Structural Engineering and Mechanics, Vol. 13, No. 3 (2002) 329-343

DOI: http://dx.doi.org/10.12989/sem.2002.13.3.329

The dynamic stability of a nonhomogeneous orthotropic elastic truncated conical shell under a time dependent external pressure

A.H. Sofiyev†

Ondokuz Mayıs University, Civil Engineering Department, 55139, Samsun, Turkey

O. Aksogan‡

Cukurova University, Civil Engineering Department, 01330, Adana, Turkey

Abstract. In this research, the dynamic stability of an orthotropic elastic conical shell, with elasticity moduli and density varying in the thickness direction, subject to a uniform external pressure which is a power function of time, has been studied. After giving the fundamental relations, the dynamic stability and compatibility equations of a nonhomogeneous elastic orthotropic conical shell, subject to a uniform external pressure, have been derived. Applying Galerkin's method, these equations have been transformed to a pair of time dependent differential equations with variable coefficients. These differential equations are solved using the method given by Sachenkov and Baktieva (1978). Thus, general formulas have been obtained for the dynamic and static critical external pressures and the pertinent wave numbers, critical time, critical pressure impulse and dynamic factor. Finally, carrying out some computations, the effects of the nonhomogeneity, the loading speed, the variation of the semi-vertex angle and the power of time in the external pressure expression on the critical parameters have been studied.

Key words: dynamic stability; nonhomogeneous; orthotropic; truncated; conical shell; external pressure; Galerkin's method; dynamic critical load; dynamic factor; wave number.

1. Introduction

In the contemporary technology, the improvement of the strength properties of materials used in producing the structural elements of construction, aims at decreasing their sizes and weights. To this end, computation methods taking the actual behaviour of materials into consideration are essential. The foregoing fact has, recently, pulled the attention of researchers to the elasticity problems of objects made of homogeneous and nonhomogeneous anisotropic materials (Lekhnitski 1980, Lomakin 1976). The aforementioned nonhomogeneity stems from the effects of humidity, heat and methods of production, which render the physical properties of materials vary from point to point. For example, Brinkman (1954) has shown that the elastic properties of metal shells, subjected to radiation, can be taken as a linear function of the coordinate in the thickness direction, as a first

† Associate Professor
‡ Professor

approximation.

Delale and Erdogan (1983), and Zhang and Hasebe (1999) have assumed the variation of the elasticity modulus to be unbounded and have used exponential functions of the radial coordinate in their expressions.

In actual engineering applications, the variation of the elastic properties of materials remains in a bounded range and small enough, necessitating a restriction on the variation functions. Researchers have given this restriction in different ways. Massalas *et al*. (1981, 1982) have, first, taken the elasticity modulus as a function of thickness coordinate and, then, as a function of longitudinal coordinate. Heyliger and Juliani (1992) have taken it to be a function of the radial coordinate in some vibration problems. Guiterrez *et al*. (1998) have taken density to be a linear, quadratic and cubic function of the radial coordinate in their work. Sofiyev and Aksogan (1999), and Aksogan and Sofiyev (2000) have solved the dynamic stability problems of single and multi-layered cylindrical shells varying the elasticity moduli in both the thickness and the longitudinal directions and elasticity modulus and density only in the thickness direction, respectively. In those studies, the functions expressing the nonhomogeneity have been assumed as continuous (linear, quadratic, cubic and exponential) functions, always less than unity.

The pioneering studies in the static buckling of shells can be mentioned as follows. Mushtari and Sachenkov (1958) have worked on the buckling of circular cylindrical and conical shells under combined external and axial pressures. Singer (1961) has studied the axisymmetric buckling problem of a circular conical shell under external pressure. Sachenkov and Aganesov (1964) have worked on the buckling and nonsymmetric vibrations of structurally orthotropic elastic cylindrical and conical shells under external and axial pressures. Singer (1966), later, proposed a procedure for the solution of the three equilibrium equations using complex series. Baruch *et al.* (1970) studied the buckling of a simply supported isotropic conical shell under axial pressure using the Donnell type shell theory.

Leissa (1973) summarized all the past literature on the vibration of conical shells. Tani (1973, 1981) examined the dynamic buckling of truncated conical shells under a periodic external pressure and a pulsating torsional loading, respectively. Irie *et al*. (1984) have studied the free vibration of truncated conical shells. Sivadas and Ganesan (1991) worked on the vibrations of laminated conical shells with variable thickness. Tong *et al*. (1992) and Tong (1993) have used the Donnell type shell theory to examine the linear vibrations and buckling of simply supported orthotropic conical shells under axial pressure. Mecitoglu (1996) has solved, numerically, the dynamic equations of a stiffened composite laminated conical thin shell under the influence of initial stresses. Lam *et al*. (1999) have studied the effects of the boundary conditions on the vibration characteristics of thick truncated conical shells.

The dynamic stability problems of thin conical shells subject to pulsating pressure loading has not been studied enough. In particular, it has been observed that, various theoretical solutions of these problems using different methods do not match well with experimental results. This is due to the difficulty of accounting for all the factors (variation of load with time, scattering of waves through the materials, etc.) affecting the behavior of systems deformed by dynamic loading. Consequently, recently, researchers are extremely more interested in the theoretical and experimental studies of thin conical shells under different loading conditions, and thus, try to obtain functional expressions for critical loads. Making use of the dynamic criterion concerning the stability of plates and shells proposed by Sachenkov (1976), Sachenkov and Klementev (1980) have examined the dynamic stability of an elastic conical shell subject to a linear time dependent external pressure and Baktieva *et al*. (1988) have studied the dynamic stability of cylindrical and conical shells. In the two latter works, some factors involved in the expressions for the critical parameters are taken from experiments and the pertinent results have been shown to be perfectly acceptable.

In practical applications, liquid and wind pressures are sometimes confronted as power functions of time, as well as, linear and periodical ones. The buckling problems under such loads have been studied by Yakushev (1990) for an homogeneous cylindrical shell, by Sofiyev and Aksogan (1999) for a nonhomogeneous orthotropic elastic cylindrical shell and by Aksogan and Sofiyev (2000) for a laminated shell composed of layers of the foregoing type.

The aim of the present work, is to study the dynamic stability of an orthotropic elastic conical shell, the elasticity moduli and density of which varies continuously with the coordinate in the thickness direction, under an external pressure which is a power function of time, employing the method presented by Sachenkov and Baktieva (1978) with some modifications.

2. Fundamental relations and governing equations

Consider a truncated conical shell of medium length with a circular cross-section made of a nonhomogeneous orthotropic elastic material. Assume that the elasticity moduli and density of the material are continuous functions of the coordinate in the thickness direction. Hence, the elasticity moduli and density can be expressed as functions of $\bar{\varsigma}$, the normalised coordinate in the thickness direction, as follows:

$$E_S(\bar{\varsigma}) = E_{0S}[1 + \mu\varphi_1(\bar{\varsigma})], \quad E_\theta(\bar{\varsigma}) = E_{0\theta}[1 + \mu\varphi_2(\bar{\varsigma})],$$

$$G(\bar{\varsigma}) = G_0[1 + \mu\varphi_3(\bar{\varsigma})], \quad \rho(\bar{\varsigma}) = \rho_0[1 + \mu\varphi_4(\bar{\varsigma})], \quad \bar{\varsigma} = \varsigma/h \tag{1}$$

where E_{0S}, $E_{0\theta}$ and G_0 are the elasticity moduli of the homogeneous orthotropic material and its shear modulus, respectively, ρ_0 is the density of the homogeneous material and μ is the variation coefficient of the elasticity moduli and density satisfying $0 \le \mu < 1$, $\varphi_i(\bar{\varsigma})$, $(i = 1, 2, 3, 4)$, are continuous functions corresponding to the variations of the elasticity moduli and density, which satisfy $\left|\varphi_i(\bar{\varsigma})\right| \le 1$, and h is the thickness of the shell.

