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Nonlinear dynamic buckling of laminated angle-ply composite spherical caps

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Abstract. This paper deals with nonlinear asymmetric dynamic buckling of clamped laminated angle-ply composite spherical shells under suddenly applied pressure loads. The formulation is based on first-order shear deformation theory and Lagrange's equation of motion. The nonlinearity due to finite deformation of the shell considering von Karman's assumptions is included in the formulation. The buckling loads are obtained through dynamic response history using Newmark's numerical integration scheme coupled with a Newton-Raphson iteration technique. An axisymmetric curved shell element is used to investigate the dynamic characteristics of the spherical caps. The pressure value beyond which the maximum average displacement response shows significant growth rate in the time history of the shell structure is considered as critical dynamic load. Detailed numerical results are presented to highlight the influence of ply-angle, shell geometric parameter and asymmetric mode on the critical load of spherical caps.

Key words: dynamic buckling; asymmetric; angle-ply; spherical caps; nonlinear response.

1. Introduction

Thin spherical shells have been interesting topics in structural engineering not only for their widespread applications but also the complicated stability problems. These shells are often subjected to snap-through buckling. Buckling analysis of such shells under dynamic loads has received considerable attention in the literature. It is known, in general, that shells subjected to dynamically applied loads usually buckle at load levels that are lower than the corresponding quasi-static buckling load.

The available work on dynamic buckling behavior of spherical shells is mainly concerned with isotropic case subjected to the step pressure load of infinite duration. Budiansky and Roth (1962) have analyzed the problem employing the Galerkin method whereas Simitses (1967) adopted Ritz-Galerkin procedure. Haung (1969), Stephens and Fulton (1969), and Ball and Burt (1973) have investigated using the finite difference scheme while Stricklin and Martinez (1969) utilized more efficient finite element procedure. The effect of geometric imperfection on the dynamic buckling load, by employing buckling criterion based on the displacement response, is examined by Kao and Perrone (1978), and Kao (1980) based on finite difference method whereas Saigal *et al.* (1987) and

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Yang and Liaw (1988) analyzed using finite element technique. Lock *et al.* (1968) have carried out experimental study on the buckling of spherical caps.

Quite often, the asymmetric modes of these shells may be excited due to the introduction of slight deviation in perfect axisymmetric loading, geometric imperfection and/or initial displacement/ velocity to the shells, leading to asymmetric type of buckling behavior. The buckling of spherical shell under such modes has received limited attention in the literature. Stricklin and Martinez (1969), Stricklin *et al.* (1971), and Ball and Burt (1973) have assumed imperfection in the step load to excite the asymmetric modes and presented results for a shell geometry with few asymmetric modes. Klosner and Longhitano (1973), while obtaining the response of dynamically loaded spherical shells, have considered an asymmetric initial velocity to the shell but the numerical results are not presented for the dynamic buckling study. Akkas (1976) has examined the asymmetric dynamic buckling behavior of spherical caps by perturbing few asymmetric modes through the initial displacement and presented very few results based on the response of asymmetric part of the displacement. It may be inferred that the effect of asymmetric modes of spherical shell with step load of infinite duration on the dynamic buckling characteristics could not be well established with a few available work in the literature in comparison with those of axisymmetric dynamic buckling case.

In recent years, advanced composite materials have found rapid growth in engineering structural applications. However, the complexity of the analysis due to the inherent directional properties of the composite materials has limited the study to axisymmetric dynamic buckling behavior of single-layered orthotropic spherical case using classical lamination theory (Alwar and Sekhar Reddy 1979, Ganapathi and Varadan 1982, Dumir *et al*. 1984, Chao and Lin 1990), except the work of Ganapathi and Varadan (1995). Alwar and Sekhar Reddy (1979), and Dumir *et al.* (1984) have examined the problem using the method of orthogonal collocation whereas Chao and Lin (1990) have obtained the critical loads based on finite difference scheme including the influence of geometric imperfection. Ganapathi and Varadan (1995) have solved the problem employing shear deformation theory coupled with finite element technique. However, to the authors' knowledge, work on the asymmetric dynamic buckling behavior of laminated composite spherical shells under externally applied pressure seems to be scarce in the literature.

In the present work, a three-noded shear flexible axisymmetric curved shell element based on semi-analytical approach and the field-consistency principle (Balakrishna and Sarma 1989, Ganapathi *et al*. 1994) is extended to analyze the asymmetric dynamic buckling of laminated composite spherical caps under externally applied pressure load. Geometric nonlinearity is assumed in the present study using von Karman's strain-displacement relations. The nonlinear governing equations derived are solved employing Newmark numerical integration method in conjunction with the modified Newton-Raphson iteration scheme. For axisymmetric case, the dynamic buckling pressure is defined as the pressure corresponding to a sudden jump in the maximum average displacement in the time history of the shell structure (Budiansky and Roth 1962, Simitses 1989). However, the load associated with the threshold value of pressure beyond which the asymmetric component of displacement response of shell shows significant growth rate with time is taken as the critical load for asymmetric buckling case (Akkas 1976, Fulton and Barton 1971). A detailed investigation is carried out to bring out the influences of ply-angle, geometric parameters, and different asymmetric modes of excitation on the dynamic buckling characteristics of clamped spherical caps.

