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Confidence region of identified parameters and optimal sensor locations based on sensitivity analysis

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Abstract. This paper presents a computational method for a confidence region of identified parameters which are affected by measurement noise and error contained in prescribed parameters. The method is based on sensitivities of the identified parameters with respect to model parameter error and measurement noise along with the law of error propagation. By conducting numerical experiments on simple models, it is confirmed that the confidence region coincides well with the results of numerical experiments. Furthermore, the optimum arrangement of sensor locations is evaluated when uncertainty exists in prescribed parameters, based on the concept that square sum of coefficients of variations of identified results attains minimum. Good agreement of the theoretical results with those of numerical simulation confirmed validity of the theory.

Key words: parameter identification; sensitivity analysis; confidence region; optimal sensor location.

1. Introduction

Identification of structural parameters to estimate dynamic behavior of the ground and structures has been found to be effective tools for seismic engineering. Numerous studies (Hoshiya and Sito 1984, Toki *et al.* 1989, Matsui and Kurita 1989, Koh *et al.* 1991, Elgamal *et al.* 1996, Zeghal *et al.* 1996) up to the present have proposed various identification methods and a valuable body of knowledge has been accumulated. Consequently, identification methods have achieved significant improvement. However, further improvement also depends on accuracy of measurement and progress in measurement technology has been greatly anticipated. Numerous uncertainties are involved in structural identification problems, which inevitably influence identified results. Observed values such as input signals and output responses are contaminated with observation noise. Parameter values given as known may also contain some error. Past studies have demonstrated that the effect of observation noise can be reduced by using some filters such as the Kalman filter and dynamic programming filter (Ott and Meder 1970, Destefano and Pena-Pardo 1996). There has been

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little investigation of the effect of prescribed parameter. Koh and See (1994) proposed to introduce system noise after they demonstrated that identification by the extended Kalman filter using the model with error in one of the masses did not converge to the correct parameter values. Yoshida and Hoshiya (1994) proposed a method of inverse analysis which took uncertainties of known conditions for static problems into consideration and discussed the effect of known and unknown conditions, including the degree of certainty in prior information. The authors presented a method of evaluating the effect of identified parameters and their sensitivity with respect to error and confirmed the validity of the method through numerical experiments (Matsui and Kurita 1990).

This paper presents a method to evaluate the effect of error in prescribed parameters on the identified results by employing the confidence region of identified parameters. Validity is confirmed by numerical experiments on simple models. The effect of observation noise on identified results is also given by a confidence region. Combining both approaches enable evaluation of the unified effect of observation noise and prescribed parameter error.

Furthermore, when identifying structural parameters, sensor locations are presumed to be important in addition to accuracy of measurement. Studies in this area have not yet fully explored the matter of sensor locations. Kiyono *et al.* (1991) presented optimum sampling locations when estimating a model parameter for a ground model considering nonuniformity. Udwadia (1994) also has proposed optimum arrangement of sensors in dynamic structural parameter identification based on the Fisher information theory.

This paper presents a method to find the optimum arrangement of measurement locations when prescribed parameter values involve uncertainties. In actual problems, the values of these parameters are not precisely known and likely to contain some error. Thus, it is presumed here that the definition of optimum sensor arrangement is a set of sensor locations which produce the least fluctuation of identified results. Monte Carlo simulation is an approach to solve this type of problem but the method, in general, requires an enormous computation time. In this paper, the method described above is utilized to evaluate the optimality of sensor locations.

2. Confidence region due to error in prescribed parameter

2.1 Theory

The motion of a linear multi degree-of-freedom system is described by the following equation:

$$\boldsymbol{M}\ddot{\boldsymbol{z}}(t) + \boldsymbol{c}\dot{\boldsymbol{z}}(t) + \boldsymbol{K}\boldsymbol{z}(t) = -\boldsymbol{M}\boldsymbol{1}\ddot{y}_0(t) \tag{1}$$

in which $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are mass, damping and stiffness matrices; $\ddot{\boldsymbol{z}}(\boldsymbol{t})$, $\dot{\boldsymbol{z}}(\boldsymbol{t})$ and $\boldsymbol{z}(\boldsymbol{t})$ are relative acceleration, velocity and displacement. **1** refers to a vector whose entities are all 1 and $\ddot{y}_0(t)$ is ground surface acceleration. Among all model parameters, unknown parameters are expressed by $\boldsymbol{X} = \{X_1,\ X_2,\ \ldots, X_M\}^T$ and known parameters by $\boldsymbol{Y} = \{Y_1,\ Y_2,\ \ldots, Y_L\}^T$. Measured values are time histories of response accelerations. Let $\ddot{u}_i(t)$ be a measured response, $\Delta\ddot{u}_i(t)$ observation noise and $\ddot{z}_i(t)$ computed acceleration at observation point *i*. Then the following relationship holds;

$$\ddot{u}_i(t) = \ddot{z}_i(t) + \Delta\ddot{u}_i(t) \qquad ,\ i \in A \tag{2}$$

Based on the least square concept, an evaluation function may be defined as,

$$J(\boldsymbol{X},\ \boldsymbol{Y}) = \frac{1}{2}\int_{t0}^{t1}\sum_{i\in A} w_i\{\ddot{u}_i - \ddot{z}_i(\boldsymbol{X},\ \boldsymbol{Y})\}^2\ dt \tag{3}$$

where $t_0 \sim t_1$ refers to a duration of analysis and w_i is a weight which is given according to importance and/or reliability of the data relative to the rest. Let known parameter Y_l be defined by the true value $\overline{Y}_l$ and error ΔY_l. If ΔY_l is sufficiently small, its effect on identified results is also small. It can be written as,

$$\boldsymbol{X} = \overline{\boldsymbol{X}} + \frac{\partial \boldsymbol{X}}{\partial Y_l}\Delta Y_l \tag{4}$$

$\overline{\boldsymbol{X}}$ is the true value of $\boldsymbol{X}$. If a covariance matrix of known parameters is assigned, a covariance matrix of identified results is presented by using the law of error propagation (Tajima and Komaki 1986).

$$_P\boldsymbol{\Sigma}_{XX} = \boldsymbol{\Lambda}_{XY}^T\boldsymbol{\Sigma}_{YY}\boldsymbol{\Lambda}_{XY} \tag{5}$$

Σ_{YY} is a matrix of the form

$$\boldsymbol{\Sigma}_{YY} = \begin{bmatrix} \sigma_{Y1Y1} & \sigma_{Y1Y2} & \cdots & \sigma_{Y1YL} \\ \sigma_{Y2Y1} & \sigma_{Y2Y2} & & \vdots \\ \vdots & & \ddots & \\ \sigma_{YLY1} & \cdots & & \sigma_{YLYL} \end{bmatrix} \tag{6}$$

and $\boldsymbol{\Lambda}_{XY}$ is a matrix which denotes sensitivity of unknown parameters with respect to known parameter error.