Let the coordinate system be chosen such that, the origin O is at the vertex of the whole cone, on the middle surface of the shell, and S axis lies on the curvilinear middle surface of the cone, S_1 and S_2 being the coordinates of the points where this axis intersects the small and large bases, respectively. The average radii of the small and large bases are R_1 and R_2, respectively, and γ is the semi-vertex angle. Furthermore, ς axis is always normal to the moving S axis, lies in the plane of the S axis and the axis of the cone and points inwards. θ is the angle of rotation around the longitudinal axis starting from a radial plane. The axes of orthotropy are parallel to the curvilinear coordinates S and θ (see Fig. 1).

In accordance with the Kirchhoff-Love hypothesis, in the case of small displacements, the strain at ς distance from the middle surface is given as follows:

$$(\varepsilon_S, \varepsilon_\theta, \varepsilon_{S\theta}) = (e_S, e_\theta, e_{S\theta}) + \varsigma(\chi_S, \chi_\theta, \chi_{S\theta}) \tag{2}$$

where e_S and e_θ are the normal strains in the curvilinear coordinate directions S and θ on the middle surface, respectively, $e_{S\theta}$ is the shear strain, χ_S and χ_θ are the curvatures of the deformed shell in the curvilinear directions S and θ, respectively, and $\chi_{S\theta}$ is the twist of the middle surface, the last three

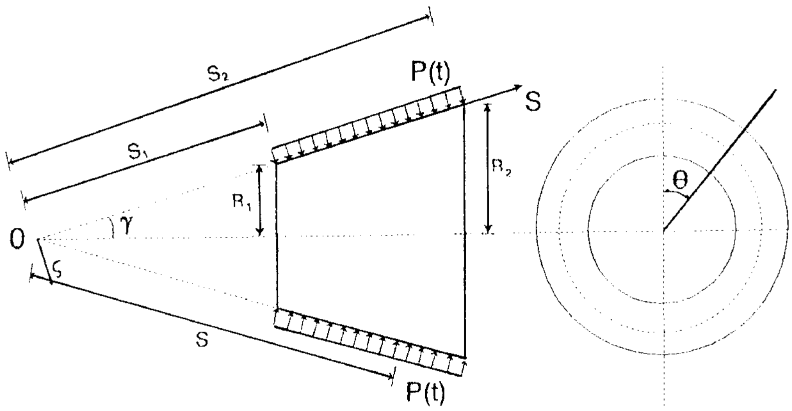

Fig. 1 The geometry of a truncated conical shell under a uniform external pressure

entities being defined as follows:

$$\chi_S = -\frac{\partial^2 u}{\partial S^2}, \quad \chi_\theta = -\frac{1}{S^2}\frac{\partial^2 u}{\partial \theta_1^2} - \frac{1}{S}\frac{\partial u}{\partial S}, \quad \chi_{S\theta} = -\frac{1}{S}\frac{\partial^2 u}{\partial S \partial \theta_1} + \frac{1}{S^2}\frac{\partial u}{\partial \theta_1} \tag{3}$$

in which $\theta_1 = \theta\ \sin\gamma$ and u is the displacement of the middle surface in the normal direction, positive towards the axis of the cone and assumed to be much smaller than the thickness (Volmir 1967, Tani 1973). For the physically linear shell described above, the stress-strain relations are as follows:

$$\begin{pmatrix} \sigma_S \\ \sigma_\theta \\ \sigma_{S\theta} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \begin{pmatrix} \varepsilon_S \\ \varepsilon_\theta \\ \varepsilon_{S\theta} \end{pmatrix} \tag{4}$$

where, σ_S, σ_θ and $\sigma_{S\theta}$ are the stresses and $\nu_{S\theta}$, $\nu_{\theta S}$ are the Poisson's ratios, which are constant, and Q_{ij} (i, j = 1,2,3) are defined as:

$$Q_{11} = \frac{E_{0S}[1 + \mu\varphi_1(\bar{\varsigma})]}{1 - \nu_{S\theta}\nu_{\theta S}}, \quad Q_{22} = \frac{E_{0\theta}[1 + \mu\varphi_2(\bar{\varsigma})]}{1 - \nu_{S\theta}\nu_{\theta S}},$$

$$Q_{12} = \nu_{\theta S} Q_{11}, \quad Q_{21} = \nu_{S\theta} Q_{22}, \quad Q_{33} = 2G_0[1 + \mu\varphi_3(\bar{\varsigma})] \tag{5}$$

The internal forces and moments per unit length of the cross-section of the shell are found using the following expressions:

$$(N_S, N_\theta, N_{S\theta}) = \int_{-h/2}^{h/2} (\sigma_S, \sigma_\theta, \sigma_{S\theta}) d\varsigma, \quad (M_S, M_\theta, M_{S\theta}) = \int_{-h/2}^{h/2} (\sigma_S, \sigma_\theta, \sigma_{S\theta}) \varsigma d\varsigma \tag{6}$$

The relations between the forces N_S, N_θ and $N_{S\theta}$ and the stress function F are as follows:

$$N_S = \frac{1}{S^2}\frac{\partial^2 F}{\partial\theta_1^2} + \frac{1}{S}\frac{\partial F}{\partial S}, \quad N_\theta = \frac{\partial^2 F}{\partial S^2}, \quad N_{S\theta} = -\frac{1}{S}\frac{\partial^2 F}{\partial S\partial\theta_1} + \frac{1}{S^2}\frac{\partial F}{\partial\theta_1} \tag{7}$$

Taking account of the radial inertia forces, the modified Donnell type stability and compatibility equations of a conical shell are found as follows (Volmir 1967):

$$\frac{\partial^2 M_S}{\partial S^2} + \frac{2}{S}\frac{\partial M_S}{\partial S} + \frac{2}{S}\frac{\partial^2 M_{S\theta}}{\partial S\partial\theta_1} - \frac{1}{S}\frac{\partial M_\theta}{\partial S} + \frac{2}{S^2}\frac{\partial M_{S\theta}}{\partial\theta_1} + \frac{1}{S^2}\frac{\partial^2 M_\theta}{\partial\theta_1^2}$$

$$+\frac{N_\theta}{S}ctg\gamma + N_S^0\frac{\partial^2 u}{\partial S^2} + \frac{N_\theta^0}{S}\left(\frac{1}{S}\frac{\partial^2 u}{\partial\theta_1^2} + \frac{\partial u}{\partial S}\right) + 2N_{S\theta}^0\frac{\partial}{\partial S}\left(\frac{1}{S}\frac{\partial u}{\partial\theta_1}\right) - \tilde{\rho}h\frac{\partial^2 u}{\partial t^2} = 0 \tag{8}$$