2. Formulation

An axisymmetric laminated composite shell of revolution is considered with the coordinates s, θ and z along the meridional, circumferential and radial/thickness directions, respectively. The displacements u, v, w at a point (s, θ, z) from the median surface are expressed as functions of middle-surface displacements u_o, v_o and w, and independent rotations β_s and β_θ of the meridional and hoop sections, respectively, as

$$u(s, \theta, z, t) = u_o(s, \theta, t) + z\beta_s(s, \theta, t)$$
$$v(s, \theta, z, t) = v_o(s, \theta, t) + z\beta_\theta(s, \theta, t)$$
$$w(s, \theta, z, t) = w(s, \theta, t) \tag{1}$$

where t is the time.

Using the semi-analytical approach, u_o, v_o, w, β_s and β_θ are represented by a Fourier series in the circumferential angle θ. For the nth harmonic, these can be written as

$$u_o(s, \theta, t) = u_o^o(s, t) + \sum_{i=1}^{4} [u_o^{c_i}(s, t)\cos(in\theta) + u_o^{s_i}(s, t)\sin(in\theta)]$$

$$v_o(s, \theta, t) = v_o^o(s, t) + \sum_{i=1}^{4} [v_o^{c_i}(s, t)\cos(in\theta) + v_o^{s_i}(s, t)\sin(in\theta)]$$

$$w(s, \theta, t) = w^o(s, t) + \sum_{i=1}^{2} [w^{c_i}(s, t)\cos(in\theta) + w^{s_i}(s, t)\sin(in\theta)]$$

$$\beta_s(s, \theta, t) = \beta_s^o(s, t) + \sum_{i=1}^{2} [\beta_s^{c_i}(s, t)\cos(in\theta) + \beta_s^{s_i}(s, t)\sin(in\theta)]$$

$$\beta_\theta(s, \theta, t) = \beta_\theta^o(s, t) + \sum_{i=1}^{2} [\beta_\theta^{c_i}(s, t)\cos(in\theta) + \beta_\theta^{s_i}(s, t)\sin(in\theta)] \tag{2}$$

where the superscripts o, c_i and s_i refer to the amplitudes associated with axisymmetric, cosine and sine terms.

The above displacement variations in the circumferential direction are chosen according to the physics of the large amplitude asymmetric vibrations of shells of revolution i.e., participation of axisymmetric mode and higher asymmetric modes (Ueda 1979, Tong and Pian 1974, Amabili *et al*. 1999). Additional terms in the in-plane displacements, compared to radial displacement, are added to keep the nonlinear membrane strains consistent.

Using von Karman's assumption for moderately large deformation, Green's strains can be written in terms of mid-plane deformations as,

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^{NL} \\ 0 \end{Bmatrix} \tag{3}$$

where, the membrane strains $\{\varepsilon_p^L\}$, bending strains $\{\varepsilon_b\}$, shear strains $\{\varepsilon_s\}$ and nonlinear in-plane strains $\{\varepsilon_p^{NL}\}$ in the Eq. (3) are written as (Kraus 1967)

$$\{\varepsilon_p^L\} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial s} + \dfrac{w}{R} \\ \dfrac{u_o \sin\phi}{r} + \dfrac{\partial v_o}{r\partial\theta} + \dfrac{w\cos\phi}{r} \\ \dfrac{\partial u_o}{r\partial\theta} - \dfrac{v_o \sin\phi}{r} + \dfrac{\partial v_o}{\partial s} \end{Bmatrix}; \qquad \{\varepsilon_b\} = \begin{Bmatrix} \dfrac{\partial\beta_s}{\partial s} + \dfrac{\partial u_o}{R\partial s} \\ \dfrac{\beta_s \sin\phi}{r} + \dfrac{\partial\beta_\theta}{r\partial\theta} + \dfrac{u_o\sin\phi}{Rr} \\ \dfrac{1}{R}\dfrac{\partial u_o}{r\partial\theta} + \dfrac{\partial v_o}{\partial s}\dfrac{\cos\phi}{r} + \dfrac{\partial\beta_s}{r\partial\theta} + \dfrac{\partial\beta_\theta}{\partial s} - \dfrac{\beta_\theta\sin\phi}{r} \end{Bmatrix};$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \beta_s + \dfrac{\partial w}{\partial s} \\ \beta_\theta + \dfrac{\partial w}{r\partial\theta} - \dfrac{v_o\cos\phi}{r} \end{Bmatrix}; \qquad \{\varepsilon_p^{NL}\} = \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w}{\partial s}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial w}{r\partial\theta}\right)^2 \\ \dfrac{\partial w}{\partial s}\dfrac{\partial w}{r\partial\theta} \end{Bmatrix} \tag{4}$$

where r, R and ϕ are the radius of the parallel circle, radius of the meridional circle and angle made by the tangent at any point in the shell with the axis of revolution.