$$\boldsymbol{\Lambda}_{XY} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1M} \\ \lambda_{21} & \lambda_{22} & & \vdots \\ \vdots & & \ddots & \\ \lambda_{L1} & \cdots & & \lambda_{LM} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial X_1}{\partial Y_1} & \dfrac{\partial X_2}{\partial Y_1} & \cdots & \dfrac{\partial X_M}{\partial Y_1} \\ \dfrac{\partial X_1}{\partial Y_2} & \dfrac{\partial X_2}{\partial Y_2} & & \vdots \\ \vdots & & \ddots & \\ \dfrac{\partial X_1}{\partial Y_L} & \cdots & & \dfrac{\partial X_M}{\partial Y_L} \end{bmatrix} \tag{7}$$

Entities in Eq. (7) can be computed from the following equation (Matsui and Kurita 1990),

$$\sum_{j=1}^{M}\left\{\int_{t0}^{t1}\sum_{i\in A} w_i\frac{\partial \ddot{z}_i}{\partial X_j}\frac{\partial \ddot{z}_i}{\partial X_k}\ dt\right\}\lambda_{jl} = -\int_{t0}^{t1}\sum_{i\in A} w_i\frac{\partial \ddot{z}_i}{\partial Y_l}\frac{\partial \ddot{z}_i}{\partial X_k}dt \tag{8}$$

$$(\ l = 1,\ \cdots,\ L\ )$$
$$(k = 1,\ \cdots,\ M)$$

If normal distribution is assumed on the identified results, the probability density function may be expressed as,

$$P(\boldsymbol{X}) = \frac{1}{(2\pi)^{\frac{M}{2}} \left|{}_P\Sigma_{XX}\right|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\boldsymbol{X}-\overline{\boldsymbol{X}})^T {}_P\Sigma_{XX}^{-1}(\boldsymbol{X}-\overline{\boldsymbol{X}})\right\} \tag{9}$$

Then, the confidence region for a degree of $1-e$ confidence is presented by

$$(\boldsymbol{X}-\overline{\boldsymbol{X}})^T {}_P\Sigma_{XX}^{-1}(\boldsymbol{X}-\overline{\boldsymbol{X}}) < \chi_{e,M}^2 \tag{10}$$

in which $\chi_{e,M}^2$ is a delimiting value for $1-e$ confidence with M degrees of freedom (Tajima and Komaki 1986). Eq. (10) denotes the inside of the region which is an ellipse when $M = 2$, an ellipsoid when $M = 3$ and hyper-ellipsoid when $M \geq 4$.

2.2 Example problem

The theory was verified as explained below, through numerical experiments with a two degree-of-freedom system shown in Fig. 1.

The input wave is the El Centro wave (NS component) of the Imperial Valley earthquake in 1940, with maximum acceleration adjusted to 300 cm/s^2. The computed response using the values of structural parameters described in Fig. 1 is taken as measured acceleration history at each mass.

The values of mass m_1 and m_2 are known and damping c_1 and c_2 as well as stiffness k_1 and k_2 are considered unknown. In the case in which the mass treated as known contains error having a statistical characteristic, its effect on the identified results is investigated. It is assumed that both masses are normally distributed with mean values of $\overline{m}_1$ and $\overline{m}_2$. $\overline{c}_1$, $\overline{c}_2$ and $\overline{k}_1$, $\overline{k}_2$ are identified results that agree with true values when the masses are $\overline{m}_1$ and $\overline{m}_2$. Weight w_i is taken as 1.0 in this example. In order to confirm validity of the theory, 1000 pairs of normally distributed random numbers with the means $\overline{m}_1$, $\overline{m}_2$ and their coefficient of variation 0.05. Damping c_1, c_2 and stiffness k_1 and k_2 are identified using the modified Marquardt method (Fletcher 1971). No correlation between the masses is presumed.

The distribution of masses given as known and that of identified results are given in Fig. 2. From

