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Efficient models for analysis of a multistory structure with flexible wings

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Abstract. This study lays emphasis on the development of efficient analytical models for a multistory structure with wings, including the in-plane deformation of floor slabs. For this purpose, a multistory structure with wings is regarded as the combination of multistory structures with rectangular plan and their junctions. In addition, a multistory structure with a rectangular plan is considered to be an assemblage of two-dimensional frames and floor slabs connecting two adjacent frames at each floor level. This modeling concept can be easily applied to multistory structures with plans in the shape of *L*, *T*, *Y*, *U*, *H*, etc. To represent the in-plane deformation of floor slabs efficiently, a two-dimensional frame and the floor slab connecting two adjacent frames at each floor level are modeled as a stick model with two degrees of freedom per floor and a stiff beam with shear deformations, respectively. Three models are used to investigate the effect of in-plane deformation of the floor slab at the junction of wings on the seismic behavior of structures. Based on the comparison of dynamic analysis results obtained using the proposed models and three-dimensional finite element models, it could be concluded that the proposed models can be used as an efficient tool for an approximate analysis of a multistory structure with wings.

Key words: multistory structure with wings; analytical models; 3-D finite element models; junction of wings; *L*-shaped plan; seismic behavior; flexible wings.

1. Introduction

In general, it is desirable to use a three-dimensional finite element model to carry out an accurate analysis of a multistory structure for the lateral loads including the in-plane deformation of floor slabs. But three-dimensional analysis of a multistory structure using a finite element model has shortcomings such as tedious input preparation, longer computational time and larger computer memory required. To overcome these defects of the three-dimensional finite element model, assumptions have to be made about the behavior of a multistory structure. One such assumption is that the floor slabs are rigid in plane. In particular, a floor slab system is usually regarded as a rigid

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diaphragm in the analysis when it is made of cast-in-place concrete, precast concrete with concrete topping, or metal deck filled with concrete (Naeim 1989). In this case, the behavior of a multistory structure can be represented by three degrees of freedom (DOF's) per floor, two translational and one rotational, due to the rigid diaphragm effect of the floor slab system. Therefore, the computational effort required to analyze the structure can be reduced significantly, especially in dynamic analysis. For this reason, some computer programs used for an efficient seismic analysis of multistory structures, such as ETABS, are based on the rigid floor diaphragm assumption. However, based on past earthquakes and dynamic experiments, it was noticed that application of this assumption to certain classes of multistory structures such as: (1) structures with long and narrow plans; (2) structures with end walls; and (3) structures with plans in the shape of *L*, *T*, *Y*, *U*, *H*, etc. may result in significant errors in seismic analyses.

To date, the effects of the in-plane floor slab flexibility on the seismic behavior of multistory structures with rectangular plans have been investigated through many studies (Boppana and Naeim 1985, Jain and Jennings 1985, Kunnath and Reinhorn 1991, Moon and Lee 1994). The effect of in-plane floor slab flexibility on the seismic behavior of a multistory structure with *V*-shaped plan was studied by Jain and Mandal (1992). However, the analytical model proposed by Jain and Mandal is only applicable to structures having a number of uniformly spaced identical frames (or walls) and floor slabs. Therefore, efficient analytical models which can be easily applied to multistory structures with plans in the shapes of *L*, *T*, *Y*, *U*, *H*, etc., are proposed in this study. The in-plane deformation of the floor slab at the junction of wings is also included in the proposed analytical models. The accuracy of the proposed analytical models is investigated by comparing the periods of natural vibration, mode shapes and response histories at several locations obtained from the proposed models to three-dimensional finite element analysis results.

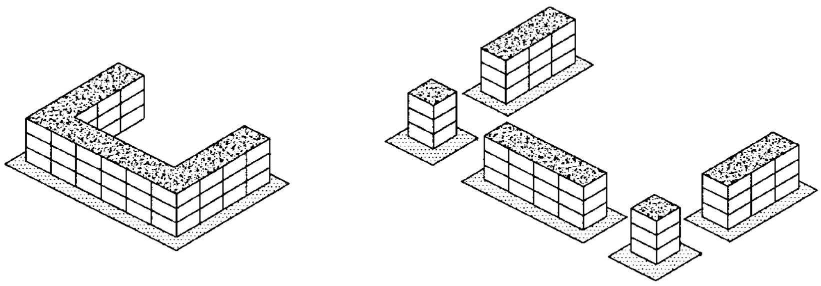
2. Development of analytical models

The proposed analytical models for multistory structures with wings, including the in-plane deformation of floor slabs, are developed based on the following assumptions.

(a) The structure has linear elastic behavior.
(b) Lateral forces are applied at each floor level.
(c) Out-of-plane and in-plane deformations of floor slabs are independent.
(d) Axial deformations in beams and the torsional rigidity of columns are ignored.
(e) The mass of a structure is lumped at each floor level.
(f) The structure is supported by a rigid foundation.