If $\{N\}$ represents the stress resultants (N_{ss}, $N_{\theta\theta}$, $N_{s\theta}$) and $\{M\}$ the moment resultants (M_{ss}, $M_{\theta\theta}$, $M_{s\theta}$), one can relate these to membrane strains $\{\varepsilon_p\}$ (= $\{\varepsilon_p^L\} + \{\varepsilon_p^{NL}\}$) and bending strains $\{\varepsilon_b\}$ through the constitutive relations as

$$\{N\} = [A]\{\varepsilon_p\} + [B]\{\varepsilon_b\} \quad \text{and} \quad \{M\} = [B]\{\varepsilon_p\} + [D]\{\varepsilon_b\} \tag{5}$$

where $[A]$, $[D]$ and $[B]$ are extensional, bending and bending-extensional coupling stiffness coefficients matrices of the composite laminate. Similarly, the transverse shear force $\{Q\}$ representing the quantities (Q_{sz}, $Q_{\theta z}$) are related to the transverse shear strains $\{\varepsilon_s\}$ through the constitutive relation as

$$\{Q\} = k^2[E]\{\varepsilon_s\} \tag{6}$$

where k^2 and $[E]$ are the shear correction factor and the transverse shear stiffness coeffiecients matrix, respectively.

For a composite laminate of thickness h, consisting of N layers with stacking angles ϕ_i (i = 1, …, N) and layer thicknesses h_i (i = 1, …, N), the necessary expressions to compute the stiffness coefficients, available in the literature (Jones 1975) are used here.

The potential energy functional $U(\delta)$ is given by,

$$U(\delta) = \frac{1}{2}\int_A [\{\varepsilon_p\}^T[A]\{\varepsilon_p\} + \{\varepsilon_p\}^T[B]\{\varepsilon_b\} + \{\varepsilon_b\}^T[B]\{\varepsilon_p\} + \{\varepsilon_b\}^T[D]\{\varepsilon_b\}$$

$$+ \{\varepsilon_s\}^T[E]\{\varepsilon_s\}]dA - \int_A qw\,dA \tag{7}$$

where δ is the vector of degrees of freedom associated to the displacement field in a finite element discretisation, A is the area of the middle surface of the shell and q is the applied external pressure load.

The kinetic energy of the shell is given by

$$T(\delta) = \frac{1}{2}\int_A [p(\dot{u}_o^2 + \dot{v}_o^2 + \dot{w}^2) + I(\dot{\beta}_s^2 + \dot{\beta}_\theta^2)]dA \tag{8}$$

where $p = \int_{-h/2}^{h/2} \rho dz \ \cdots$ and $\cdots \ I = \int_{-h/2}^{h/2} \rho z^2 dz$ and ρ is the mass density. The dot over the variable denotes derivative with respect to time.

Following the procedure given in the work of Rajasekaran *et al.* (1973), the potential energy functional U given in Eq. (7) is rewritten as

$$U(\delta) = \{\delta\}^T[(1/2)[K] + (1/6)[N_1(\delta)] + (1/12)[N_2(\delta)]\ ]\{\delta\} + \{\delta\}^T\{F\} \tag{9}$$

where $[K]$ is the linear stiffness matrix, $[N_1]$ and $[N_2]$ are non-linear stiffness matrices linearly and quadratically dependent on the field variables, respectively and $\{F\}$ is the load vector. Substituting Eqs. (8) and (9) in Lagrange's equation of motion, the governing equation for the shell are obtained as:

$$[M]\{\ddot{\delta}\} + \left[[K] + \frac{1}{2}[N_1(\delta)] + \frac{1}{3}[N_2(\delta)]\right]\{\delta\} = \{F\} \tag{10}$$

where $[M]$ is the mass matrix.

The Eq. (10) is solved using the implicit method (Subbaraj and Dokainish 1989). In this method, equilibrium conditions are considered at the same time step for which solution is sought. If the solution is known at time t and one wishes to obtain the displacements, etc., at time $t + \Delta t$, then the equilibrium equations considered at time $t + \Delta t$ are given as

$$[M]\{\ddot{\delta}\}_{t+\Delta t} + [[N(\delta)]\{\delta\}]_{t+\Delta t} = \{F\}_{t+\Delta t} \tag{11}$$

where $\{\ddot{\delta}\}_{t+\Delta t}$ and $\{\delta\}_{t+\Delta t}$ are the vectors of the nodal accelerations and displacements at time $t + \Delta t$, respectively. $[[N(\delta)]\{\delta\}]_{t+\Delta t}$ is the internal force vector at time $t + \Delta t$ and is given as

$$[[N(\delta)]\{\delta\}]_{t+\Delta t} = ([[K] + (1/2)[N_1(\delta)] + (1/3)[N_2(\delta)]]\{\delta\})_{t+\Delta t} \tag{12}$$

