*Structural Engineering and Mechanics, Vol. 15, No. 2 (2003) 239-248*

DOI: http://dx.doi.org/10.12989/sem.2003.15.2.239

# Dynamic response analysis of closed loop control system for intelligent truss structures based on probability

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*(Received June 17, 2002, Accepted January 6, 2003)*

**Abstract.** The dynamic response analysis of closed loop control system based on probability for the intelligent truss structures with random parameters is presented. The expressions of numerical characteristics of structural dynamic response of closed loop control system are derived by means of the mode superposition method, in which the randomness of physical parameters of structural materials, geometric dimensions of active bars and passive bars, applied loads and control forces are considered simultaneously. The influences of the randomness of them on structural dynamic response are inspected by several engineering examples and some significant conclusions are obtained.

**Key words:** piezoelectric intelligent truss structures; closed loop control; random forces; random parameters; dynamic response analysis.

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## 1. Introduction

Dynamic response analysis of closed loop control system for intelligent structures is an important segment in both the process of intelligent structural design and its shape and vibration control. The results of the dynamic response analysis are the important base of the determination of the active bars' optimal location. The location of active bars in intelligent truss structures affects the validity of its shape and vibration control directly. In recently years, some results of intelligent truss structure have been published. Such as, Chen and Bruno *et al.* (1991) studied the problem of the optimal placement of active and passive members in complex truss structures and the maximization of the cumulative energy dissipated over finite time intervals as the measure of optimality. Lu and Utku (1993) studied the vibration suppression for large scale adaptive truss structures by using of direct output feedback control. Won and Sulla *et al.* (1994), in which the piezoelectric films and strain gauges are used as the control sensors and for active control, linear quadratic Gaussian (LQG) and second-order controllers are designed and compared with direct-rate feedback to validate the modeling and control schemes. Lammering and Jia *et al.* (1994) utilized the optimal placement strategies developed in conjunction with the independent modal space control method (IMSC) to find the optimal placements of piezoelectric actuators in adaptive truss structures. In this paper the

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optimization algorithm calculating the optimal actuator placements is utilized with the electric potentials to minimize the control effort. Li and Zhang (1997) investigated the principle and effectiveness of active vibration control of adaptive truss structures using their active members and optimal placements of the active members. The optimization criterion based on modal dissipation energy factor and modal strain energy factor are developed in this paper to find the optimal placement of the active member in adaptive truss structures. Peng *et al.* (1998) studied the active position control and vibration control of composite beams with distributed piezoelectric sensors and actuators with a finite element modal based on third order laminate theory. Tang *et al.* (2000) developed a robust controller to compensate for the nonlinearities as uncertainties in the integrated piezoelectric intelligent system and demonstrated the effectiveness of their approach by numerical and experimental analyses. So far, however, most of the modeling on intelligent structural dynamic analysis basically belongs to the determinate models, that is, all structural parameters are regarded as determinate ones. Apparently, this kind of model can't reflect the influence of the randomness of intelligent structure on the structural dynamic response.

Structural dynamic analysis includes structural dynamic character and response analysis. Some research works about the problem of structure with random parameters has been published. Such as, Wu & Yang (1996) introduced the randomness of structural parameter as the perturbation quantity nearby the mean value of random variables, and obtained the perturbing expression of structural dynamic characteristic by using the variation principle of Rayleigh's quotient. Lin & Yi (2001) studied the dynamic characteristics of structure with random parameter, in which the random factors in structures are described with small parameter, and the problem is solved by means of FEM or matrix perturbation method. Chen & Che (2002) studied the structural dynamic characteristic, in which the randomness of both physical parameters (elastic module and mass density) of structural materials and geometric dimension of bars were considered.

In this paper, the intelligent truss structures are taken as analyzing objects and their structural dynamic modeling and analyzing based on probability are studied. The computational expressions of numerical characteristics of structural dynamic response of closed loop control system are derived by means of the mode superposition method, in which the randomness of physical parameters (elastic module and mass density) of structural materials, geometric dimensions (length and cross-section area) of active bars and passive bars, applied loads and control forces are considered simultaneously. Through engineering examples, the influences of them on structural dynamic response are inspected and some significant conclusions are obtained.