$$\frac{ctg\gamma}{S}\frac{\partial^2 u}{\partial S^2} - \frac{2}{S}\frac{\partial^2 e_{S\theta}}{\partial S\partial\theta_1} - \frac{2}{S^2}\frac{\partial e_{S\theta}}{\partial\theta_1} + \frac{\partial^2 e_\theta}{\partial S^2} + \frac{1}{S^2}\frac{\partial^2 e_S}{\partial\theta_1^2} + \frac{2}{S}\frac{\partial e_\theta}{\partial S} - \frac{1}{S}\frac{\partial e_S}{\partial S} = 0 \tag{9}$$

where N_S^0, N_θ^0 and $N_{S\theta}^0$ are the membrane forces in the fundamental configuration, $\tilde{\rho} = \rho_0 \int_{-1/2}^{1/2} [1 + \mu\varphi_4(\bar{\varsigma})]d\bar{\varsigma}$ and t is time. The shell is subject to a uniform external pressure (see Fig. 1), varying as a power function of time as follows (Yakushev 1990, Aksogan and Sofiyev 2000):

$$N_S^0 = 0, \quad N_\theta^0 = -S(P_1 + P_0 t^\alpha)tg\gamma, \quad N_{S\theta}^0 = 0 \tag{10}$$

where P_0 is the loading speed, P_1 is the static external pressure, α is a positive whole number power which expresses the time dependence of the external pressure satisfying $\alpha \geq 1$. Substituting expressions (2, 4) in (6) and considering the resulting expressions together with relations (7), after some rearrangements the relations found for moments and strains, being substituted in (8-9) together with relations (7, 10), one gets the following pair of equations, written in matrix from, for u and F:

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{Bmatrix} F \\ u \end{Bmatrix} = 0 \tag{11}$$

where L_{ij} (i, j = 1,2) are the partial differential operators, which are defined in the Appendix.

3. Solution of the differential equations

Considering the shell to be simply supported along the peripheries of both bases, the displacement and stress functions u and F, can be chosen as follows (Baktieva *et al*. 1988):

$$u = \xi(t)e^{\lambda r}\sin m_1 r\cos n_1\theta_1, \quad F = \zeta(t)S_2 e^{\lambda_1 r}\sin m_1 r\cos n_1\theta_1 \tag{12}$$

where $\xi(t)$ and $\zeta(t)$ are time dependent amplitudes, $m_1 = \pi/l$, $l = \ln(S_2/S_1)$, $n_1 = n/\sin\gamma$, $\lambda_1 = \lambda + 1$ and n is the wave number in the circumferential direction. For a truncated cone, the parameter λ

varies with the geometric parameter l as follows (Sachenkov and Aganesov 1964):

$$\lambda = 1.2 \text{ when } l < 2.7,\ \lambda = 1.6 \text{ when } 2.7 \leq l \leq 3.5 \text{ and } \lambda = 2.0 \text{ when } l < 3.5 \tag{13}$$

After applying $r = \ln(S/S_2)$ transformation to this system of equations and taking derivatives with respect to variables θ_1 and S, each at a time, it is noted that, the functions involved in them should be steeply increasing with respect to θ_1 and varying slowly with respect to S. Taking these properties into consideration, neglecting small terms, multiplying the first equation by $uS_2^2 e^{2r} dr d\theta_1$ and the second by $FS_2^2 e^{2r} dr d\theta_1$, considering Eq. (12), for $0 \leq \theta_1 \leq 2\pi \sin\gamma$ and $-l \leq r \leq 0$, applying Galerkin's method and eliminating $\zeta(t)$ from the equations, thus obtained, one gets

$$\frac{d^2\xi(\tau)}{d\tau^2} + \left[\Lambda - (P_1 + P_0\tau^{\alpha} t_{kr}^{\alpha})\frac{t_{kr}^2 \delta_{1/2} tg\gamma n_1^2}{\tilde{\rho} h S_2}\right]\xi(\tau) = 0 \tag{14}$$

in which $t = t_{kr}\tau$, t_{kr} being the critical time and τ being the dimensionless time parameter such that $0 \leq \tau \leq 1$. In Eq. (14) the following definitions apply:

$$\Lambda = \frac{t_{kr}^2}{\tilde{\rho} h S_2^2}\left[\frac{q}{b_{11} S_2^2} n_1^4 + \frac{m_2^2}{n_1^4}\frac{\delta_0}{b_{11}} ctg^2\gamma\right] \tag{15}$$

$$q = (b_{11}c_{24} - c_{21}b_{14})\delta_{-1} \tag{16}$$

$$m_2^2 = (m_1^2 + \lambda^2)(m_1^2 + \lambda^2 - 1) \tag{17}$$

$$\delta_k = \frac{[1 - e^{-2(\lambda + k)l}][m_1^2 + (\lambda + 1)^2](\lambda + 1)}{[1 - e^{-2(\lambda + 1)l}][m_1^2 + (\lambda + k)^2](\lambda + k)},\ k = -1,\ 0,\ 1/2 \tag{18}$$

In pertinent literature the approximating function satisfying Eq. (14) has been chosen as a first approximation in the following form (Sachenkov and Baktieva 1978):

$$\xi(\tau) = Ae^{\beta\tau}\tau[(\beta + 2)(\beta + 1)^{-1} - \tau] \tag{19}$$

and satisfies the initial conditions $\xi(0) = 0$, $\dfrac{\partial\xi(1)}{\partial\tau} = 0$. Here, β is an unknown coefficient and the displacement amplitude A is found from the condition of transition to static condition.

Applying the method given by Sachenkov and Baktieva (1978) to Eq. (14), i.e., multiplying it by $\xi'(\tau)$ and integrating with respect to τ, first from 0 to τ and then from 0 to 1, one finds the following characteristic equation:

$$P_0 t_{kr}^{\alpha} = B_0(\alpha, \beta)\left[\frac{q}{S_2^3}\frac{ctg\gamma}{\delta_{1/2} b_{11}} n_1^2 + \frac{1}{S_2}\frac{m_2^2}{b_{11}}\frac{\delta_0}{\delta_{1/2}}\frac{ctg^3\gamma}{n_1^6} - P_1\right] + \frac{B_1(\alpha, \beta)\tilde{\rho} h S_2}{t_{kr}^2 \delta_{1/2}}\frac{ctg\gamma}{n_1^2} \tag{20}$$

where the following definitions apply:

$$B_0(\alpha,\beta) = \frac{\int_0^1 [\xi(\tau)]^2 d\tau}{2\int_0^1\int_0^\tau \eta^\alpha \xi'(\eta)\xi(\eta)d\eta d\tau}, \quad B_1(\alpha,\beta) = \frac{\int_0^1 [\xi'(\tau)]^2 d\tau}{2\int_0^1\int_0^\tau \eta^\alpha \xi'(\eta)\xi(\eta)d\eta d\tau} \tag{21}$$

Minimizing $(P_0 t_{kr}^\alpha)$ with respect to n_1^2 and considering the result in Eq. (20), after some mathematical operations, the following equation is found to determine the minimum critical load:

$$P_0 t_{kr}^\alpha = 2B_0(\alpha,\beta)\left[\frac{q}{S_2^3}\frac{ctg\gamma}{\delta_{1/2}b_{11}}n_1^2 - \frac{m_2^2}{S_2 b_{11}}\frac{\delta_0}{\delta_{1/2}}\frac{ctg^3\gamma}{n_1^6} - \frac{P_1}{2}\right] \tag{22}$$