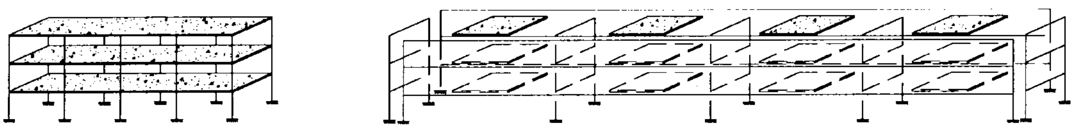
2.1 Formation of proposed analytical models

In this study, a multistory structure with wings is regarded as a combination of multistory structures with rectangular plan and their junctions in the development of efficient analytical models as shown in Fig. 1. In addition, a multistory structure with rectangular plan is considered to be an assemblage of two-dimensional frames in each direction and floor slabs as illustrated in Fig. 2. Finally, a multistory structure with wings is idealized as an assemblage of a series of frames, floor slabs and the junctions of wings. This modeling concept can be applied to multistory structures with



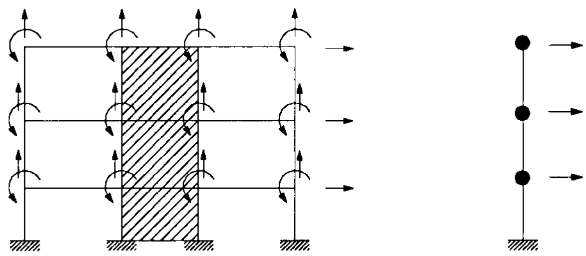
(a) a multistory structure with wings; (b) three rectangular multistory structures and two junctions

Fig. 1 Idealization of a multistory structure with wings



(a) prototype structure; (b) two-dimensional frames in each direction and floor slabs

Fig. 2 Idealization of a multistory structure with rectangular plan



(a) typical two-dimensional frame; (b) stick model

Fig. 3 Reduction of a two-dimensional frame

plans in the shape of *L, T, Y, U, H*, etc.

2.1.1 Modeling of a frame in each direction

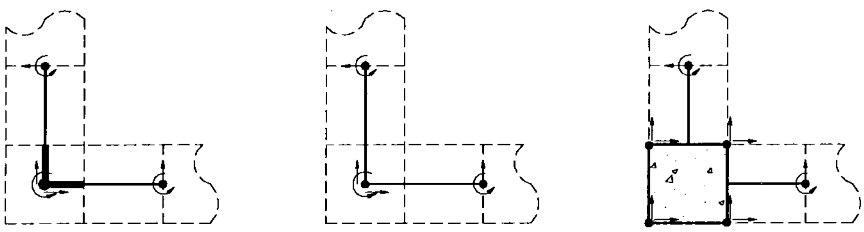
For a typical two-dimensional frame as shown in Fig. 3(a), three DOF's per node are needed to represent its behavior. However, based on the aforementioned assumption (d), the axial deformations in beams are ignored and lateral displacements of the nodes on a floor can be represented by one lateral displacement. Thus, a frame has one translational DOF per floor and two DOF's per node as shown in Fig. 3(a). To develop a stick model with one translational DOF per floor, two DOF's at each node should be eliminated. In order to consider the effects of flexural deformation in beams and axial deformation in columns on lateral displacement, the static condensation technique (Weaver and Johnston 1987) was employed to eliminate two DOF's per node. As a result, a frame can be simplified to a stick model with one translational DOF per floor as shown in Fig. 3(b). For modeling of shear walls, a rectangular plane stress element with rotational DOF's (Weaver and Lee 1981) was used in this study. Since this element fully satisfies the compatibility at a beam-shear wall joint, the use of fictitious rigid beam elements are not necessary. Thus, a more efficient analysis of a structure with shear walls was possible.

2.1.2 Modeling of floor slabs

Through previous experiments (Karadogan and Huang 1982, Nakashima and Huang 1984), it is well known that the influence of the in-plane and out-of-plane forces on the deformation of a floor slab can be uncoupled in a linear elastic analysis. Therefore, the effect of the in-plane floor slab flexibility can be obtained by the analysis of a floor slab subjected to in-plane forces. With this fact, the analysis of a floor slab can be simplified to a two-dimensional plane stress problem. Although a floor slab can be modeled by plane stress elements, a stiff beam element with shear deformations (Prezemieniecki 1968) was used for an efficient analytical model for a floor slab connecting two adjacent frames at each floor level. A stiff beam element has two DOF's per node for the lateral displacement and the in-plane rotation to consider the in-plane deformation of floor slabs (Moon and Lee 1994).