## 2. Structural dynamic response analysis of closed loop control system

Suppose that there are *ne* elements in the analyzed intelligent truss structure. In the structure, any element can be taken as passive bar or active bar. A piezoelectric bar is utilized as active bar. In order to utilize the unite form to express the structural stiffness and mass matrix, a kind of mixed element have been constructed. A Boolean algebra value named as $\theta$ is introduced in the mixed element, when $\theta = 0$, the mixed element is active element bar and when $\theta = 1$, the mixed element is passive element bar. In the following, expressions of the stiff matrix $[K]$ and mass matrix $[M]$ of intelligent truss structure in global coordinate will be developed by utilizing this kind of mixed element.

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left\{\left[\theta\frac{E_m^{(e)}A_m^{(e)}}{l_m^{(e)}} + (1-\theta)\frac{c_{33}^{(e)} + (e_{33}^{(e)})^2/\varepsilon_{33}^{(e)}}{l_P^{(e)}}A_P^{(e)}\right][G]\right\} \tag{1}$$

$$[M] = \sum_{e=1}^{ne}[M^{(e)}] = \sum_{e=1}^{ne}\left\{\frac{1}{2}(\theta\rho_m^{(e)}A_m^{(e)}l_m^{(e)} + (1-\theta)\rho_P^{(e)}A_P^{(e)}l_P^{(e)})[I]\right\} \tag{2}$$

where $[K^{(e)}]$ is the $e$th element's stiffness matrix, $[M^{(e)}]$ is the $e$th element's mass matrix. $[I]$ is a 6 order identity matrix. $\rho_m^{(e)}, A_m^{(e)}$ and $l_m^{(e)}$ are the $e$th passive bars' mass density, cross-section area and length, respectively. $\rho_P^{(e)}, A_P^{(e)}$ and $l_P^{(e)}$ are the $e$th active bars' mass density, cross-section area and length, respectively. $E_m^{(e)}$ is the $e$th passive bars' elastic module. $c_{33}^{(e)}, e_{33}^{(e)}$ and $\varepsilon_{33}^{(e)}$ are elastic module, piezoelectric force/electrical constant and dielectric constant respectively. $[G]$ is a $6\times 6$ matrix, where $g_{11} = g_{44} = 1$, $g_{14} = g_{41} = -1$, other elements of $[G]$ are all equal to 0.

Here introduce another expression as follow:

$$E_P^{(e)} = c_{33}^{(e)} + (e_{33}^{(e)})^2/\varepsilon_{33}^{(e)} \tag{3}$$

$E_P^{(e)}$ just is the generalized elastic module of piezoelectric active bars while considering the mechanic-electronic coupling effect.

Substituting Eq. (3) into Eq. (1), then

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left\{\left[\theta\frac{E_m^{(e)}A_m}{l_m} + (1-\theta)\frac{E_P^{(e)}A_P}{l_P}\right][G]\right\} \tag{4}$$

Following the finite element formulation, the equation of motion for an intelligent structure is given by

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} + [B_1]\{F_P(t)\} \tag{5}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $\{x(t)\}$, $\{\dot{x}(t)\}$ and $\{\ddot{x}(t)\}$ are structural displacement vector, velocity vector and acceleration vector, respectively. $\{F(t)\}$ is the load force vector. The $\{F_P(t)\}$ term represents the initial strain effect of piezoelectric active bar. Since $\{F_P(t)\}$ appears on the right-hand side of the equation, it is referred to as the apparent active bar force, that is the control force vector. The $[B_1]$ matrix consists of the active bar's direction cosines.