For $P_1 = 0$ and $P_0 \geq 200$ MPa/s$^\alpha$, eliminating t_{kr} from Eqs. (20) and (22), solving the resulting equation for n_1 wave parameter and taking the relation $n_1 = n/\sin\gamma$ into consideration, one finds (Aksogan and Sofiyev 2000).

$$n_d^2 = [m_2^2 S_2^2 \delta_0 q^{-1} ctg^2\gamma]^{1/4} C^{\alpha/(2+2\alpha)} \sin^2\gamma \tag{23}$$

where the wave number n_d, which is dependent on the way the dynamic load varies, characterizes the form of loss of stability of the shell under the dynamic load and the following definition applies:

$$C = \frac{B_1(\alpha,\beta)\tilde{\rho}h(P_0\delta_{1/2})^{2/\alpha} S_2^{(3\alpha+5)/\alpha} b_{11}^{(2+\alpha)/\alpha}}{2^{2/\alpha} B_0^{(2+\alpha)/\alpha}(\alpha,\beta)[m_2^2\delta_0]^{(1+\alpha)/(2\alpha)}[qctg^2\gamma]^{(3+\alpha)/(2\alpha)}} \tag{24}$$

When $P_1 = 0$, substituting expression (23) in Eq. (22), the dynamic critical load is found as follows:

$$P_{kr}^d = P_0 t_{kr}^\alpha = \frac{2B_0(\alpha,\beta)\delta_0^{1/4}}{\delta_{1/2}}\frac{m_2^{1/2} q^{3/4} ctg^{3/2}\gamma}{b_{11} S_2^{5/2}} C^{\alpha/(2+2\alpha)} \tag{25}$$

The β coefficient, for which this dynamic critical load takes its minimum value, is found as the ordinate of the minimum point of (P_{kr}^d, β) parabola and, for external pressures given as a power function of time, it can be shown by numerical computations that it corresponds to $\beta = \alpha + 1$. In the static case ($t_{kr} \to \infty$, $P_0 \to 0$) the following equation is found for the wave number corresponding to the static critical load:

$$n_{st}^2 = [3m_2^2 S_2^2 \delta_0 q^{-1} ctg^2\gamma]^{1/4} \sin^2\gamma \tag{26}$$

When $P_1 = 0$, substituting expression (26) in Eq. (22), the static critical load, which refers to $P_0 t_{kr}^\alpha / B_0(\alpha, \beta)$, is given as

$$P_{kr}^{st} = \frac{4}{3^{3/4}}\frac{\delta_0^{1/4}}{\delta_{1/2}}\frac{m_2^{1/2} q^{3/4} ctg^{3/2}\gamma}{b_{11} S_2^{5/2}} \tag{27}$$

and, from the definition $K_d = P_{kr}^{d} / P_{kr}^{st}$, the dynamic factor is given as

$$K_d = \frac{3^{3/4} B_0(\alpha, \beta)}{2} C^{\alpha/(2+2\alpha)} \tag{28}$$

The critical time can be found from Eq. (25) as

$$t_{kr} = \left[\frac{2B_0(\alpha, \beta)\delta_0^{1/4} m_2^{1/2} q^{3/4} ctg^{3/2}\gamma}{P_0 \delta_{1/2} b_{11} S_2^{5/2}}\right]^{1/\alpha} C^{1/(2+2\alpha)} \tag{29}$$

The corresponding critical impulse is obtained as

$$I_{kr} = \int_0^{t_{kr}} P_0 t^{\alpha} dt = \frac{C^{1/2}}{(1+\alpha)P_0^{1/\alpha}}\left[\frac{2B_0(\alpha, \beta)\delta_0^{1/4} m_2^{1/2} q^{3/4} ctg^{3/2}\gamma}{\delta_{1/2} b_{11} S_2^{5/2}}\right]^{(1+\alpha)/\alpha} \tag{30}$$

Remembering that, for a conical shell made of an homogeneous orthotropic elastic material, $\mu = 0$ and the counterparts of (27-30) formulas are found as a special case.

4. Numerical computations and results

Computations have been carried out for the case where the material properties are $E_{0S} = 2.225\times 10^4$ MPa, $E_{0\theta} = 1.085\times10^4$ MPa, $\nu_{S\theta} = 0.117$, $\nu_{\theta S} = 0.057$, $\rho_0 = 1.84\times10^2$ kgs^2/m^4 (Yakushev 1990) and the shell parameters are $R_2 = 8\times10^{-2}$ m, $R_1 = 2.25\times10^{-2}$ m, $h = 2.5\times10^{-4}$ m (Sachenkov and Klementev 1980). For comparison the following material and shell properties have been used $E_0 = 2.11\times10^5$ MPa, $\nu = 0.3$, $\rho_0 = 8\times10^2$ kgs^2/m^4 and $R_2 = 8\times10^{-2}$ m, $R_1 = 2.25\times10^{-2}$ m, $h = 1.3\times10^{-4}$ m (Sachenkov and Klementev 1980) (Table 6). A computer program called MAPLEV2 has been used to compute numerical values from the formulas obtained.

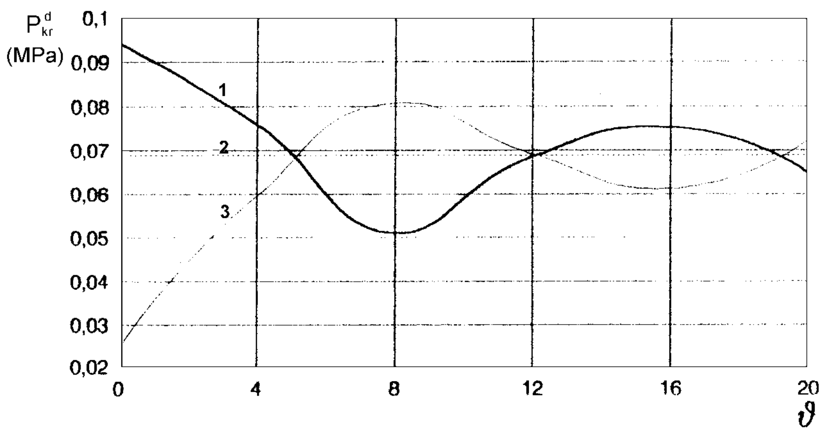

Fig. 2 Variation of dynamic critical load with the parameter ϑ ($\alpha = 1$, $\beta = 2$, $P_0 = 200$ MPa/s)

Table 1 Variation of critical parameters with elasticity moduli ($\alpha = 1$, $\beta = 2$, $P_0 = 200$ MPa/s)

		$\mu = 0$				$\varphi_i(\bar{\varsigma}) = e^{-0.1\|\bar{\varsigma}\|}\cos(0.8\bar{\varsigma})(i=1,2,4), \mu = 0.9$			
γ	$E_{0S}/E_{0\theta}$	n_{st}	n_d	P_{kr}^d (MPa)	K_d	n_{st}	n_d	P_{kr}^d (MPa)	K_d
30°	2	11	10	0.0911	2.0132	11	8	0.1236	1.4899
	5	13	14	0.0724	3.1897	13	12	0.0982	2.3606
45°	2	9	7	0.0824	1.7435	9	6	0.1117	1.2903
	5	10	10	0.0654	2.7624	10	9	0.0888	2.0443
60°	2	7	6	0.0693	2.0132	7	5	0.0940	1.4899
	5	8	9	0.0550	3.1897	8	8	0.0746	2.3606

