\
&\cos\lambda_n l_1 \cdot \cos\lambda_n l_3 \cdot \sin\lambda_n l_2 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_2 \cdot \sinh\lambda_n l_3 + \\
&\cos\lambda_n l_1 \cdot \cos\lambda_n l_2 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh\lambda_n l_2 \cdot \sinh\lambda_n l_3 - \\
&\sin\lambda_n l_1 \cdot \sin\lambda_n l_3 \cdot \sinh\lambda_n l_1 \cdot \sinh^2\lambda_n l_2 \cdot \sinh\lambda_n l_3 \;=\; 0
\end{aligned}
$$

Shape-functions equation:

$$\Psi_n(x_1) = \frac{1}{\sin\lambda_n l_1}\sin\lambda_n x_1 - \frac{1}{\sinh\lambda_n l_1}\sinh\lambda_n x_1 \qquad \text{for} \quad 0 \le x_1 \le l_1$$

$$
\begin{aligned}
\Psi_n(x_2) = {} & \left(-\cot\lambda_n l_2 + \frac{C}{\sin\lambda_n l_2}\right)\sin\lambda_n x_2 + \cos\lambda_n x_2 \\
& + \left(\coth\lambda_n l_2 - \frac{C}{\sinh\lambda_n l_2}\right)\sinh\lambda_n x_2 - \cosh\lambda_n x_2 \qquad \text{for} \quad 0 \le x_2 \le l_2
\end{aligned}
$$

$$
\begin{aligned}
\Psi_n(x_3) = {} & -C \cdot \cot\lambda_n l_3 \cdot \sin\lambda_n l_3 + C \cdot \cos\lambda_n x_3 \\
& + C \cdot \coth\lambda_n l_3 \cdot \sinh\lambda_n x_3 - C \cdot \cosh\lambda_n x_3 \qquad \text{for} \quad 0 \le x_3 \le l_3
\end{aligned}
$$

where: $\lambda_n = \left(\frac{m\,\omega_n^2}{EI}\right)^{0.25}$

and $C = \dfrac{\sin\lambda_n l_2 - \sinh\lambda_n l_2}{\sin\lambda_n l_2 \cdot \sinh\lambda_n l_2(\coth\lambda_n l_2 + \coth\lambda_n l_3 - \cot\lambda_n l_2 - \cot\lambda_n l_3)}$