By means of the mode superposition method, the structural displacement response can be expressed as

$$\{x(t)\} = \sum_{i=1}^{n}\{\phi\}_i z_i(t) \tag{6}$$

where the displacement response of $j$th degree of freedom is

$$x_j(t) = \sum_{i=1}^{n}\phi_{ji}z_i(t) \quad (j = 1, 2, ..., n) \tag{7}$$

$$z_i(t) = \frac{1}{\overline{\omega}_i}\int_0^t(R_i(\tau) + H_i(\tau))\exp[-\xi_i\omega_i(t-\tau)]\sin\overline{\omega}_i(t-\tau)d\tau \quad (i = 1, 2, ..., n) \tag{8}$$

$$R_i(t) = \{\phi\}_i^T\{F(t)\},\ \ H_i(t) = \{\phi\}_i^T([B_1]\{F_P(t)\}) \quad (i = 1, 2, ..., n) \tag{9}$$

where $R_i(t)$, $H_i(t)$ and $z_i(t)$ are load, control force and displacement response of $i$th degree of freedom in principal coordinates, respectively. $\omega_i$, $\{\phi\}_i$ and $\xi_i$ are $i$th order inherence frequency, mode shape and mode damping of structure, respectively. $\overline{\omega}_i = \omega_i(1-\xi_i^2)^{1/2}$. Since $\xi_i \ll 1$, $\overline{\omega}_i = \omega_i$ can be obtained.

After the structural displacement response of closed loop system obtained, the stress response of the $e$th element can be expressed as

$$\{\sigma(t)^{(e)}\} = E \cdot [B] \cdot \{x(t)^{(e)}\} \quad (e = 1, 2, ..., n_e) \tag{10}$$

where $\{x(t)^{(e)}\}$ is the displacement response of the nodal point of the $e$th element, $\{\sigma(t)^{(e)}\}$ is the stress response of the $e$th element, $[B]$ is the geometric matrix of the $e$th element.

## 3. Probabilistic structural dynamic response analysis of closed loop system

Here, the randomness of $\rho_m^{(e)}, \rho_P^{(e)}, A_m^{(e)}, A_P^{(e)}, l_m^{(e)}, l_P^{(e)}, E_m^{(e)}, c_{33}^{(e)}$ and $\{F(t)\}$ are considered simultaneously. From the Eq. (3), it can be obtained easily that $E_P^{(e)}$ is a random variable. In the closed loop system, because the production and response process of $\{F_P(t)\}$ determined by $\{F(t)\}$, $\{F_P(t)\}$ is random variable too. And these two variables are full positive correlation. The engineering background of this case is the stochastic loads and control forces act on the intelligent structure with random parameters. This case is the most general one of random model of intelligent structural dynamic response analysis.

Suppose that the length and cross-section areas of bars are two kinds of random variables and the dispersivity of each kind of random variables of each bar is equal respectively. Then the length and cross-section area of the $e$th element can be written respectively as: $l^{(e)} = \lambda^{(e)} \cdot l$ ; $A^{(e)} = \eta^{(e)} \cdot A (e = 1, \ldots, ne)$. Where $\lambda^{(e)}$ and $\eta^{(e)}$ are the determinate quantities that denote the nominal length and nominal cross section area of $e$th bar, respectively. $l$ is the random variable factor of all bars' length, its mean value is 1.0 and variance is $v_l^2$, $A$ is the random variable factor of all bars' cross-section area, its mean value is 1.0 and variance is $v_A^2$. If the physical parameters of each active element are equal, the physical parameters of each passive element are also equal, $\rho_m = \rho_m^{(e)}$, $\rho_P = \rho_P^{(e)}$, $E_m = E_m^{(e)}, E_P = E_P^{(e)}$ can be obtained easily. Therefore, the matrices $[K]$ and $[M]$ can be expressed respectively as

$$[K] = \sum_{e=1}^{ne} [K^{(e)}] = \sum_{e=1}^{ne} \left( \theta \frac{E_m^{(e)} A}{l} [K_m^{(e)}]^{\#} + (1-\theta) \frac{E_P^{(e)} A}{l} [K_P^{(e)}]^{\#} \right) \tag{11}$$

where $[K_m^{(e)}]^{\#}$ and $[K_P^{(e)}]^{\#}$ are the determinate part in stiff matrices $[K_m^{(e)}]$ and $[K_P^{(e)}]$ respectively, just taking the parameters as: $l_m^{(e)} = \lambda^{(e)}, A^{(e)} = \eta^{(e)}, E_m^{(e)} = 1$ and $E_P^{(e)} = 1$.

$$[M] = \sum_{e=1}^{ne} [M^{(e)}] = \sum_{e=1}^{ne} (\theta \rho_m^{(e)} \cdot A \cdot l [M_m^{(e)}]^{\#} + (1-\theta) \rho_P^{(e)} \cdot A \cdot l [M_P^{(e)}]^{\#}) \tag{12}$$

Where $[M_m^{(e)}]^{\#}$ and $[M_P^{(e)}]^{\#}$ are the determinate part in mass matrices $[M_m^{(e)}]$ and $[M_P^{(e)}]$ respectively, just taking the parameters as: $l_m^{(e)} = \lambda^{(e)}, A^{(e)} = \eta^{(e)}, \rho_m^{(e)} = 1$ and $\rho_P^{(e)} = 1$.