2.1.3 Modeling of the junction of wings

The floor flexibility of a multistory structure with wings is mainly influenced by the in-plane deformation of the floor slab at the junction of wings. Three models (Models A, B and C) are considered in this study to investigate the effect of the in-plane deformation of the floor slab at the junction of wings on the seismic behavior of structures. The in-plane deformation of the floor slab at the junction of wings is represented by a rigid beam for Model A, a stiff beam with shear deformations for Model B and a four node plane stress element for Model C. Fig. 4 illustrates the modeling of the floor slab at the junction of two wings and the neighboring floor slabs at a floor level for each model. For Models A and B, one node is located at the center of the floor slab at the junction of wings (see Figs. 4(a) and (b)) and nodes are located at the corners of a floor slab for Model C (see Fig. 4(c)). To connect the floor slab at the junction of wings with the neighboring floor slab directly, for Models A and B, a stiff beam element having three DOF's at one end and two DOF's at the other end with or without a rigid beam at one end is used as shown in Figs. 4(a)



(a) Model A; (b) Model B; (c) Model C

Fig. 4 Modeling of the floor slab at the junction of two wings

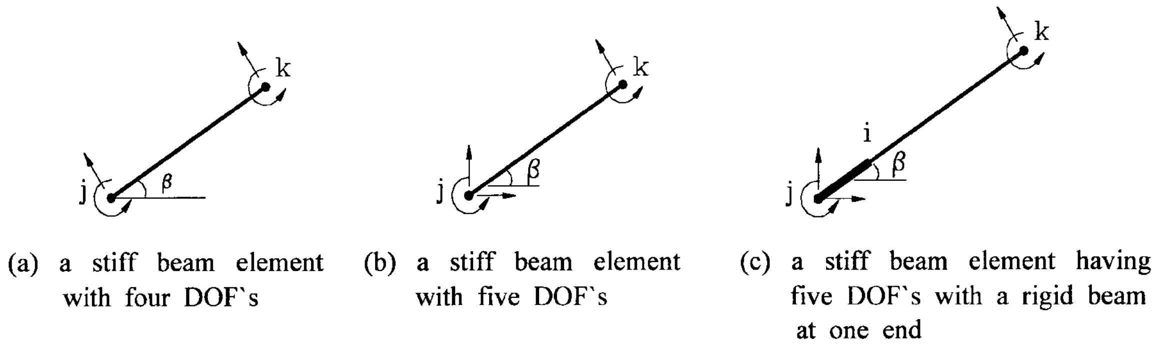


Fig. 5 Stiff beam elements used in Models A and B

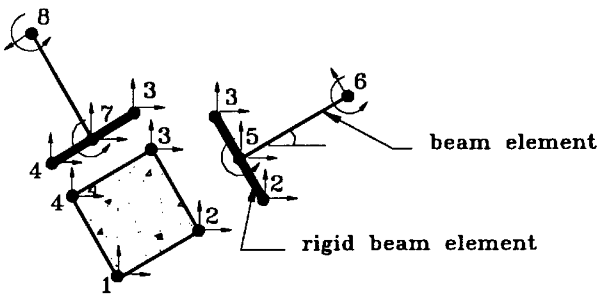


Fig. 6 Connection of a plane stress element with a stiff beam element

and 4(b). For Model C, the stiff beam element having six DOF's with a rigid beam at one end is used as shown in Fig. 6. The stiffness matrix for each element used in Models A, B, and C can be obtained through the following procedures.

· A stiff beam with five DOF's

The stiffness matrix $[k_b^*]$ for a stiff beam with three DOF's at one end and two DOF's at the other end as shown in Fig. 5(b) can be derived by rotating the DOF at the joint j (see Fig. 5(a)) as follows:

$$[k_b^*] = [T_b]^T[k_b][T_b] \tag{1}$$

The matrix $[k_b]$ is the stiffness matrix for a stiff beam element shown in Fig. 5(a). It's size is 5×5 and elements in the first row and the first column are zero. The transformation matrix $[T_b]$ is given by

$$[T_b] = \begin{pmatrix} \cos\beta & \sin\beta & 0 & 0 & 0 \\ -\sin\beta & \cos\beta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where β represents the angle between horizontal axis and the stiff beam element (see Fig. 5).

· A stiff beam having five DOF's with a rigid beam at one end

The stiffness matrix $[k_r]$ for a stiff beam with a rigid beam at one end as shown in Fig. 5(c) is derived by the following formula from the stiffness matrix $[k_b^*]$ for a stiff beam with five DOF's:

$$[k_r] = [T_r]^T[k_b^*][T_r] \tag{3}$$

where $[T_r]$ is a transformation matrix used to transfer the DOF's from the node i to the node j and is given as follows:

$$[T_r] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -a\sin\beta & a\cos\beta & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

where a is the length of the rigid beam.

· A stiff beam having six DOF's with a rigid beam at one end

Because the floor slab at the junction of wings is modeled by a four node plane stress element with two DOF's per node in Model C, there are problems in the connection between the plane stress element and the stiff beam element shown in Fig. 6. For instance, node 5 of the stiff beam element does not coincide with any one of the four nodes in the plane stress element. Moreover, the rotational DOF appears at node 5, but nodes 1 through 4 have translational DOF's only. These problems could be solved by the transformation of the stiffness matrix for the stiff beam element relating to nodes 5 and 6 into that associated with nodes 2, 3 and 6. The stiffness matrix $[k_b^\dagger]$ for nodes 2, 3 and 6 can be obtained as follows (Cook and Malkus 1989):