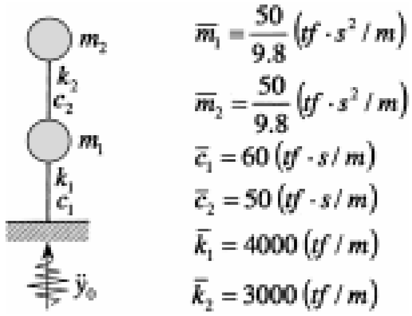

Fig. 1 Two degree-of-freedom system

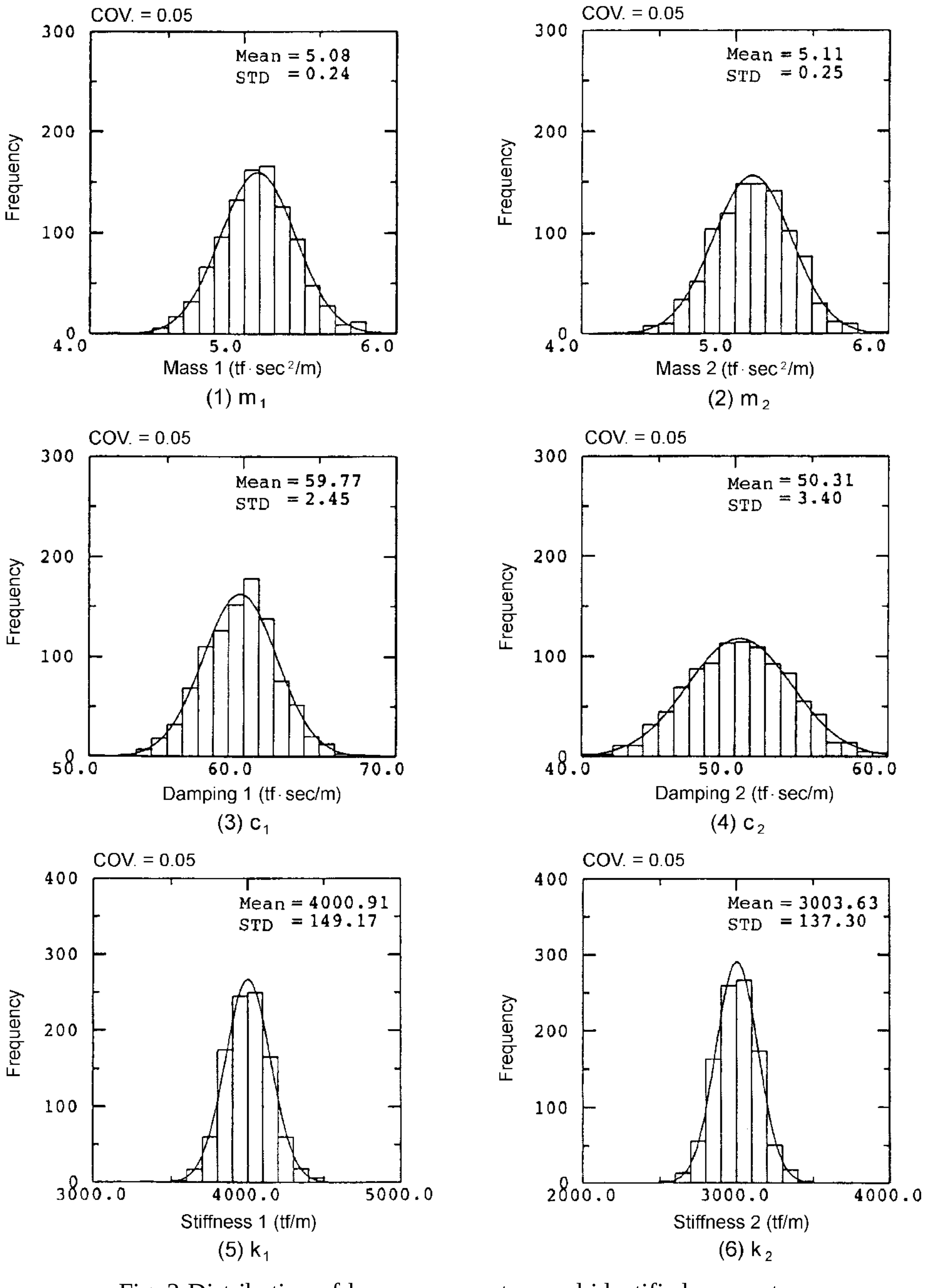

Fig. 2 Distribution of known parameters and identified parameters

the figure it is confirmed that the masses have the stated means and variations. Also the figure shows that the coefficients of variation for the identified results vary in the range from 0.03 to 0.07. The results also manifest normal distribution. The theoretical results and the results from numerical simulation are compared in Fig. 3. The ellipse is called a probability ellipse of 95% confidence

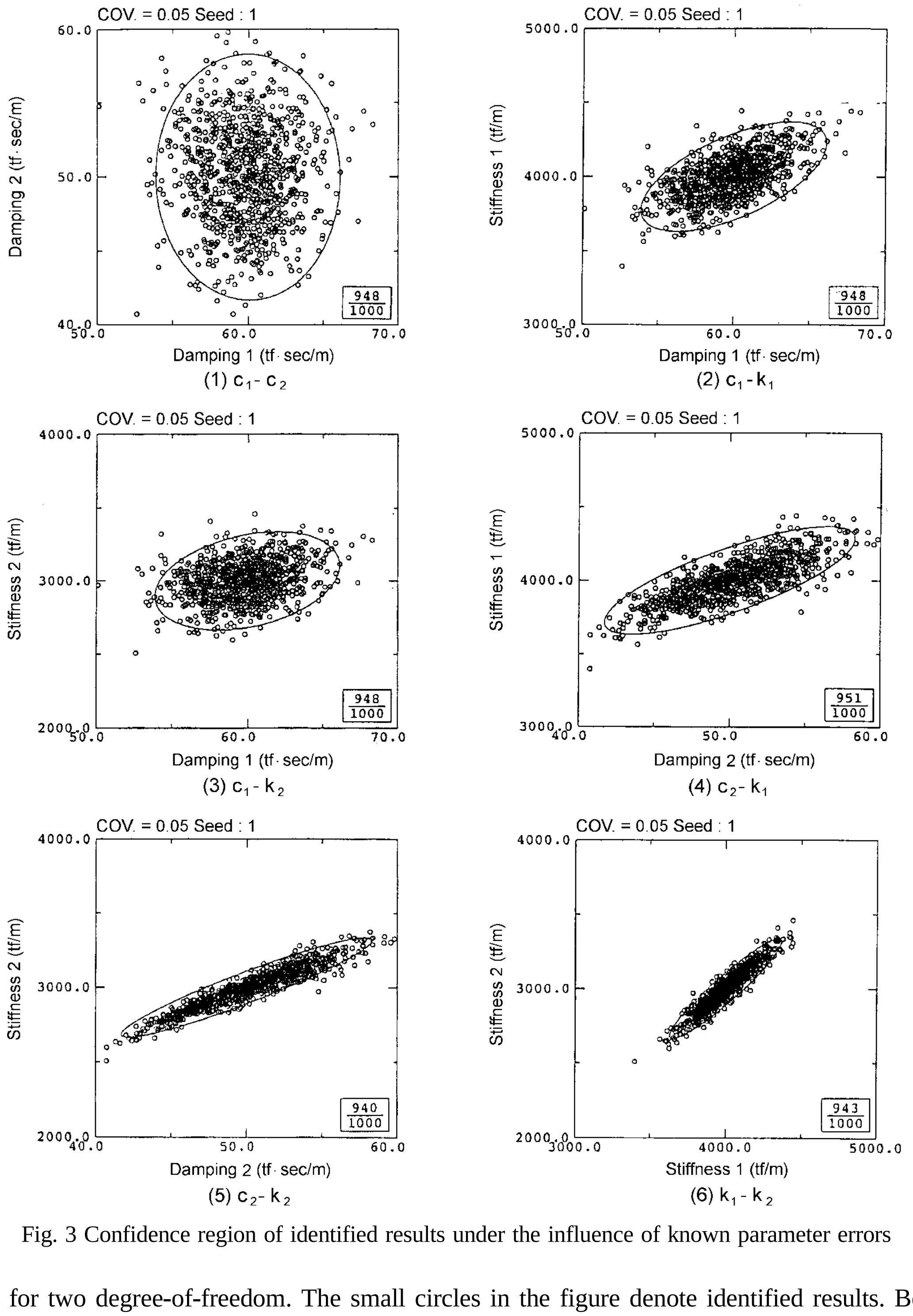

Fig. 3 Confidence region of identified results under the influence of known parameter errors

region for two degree-of-freedom. The small circles in the figure denote identified results. Because the number of unknowns are four, Eq. (10) results in a four dimensional hyper-ellipsoid. However, since it is impossible to illustrate in a plane, the ellipsoid is projected onto a plane as shown in the

Table 1 Sensitivities of identified parameters with respect to mass errors

Y_l	$\frac{\partial c_1}{\partial Y_l}$	$\frac{\partial c_2}{\partial Y_l}$	$\frac{\partial k_1}{\partial Y_l}$	$\frac{\partial k_2}{\partial Y_l}$
m_1	9.56	–3.14	248.26	51.18
m_2	2.20	12.94	535.74	536.82

(Units) $\frac{\partial c_i}{\partial m_j}$: (1/*s*), $\frac{\partial k_i}{\partial m_j}$: (1/s^2)