If the physical parameters' variation of active elements and passive elements are equal each other, $\rho_P = k_1 \rho_m = k_1 \rho$ and $E_P = k_2 E_m = k_2 E$ ($k_1$ and $k_2$ are constant ratio factors) can be easily obtained. Therefore, the matrices $[K]$ and $[M]$ can be expressed respectively as

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left(\theta\frac{EA}{l}[K_m^{(e)}]^{\#} + (1-\theta)k_2\frac{EA}{l}[K_m^{(e)}]^{\#}\right)$$
$$= \frac{EA}{l}[\theta + (1-\theta)k_2][K_m]^{\#} \tag{13}$$

$$[M] = \sum_{e=1}^{ne}[M^{(e)}] = \sum_{e=1}^{ne}(\theta\rho\cdot A\cdot l\cdot[M_m^{(e)}]^{\#} + (1-\theta)k_1\rho\cdot A\cdot l\cdot[M_m^{(e)}]^{\#})$$
$$= \rho\cdot A\cdot l\cdot[\theta + (1-\theta)k_1][M_m]^{\#} \tag{14}$$

The randomness of physical parameters and geometric dimension will lead the structural dynamic characteristics (inherence frequency $\omega$ and mode shape $\{\phi\}$) having randomness. Therefore, the randomness of the structural dynamic characteristics, loads and control forces will lead the structural dynamic response (dynamic displacement and dynamic stress) having randomness too. The statistical descriptions of random variables are represented by utilizing its numerical characteristic. In the following, the expressions of numerical characteristics of dynamic response random variables will be derived.

### 3.1 Numerical characteristics of dynamic displacement response of closed loop system

From Eq. (9), the mean value and variance of $R_i(t)$ and the mean value and variance of $H_i(t)$ in principal coordinates can be deduced by means of the algebra synthesis method.

$$\mu_{R_i(t)} = \{\mu_\phi\}_i^T\{\mu_{F(t)}\} \tag{15}$$

$$\sigma_{Ri(t)}^2 = \{\mu_\phi^2\}_i^T\{\mu_{F(t)}^2\}(v_{F(t)}^2 + v_\omega^2 + v_{F(t)}^2\cdot v_\omega^2) \tag{16}$$

$$\mu_{H_i(t)} = \{\mu_\phi\}_i^T\{\mu_{F_P(t)}\} \tag{17}$$

$$\sigma_{Hi(t)}^2 = \{\mu_\phi^2\}_i^T\{\mu_{F_P(t)}^2\}(v_{F_P(t)}^2 + v_\omega^2 + v_{F_P(t)}^2\cdot v_\omega^2) \tag{18}$$

where $\mu_{R_i}(t)$ and $\sigma_{Ri(t)}^2$ are the mean value and variance of $R_i(t)$, respectively; $\mu_{H_i(t)}$ and $\sigma_{Hi(t)}^2$ are the mean value and variance of $H_i(t)$, respectively; $v_{\omega i}$, $v_{F(t)}$ and $v_{F_P(t)}$ are the variation coefficient of $\omega_i$, $\{F(t)\}$ and $\{F_P(t)\}$, respectively.