In Fig. 2 are seen three curves pertaining to the critical dynamic loads for three different forms of the elasticity moduli and density variation functions. In curve 1, the case of $\varphi_i(\bar{\varsigma}) = e^{-0.1|\bar{\varsigma}|}\cos(\vartheta\bar{\varsigma})$, ($i = 1, 2, 4$) and $\mu = 0.90$ is considered. For $0 \le \vartheta \le 5.15$, the dynamic critical load is higher than that for the homogeneous case and takes its maximum value for $\vartheta = 0$ as (0.0942 MPa). For $5.15 < \vartheta \le 12.10$, it is lower than the value for the homogeneous case and takes its minimum value for $\vartheta = 8$ as (0.0510 MPa). Curve 2 corresponds to the homogeneous case and the pertinent value of the dynamic critical load is 0.0688 Mpa. In curve 3, the case of $\varphi_i(\bar{\varsigma}) = -e^{-0.1|\bar{\varsigma}|}\cos(\vartheta\bar{\varsigma})$, ($i = 1, 2, 4$) and $\mu = 0.90$ is considered. For $0 \le \vartheta \le 5.15$, the dynamic critical load is lower than that for the homogeneous case and takes its minimum value for $\vartheta = 0$ as (0.0246 MPa), whereas, for $5.15 < \vartheta \le 12.10$, it takes higher values and is a maximum for $\vartheta = 8$, taking the value (0.0808 MPa). For $\vartheta > 12.10$, the effect of the variation of the elasticity moduli and density on the dynamic critical load is very little (Fig. 2).

When E_{0S} is kept constant and the ratio $E_{0S}/E_{0\theta}$ is increased, the dynamic critical load decreases for both homogeneous and nonhomogeneous cases, whereas, the dynamic factor increases and the effect of the variation of the elasticity moduli and density is relevant. As the ratio $E_{0S}/E_{0\theta}$ increases, the difference between n_{st} and n_d increases, also. Hence, for materials with high degree of anisotropy, an increase in the the difference between n_{st} and n_d can cause the loss of the stability of the conical shell (Table 1).

When $\varphi_i(\bar{\varsigma}) = \pm e^{-0.1|\bar{\varsigma}|}\cos(0.2\bar{\varsigma})$, ($i = 1, 2, 4$) the dynamic critical load, critical time and critical impulse decrease as γ increases. The values of the static critical load and dynamic factor decrease for $20^\circ \le \gamma \le 45^\circ$ and increase for $45^\circ < \gamma \le 80^\circ$. When $\varphi_i(\bar{\varsigma}) = e^{-0.1|\bar{\varsigma}|}\cos(0.2\bar{\varsigma})$, ($i = 1, 2, 4$), the difference between the wave numbers corresponding to the dynamic and static critical loads take values lower than that for the homogeneous case, whereas, when $\varphi_i(\bar{\varsigma}) = -e^{-0.1|\bar{\varsigma}|}\cos(0.2\bar{\varsigma})$, ($i = 1, 2, 4$), it takes higher values. When the angle γ increases the effect of the variations of elasticity moduli and density on the dynamic critical load and dynamic factor does not change appreciably (Table 2).

When α increases, the dynamic critical load and dynamic factor decrease. When the variation of the elasticity moduli and density are given by linear, quadratic and exponential functions, the effect on the critical parameters is more pronounced in the second case than in the first and even more so in the third case. When $\varphi_i(\bar{\varsigma}) = -e^{-0.1|\bar{\varsigma}|}\cos(0.8\bar{\varsigma})$, ($i = 1, 2, 4$), $\mu = 0.9$, compared to the homogeneous case, the dynamic critical load takes 60.15% lower values for $\alpha = 1$ and 75.00% lower values for $\alpha = 4$, whereas, the dynamic factor takes 134% higher values for $\alpha = 1$ and 37.95% higher values for $\alpha = 4$. Consequently, when α increases, the effect of the variation of the elasticity moduli and density on the dynamic critical load increases, whereas, that on the dynamic factor decreases (Table 3).

As the loading speed increases, the dynamic critical load and dynamic factor values increase, too.

Table 2 Variation of critical parameters with $\gamma\,(\alpha = 1,\ \beta = 2,\ P_0 = 200$ MPa/s)

γ	μ	n_d	n_{st}	P_{kr}^{d} (MPa)	P_{kr}^{st} (MPa)	K_d	t_{kr} (s)	I_{kr} (MPa s)
$\varphi_i(\bar{\varsigma}) = e^{-0.1\lvert\bar{\varsigma}\rvert}\cos(0.2\bar{\varsigma})(i=1,2,4)$								
20°	0	14	14	0.0943	0.0344	2.7468	0.0013	0.0017
	0.9	12	14	0.1290	0.0641	2.0117	0.0013	0.0017
30°	0	10	11	0.0906	0.0444	2.0387	0.0012	0.0014
	0.9	8	11	0.1239	0.0830	1.4931	0.0012	0.0014
40°	0	8	10	0.0852	0.0475	1.7928	0.0012	0.0014
	0.9	6	10	0.1165	0.0887	1.3130	0.0012	0.0014
45°	0	7	9	0.0818	0.0464	1.7656	0.0011	0.0012
	0.9	6	9	0.1119	0.0866	1.2931	0.0011	0.0012
50°	0	7	8	0.0780	0.0435	1.7928	0.0011	0.0012
	0.9	6	8	0.1067	0.0813	1.3130	0.0011	0.0012
60°	0	6	7	0.0688	0.0338	2.0387	0.0009	0.0008
	0.9	5	7	0.0941	0.0630	1.4931	0.0009	0.0008
70°	0	6	6	0.0569	0.0207	2.7468	0.0008	0.0006
	0.9	5	6	0.0778	0.0387	2.0117	0.0008	0.0006
80°	0	7	5	0.0406	0.0079	5.1622	0.0006	0.0004
	0.9	6	5	0.0555	0.0147	3.7807	0.0005	0.0003
$\varphi_i(\bar{\varsigma}) = -e^{-0.1\lvert\bar{\varsigma}\rvert}\cos(0.2\bar{\varsigma})(i=1,2,4)$								
20°	0.9	23	14	0.0339	0.0046	7.4572	0.0013	0.0017
30°	0.9	16	11	0.0326	0.0059	5.5349	0.0012	0.0014
40°	0.9	13	9	0.0307	0.0063	4.8673	0.0011	0.0012
45°	0.9	12	9	0.0294	0.0061	4.7934	0.0011	0.0012
60°	0.9	11	8	0.0281	0.0058	4.8673	0.0011	0.0012
70°	0.9	10	7	0.0248	0.0045	5.5349	0.0009	0.0008
80°	0.9	11	6	0.0205	0.0028	7.4572	0.0008	0.0006
80°	0.9	12	5	0.0146	0.0010	14.0149	0.0006	0.0004

Moreover, the values of the wave numbers corresponding to the the dynamic critical loads, increase, also. The difference between the wave numbers corresponding to the dynamic and static critical loads increase as the loading speed increases (Table 4).