Table 2 Comparison of variance-covariances and correlation coefficients

		Estimated	Identified
Variance-covariance	σ_{c1c1}	6.3	6.0
	σ_{c1c2}	–0.1	–0.2
	σ_{c1k1}	231.1	219.9
	σ_{c1k2}	108.7	103.1
	σ_{c2c2}	11.5	11.6
	σ_{c2k1}	400.4	399.4
	σ_{c2k2}	441.6	441.5
	σ_{k1k1}	22689.1	22240.6
	σ_{k1k2}	19542.7	19334.1
	σ_{k2k2}	18924.1	18846.2
	trace$({}_P\Sigma_{XX})$	41631.0	41104.4
Correlation coefficient	ρ_{c1c2}	–0.0120	–0.0192
	ρ_{c1k1}	0.6131	0.6018
	σ_{c1k2}	0.3156	0.3065
	ρ_{c2k1}	0.7826	0.7869
	ρ_{c2k2}	0.9450	0.9448
	ρ_{k1k2}	0.9431	0.9444

figure. Fig. 3(1) shows no correlation between the identified c_1 and c_2. Positive correlation is observed in Fig. 3(2-6). The number indicated in each figure implies the number of circles falling within the ellipse over the total number of circles 1000. This proves that the identified results are in good agreement with the theory. Table 1 shows the sensitivity of the unknown parameters with respect to the mass error which is used to calculate the probability ellipse. The values of variance-covariance and coefficients of correlation of unknown parameters, from numerical simulation and theoretical results, are compared in Table 2. They are also in good agreement. The trace$({}_P\Sigma_{XX})$ in the table refers to a trace of the variance-covariance matrix. Coefficients of correlation between two different parameters indicate that c_1 and c_2 are slightly correlated, while other parameters are more clearly correlated. The above evidence proves that confidence region expressed by a probability ellipse can evaluate the effects of known parameter errors on unknown parameters.

3. Confidence region due to measurement noise

3.1 Theory

Measurable quantities considered here are time histories of acceleration. Let $\ddot{u}_i(t)$ be observed acceleration at point i, $\Delta\ddot{u}_i(t)$ be noise contained in $\ddot{u}_i(t)$ and $\ddot{z}_i(t)$ be computed response corresponding to $\ddot{u}_i(t)$. Then the following relationship holds:

$$\Delta\ddot{u}_i(t) = \varepsilon_i \eta_i(t) \tag{11}$$

in which $\eta_i(t)$ is a noise wave form, in which its absolute maximum is adjusted to 1.0 and ε_i is a scalar quantity indicating the maximum value of noise.

It is assumed here that the true values $\overline{\boldsymbol{Y}}$ are known for prescribed parameters. If the effect of noise on identified parameters is relatively small, the values of unknown parameters that the noise at observation point l affects are expressed by

$$\begin{aligned}\boldsymbol{X} &= \overline{\boldsymbol{X}} + \frac{\partial \boldsymbol{X}}{\partial \ddot{u}_l}\Delta\ddot{u}_l \\ &= \overline{\boldsymbol{X}} + \frac{\partial \boldsymbol{X}}{\partial \varepsilon_l}\Delta\varepsilon_l\end{aligned} \tag{12}$$

in which $\overline{\boldsymbol{X}}$ is the true value of $\boldsymbol{X}$. If variance-covariance of noise is known, by employing the law of error propagation, the variance-covariance of unknown parameters can be obtained from

$$_N\Sigma_{XX} = \Gamma_{X\varepsilon}^T \Sigma_{\varepsilon\varepsilon} \Gamma_{X\varepsilon} \tag{13}$$

where $\Sigma_{\varepsilon\varepsilon}$ is the variance-covariance matrix of noise.

$$\Sigma_{\varepsilon\varepsilon} = \begin{bmatrix} \sigma_{\varepsilon1\varepsilon1} & \sigma_{\varepsilon1\varepsilon2} & \cdots & \sigma_{\varepsilon1\varepsilon L} \\ \sigma_{\varepsilon2\varepsilon1} & \sigma_{\varepsilon2\varepsilon2} & & \vdots \\ \vdots & & \ddots & \\ \sigma_{\varepsilon L\varepsilon1} & \cdots & & \sigma_{\varepsilon L\varepsilon L} \end{bmatrix} \tag{14}$$

$\Gamma_{X\varepsilon}$ is sensitivity of unknown parameters with respect to measurement noise.

$$\Gamma_{X\varepsilon} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1M} \\ \Gamma_{21} & \Gamma_{22} & & \vdots \\ \vdots & & \ddots & \\ \Gamma_{L1} & \cdots & & \Gamma_{LM} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial X_1}{\partial \varepsilon_1} & \dfrac{\partial X_2}{\partial \varepsilon_1} & \cdots & \dfrac{\partial X_M}{\partial \varepsilon_1} \\ \dfrac{\partial X_1}{\partial \varepsilon_2} & \dfrac{\partial X_2}{\partial \varepsilon_2} & & \vdots \\ \vdots & & \ddots & \\ \dfrac{\partial X_1}{\partial \varepsilon_L} & \cdots & & \dfrac{\partial X_M}{\partial \varepsilon_L} \end{bmatrix} \tag{15}$$