$$[k_b^\dagger] = [T_b^\dagger]^T[k_b^*][T_b^\dagger] \tag{5}$$

where the transformation matrix $[T_b^\dagger]$ is defined as follows:

$$[T_b^\dagger] = \frac{1}{d}\begin{bmatrix} 0.5d & 0 & 0.5d & 0 & 0 & 0 \\ 0 & 0.5d & 0 & 0.5d & 0 & 0 \\ \cos\beta & \sin\beta & -\cos\beta & -\sin\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

in which d is the width of the floor slab at the junction of wings. The modified stiffness matrix $[k_b^\dagger]$ in Eq. (5) is the 6×6 matrix and the corresponding displacement vector $\{d_b\}$ is as follows:

$$\{d_b\} = \{u_2 \quad v_2 \quad u_3 \quad v_3 \quad u_6 \quad v_6\} \tag{7}$$

2.2 Formation of stiffness matrix

A multistory structure with wings is regarded as a combination of multistory structures with rectangular plan, namely, wing structures and their junction in this study. Therefore, the total stiffness matrix $[K]$ for a multistory structure with wings is composed of the stiffness matrices for wing structures and their junction.

$$[K] = \begin{bmatrix} [K_w] & [K_{wj}] \\ symm. & [K_j] \end{bmatrix} \tag{8}$$

where the submatrices $[K_w]$ and $[K_j]$ are the stiffness matrix for wing structures and their junction, respectively. The submatrix $[K_{wj}]$ represents the interaction between wing structures and their junction. When the number of wings is N_w, the total stiffness matrix $[K]$ for a given structure is as follows:

$$[K] = \begin{bmatrix} [k_w]_1 & [0] & \cdots & [0] & \cdots & [0] & [k_{wj}]_1 \\ & [k_w]_2 & \cdots & [0] & \cdots & [0] & [k_{wj}]_2 \\ & & & \vdots & & \vdots & \vdots \\ & & & [k_w]_i & \cdots & [0] & [k_{wj}]_i \\ & & & & & \vdots & \vdots \\ & symm. & & & & [k_w]_{N_w} & [k_{wj}]_{N_w} \\ & & & & & & [K_j] \end{bmatrix} \tag{9}$$

where $[k_w]_i$ represents the stiffness matrix for the i-th wing structure and $[k_{wj}]_i$ means the interaction between the i-th wing structure and the junction of wings. To develop an efficient analytical model for a multistory structure with wings, two coordinates are used in this study, which are the local coordinate used for each wing structure and the global coordinate used for their junction. Fig. 7 illustrates the modeling of a one-story structure with two wings using the proposed modeling concepts.

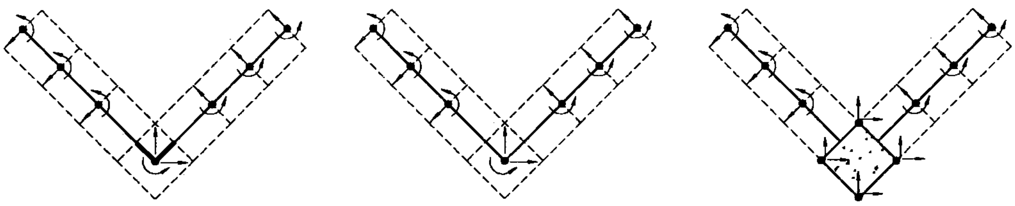

(a) Model A (b) Model B (c) Model C

Fig. 7 Modeling of a one-story structure with two wings

2.2.1 Stiffness matrix for a wing structure

Each wing structure is composed of floor slabs and two-dimensional frames in each direction, that is, transverse and longitudinal directions. In this study, it is assumed that the stiffness for longitudinal frames is concentrated on the junction of wings to develop an efficient analytical model. Thus the stiffness matrix for a wing structure consists of the stiffness matrices for floor slabs and transverse frames. The stiffness matrix for the i-th wing structure, $[k_w]_i$, is as follows:

$$[k_w]_i = \begin{bmatrix} [k_t]_i & [k_{tf}]_i \\ symm. & [k_f]_i \end{bmatrix} \tag{10}$$

where the submatrices $[k_t]_i$ and $[k_f]_i$ are the stiffness matrix for transverse frames and floor slabs, respectively and the submatrix $[k_{tf}]_i$ represents the interaction between transverse frames and floor slabs.

2.2.2 Stiffness matrix for the junction of wings

The stiffness matrices $[K_j]$ and $[K_{wj}]$ in Eq. (8) associated with the junction of wings are obtained using the stiffness matrix $[k_l]$ for longitudinal frames of the each wing and that for the floor slab at the junction of wings. The stiffness matrix $[K_j]$ for the junction of wings is determined by the following two formulas:

$$[K_j] = \sum_{i=1}^{N_w} \sum_{k=1}^{N_l} [k_l]_{ik} \qquad \text{(for Models A and B)} \tag{11a}$$

$$[K_j] = \sum_{i=1}^{N_w} \sum_{k=1}^{N_l} [k_l]_{ik} + [k_{fj}] \qquad \text{(for Model C)} \tag{11b}$$

where N_w is the number of wings and N_l is the number of longitudinal frames of each wing. In Eq. (11b), the stiffness matrix $[k_{fj}]$ is a 4×4 matrix, which is associated with a four node plane stress element representing the floor slab at the junction of wings. Although the stiffness for the floor slab at the junction of wings is considered in stiffness matrix $[K_j]$ in Model C, the floor slab stiffness is considered in the stiffness matrix $[K_{wj}]$ for the convenience of formation of stiffness matrix for Models A and B.