When the elasticity moduli vary together with the density in the thickness direction, the effect on the dynamic critical load is higher than when the density is kept constant, whereas, that on the dynamic factor is less. Table 5 shows such results for $\varphi_i(\bar{\varsigma}) = -e^{-0.1\lvert\bar{\varsigma}\rvert}$ cos $(2.1\bar{\varsigma})$. When the density is kept constant and only the elasticity moduli are changed, if the variation function is negative, the increase in the difference between n_{st} and n_d may also contribute to the loss of stability of the shell (Table 5).

As seen from Table 6, the results of the present work have been compared with the theoretical-experimental and purely experimental results of Sachenkov and Klementev (1980) for an homogeneous elastic conical shell subject to an external pressure varying linearly with time and a good match has been observed.

Table 3 Variation of the critical dynamic load and dynamic factor with α and $\varphi_i(\bar{\varsigma})$($\gamma = 30^{\circ}$, $\beta = \alpha + 1$, $P_0 = 200$ MPa/s$^{\alpha}$)

	$\mu = 0$		$\varphi_i(\bar{\varsigma}) = -\bar{\varsigma}$ (i = 1, 2, 4), $\mu = 0.9$		$\varphi_i(\bar{\varsigma}) = -\bar{\varsigma}^2$ (i = 1, 2, 4), $\mu = 0.9$		$\varphi_i(\bar{\varsigma}) = -e^{-0.1\lvert\bar{\varsigma}\rvert}\cos(0.8\bar{\varsigma})$ (i = 1, 2, 4), $\mu = 0.9$	
α	P_{kr}^{d} (MPa)	K_d	P_{kr}^{d} (MPa)	K_d	P_{kr}^{d} (MPa)	K_d	P_{kr}^{d} (MPa)	K_d
1.0	0.0906	2.0387	0.0890	2.1112	0.0857	2.1920	0.0361	4.7756
2.0	0.0093	0.2089	0.0091	0.2151	0.0086	0.2205	0.0027	0.3684
3.0	0.0034	0.0756	0.0033	0.0777	0.0031	0.0791	0.0009	0.1145
4.0	0.0020	0.0440	0.0019	0.0451	0.0018	0.0458	0.0005	0.0607

Table 4 The variation of the critical parameters at different loading speeds ($\alpha = 1$, $\beta = 2$)

		$\mu = 0$				$\varphi_i(\bar{\varsigma}) = e^{-0.1\lvert\bar{\varsigma}\rvert}\cos(2.1\bar{\varsigma})$ (i = 1, 2, 4), $\mu = 0.9$			
γ	P_o (MPa/s)	n_{st}	n_d	P_{kr}^{d} (MPa)	K_d	n_{st}	n_d	P_{kr}^{d} (MPa)	K_d
30°	200	11	10	0.0906	2.0387	11	12	0.0521	3.2217
	470	11	12	0.1388	3.1253	11	15	0.0798	4.9388
	650	11	13	0.1633	3.6754	11	16	0.0939	5.8080
60°	200	7	6	0.0688	2.0387	7	8	0.0396	3.2217
	470	7	8	0.1055	3.1253	7	9	0.0607	4.9388
	650	7	9	0.1241	3.6754	7	10	0.0713	5.8080

Table 5 The variation of the critical parameters with the elasticity moduli and density ($\alpha = 1$, $\beta = 2$, $P_0 = 200$ MPa/s, $\varphi_i(\bar{\varsigma}) = -e^{-0.1\lvert\bar{\varsigma}\rvert}\cos(2.1\bar{\varsigma})$)

		(i = 1, 2, 4)				(i = 1, 2)				(i = 4)			
P_{kr}^{d} (MPa)	μ	n_{st}	n_d	P_{kr}^{d} (MPa)	K_d	n_{st}	n_d	P_{kr}^{d} (MPa)	K_d	n_{st}	n_d	P_{kr}^{d} (MPa)	K_d
30°	0	11	10	0.0906	2.0387	11	10	0.0906	2.0387	11	10	0.0906	2.0387
	0.9	11	12	0.0521	3.2217	11	14	0.0721	4.4574	11	8	0.0655	1.4735
45°	0	9	7	0.0818	1.7656	9	7	0.0818	1.7656	9	7	0.0818	1.7656
	0.9	8	9	0.0471	2.7901	8	10	0.0651	3.8602	9	6	0.0592	1.2761
60°	0	7	6	0.0688	2.0382	7	6	0.0688	2.0387	7	6	0.0688	2.0387
	0.9	7	8	0.0396	3.2217	7	9	0.0547	4.4574	7	5	0.0497	1.4735

Table 6 Comparison of critical parameters with those of Sachenkov and Klementev (1980) ($\alpha = 1$, $\beta = 2$, $P_0 = 225$ MPa/s)

	Theoretical-experimental			Experimental			Present work ($\mu = 0$)		
γ	P_{kr}^{st} (MPa)	P_{kr}^{d} (MPa)	K_d	P_{kr}^{st} (MPa)	P_{kr}^{d} (MPa)	K_d	P_{kr}^{st} (MPa)	P_{kr}^{d} (MPa)	K_d
20°	0.0208	0.0837	4.0240	0.0200	0.0575	2.8800	0.0205	0.0806	3.9248
30°	0.0269	0.0720	2.6755	0.0270	0.0726	2.6900	0.0266	0.0774	2.9131
40°	0.0288	0.0699	2.4320	0.0300	0.0810	2.7000	0.0284	0.0728	2.5617

5. Conclusions

The dynamic stability of an orthotropic elastic truncated conical shell, with elasticity moduli and density varying continuously in the thickness direction, subject to a uniform external pressure which is a power function of time, has been studied and general formulas have been obtained for the critical parameters. The following conclusions have been drawn from the numerical analysis carried out using the general formulas obtained from the analytical study:

(a) When the semi-vertex angle γ of the conical shell increases, the values of the dynamic critical load, the critical time and the critical impulse decrease. Furthermore, the static critical load and dynamic factor get lower for $20^o \leq \gamma \leq 45^o$ and higher for $45^o \leq \gamma \leq 80^o$.

(b) When the power of time, α, in the external pressure expression increases, the effect of nonhomogeneity on the dynamic critical load increases, whereas, that on the dynamic factor decreases.

(c) When the variation of the elasticity moduli and the density are given by linear, quadratic and exponential functions, it is observed that the effect of this nonhomogeneity on the critical parameters is relatively more for the exponential functions. Furthermore, when the variation function is negative the conical shell gets more unstable.

(d) When the elasticity moduli and the density vary continuously in the thickness direction, the effect on the dynamic critical load is higher than when the density is kept constant, whereas, the effect on the dynamic factor is less.

(e) An increase in the loading speed P_o causes the dynamic critical load, the dynamic factor and the wave number corresponding to the dynamic critical load to increase, also.

(f) When the density is kept constant and only the elasticity moduli are changed, for negative variation functions and with materials having high degrees of anisotropy, the difference between n_{st} and n_d increases.

These observations have shown that the effects of the variations of the elasticity moduli, the power α, the semi-vertex angle γ and the loading speed P_0, on the critical parameters of the problem in the heading, are rather important.

References

Aksogan, O., and Sofiyev, A. (2000), "The dynamic stability of a laminated nonhomogeneous orthotropic elastic cylindrical shell under a time dependent external pressure", *Int. Conf. on Modern Practice in Stress and Vibration Analysis*, Nottingham, UK, 349-360.