From Eq. (8), the mean value $\mu_{Z_i(t)}$ and mean variance $\sigma_{Z_i(t)}$ of the displacement of the ith degree of freedom in principal coordinate can be deduced by means of the random variable's functional moment method.

$$\mu_{z_i(t)} = \frac{1}{\mu_{\omega_i}}\int_0^t(\mu_{R_i(\tau)} + \mu_{H_i(\tau)})\exp(-\xi\mu_{\omega_i}(t-\tau))\sin\mu_{\omega_i}(t-\tau)d\tau \tag{19}$$

$$\sigma_{z_i(t)} = \left\{v_\omega^2\left[-\frac{1}{\mu_{\omega_i}}\int_0^t(\mu_{R_i(\tau)} + \mu_{H_i(\tau)})\exp(-\xi\mu_{\omega_i}(t-\tau))\sin\mu_{\omega_i}(t-\tau)d\tau\right.\right.$$
$$-\xi\int_0^t(\mu_{R_i(\tau)} + \mu_{H_i(\tau)})(t-\tau)\exp(-\xi\mu_{\omega_i}(t-\tau))\sin\mu_{\omega_i}(t-\tau)d\tau$$
$$+\int_0^t(\mu_{R_i(\tau)} + \mu_{H_i(\tau)})(t-\tau)\exp(-\xi\mu_{\omega_i}(t-\tau))\cos\mu_{\omega_i}(t-\tau)d\tau]^2$$

$$+\frac{1}{\mu_{\omega_i}^2}\int_0^t\{\phi^2\}_i^T[\{\mu_{F(t)}^2\}(\nu_{F(t)}^2+\nu_{\omega_i}^2+\nu_{F(t)}^2\cdot\nu_{\omega_i}^2)$$
$$+\{\mu_{F_P(t)}^2\}(\nu_{F_P(t)}^2+\nu_{\omega_i}^2+\nu_{F_P(t)}^2\cdot\nu_{\omega_i}^2)+2\cdot\{\mu_{F(t)}\}\cdot\{\mu_{F_P(t)}\}\cdot$$
$$((\nu_{F(t)}^2+\nu_{\omega_i}^2+\nu_{F(t)}^2\cdot\nu_{\omega_i}^2)\cdot(\nu_{F_P(t)}^2+\nu_{\omega_i}^2+\nu_{F_P(t)}^2\cdot\nu_{\omega_i}^2))^{1/2}]\cdot$$
$$\exp(-2\xi\omega_i(t-\tau))\cdot\sin^2\mu\omega_i(t-\tau)d\tau\}^{1/2} \tag{20}$$

where the symbol $\mu$, $\sigma$ and $\nu$ denote the random variable's mean values, mean variances and variation coefficients, respectively. $\mu_{F(t)}, \nu_{F(t)}, \mu_{F_P(t)}$ and $\nu_{F_P(t)}$ are given by the statistical information of the loads and control forces. The $\{\mu_\phi\}_i$ term can be gained from the structural dynamic characteristic computation. $\mu_{\omega i}$, $\sigma_{\omega_i}^2$ and $\nu_{\omega i}$ can be obtained from the Rayleigh's quotient by means of the algebra synthesis method

$$\mu_{\omega_i}=\omega_i^{\#}\left(\frac{\mu_E}{\mu_\rho\mu_Z}\right)^{1/2}\{[1+\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2$$
$$-C_{E\rho}\cdot(\nu_A^2+\nu_E^2+\nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]^2$$
$$-\frac{1}{2}[2\nu_A^2+\nu_E^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_E^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2$$
$$-2C_{E\rho}\cdot(\nu_A^2+\nu_E^2+\nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]\}^{1/4} \tag{21}$$

$$\sigma_{\omega_i}^2=(\omega_i^{\#})^2\left(\frac{\mu_E}{\mu_\rho\mu_Z}\right)\{[1+\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2$$
$$-C_{E\rho}\cdot(\nu_A^2+\nu_E^2+\nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]$$
$$-\{[1+\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2$$
$$-C_{E\rho}\cdot(\nu_A^2+\nu_E^2+\nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]^2$$
$$-\frac{1}{2}[2\nu_A^2+\nu_E^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_E^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2$$
$$-2C_{E\rho}\cdot(\nu_A^2+\nu_E^2+\nu_A^2\nu_E^2)^{1/2}\cdot(\nu_A^2+\nu_Z^2+\nu_\rho^2+\nu_A^2\nu_Z^2+\nu_A^2\nu_\rho^2+\nu_Z^2\nu_\rho^2+\nu_A^2\nu_Z^2\nu_\rho^2)^{1/2}]\}^{1/2}\} \tag{22}$$

$$\nu_z=\nu_l^2=\frac{\sqrt{4\nu_l^2+2\nu_l^4}}{1+\nu_l^2} \tag{23}$$

$$\nu_{\omega_i}=\frac{\sigma_{\omega_i}}{\mu_{\omega_i}} \tag{24}$$

where $\nu_E$ and $\nu_\rho$ are variation coefficients of $E$ and $\rho$, respectively. $C_{E\rho}$ is the correlation coefficient of variables $E$ and $\rho$. $\omega_i^{\#}$ can be obtained from the structural dynamic characteristic computation.