For Models A and B, the stiffness for longitudinal frames is concentrated on one node located in the center of the floor slab at the junction of wings while for Model C that is distributed into four nodes. The stiffness matrix $[k_{wj}]_i$ related to the i-th wing structure is equal to the stiffness matrix $[k_r]$ in Eq. (3) or $[k_b^*]$ in Eq. (1) for Models A and B respectively. For Model C, it is represented by the stiffness matrix $[k_b^\dagger]$ in Eq. (5) for the modified stiff beam element. For a multistory structure with a rectangular plan, the total stiffness matrix $[K]$ of a given structure can be obtained by considering the submatrix $[K_w]$ only except the submatrices associated with the junction of wings such as $[K_j]$ and $[K_{wj}]$ in Eq. (8), as follows:

$$[K] = [K_w] = \begin{bmatrix} [K_t] & [K_{tf}] \\ symm. & [K_f] \end{bmatrix} \tag{12}$$

3. Performance of the proposed models

3.1 Numerical examples

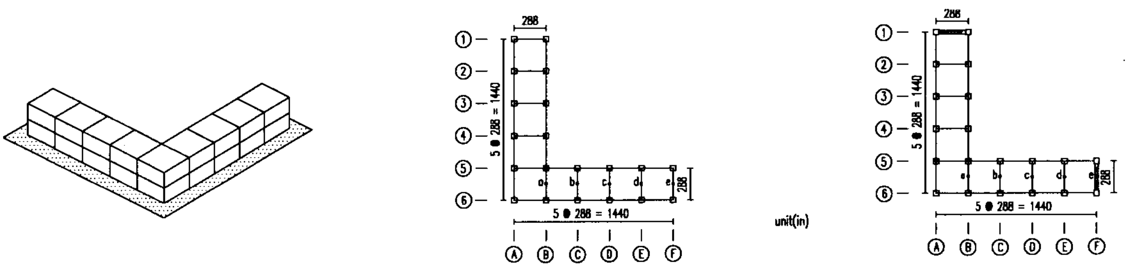
To investigate the accuracy and the computational efficiency of the proposed models in this study, the analysis results obtained by the proposed model are compared with those acquired using three-dimensional finite element models. Example structures are *L*-shaped 4-bay, 2-story structures with two wings as shown in Fig. 8(a). Two types of structural systems, the frame system (F-type) and the frame-shear wall system (FW-type), are considered to examine the in-plane behavior of floor slabs due to the difference in stiffness distribution. For the frame-shear wall system, the shear wall is added at the end frames ① and Ⓕ as shown in Fig. 8(c). The beams are 12 inches wide and 18 inches deep while the columns are 16 inches by 16 inches. The thicknesses of slabs and shear walls are 5 inches and 6 inches, respectively. The modulus of elasticity is 3000 psi and the Poisson's ratio is 0.167. The N-S component of the El Centro earthquake (1940) record is used to perform the dynamic analysis and the ground motion is applied along the axis of symmetry of a plan. The damping ratio is assumed to be 5% for each vibrational modes.

3.1.1 Three-dimensional finite element models

The floor slab connecting two adjacent frames and the shear wall are modeled using plane stress elements because the main interest is placed on the in-plane behavior of floor slabs. Fig. 9 illustrates three-dimensional finite element models F-1, F-2, F-3, FW-1, FW-2 and FW-3 with different structural system and a number of elements used in the modeling of a floor slab and shear wall, respectively. The number of nodes, elements and DOF's used in the proposed models and three-dimensional finite element models are listed in Table 1.

3.1.2 Results of analysis

Natural vibration periods for the first four modes obtained using the proposed models and three-dimensional finite element models are shown in Table 2 and the corresponding mode shapes for the first four modes are shown in Figs. 10 and 11. Fig. 12 represents the displacement time histories at points *e* (see Fig. 8(b)) and *b* (see Fig. 8(c)). Table 2 shows that natural vibration periods obtained from the proposed models are in good agreement with those acquired using three-dimensional finite



(a) perspective view (b) frame system(F-type) (c) frame-shear wall system(FW-type)