Baktieva, L.U., Jigalko, YU. P., Konoplev, YU. G., Mitryaikin, V.I., Sachenkov, A.V., and Filippov, E.B. (1988), "The stability and vibrations of shells under impulsive distribution and local loads", *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **l**, 113-130. (in Russian)

Baruch, M., Harari, O., and Singer, J. (1970), "Low buckling loads of axially compressed conical shells", *J. Appl. Mech.*, **37**, 384-392.

Brinkman, J.A. (1954), "On the nature of radiation damage in metals", *J. Applied Physics*, **25**, 961-970.

Delale, F., and Erdogan, F. (1983), "The crack problem for a nonhomogeneous plane", *J. Appl. Mech.*, ASME, **50**, 609-614.

Gutierrez, R.H., Laura, P.A.A., Bambill, D.V., Jederlinic, V.A., and Hodges, D.H. (1998), "Axisymmetric vibrations of solid circular and annular membranes with continuously varying density", *J. Sound and Vibrations*, **212**(4), 611-622.

Heyliger, P.R., and Julani, A. (1992), "The free vibrations of inhomogeneous elastic cylinders and spheres", *Int.*

J. Solids and Struct., **29**, 2689-2708.

Irie, T., Yamada, G., and Kaneko,Y. (1984), "Natural frequencies of truncated conical shells", *J. Sound and Vibrations*, **92**, 447-453.

Lam, K.Y., and Hua, L. (1999), "Influence of boundary conditions on the frequency characteristics of a rotating truncated circular conical shell", *J. Sound and Vibrations*, **223**, 171-195.

Leissa, A.W. (1973), *Vibration of Shells*, NASA SP-288.

Lekhnitski, S.G. (1980), *Theory of Elasticity of an Anisotropic Elastic Body,* Holden Day, San Francisco, Also Mir Publishers, Moscow.

Lomakin, V.A. (1976), *The Elasticity Theory of Nonhomogeneous Materials*, Moscow, Nauka. (in Russian)

Massalas, C., Dalamanagas, D., and Raptis, A. (1982), "Dynamic characteristics of conical shell with variable modulus of elasticity", Review Roumanie Sciences Techniques, Mechanics Applications, Bucharest, **27**, 609-628.

Massalas, C., Dalamanagas, D., and Tzivanidis, G. (1981), "Dynamic instability of truncated conical shells with variable modulus of elasticity under periodic compressive forces", *J. Sound and Vibrations*, **79**, 519-528.

Mecitoglu, Z. (1996), "Governing equations of a stiffened laminated inhomogeneous conical shell", American Institute of Aeronautics and Astronautics J., **34**, 2118-2125.

Mushtari, K.M., and Sachenkov, A.V. (1958), Stability of Cylindrical and Conical Shells of Circular Cross Section with Simultaneous Action of Axial Compression and External Normal Pressure, NASA TM-1433.

Sachenkov, A.V. (1976), "The dynamic criterion of the stability of plates and shells", *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **12**, 281-293. (in Russian)

Sachenkov, A.V., and Aganesov, L.G (1964), "The stability and vibration of circular conical and cylindrical shells at different boundary conditions", *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **2**, 111-126. (in Russian)

Sachenkov, A.V., and Baktieva, L.U. (1978), *Research on the theory of plates and shells*, "Approach to the Solution of Dynamic Stability Problems of Thin Shells", Kazan State University, Kazan, **13**, 137-152. (in Russian)

Sachenkov, A.V., and Klementev, G.G. (1980), "Research of the stability of conical shells by theoretical-experimental method under impulsive load", *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **15**, 115-125. (in Russian)

Singer, J. (1966), "Buckling of damped conical shells under external pressure", American Institute of Aeronautics and Astronautics J., **4**, 328-337.

Singer, J. (1961), "Buckling of circular conical shells under axisymmetrical external pressure", *J. Mech. Eng. Sci.*, **3**, 330-339.

Sivadas, K.R., and Ganesan, N. (1991), "Vibration analysis of laminated conical shells with variable thickness", *J. Sound and Vibrations*, **148**, 477-491.

Sofiyev, A., and Aksogan, O. (1999), "Dynamic stability of a nonhomogeneous orthotropic elastic cylindrical shell under a time dependent external pressure", Technical Journal, Chamber of Civil Engineers of Turkey, **10**, 2011-2028.

Tani, J. (1973), "Dynamic stability of truncated conical shells under periodic external pressure", *The Report of the Institute of High Speed Mechanics*, Tohoku University, Japan, **28**, 135-147.

Tani, J. (1981), "Dynamic stability of truncated conical shells under pulsating torsion", Transactions of the *J. Appl. Mech.*, ASME, **48**, 391-398.

Tong, L. (1993), "Free vibration of orthotropic conical shells", *Int. J. Eng. Sci.*, **31**, 719-733.

Tong, L., Tabarrok, B., and Wang, T.K. (1992), "Simple solution for buckling of orthotropic conical shells", *J. Solids and Struct.*, **29**, 933-946.

Volmir, A.S. (1967), *The Stability of Deformable Systems*, Nauka, Moscow. (in Russian)

Yakushev, A.N. (1990), "The stability of orthotropic cylindrical shells under dynamic pressure", *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **20**, 215-222. (in Russian)

Zhang, X., and Hasebe, N. (1999), "Elasticity solution for a radially nonhomogeneous hollow circular cylinder", *J. Appl. Mech.*, ASME, **66**, 598-606.

Appendix

The coefficients a_{ij}, b_{ij} and c_{ij}, (i, j = 1, 2, 3, 4) and L_{ij}, (i, j = 1, 2) appearing in some equations in this paper are defined as follows:

$$L_{11} = c_{12}\frac{\partial^4}{\partial S^4} + \frac{c_{11}+2c_{12}-c_{22}}{S}\frac{\partial^3}{\partial S^3} + \left(\frac{ctg\gamma}{S} - \frac{c_{21}}{S^2}\right)\frac{\partial^2}{\partial S^2} + \frac{c_{21}}{S^3}\frac{\partial}{\partial S} +$$

$$\frac{c_{21}}{S^4}\frac{\partial^4}{\partial\theta_1^4} + \frac{c_{11}-2c_{31}+c_{22}}{S^2}\frac{\partial^4}{\partial S^2\partial\theta_1^2} + \frac{2(c_{31}-c_{11})}{S^3}\frac{\partial^3}{\partial S\partial\theta_1^2} + \frac{2(c_{11}-c_{31}+c_{21})}{S^4}\frac{\partial^2}{\partial\theta_1^2},$$

$$L_{12} = -\frac{c_{24}}{S^4}\frac{\partial^4}{\partial\theta_1^4} - \frac{c_{14}+c_{23}+2c_{32}}{S^2}\frac{\partial^4}{\partial S^2\partial\theta_1^2} + \frac{2(c_{14}+c_{32})}{S^3}\frac{\partial^3}{\partial S\partial\theta_1^2} -$$

$$\left[\frac{P_1+P_0t^\alpha}{S ctg\gamma} + \frac{2(c_{14}+c_{32}+c_{24})}{S^4}\right]\frac{\partial^2}{\partial\theta_1^2} - c_{13}\frac{\partial^4}{\partial S^4} + \frac{c_{23}-c_{14}-2c_{13}}{S}\frac{\partial^3}{\partial S^3} +$$