In developing equations for the implicit integration, the internal forces $[N(\delta)]\{\delta\}$ at the time $t + \Delta t$ are written in terms of the internal forces at time t, by using the tangent stiffness approach, as

$$[[N(\delta)]\{\delta\}]_{t+\Delta t} = [[N(\delta)]\{\delta\}]_t + [K_T(\delta)]_t\{\Delta\delta\}, \tag{13}$$

where $[K_T(\delta)]_t = [[K] + [N_1] + [N_2]]$ is the tangent stiffness matrix and $\{\Delta\delta\} = \{\delta\}_{t+\Delta t} - \{\delta\}_t$. Substituting Eq. (13) into Eq. (11), one obtains the governing equation at $t + \Delta t$ as

$$[M]\{\ddot{\delta}\}_{t+\Delta t} + [K_T(\delta)]_t\{\Delta\delta\} = \{F\}_{t+\Delta t} - \{[N(\delta)]\{\delta\}\}_t, \tag{14}$$

To improve the solution accuracy and to avoid numerical instabilities, it is necessary to employ iteration within each time, thus maintaining the equilibrium.

The non-linear equations obtained by the above procedure are solved by the Newmark's numerical integration method. Equilibrium is achieved for each time step through modified Newton-Raphson iteration until the convergence criteria (Bergan and Clough 1972) are satisfied within the specific tolerance limit of less than one percent.

3. Dynamic buckling criterion

Criteria for the static buckling of axisymmetric shallow spherical shell are well defined whereas it is not so for the dynamic case. It requires the evaluation of the transient response of the shell for different load amplitudes. However, the dynamic buckling criterion suggested by Budiansky and Roth (1962) is generally accepted because the results obtained by various investigators by different numerical techniques using the criterion are in reasonable agreement with each other. This criterion is based on the plots of the peak nondimensional average displacement in the time history of the structure with respect to the amplitude of the pressure load. The average displacement Δ is defined as

$$\Delta = \frac{\int_0^a r w \, dr}{\int_0^a r Z \, dr}$$

The numerator is the volume generated by the shell deformation and the denominator corresponds to the original volume under the spherical cap. Z is the height of a point on the middle-surface of the shell. There is a load range where a sharp jump in peak average displacement occurs for a small change in load magnitude. The inflection point of the load-deflection curve is considered as the dynamic buckling load.

For asymmetric dynamic buckling analysis of spherical shells, there is no well-understood and generally accepted criterion available so far. Furthermore, the available numerical results are very few to obtain a reasonable conclusion on the criterion, compared to those of axisymmetric case. The criterion adopted by Fulton and Barton (1971) and Akkas (1976) is somewhat similar to that of axisymmetric case. It is based on the peak values of the nondimensional average asymmetric component of the displacement in the time history of the structure against the amplitude of the pressure load plots. However, there is no occurrence of a sudden jump in peak average displacement associated with asymmetric part of the deformation over a load range. Hence, the load corresponding to the inflection point on the load-deflection curve beyond which the asymmetric part of the displacement response reveals significant growth rate is considered as dynamic buckling load.

4. Element description

The element employed here is a three-noded C^0 continuous shear flexible laminated axi-symmetric element with thirty-three degrees of freedom (u_o^o, $u_o^{c_1}$, $u_o^{s_1}$, $u_o^{c_2}$, $u_o^{s_2}$, $u_o^{c_3}$, $u_o^{s_3}$, $u_o^{c_4}$, $u_o^{s_4}$, v_o^o, $v_o^{c_1}$,

$v_o^{s_1}, v_o^{c_2}, v_o^{s_2}, v_o^{c_3}, v_o^{s_3}, v_o^{c_4}, v_o^{s_4}, w^o, w^{c_1}, w^{s_1}, w^{c_2}, w^{s_2}, \beta_s^o, \beta_s^{c_1}, \beta_s^{s_1}, \beta_s^{c_2}, \beta_s^{s_2}, \beta_\theta^o, \beta_\theta^{c_1}, \beta_\theta^{s_1}, \beta_\theta^{c_2}, \beta_\theta^{s_2}$) per node as per the kinematics given in Eq. (2). The quadratic interpolation functions defined based on the three-nodes of the element are used for the interpolation of the field variables in the meridional direction.