From Eqs. (19) and (20), the variation coefficient of $z_i(t)$ can be obtained as

$$\nu_{z_i(t)}=\sigma_{z_i(t)}/\mu_{z_i(t)} \tag{25}$$

From Eq. (6), the mean value and mean variance of displacement response in nature coordinate can be deduced by means of the algebra synthesis method.

$$\{\mu_{x(t)}\} = \sum_{i=1}^{n}\{\mu_{\phi}\}_i \mu_{z_i}(t) \tag{26}$$

$$\{\sigma_{x(t)}\} = \left\{\sum_{i=1}^{n}\left(\{\mu_{\phi}^2\}_i\, \mu_{z_i(t)}^2(\nu_{z_i(t)}^2 + \nu_{\omega i}^2 + \nu_{\omega i}^2 \cdot \nu_{z_i(t)}^2)\right)\right\}^{1/2} \tag{27}$$

From above the formulas, it can be seen easily that the randomness of the structural displacement response are dependent not only on the structural physical parameters $E$, $\rho$ and the geometric dimension $\bar{L}, \bar{A}$ but also on the randomness of loads and control forces. In addition, they are determined by the values of the inherence frequency, mode shape, loads and control forces.

### *3.2 Numerical characteristics of dynamic stress response of closed loop system*

After the displacement response of the element's nodal were obtained, the mean value and mean variance of the $e$th element's stress response can be obtained by means of the algebra synthesis method from Eq. (10).

$$\{\mu_{\sigma(t)}^{(e)}\} = [B]\mu_E\{\mu_{x(t)}^{(e)}\} \quad (e = 1, 2, ..., n_e) \tag{28}$$

$$\{\mu_{\sigma(t)}^{(e)}\} = [B]\mu_E\{\mu_{x(t)}^{(e)}\}[\nu_{\{x(t)^{(e)}\}}^2 + \nu_E^2 + \nu_E^2 \nu_{\{x(t)^{(e)}\}}^2]^{1/2} \quad (e = 1, 2, ..., n_e) \tag{29}$$

## 4. Examples

In the following examples, for the intelligent structure's active bars, a constant output velocity feedback control law is selected. Active bar's and passive bar's materials and their parameters' value are given in Table 1.

### *Example 1 16-bar planar truss structure (Fig. 1)*

The elastic module $E$, mass density $\rho$, bars' length $\bar{L}$ and bars' cross-section area $\bar{A}$ are all random variables. A step load along with the positive direction of X-axis acts on the nodal 9 of the structure. The amplitude of the step load is a normal random variable. Its mean value is $\mu_{F(t)}$ = 7500($N$). The gain of the closed loop control system is $g$ = –100. Now, the 4th element and 16th

Table 1 Intelligent truss structure's physical parameters

| | Active bar(PZT-4) | Passive bar(steel) |
|---|---|---|
| Mean value of mass density $\rho$ | 7600 kg/m$^3$ | 7800 kg/m$^3$ |
| Mean value of elastic module $c_{33}$ | $8.807 * 10^{10}$ N/m$^2$ | $2.1 * 10^{11}$ N/m$^2$ |
| Piezoelectric force/electric constant $e_{33}$ | 18.62 C/m$^2$ | - |
| Dielectric impermeability constant $\varepsilon_{33}$ | $5.92 * 10^{-9}$ C/Vm | - |
| Cross section area A | $3.0 * 10^{-4}$ m$^2$ | $3.0 * 10^{-4}$ m$^2$ |