Fig. 8 Example structures

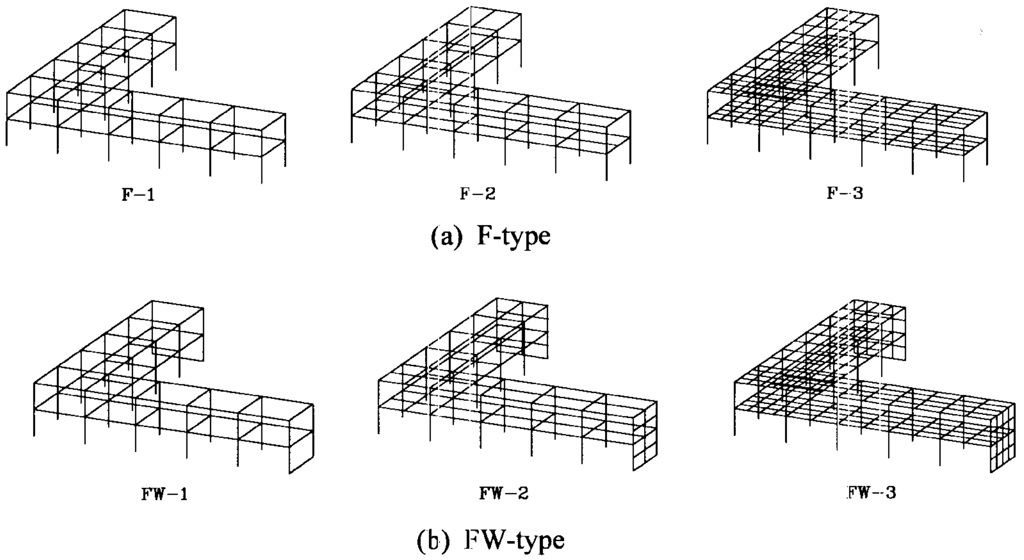


Fig. 9 Three-dimensional finite element models for example structures

Table 1 Comparison of the number of nodes, elements and DOF's for example structures

Type	Frame system (F-type)						Frame-shear wall system (FW-type)					
Model	3D FE models			Proposed model			3D FE models			Proposed model		
	F-1	F-2	F-3	A	B	C	FW-1	FW-2	FW-3	A	B	C
Node	77	103	247	18	18	24	77	117	273	18	18	24
Element	114	160	360	34	34	42	118	184	400	34	34	42
DOF's	360	508	1152	38	38	48	360	522	1204	38	38	48

Table 2 Comparison of natural vibration periods for the first four modes of example structures

unit(sec)

Type	Frame system (F-type)						Frame-shear wall system (FW-type)					
Model	3D FE models			Proposed model			3D FE models			Proposed model		
	F-1	F-2	F-3	A	B	C	FW-1	FW-2	FW-3	A	B	C
Mode 1	0.489	0.489	0.489	0.489	0.499	0.489	0.490	0.490	0.490	0.490	0.505	0.490
Mode 2	0.488	0.488	0.488	0.488	0.498	0.488	*0.405*	*0.406*	*0.408*	*0.397*	*0.428*	*0.413*
Mode 3	0.469	0.469	0.469	0.469	0.475	0.470	0.153	0.153	0.153	0.153	0.157	0.153
Mode 4	*0.215*	*0.217*	*0.220*	*0.197*	*0.233*	*0.222*	0.149	0.149	0.149	0.149	0.154	0.150

element models. However, Model B results in longer natural vibration periods than three-dimensional finite element models, which is attributed to the overestimation of the in-plane deformation of the floor slab at the junction of wings. It is noteworthy that the in-plane deformation