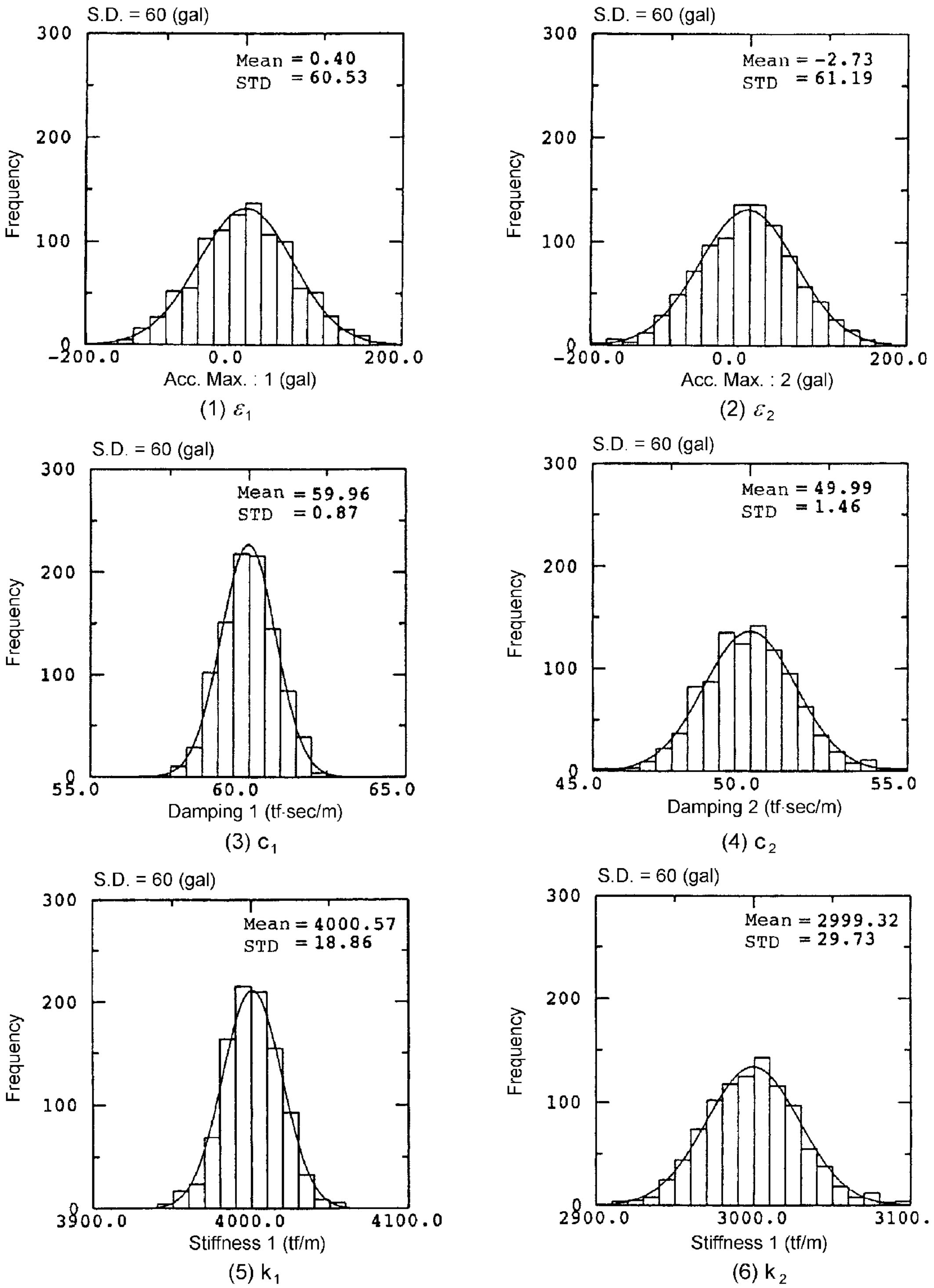

Fig. 4 Distribution of peak accelerations and identified parameters

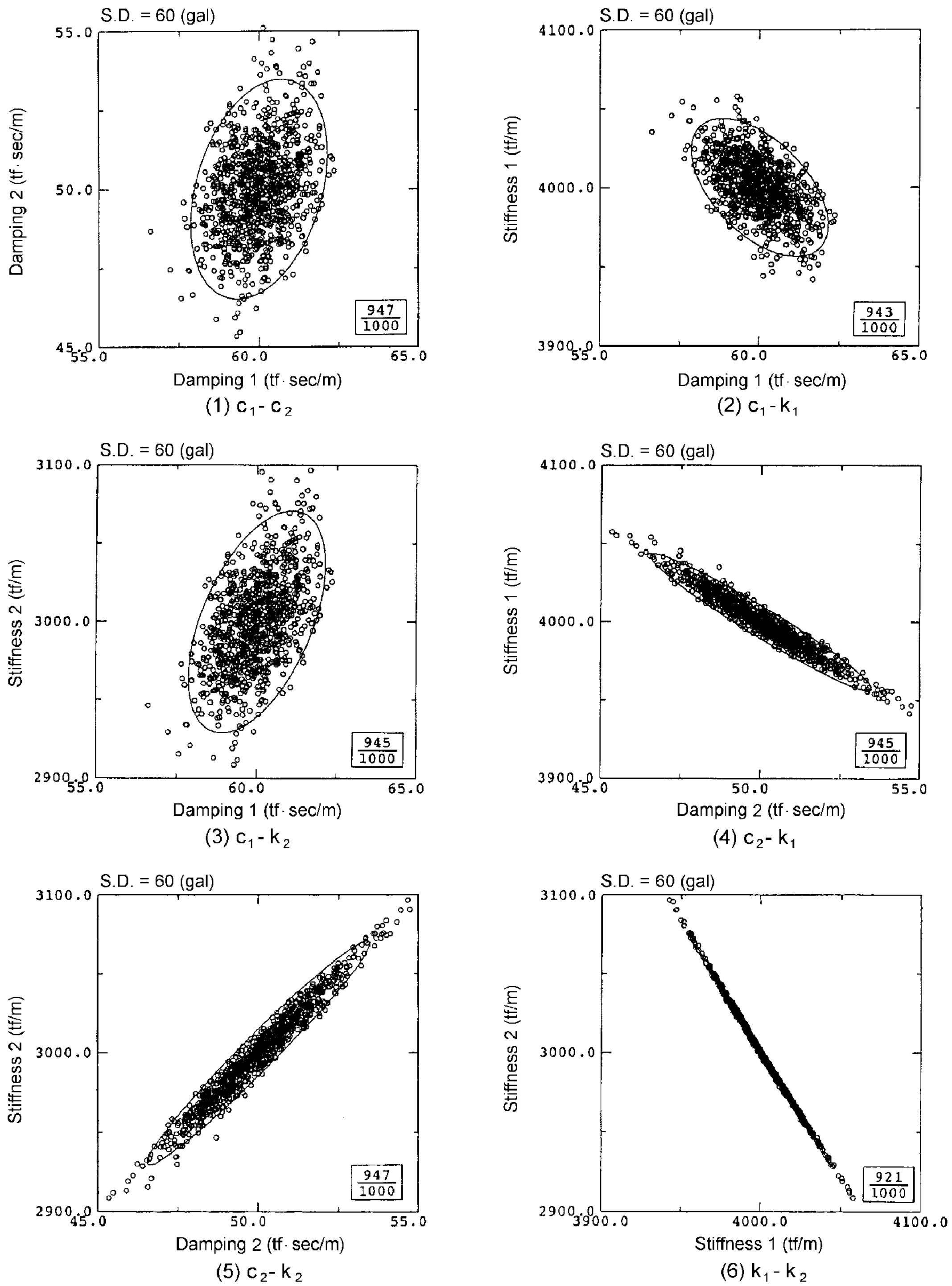

Fig. 5 Confidence region of identified results under the influence of measurement noise

The sensitivity given by Eq. (15) can be computed from the following sensitivity equation (Matsui and Kurita 1990),

$$\sum_{j=1}^{M}\left\{\int_{t0}^{t1}\sum_{i\in A} w_i \frac{\partial \ddot{z}_i}{\partial X_j}\frac{\partial \ddot{z}_i}{\partial X_k}\,dt\right\}\Gamma_{jl} = -\int_{t0}^{t1} w_l \eta_l(t)\frac{\partial \ddot{z}_l}{\partial X_k}dt \qquad (16)$$

$$(l \in A)$$
$$(k = 1, \ldots, M)$$

If ε_l is normally distributed and the identified results are also normally distributed, the probability density function of unknown parameters becomes Eq. (10).