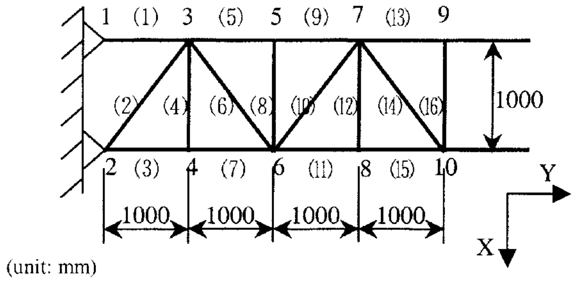


Fig. 1 16-bar planar intelligent truss structure

Table 2 The computational results of dynamic response of 16-bar planar structure

| Model \ Dynamic response | Maximal displacement response(mm) | | Maximal stress response(Mpa) | |
|---|---|---|---|---|
| | 4th element act as active bar | 16th element act as active bar | 4th element act as active bar | 16th element act as active bar |
| Determinate model | 5.811 | 8.797 | 60.334 | 91.427 |
| Random model (I) $\nu_E = \nu_\rho = \nu_l = \nu_A = 0.1$ $\nu_{F(t)} = \nu_{F_p(t)} = 0$ | $\mu_{z_{\max}(t)} = 5.811$ $\sigma_{z_{\max}(t)} = 0.203$ | $\mu_{z_{\max}(t)} = 8.797$ $\sigma_{z_{\max}(t)} = 0.307$ | $\mu_{\sigma_{\max}(t)} = 60.334$ $\sigma_{\sigma_{\max}(t)} = 7.240$ | $\mu_{\sigma_{\max}(t)} = 91.427$ $\sigma_{\sigma_{\max}(t)} = 10.971$ |
| Random model (II) $\nu_E = \nu_\rho = \nu_l = \nu_A = 0$ $\nu_{F(t)} = \nu_{F_p(t)} = 0.1$ | $\mu_{z_{\max}(t)} = 5.811$ $\sigma_{z_{\max}(t)} = 0.464$ | $\mu_{z_{\max}(t)} = 8.797$ $\sigma_{z_{\max}(t)} = 0.703$ | $\mu_{\sigma_{\max}(t)} = 60.334$ $\sigma_{\sigma_{\max}(t)} = 4.766$ | $\mu_{\sigma_{\max}(t)} = 91.427$ $\sigma_{\sigma_{\max}(t)} = 7.222$ |
| Random model (III) $\nu_E = \nu_\rho = \nu_l = \nu_A$ $= \nu_{F(t)} = \nu_{F_p(t)} = 0.1$ | $\mu_{z_{\max}(t)} = 5.811$ $\sigma_{z_{\max}(t)} = 0.490$ | $\mu_{z_{\max}(t)} = 8.797$ $\sigma_{z_{\max}(t)} = 0.742$ | $\mu_{\sigma_{\max}(t)} = 60.334$ $\sigma_{\sigma_{\max}(t)} = 9.774$ | $\mu_{\sigma_{\max}(t)} = 91.427$ $\sigma_{\sigma_{\max}(t)} = 14.811$ |

element are utilized as piezoelectric active bar, respectively. The corresponding computational results of the structural dynamic displacement response and dynamic stress response are given in Table 2. In this Table, the dynamic displacement response is that of the nodal 10, the dynamic stress response is that of the 1st element.

In order to compare, two kinds of models, the determinate model and random model, are adopted respectively in computational process. In the determinate model, the mean values of all random variables are regarded as determinate quantity, and their variation coefficients are taken as 0. In the random model, three situations are considered respectively. They are: (1) the physical parameters and geometric dimensions of the structure are random variables; (2) the loads and the control forces of the closed loop system for the structure are random variables; (3) the physical parameters, geometric dimensions, loads and control forces are simultaneously random variables.

It can be seen from Table 2 that under the conditions that the variation coefficients of physical parameters and geometric dimensions are equal to the variation coefficients of the loads and control forces, the randomness of physical parameters and geometric dimensions will produce greater effect on the randomness of structural stress response than the one produced by the randomness of the loads and control forces. However, the randomness of the loads and control forces will produce greater effect on the randomness of structural displacement response than the randomness of physical parameters and geometric dimensions. In addition, comparing with the conditions that only the randomness of the physical parameters and geometric dimensions or the loads and control forces are considered, the randomness of the structural displacement and stress response are greater under the condition that their randomness are all considered.