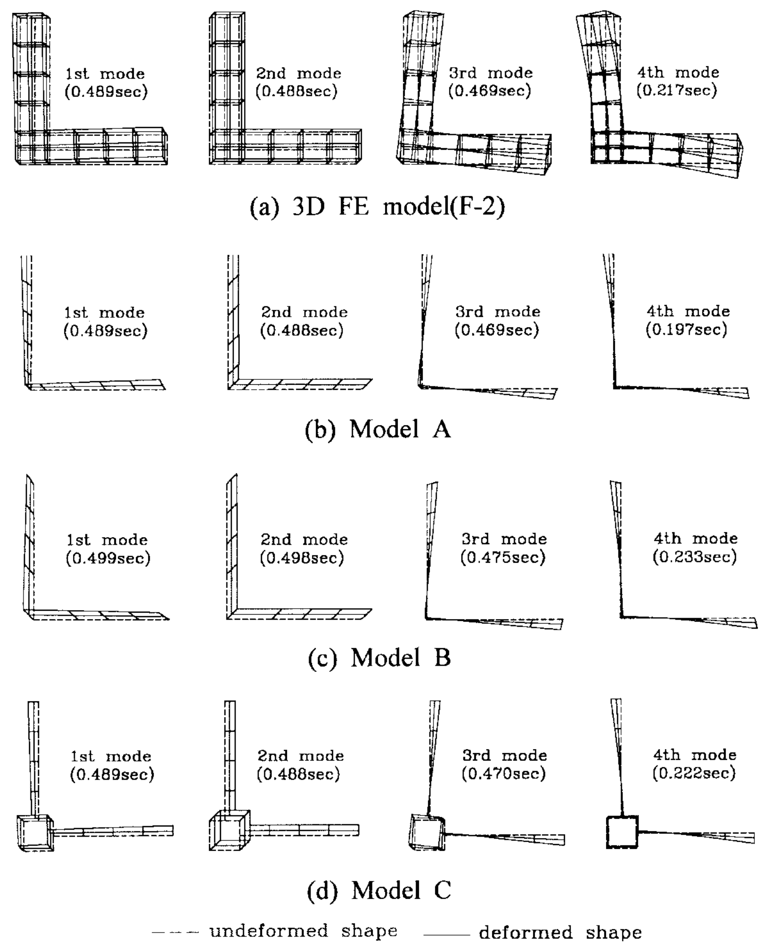


Fig. 10 Plan view of mode shapes of the F-type example structure

of the floor slabs occurred in the lower modes such as the 4th mode for F-type structure (see Fig. 10) and the 2nd mode for FW-type structure (see Fig. 11). Comparison of the natural vibration periods obtained from Models A and C for such modes shows that the neglect of in-plane deformation of the floor slab at the junction of wings results in shorter periods. From Fig. 12, Model B results in a somewhat different displacement time histories for F-type and FW-type structures. It is observed that for the F-type structure the displacement time histories obtained using Models A and C are in good agreement with those acquired using three-dimensional finite element models. However, Model A shows somewhat different displacement time histories for the FW-type structure. This means that the displacement time histories at any point may be affected by the in-plane deformation of the floor slab at the junction of the wing when the difference in stiffness among the frames is expected to be large. From the above mentioned analysis results, it seems that the proposed models have the satisfactory accuracy and computational efficiency for a multistory structure with wings, including the in-plane floor slab flexibility.

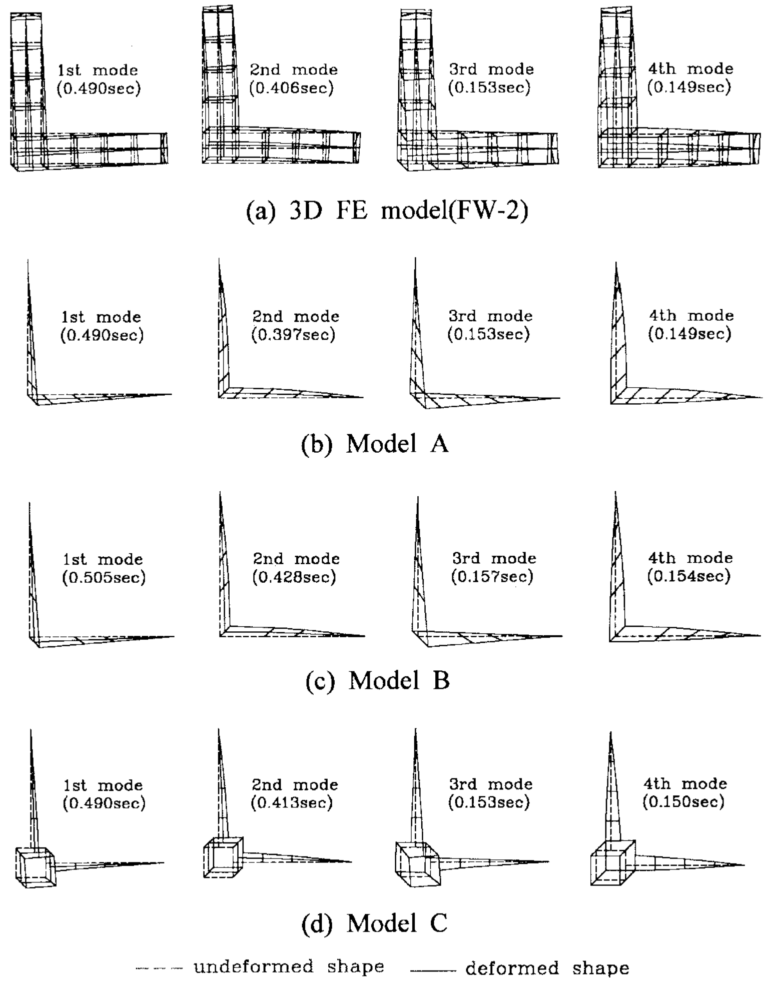
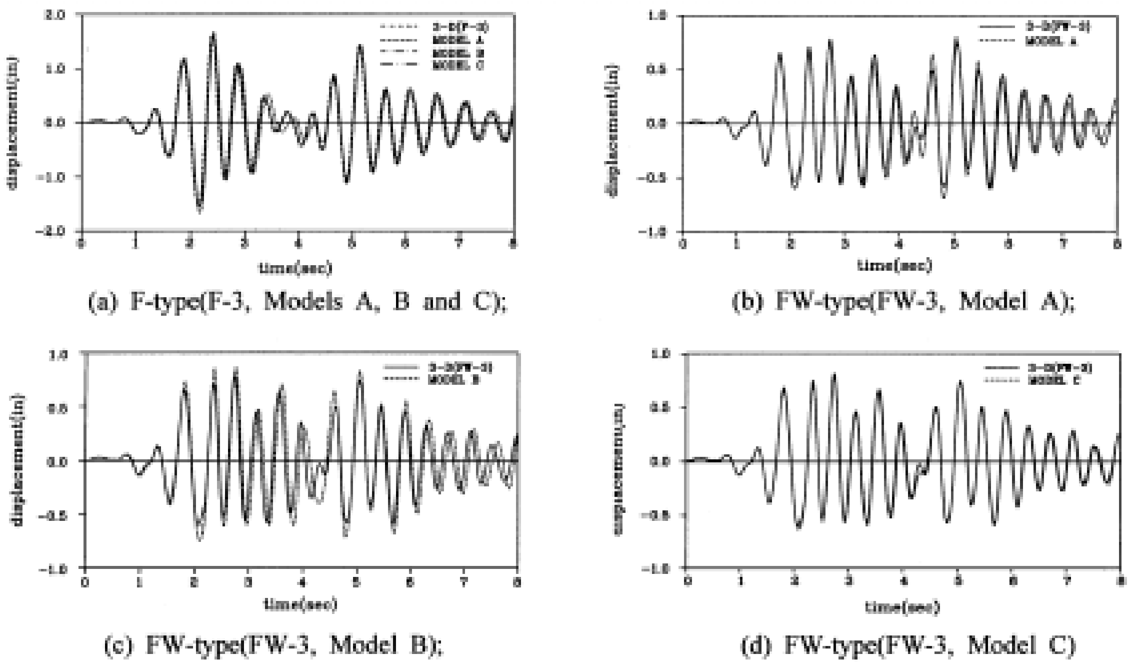