3.2 Example problem

A numerical experiment using a simple model was conducted to confirm the theory stated above.

Table 3 Sensitivities of identified parameters with respect to measurement noise

ε_l	$\frac{\partial c_1}{\partial \varepsilon_l}$	$\frac{\partial c_2}{\partial \varepsilon_l}$	$\frac{\partial k_1}{\partial \varepsilon_l}$	$\frac{\partial k_2}{\partial \varepsilon_l}$
ε_1	–0.009	0.013	-0.093	0.167
ε_2	0.011	0.020	-0.283	0.453

(Units) $\frac{\partial c_i}{\partial \varepsilon_l}$: (tf · s^3/m^2), $\frac{\partial k_i}{\partial \varepsilon_l}$: (tf · s^2/m^2)

Table 4 Comparison of variance-covariances and correlation coefficients

		Estimated	Identified
Variance-covariance	σ_{c1c1}	0.8	0.8
	σ_{c1c2}	0.4	0.4
	σ_{c1k1}	–8.5	–8.8
	σ_{c1k2}	13.0	13.4
	σ_{c2c2}	2.0	2.1
	σ_{c2k1}	–24.6	–25.9
	σ_{c2k2}	40.3	42.6
	σ_{k1k1}	318.3	334.5
	σ_{k1k2}	–517.1	–544.0
	σ_{k2k2}	841.0	886.0
	trace($_N\Sigma_{XX}$)	1162.1	1223.4
Correlation coefficient	ρ_{c1c2}	0.3210	0.3275
	ρ_{c1k1}	–0.5438	–0.5457
	σ_{c1k2}	0.5124	0.5145
	ρ_{c2k1}	–0.9694	–0.9702
	ρ_{c2k2}	0.9778	0.9786
	ρ_{k1k2}	–0.9993	–0.9992

The model used herein is illustrated in Fig. 1. The input earthquake wave is the El Centro wave (NS component, 1940) with maximum acceleration adjusted to 300 cm/s^2. Acceleration response of each mass computed using the true values of structural parameters is regarded as the true measurement data. Noise contaminating the measured response is band limited white noise with frequency from 0.02 Hz to 25 Hz. Mass m_1 and m_2 are prescribed and the true values given in the figure is assigned. Damping c_1 and c_2 and stiffness k_1 and k_2 are considered unknown parameters. In order to verify the theory, 1000 sets of normally distributed random variables with means equal to 0 are generated. The sums of computed responses and the noise generated for each are considered measured responses. Using the 1000 sets of measured responses, the unknown parameters are identified. The identified results are compared with the true values. The standard deviation of maximum noise is taken as 60 cm/s^2. It is also assumed that there is no correlation between the noise in measured responses. The modified Marquardt method is employed for parameter identification.

Fig. 4 illustrates the histogram of maximum noise and identified results. The curves in the figure denote probability density functions for normal distribution. It is confirmed that the distribution of maximum noise has the properties stated above. It can be stated that the identified results also show normal distribution. The coefficients of variation of the results vary within a range between 0.005 to 0.015. The effect of measurement noise on identified results is relatively small. The results from numerical experiment and theory are presented in Fig. 5. The probability ellipse in the figure is 95% confidence region for two degree-of-freedom. The identified results are plotted with small circles. Since the number of unknowns is four, Eq. (10) results in a four dimensional hyper-ellipsoid. However, since it is impossible to illustrate in a plane, the ellipsoid is projected onto a plane as shown in the figure. The number of small circles in the ellipse over the total number of circles of 1000 is denoted in the lower write corner of each figure. The result of nearly 950 circles confirms the theory. Sensitivity of unknown parameters with respect to measurement noise is given in Table 3. Variance-covariance matrix of unknown parameters and coefficients of correlation are presented in Table 4. Estimated in the table means the theoretical results computed from Eq. (13). The results from the numerical experiment and the theory show excellent agreement. The trace$({}_N\Sigma_{XX})$ in the table refers to a trace of the variance-covariance matrix. The table also shows a strong negative correlation between two stiffness parameters k_1 and k_2 and the coefficient of correlation is nearly -1.

It is clear from the above that the effect of measurement noise on identified results is relatively small and that the effect is evaluated by the form of confidence region.

4. Optimum arrangement of observation points

4.1 Theory

Uncertainties in known parameters may lead to uncertainties in the identified results. Probability theory is suitable for evaluating potential arrangements of measurement points for such identification. In this study, several candidates for arrangements of measurement points are selected and evaluated using variations of identified results as measures of goodness. In parameter identification using known information involving uncertainties, the fluctuation in identified parameter value may be evaluated from the following coefficient of variation:

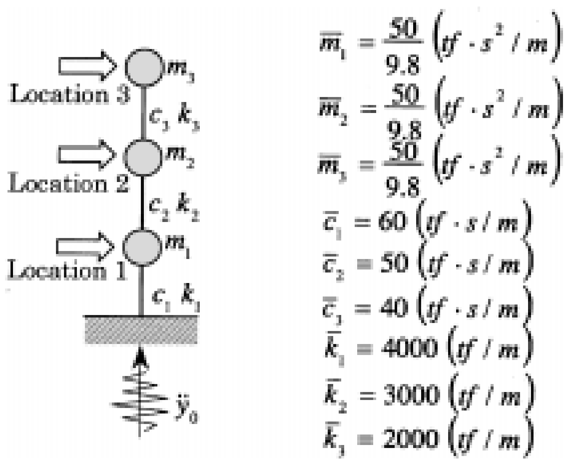

Fig. 6 Three degree-of-freedom system

$$\mathrm{COV}(X_i) = \frac{\sqrt{\sigma_{XiXi}}}{\overline{X}_i} \tag{17}$$

in which $\overline{X}_i$ and σ_{XiXi} is the mean of X_i and its variance respectively. Because a coefficient of variation is a nondimensional quantity, comparison between quantities with different dimensions can be made. The various candidates for arrangement of observation points are compared using the average of the coefficients for all the identified results:

$$\text{Average of COV} = \frac{1}{M}\sum_{i=1}^{M} \mathrm{COV}(X_i) \tag{18}$$

Computing the average coefficients of all the candidates, the one that minimizes the coefficient is the optimum arrangement of observation points. The coefficient of variation for identified results can be computed through numerical experiment by using randomly generated values of mass. In order to obtain results with good accuracy, a large number of sample data is necessary, which requires considerable computation time. Hence the analytical approach stated in 2.1 is utilized and compared to the numerical experiment.