*Example 2 4-bar space truss structure (Fig. 2)*

A step load along with the negative direction of Y-axis acts on the nodal 5 of the structure. The amplitude of the step load is a normal random variable. Its mean value is $\mu_{F(t)} = 50000$(N). The gain of the closed loop control system is $g = -100$. Now, the 4th element is used as piezoelectric active bar. The computational results of the dynamic displacement response and dynamic stress response of the structure are given in Table 3. In Table 3, the dynamic displacement response is that of nodal 5, the dynamic stress response is that of 1th element.

In order to investigate the effect of the dispersal degree of random variables $E$, $\rho$ and the dimension random variable factor $l$, $A$ and loads, control forces on the structural dynamic characteristic, the values of variation coefficients of parameters $E$, $\rho$, $l$, $A$, $F$, $F_P$ are taken as two groups respectively. Group I : $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_F = \nu_{F_P} = 0.01$. Group II : $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_F = \nu_{F_P} = 0.1$. In order to compare, two kinds of models, the determinate model and random model, are adopted respectively in computational process too.

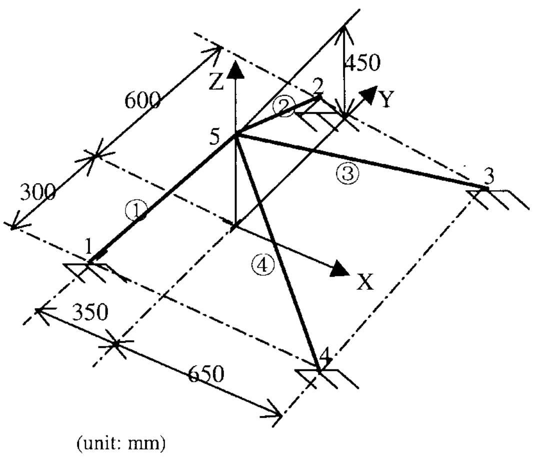


Fig. 2 4-bar space intelligent truss structure

Table 3 The computational results of dynamic response of 4-bar space structure

| Model \ Dynamic response | Maximal displacement response (mm) | Maximal stress response (Mpa) |
|---|---|---|
| Determinate model | 2.74 | 46.51 |
| Random model (I) $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_F = \nu_{F_P} = 0.01$ | $\mu_{x_{\max}(t)} = 2.74$<br>$\sigma_{x_{\max}(t)} = 0.016$ | $\mu_{\sigma_{\max}(t)} = 46.51$<br>$\sigma_{\sigma_{\max}(t)} = 0.533$ |
| Random model (II) $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_F = \nu_{F_P} = 0.1$ | $\mu_{x_{\max}(t)} = 2.74$<br>$\sigma_{x_{\max}(t)} = 0.167$ | $\mu_{\sigma_{\max}(t)} = 46.51$<br>$\sigma_{\sigma_{\max}(t)} = 5.395$ |

It can be seen from Table 3 easily that changing the variation coefficients of physical parameters, geometric dimensions, loads and control forces will produce considerable effects on the computational results of structural dynamic response. The dispersal degree of structural dynamic response will notably increase along with the increase of the variation coefficients of physical parameters, geometric dimensions, loads and control forces.

## 5. Conclusions

(1) Under the conditions that the values of the physical parameters, geometric dimensions, loads and gain are given and taking different elements as active bar, the corresponding results of the structural dynamic response are remarkably different.
(2) The examples show that the analyzing results of intelligent structural dynamic response of the determinate model are different from the ones of the random model. So that when one of the physical parameters, geometric dimensions, loads and control forces is random variable, the conventional determinate analysis method of structural dynamic response will not reflect the effect of the randomness. It is only dependent on the structural dynamic response analysis method based on probability.
(3) The results of examples show that the dispersal degree of structural dynamic response will increase along with the increase of the variation coefficients of physical parameters, geometric dimensions, loads and control forces. This conclusion is completely coincident with the conclusion that has been obtained in the structural reliability.
(4) The examples show that the model of dynamic response analysis of intelligent truss structure based on probability presented in this paper are rational.

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