Fig. 11 Plan view of mode shapes of FW-type example structure

4. Conclusions

Efficient models for analysis of a multistory structure with wings, including in-plane deformation of floor slabs, are proposed. For this purpose, a multistory structure with wings is regarded as a combination of multistory structures with rectangular plan and their junctions. In addition, a multistory structure with rectangular plan is considered to be an assemblage of two-dimensional frames in each direction and floor slabs. To represent the in-plane deformation of floor slabs efficiently, a two-dimensional frame and the floor slab connecting two adjacent frames at each floor level is modelled as a stick model and a stiff beam with shear deformations, respectively. Three models such as Models A, B and C are proposed to investigate the effect of in-plane deformation of the floor slab at the junction of wings on the seismic behavior of structures. The accuracy of the proposed models is examined by comparison of analysis results obtained from the proposed models and three-dimensional finite element models in terms of natural vibration periods, mode shapes and displacement time histories. Based on the analysis results, the following conclusions can be made.



(a) F-type(F-3, Models A, B and C); (b) FW-type(FW-3, Model A); (c) FW-type(FW-3, Model B); (d) FW-type(FW-3, Model C)

Fig. 12 Comparison of displacement time histories at points *e* (F-type) and *b* (FW-type)

1. Since a multistory structure with wings is idealized as a combination of multistory structures with rectangular plan and their junctions in this study, the proposed modeling concept can be easily applied to multistory structures with plans in the shapes of *L*, *T*, *Y*, *U*, *H*, etc.
2. Comparison of the analysis results obtained from the proposed models and three-dimensional finite element models for example structures shows that the proposed models can be used as an efficient tool for an analysis of multistory structures with wings. In particular, the in-plane deformation of the floor slabs at the junction of wings was significant in the lower modes in the case of the FW-type structure. In this case, the effects of the in-plane deformation of the junction on the seismic behavior of structures can be investigated by the proposed Model C in which the floor slab at the junction is modelled by a plane stress element with two DOF's per node.

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Notation

a	: length of a rigid beam element
β	: angle between horizontal axis and a stiff beam element
d	: width of the floor slab at the junction of wings
N_l	: number of longitudinal frames of each wing
N_w	: number of wings
$[k_b]$	: stiffness matrix for a stiff beam element with shear deformations
$[k_b^*]$	: stiffness matrix for a stiff beam element with five DOF's
$[k_r]$	: stiffness matrix for a stiff beam element having five DOF's with a rigid beam at one end
$[k_b^\dagger]$	: stiffness matrix for a stiff beam element having six DOF's with a rigid beam at one end
$[I]$	: 2 × 2 identity matrix
$\{d_b\}$	: displacement vector
$[K]$	: total stiffness matrix for a multistory structure with wings
$[K_w]$	: stiffness matrix for wing structures
$[K_j]$	: stiffness matrix for the junction of wings
$[K_{wj}]$	: stiffness matrix representing the interaction between wing structures and their junctions
$[k_w]_i$	: stiffness matrix for the *i*-th wing structure
$[k_{wj}]_i$	: stiffness matrix representing the interaction between the *i*-th wing structure and the junction of wings
$[k_t]_i$	: stiffness matrix for transverse frames of the *i*-th wing structure
$[k_{tf}]_i$	: stiffness matrix representing the interaction between transverse frames and floor slabs of the *i*-th wing structure
$[k_f]_i$	: stiffness matrix for floor slabs of the *i*-th wing structure
$[k_l]$	: stiffness matrix for longitudinal frames of each wing
$[k_{fj}]$	: stiffness matrix for the floor slab at the junction of wings