4.2 Example problem

To examine the effectiveness of the present theory, numerical simulation was performed. Fig. 6 shows the three degree-of-freedom system used for the simulation. The input wave is the El Centro wave of the Imperial Valley earthquake in 1940 with the maximum acceleration adjusted to 300 cm/s^2.

Table 5 Arrangements of measurement positions

Location Number	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
3	O		O	O			O
2	O	O	O			O	
1	O	O		O	O		

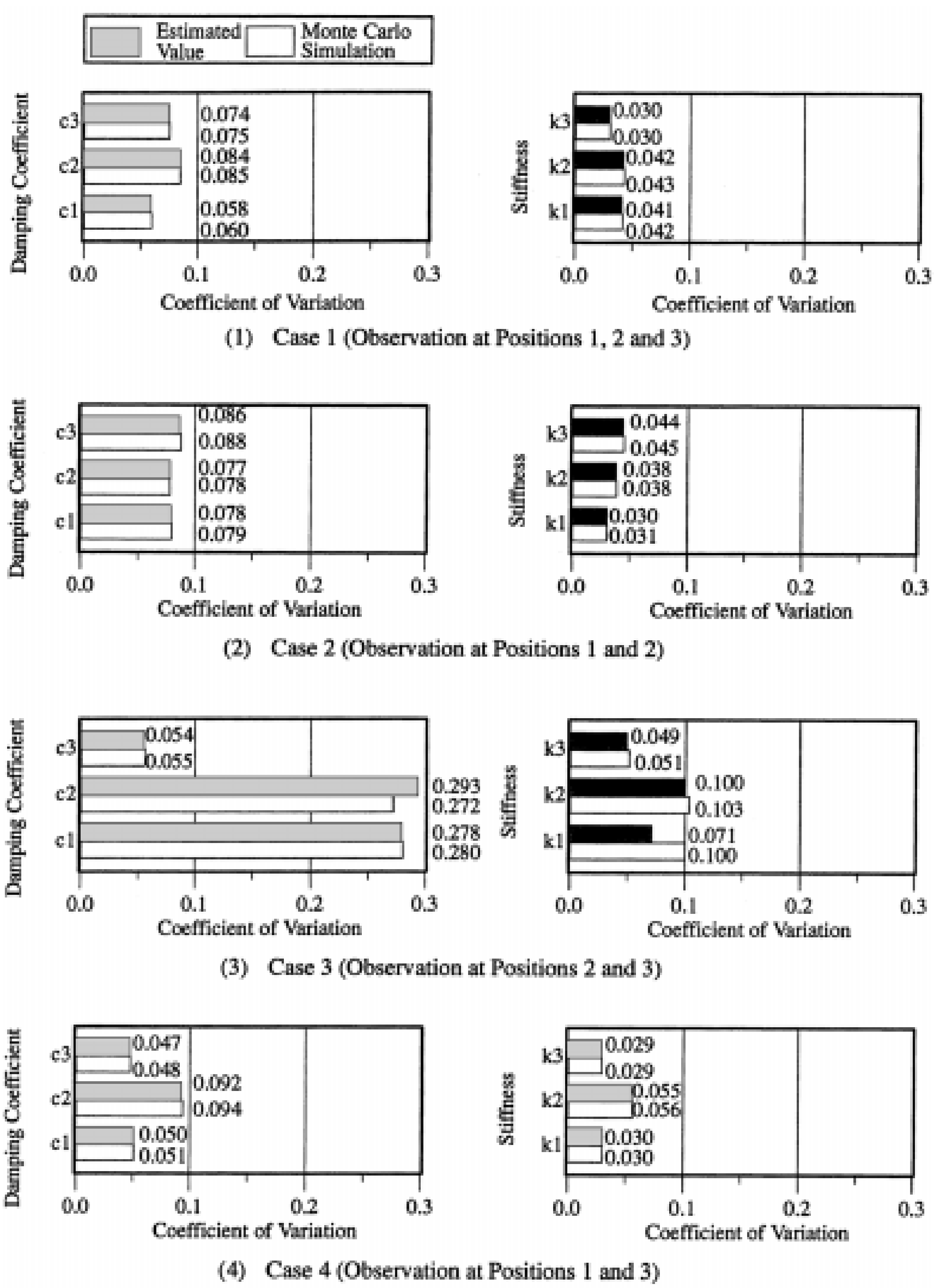

Fig. 7 Coefficient of variation for each identified parameter

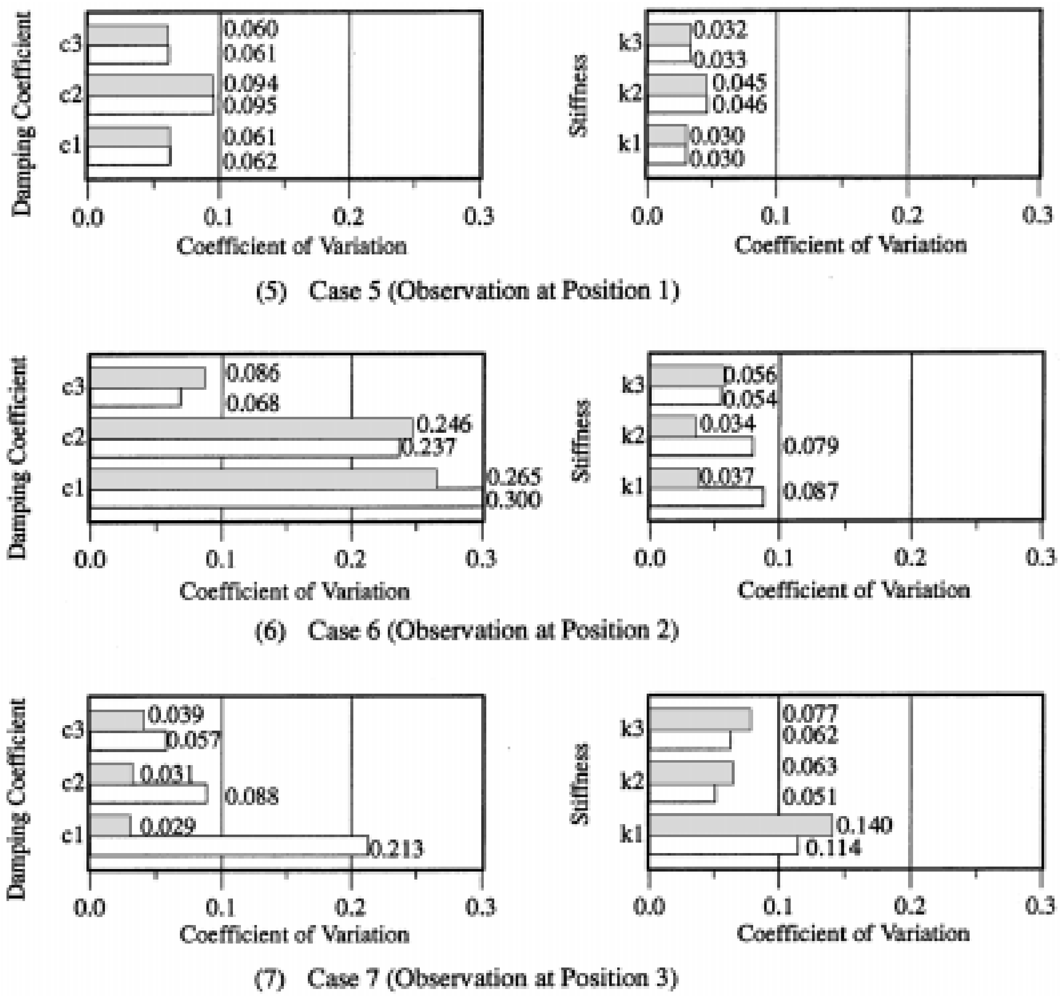

Fig. 7 Continued

This figure also shows the structural parameters used by the authors in analyzing the responses in order to obtain a time history of each mass point. This time history is used as observation data in this simulation and is assumed to be free from noise. Here, the masses m_1, m_2 and m_3 are known structural parameters. The damping coefficients c_1, c_2 and c_3 and the stiffness k_1, k_2 and k_3 are unknown parameters. We assume that the masses are in normal distribution with an average $\overline{m}_i$ of 50 ton and coefficient of variation COV(m_i) of 0.05 ($i = 1, 2, 3$). For such uncertainties in the known parameters, identified results are assumed to yield fluctuations having normal distribution. In this case, the value shown in Fig. 6 is an average. In the following analysis, every weight factor w_i contained in the evaluation function is set to 1.0. In order to confirm the proposed theory, 1000 sets of normal random numbers that would give $\overline{m}_1$, $\overline{m}_2$ and $\overline{m}_3$ as the averages and 0.05 as the coefficient of variation are generated. Then using the data, the structural parameters are identified.

Table 5 shows the seven arrangements used for the measurement positions. The time history at each marked position is assumed as known as well as the input earthquake motion. Fig. 7 shows a

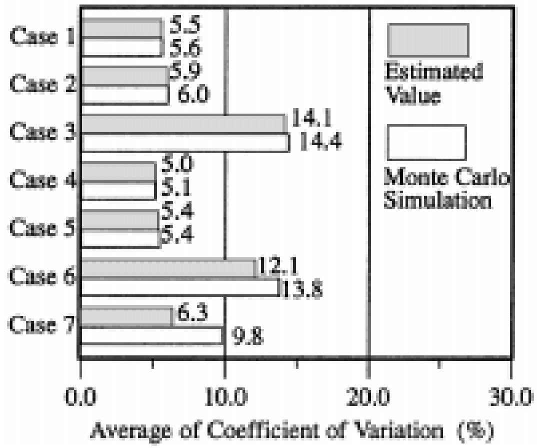

Fig. 8 Average of the coefficients of variation for all the parameters

Table 6 Participation vectors of analytical model

		1st mode	2nd mode	3rd mode
Eigenfrequency (Hz)		1.78	4.46	6.82
Participation Vector	Mass Point			
	3	1.28	–0.33	0.05
	2	0.87	0.33	–0.20
	1	0.41	0.33	0.26

comparison of the estimated coefficients of variation obtained from the proposed theory and the Monte Carlo simulation. They agree very well for Cases 1, 2, 4 and 5, differ slightly for Case 3, and manifest a significant difference for Cases 6 and 7. The damping coefficients exhibit particularly large disagreement in Case 7. This is probably because some of 1000 sets produce incomplete convergent results, because number of iteration is terminated if it exceeds 100. The incomplete convergence is caused by a poor selection of measurement points that can not provide sufficient information for identification. It is evident that this arrangement of Case 7 is inferior to any of the others with respect to accuracy.