If the interpolation functions for three-noded element are used directly to interpolate the five field variables u_o, v_o, w, β_s and β_θ in deriving the transverse shear and membrane strains, the element will lock and show oscillations in the shear and membrane stresses. Field consistency requires that the membrane and transverse shear strains must be interpolated in a consistent manner. Thus, β_s term in the expression for $\{\varepsilon_s\}$ given in Eq. (4) has to be consistent with field function $\partial w/\partial s$ as shown in the works of Balakrishna and Sarma (1989), Ganapathi *et al.* (1994), and Prathap and Ramesh Babu (1986). Similarly the w and (u_o, v_o) terms in the expression of $\{\varepsilon_p^L\}$ (first and third strain components) have to be consistent with the field functions $\partial u_o/\partial s$ and $\partial v_o/\partial s$, respectively. This is achieved by using the field redistributed substitute shape functions to interpolate those specific terms that must be consistent as described by Prathap and Ramesh Babu (1986). The element derived in this fashion behaves very well for both thick and thin situations, and permits the greater flexibility in the choice of integration order for the energy terms. It has good convergence and has no spurious rigid modes. For the sake of brevity, the development and the performance of the element are omitted, as they are available in the literature (Balakrishna and Sarma 1989, Ganapathi *et al.* 1994, Prathap and Ramesh Babu 1986).

5. Results and discussion

The study here deals with asymmetric dynamic buckling behavior of clamped laminated angle-ply composite spherical caps. Since the finite element used here is based on the field consistency approach, an exact integration is employed to evaluate all the strain energy terms. The shear correction factor (k^2) which is required in a first-order theory to account for the variation of transverse shear stresses, is taken as 5/6. For the present analysis, based on progressive mesh refinement, 15 elements idealization is found to be adequate in modeling the spherical caps. The initial conditions for the nonlinear asymmetric dynamic response analysis are considered as non-zero values for displacements and zero values for the velocities. The initial displacement vectors are assumed to be proportional to the normalized linear flexural asymmetric mode vectors and then scaled up by multiplying the mode vectors with a very small value for the perturbation of asymmetric mode of the spherical shell. From the dynamic response curves, the load amplitudes and the corresponding maximum average displacements are obtained for applying the buckling criterion. The constants α and β (controlling parameters for stability and accuracy of the solution) in the Newmark's integration are taken as 0.5 and 0.25, which correspond to the unconditionally stable scheme in linear analysis. Since there is no estimate of the time step for the non-linear dynamic analysis available in the literature, the critical time step of a conditionally stable finite difference schemes (Leech 1965, Tsui and Tong 1971) is introduced as a guide and a convergence study was conducted to select a time step which yields a stable and accurate solution.

The material properties assumed in the present analysis are

$$E_L/E_T = 25.0,\ G_{LT}/E_T = 0.5,\ G_{TT}/E_T = 0.2,\ \nu_{LT} = 0.25,\ E_T = 1 \text{ GPa},\ \rho = 1600 \text{ Kg/m}^3$$

where E, G and ν are Young's modulus, shear modulus and Poisson's ratio. Subscripts L and T are the longitudinal and transverse directions respectively with respect to the fibres. All the layers are of equal thickness. The ply-angles are measured with respect to the meridional axis.

The clamped boundary conditions at the base of the spherical cap are taken as

$$u_o^o = u_o^{c_1} = u_o^{s_1} = u_o^{c_2} = u_o^{s_2} = u_o^{c_3} = u_o^{s_3} = u_o^{c_4} = u_o^{s_4} = v_o^o = v_o^{c_1} = v_o^{s_1} = v_o^{c_2} = v_o^{s_2} = v_o^{c_3} = v_o^{s_3} = v_o^{c_4} = v_o^{s_4} = 0$$

$$w^o = w^{c_1} = w^{s_1} = w^{c_2} = w^{s_2} = \beta_s^o = \beta_s^{c_1} = \beta_s^{s_1} = \beta_s^{c_2} = \beta_s^{s_2} = \beta_\theta^o = \beta_\theta^{c_1} = \beta_\theta^{s_1} = \beta_\theta^{c_2} = \beta_\theta^{s_2} = 0\,. \qquad (20)$$

The results of non-dimensional dynamic pressure, P_{cr}, are presented for isotropic, cross- and angle-ply shells for different values of the geometrical parameter λ. P_{cr} and λ are given by

$$P_{cr} = \frac{1}{8}[3(1-\nu_{LT}\nu_{TL})]^{1/2}\left(\frac{h}{H}\right)^2 \frac{qa^4}{E_L h^4}$$

$$\lambda = 2[3(1-\nu_{LT}\nu_{TL})^{1/4}]\left(\frac{H}{h}\right)^{1/2}$$

Here, H, a are the central shell rise and base radius, respectively. For the chosen shell parameter and lamination scheme, the dynamic buckling study is conducted for step loading of infinite duration. The length of response calculation time $\tau\left[= \left(\frac{D_{11}}{\rho h a^4}\right)^{1/2} t\right]$ in the present study is varied between 1 and 2 with the criterion that in the neighborhood of the buckling, τ is large enough to allow deflection-time curves to fully develop. The time step selected, based on the convergence study, is $\delta\tau$ = 0. 002. The values selected for τ & $\delta\tau$ are of same order as considered in the work of Ball and Burt (1973), Chao and Lin (1990) and Kao and Perrone (1978).