$$+\frac{c_{24}}{S^2}\frac{\partial^2}{\partial S^2} - \left(\frac{P_1+P_0t^\alpha}{ctg\gamma} + \frac{c_{24}}{S^3}\right)\frac{\partial}{\partial S} - \tilde{\rho}h\frac{\partial^2}{\partial t^2},$$

$$L_{21} = \frac{b_{11}}{S^4}\frac{\partial^4}{\partial\theta_1^4} + \frac{b_{31}+b_{21}+b_{12}}{S^2}\frac{\partial^4}{\partial S^2\partial\theta_1^2} - \frac{b_{31}+2b_{21}}{S^3}\frac{\partial^3}{\partial S\partial\theta_1^2} + \frac{b_{31}+2b_{21}+2b_{11}}{S^4}\frac{\partial^2}{\partial\theta_1^2} +$$

$$\frac{b_{14}}{S^3}\frac{\partial}{\partial S} + \frac{b_{21}-b_{11}}{S^2}\frac{\partial^2}{\partial S^2} + \frac{b_{21}+2b_{22}-b_{12}}{S}\frac{\partial^3}{\partial S^3} + b_{22}\frac{\partial^4}{\partial S^4},$$

$$L_{22} = -\frac{b_{14}}{S^4}\frac{\partial^4}{\partial\theta_1^4} + \frac{b_{32}-b_{13}-b_{24}}{S^2}\frac{\partial^4}{\partial S^2\partial\theta_1^2} + \frac{2b_{24}-b_{32}}{S^3}\frac{\partial^3}{\partial S\partial\theta_1^2} + \frac{b_{32}-2b_{24}-2b_{14}}{S^4}\frac{\partial^2}{\partial\theta_1^2} -$$

$$\frac{b_{11}}{S^3}\frac{\partial}{\partial S} + \left(\frac{b_{14}}{S^2} + \frac{ctg\gamma}{S}\right)\frac{\partial^2}{\partial S^2} + \frac{b_{13}-b_{24}-2b_{23}}{S}\frac{\partial^3}{\partial S^3} - b_{23}\frac{\partial^4}{\partial S^4},$$

$$c_{11} = a_{11}^1 b_{11} + a_{12}^1 b_{21}, \quad c_{12} = a_{11}^1 b_{12} + a_{12}^1 b_{22}, \quad c_{13} = a_{11}^1 b_{13} + a_{12}^1 b_{23} + a_{11}^2,$$

$$c_{14} = a_{11}^1 b_{14} + a_{12}^1 b_{24} + a_{12}^2, \quad c_{21} = a_{21}^1 b_{11} + a_{22}^1 b_{21}, \quad c_{22} = a_{21}^1 b_{12} + a_{22}^1 b_{22},$$

$$c_{23} = a_{21}^1 b_{13} + a_{22}^1 b_{23} + a_{21}^2, \quad c_{24} = a_{21}^1 b_{14} + a_{22}^1 b_{24} + a_{22}^2, \quad c_{31} = a_{33}^1 b_{31},$$

$$c_{32} = a_{33}^1 b_{32} + 2a_{33}^2, \quad b_{11} = a_{22}^0 L_0^{-1}, \quad b_{12} = -a_{12}^0 L_0^{-1}, \quad b_{13} = (a_{12}^0 a_{21}^1 - a_{11}^1 a_{22}^0)L_0^{-1},$$

$$b_{14} = (a_{12}^0 a_{22}^1 - a_{12}^1 a_{22}^0)L_0^{-1}, \quad b_{21} = -a_{21}^0 L_0^{-1}, \quad b_{22} = a_{11}^0 L_0^{-1}, \quad b_{23} = (a_{21}^0 a_{11}^1 - a_{21}^1 a_{11}^0)L_0^{-1},$$

$$b_{24} = (a_{21}^0 a_{12}^1 - a_{22}^1 a_{11}^0)L_0^{-1}, \quad b_{31} = 1/a_{33}^0, \quad b_{32} = -2a_{33}^1/a_{33}^0, \quad L_0 = a_{11}^0 a_{22}^0 - a_{21}^0 a_{12}^0,$$

$$a_{11}^k = \frac{E_{0S}h^{k+1}}{1-\nu_{S\theta}\nu_{\theta S}}\int_{-1/2}^{1/2}\bar{\varsigma}^k[1+\mu\varphi_1(\bar{\varsigma})]d\bar{\varsigma}, \quad a_{22}^k = \frac{E_{0\theta}h^{k+1}}{1-\nu_{S\theta}\nu_{\theta S}}\int_{-1/2}^{1/2}\bar{\varsigma}^k[1+\mu\varphi_2(\bar{\varsigma})]d\bar{\varsigma},$$

$$a_{12}^k = \nu_{\theta S}a_{11}^k, \quad a_{21}^k = \nu_{S\theta}a_{22}^k, \quad a_{33}^k = 2G_0h^{k+1}\int_{-1/2}^{1/2}\bar{\varsigma}^k[1+\mu\varphi_3(\bar{\varsigma})]d\bar{\varsigma}, \quad k = 0, 1, 2$$

Notation

E_{0S}, $E_{0\theta}$ Elasticity moduli of the homogeneous orthotropic materials

E_0	Elasticity moduli of the homogeneous isotropic materials
G_0	Shear moduli of the homogeneous materials
e_S, e_θ, $e_{S\theta}$	Strain components on the middle surface of the conical shell
F	Stress Function
h	Thickness of the shell
K_d	Dynamic factor
M_S, M_θ, $M_{S\theta}$	Internal moments per unit length of the cross-section of the shell
N_S, N_θ, $N_{S\theta}$	Internal forces per unit length of the cross-section of the shell
N_S^0, N_θ^0, $N_{S\theta}^0$	Membrane forces prior to buckling
n	Wave number in the circumferential direction
n_{st}	Wave number corresponding to the static critical load
n_d	Wave number corresponding to the dynamic critical load
p_{kr}^{st}	Static critical load
p_{kr}^{d}	Dynamic critical load
P_0	Loading speed
P_1	Static external pressure
R_1 and R_2	Average radii of the small and large bases of the conical shell
$S\theta\varsigma$	Curvilinear coordinate system on the middle surface of the conical shell
S	The coordinate axis through the vertex on the curvilinear middle surface of the cone
0	The angle of rotation around the longitudinal axis starting from a radial plane
S_1 and S_2	The inclined distances of the bases of the cone from the vertex
t	Time
t_{kr}	Critical time
u	Displacement of the middle surface in the inwards normal direction ς
α	Power of time in the external pressure expression
χ_S, χ_θ, $\chi_{S\theta}$	Curvatures of the middle surface
ε_S, ε_θ, $\varepsilon_{S\theta}$	Strains in the curvilinear coordinate directions
γ	Semi-vertex angle of the cone
τ	Dimensionless time parameter
ρ_0	Density of the homogeneous materials
$v_{S\theta}$, $v_{\theta S}$	Poisson's ratios of the homogeneous orthotropic materials
v_0	Poisson's ratios of the homogeneous materials
μ	Elasticity moduli and densities variation coefficient
λ	A parameter that depends on the geometry of the shell
σ_S, σ_θ, $\sigma_{S\theta}$	Stress components
$\varphi_i(\varsigma)$	Variation functions of the elasticity moduli and densities
$\xi(t)$, $\zeta(t)$	Time dependent amplitudes
ς	The coordinate axis in the inwards normal direction of the middle surface of the cone