In all cases, the damping coefficients tend to have a larger fluctuation than stiffness. This tendency is particularly prominent in Cases 3 and 6. Fig. 8 shows that the average of the coefficients of variation for all parameters. In Case 7, the theory-derived findings disagree substantially with those from the simulation. Fig. 8 reveals that Cases 3, 6 and 7 are not very suitable as arrangements because of the large fluctuation produced. In the other cases, the average coefficients of variation are similar and sufficiently small. These cases seem suitable as arrangements. Case 4 gives the best accuracy of identification. It should be noted that the one point measurement in Case 5 produced accuracy equal to the three point measurement in Case 1. Measurements at two points, like Case 3, may not necessarily lead to good accuracy. Also noteworthy was that all the cases that included the measurement point at mass 1 showed good accuracy. Thus measurement at mass 1 is crucially important. In order to investigate the reason, participation factors for three modes without damping are computed to examine the contribution of the response at each mass to the three modes. The participation factors are summarized in Table 6. At mass 3 the participation factor for 1st mode is very large compared with 2nd and 3rd modes. At mass 1, the participation factors appear in the same magnitude. Thus, mass 1 contains the information for the three modes in an equal amount. Identification using the response data at masses 2 and 3 results in poor estimates because the data lack the information for the higher modes which becomes obscure in noise contaminated data.

5. Conclusions

This paper proposed methods to analytically evaluate the confidence region of the identified results due to known parameters error and measurement noise, and verified the theory through the numerical simulation. Furthermore, a method to evaluate the optimum arrangement of measurement points is proposed. The following conclusions can be stated:

(1) The confidence region of identified parameters under the influence of known parameter errors and measurement noise can be evaluated as a probability ellipse.

(2) Confidence region based on a linear approximation demonstrates good agreement with numerical simulation results. This proves that the analytical approach enables evaluation of the effect of known parameter error and measurement noise on identified results with sufficient accuracy.

(3) The analytical approach reduces computational time in evaluating the effect of known parameter error and measurement noise.

(4) Regarding arrangement of measurement points, it is made clear that there exists a key measurement point. Although the more measurement points, the better, it is more important to include the key measurement point in the arrangement.

Appendix. Conversion factors

1 tf $\cdot$ s^2/m = 9.81 ton; 1 tf $\cdot$ s/m = 9.81 kN $\cdot$ s/m; 1 tf/m = 9.81 kN/m;
1 tf $\cdot$ s^3/m^2 = 9.81 kN $\cdot$ s^3/m^2; and 1 tf $\cdot$ s^2/m^2 = 9.81 kN $\cdot$ s^2/m^2.

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