Before proceeding for the analysis of nonlinear asymmetric dynamic buckling characteristics of laminated angle-ply cases, the present formulation is tested considering the problems of axisymmetric/asymmetric buckling of isotropic spherical shells for which the solutions are available

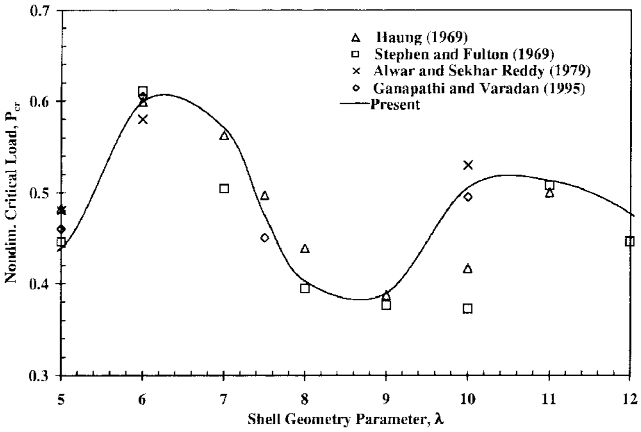

Fig. 1 Comparision of axisymmetric nondimensional critical load for isotropic spherical caps

Table 1 Comparison of asymmetric nondimensional critical load for isotropic spherical cap

λ	Circum. Wave No., n = 2			3		5	
	Present	Akkas (1976)	Ball and Burt (1973)	Present	Akkas (1976)	Present	Ball and Burt (1973)
6	0.459	0.42	0.52	0.534	0.52	1.075	-
7	0.443	0.42	-	0.431	0.438	0.731	-
12	0.579	-	-	0.51	-	0.49	0.51

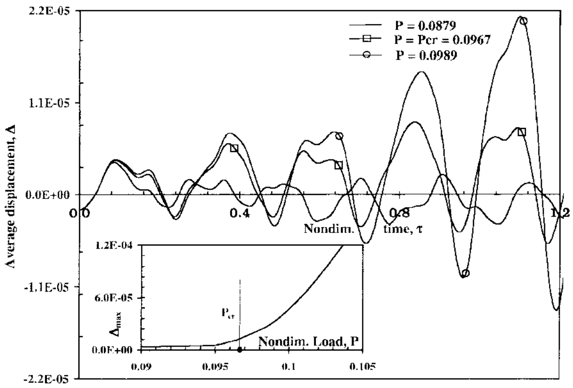

Fig. 2 Average displacement versus nondimensional time for eight-layered angle-ply $(15^0/-15^0)_4$ spherical cap (λ = 7.16, Asymmetric case n = 3)

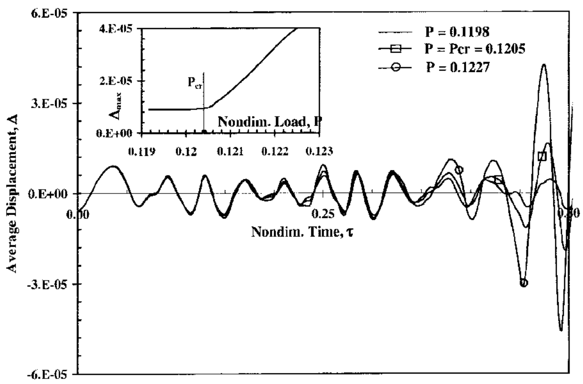

Fig. 3 Average displacement versus nondimensional time for eight-layered angle-ply $(60^0/-60^0)_4$ spherical cap (λ = 7.16, Asymmetric case n = 3)

in the literature. The results obtained for axisymmetric and asymmetric cases are shown in Fig. 1 and Table 1, respectively. It is seen from these results that the critical dynamic pressures calculated for various geometrical parameter values are, in general, found to be in good agreement with those of available analytical/numerical results (Haung 1969, Stephens and Fulton 1969, Alwar and Sekhar Reddy 1979, Ganapathi and Varadan 1995, Ball and Burt 1973, Akkas 1976).

Next, for the laminated eight-layered angle-ply $(\phi^0/-\phi^0)_4$ spherical caps considering different values for the geometrical parameter λ, the dynamic buckling loads are evaluated based on asymmetric nonlinear dynamic response of shells subjected to externally applied pressure. This study is carried out perturbing the different asymmetric modes (circumferential wave number, n). Figs. 2 and 3 exhibit the dynamic response pattern of average asymmetric displacement components of two angle-ply shells [λ = 7.16, n = 3; $(15^0/-15^0)_4$ & $(60^0/-60^0)_4$] with time and, in turn, the variation of maximum average displacement with applied loads. The critical pressure is taken as the load in the plot of maximum average displacement with applied load that yields significant growth rate of asymmetric response. The influence of various asymmetric modes on the critical dynamic load is investigated and highlighted in Figs. 4-7 for various angle-ply laminates. It is observed from Fig. 4 that, for low angle-ply case (15^0) considered here, the lowest critical load of very shallow shell corresponds to axisymmetric mode. But, the asymmetric mode of buckling behavior dominates the stability of deep spherical caps. However, the differences in the critical loads predicted among

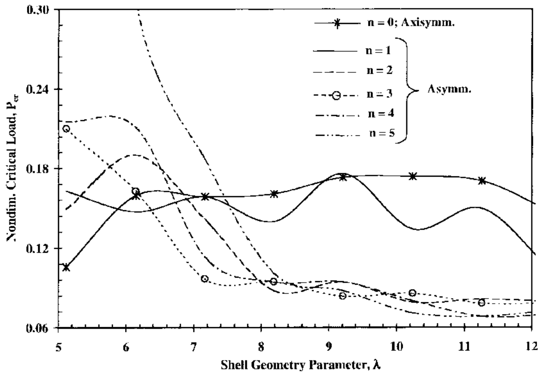

Fig. 4 Nondimensional critical load versus shell geometry parameter for eight-layered angle-ply $(15^0/–15^0)_4$ spherical caps

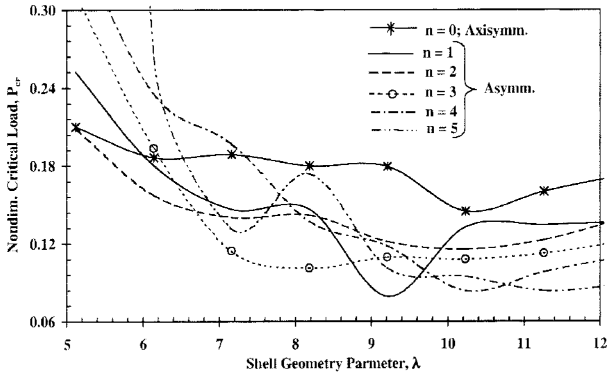

Fig. 5 Nondimensional critical load versus shell geometry parameter for eight-layered angle-ply $(30^0/–30^0)_4$ spherical caps

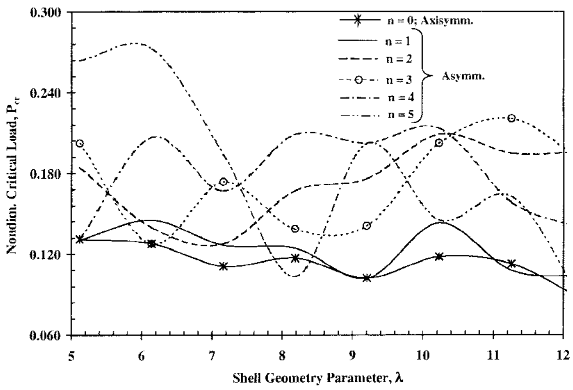

Fig. 6 Nondimensional critical load versus shell geometry parameter for eight-layered angle-ply $(45^0/–45^0)_4$ spherical caps

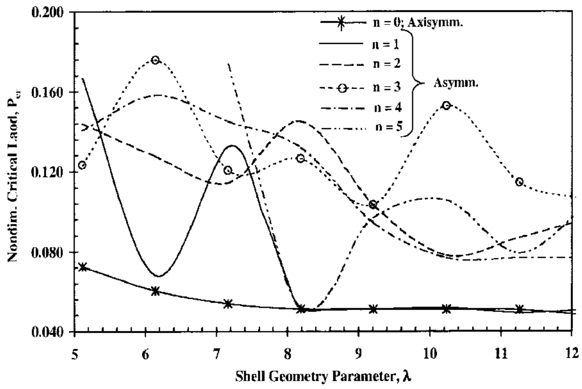

Fig. 7 Nondimensional critical load versus shell geometry parameter for eight-layered angle-ply $(60^0/–60^0)_4$ spherical caps

different asymmetric mode cases decrease with increase in the geometric parameter value. For 30^0 angle-ply case, it is viewed from Fig. 5 that, irrespective of geometric shell parameter values, the asymmetric mode produces the lowest critical load. It is also noticed from Fig. 5 that the value of critical pressure significantly depends on the type of asymmetric disturbance. Furthermore, it can be noted that the shallow shells buckle at less load corresponding to lower circumferential mode numbers whereas the deep shells exhibit loss of stability easily at higher mode numbers. For spherical caps with higher ply-angle cases (45^0 & 60^0), it is inferred from Figs. 6 and 7 that the failure mode pertaining to the least critical pressure corresponds to axisymmetric one. It is also revealed from these figures that the variation of critical loads related to different asymmetric modes of perturbation is significant, irrespective of the type of spherical caps. For the layered angle-ply shells considered here, it is brought out from Figs. 4-7 that the shell geometries with high ply-angle

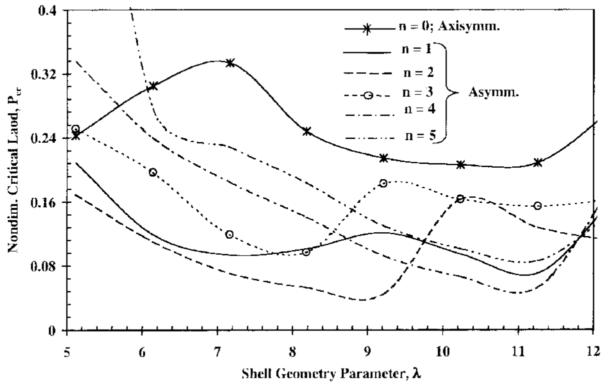

Fig. 8 Nondimensional critical load versus shell geometry parameter for eight-layered cross-ply $(0^0/90^0)_4$ spherical caps

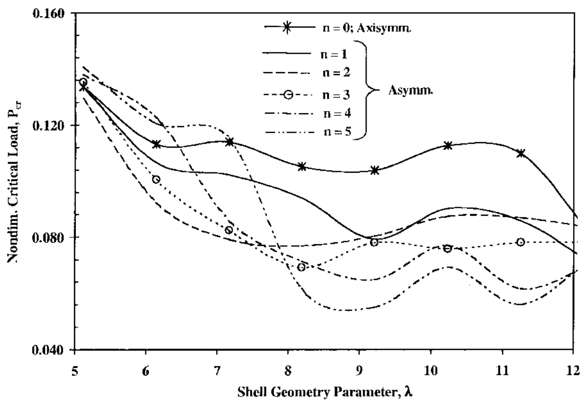

Fig. 9 Nondimensional critical load versus shell geometry parameter for two-layered angle-ply $(30^0/–30^0)$ spherical caps

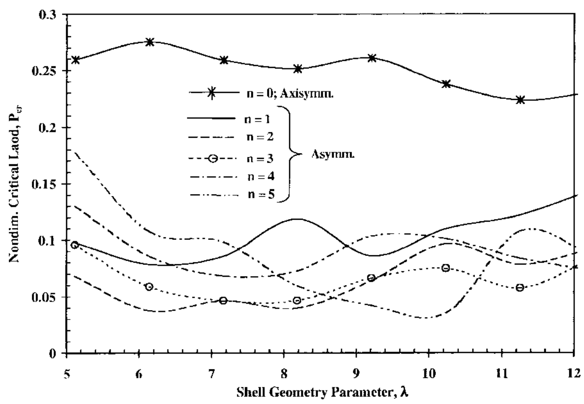

Fig. 10 Nondimensional critical load versus shell geometry parameter for two-layered cross-ply $(0^0/90^0)$ spherical caps

are prone to buckle at lowest critical loads. The difference in the buckling characteristics of higher ply-angle cases compared to lower ply-angles may be attributed to the changes in relative contributions of the membrane, bending and membrane-bending coupling energy terms due to the stiffening of the shell in the circumferential direction at higher ply-angles. It is worthwhile to mention here that the shallow shells with certain ply-angles may not fail under asymmetric modes of large circumferential wave number, as the shell response characteristics do not reveal any significant growth with time. Similar study is carried out for cross-ply case. The dynamic buckling values obtained for eight-layered cross-ply shells are presented in Fig. 8. It can be seen from these investigations that the variation of buckling load of cross-ply shells is qualitatively similar to those of low angle-ply laminates.

The effect of bending-stretching coupling due to lamination scheme is also examined and brought out in Figs. 9 and 10 for angle- and cross-ply spherical caps. The predicted asymmetric dynamic critical loads for two-layered case are, in general, less and the buckling behavior is qualitative similar to those of eight-layered shells. In general, it can be opined that the lowest critical dynamic buckling load of a given spherical cap significantly depends on its geometrical parameter value, ply-angle and type of mode of excitation.

6. Conclusions

Asymmetric dynamic buckling of clamped laminated angle-ply composite spherical caps subjected to externally applied pressure has been investigated through nonlinear transient dynamic response analysis. A three-noded axisymmetric curved shell element based on field consistency principle has been employed for this purpose. Numerical results obtained here for an isotropic case are found to be fairly in good agreement with the previous findings. From the detailed study, the following observations can be made:

1. For low angle-ply case, the difference in the buckling values predicted among various asymmetric modes decreases rapidly with increase in the value of the shell parameter and the lowest critical pressure corresponds to either axisymmetric or asymmetric modes depending on the shell parameter value.
2. With increase in ply-angle, the influence of asymmetric mode on the dynamic buckling load is significant for all the shell parameter but the axisymmetric buckling mode yields the lowest critical values.
3. Asymmetric buckling at large circumferential wave numbers is not observed for shallow shells with certain ply-angles.
4. The shells with higher lamination angles are prone to buckle at lower critical loads.
5. The bending-stretching coupling due to lay-up, in general, reduces the buckling loads and the asymmetric buckling mode dominates the failure of various shell geometric parameters considered here.
6. Shallow shells exhibit the onset of asymmetric buckling at lower circumferential mode numbers whereas the occurrence of instability of deep shells associates with the higher mode number.
7. For a given spherical cap, the critical dynamic buckling load depends significantly on the initial conditions, geometrical parameter and ply-angle.

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