Evaluation of torsional natural frequencies for non-tubular bonded joints

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Abstract. During the last several years, research activity on non-tubular bonded joints has concentrated on the effects of normal stress, bending moments and shear. Nevertheless, in certain situations, the structure may be subjected to twisting moments, so that the evaluation of its dynamic behaviour to torsional vibrations becomes of great importance even though evaluations of such loading conditions is entirely lacking in the literature. The aim of this article is to show that torsional natural frequencies of the non-tubular joint can be evaluated by determining the roots of a determinantal equation, derived by taking advantage of some analytical results obtained in a previous paper dealing with the analysis of the state of stress in the adhesive. Numerical results related to clamped-free and clamped-clamped joints complete the article.

Key words: bonded joint; dynamic properties; torsional oscillation.

1. Introduction

Adhesively bonded joints have long been recognized as attractive alternatives to conventional mechanical joining techniques, due to a greater uniformity in load distribution as well as a reduced weight and processing case. Furthermore, they do not require holes and the load is distributed over a larger area than for mechanical joints. Moreover, they are excellent electrical and thermal insulators. However, bonded joints are very sensitive to the adherend geometry, the quality of surface treatment, the service temperature, humidity and other environmental conditions.

Considerable research in this area has been conducted regarding the torsion of adhesively bonded tubular joints but only recently non-tubular bonded structures under torsion have also been studied.

Since the two pioneer papers (Goland *et al.* 1994), (Lukbin *et al.* 1956), tubular joints subject to torsion have been studied from many different points of view. Theoretical approaches have been validated both experimentally (Reddy *et al.* 1993), (Kim *et al.* 1992) (Gent *et al.* 1982), (Choi 1994) and numerically (Choi 1944), (Hipol 1984), (Rao *et al.* 1994) and the the importance of

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bonded joints in composite structures has been widely emphasized (Reddy *et al.* 1993), (Hipol 1984), (Graves *et al.* 1981), (Nayebhashemi *et al.* 1997). Fracture mechanics has been used to solve the problem of the joint's strength in the case of brittle collapse with static (Reddy *et al.* 1993), (Gent *et al.* 1982), (Lee *et al.* 1992) or fatigue loading (Reddy *et al.* 1993), (Gent *et al.* 1982), (Choi, 1994), (Nayebhashemi *et al.* 1997). The non-linear and viscoelastic adhesive's behavior have been considered in (Choi 1944), (Lee *et al.* 1992), (Lee *et al.* 1995), (Alwar *et al.* 1976), (Medri 1988), (Zhou *et al.* 1993), and only recently also the dynamic analysis for tubular bonded joints has been addressed (Rao *et al.* 1994), (Ko *et al.* 1995).

In contrast only recently have non tubular bonded joints been studied. The old gap in the literature about non tubular adhesive bonded joints can perhaps be explained by noting that these kind of joints are not designed to withstand a torsional moment, which can thus induce a non-shearing stress state in the adhesive of such joints. In fact, it is well known that adhesives are by nature less effective when subjected to normal stresses (as illustrated by the differences encountered when attempting to separate two pieces of adhesive tape by applying tensile or shear stresses). Though this is likely to be the major reason that little work has been done with non-tubular joints, it cannot be considered a justification. During its service life, in fact, a non-tubular adhesive bonded joint can find itself called upon to withstand accidental torsional loading: as the joint is not designed for this type of characteristic of internal reaction (the joint should be designed to tensile loading), even modest torsional loads can prove to be critical.

The indicated old lack of work on non-tubular joints motivated the investigation presented in Pugno (1998, 1999), Pugno *et al.* (2000a & 2000b).

In (Pugno *et al.* 2000b) a non-tubular joint subjected to torsion was studied from a theoretical point of view, and the mathematical model was validated numerically through a three dimensional finite element analysis. In that article the predominant stress and strain fields in the adhesive as well as the torsional moment transmission section by section along the overlap are emphasized.

In (Pugno 1999) the non-tubular joint, streamlined for uniform torsional strength (UTS), is considered: starting from a non-tapered joint, the optimization is achieved by chamfering the edges, which are in any case not involved in the stress flow induced by the load for which the joint should be designed. The resulting optimized joint shape is thus both lighter and stronger (both analysis and optimization were carried out also for tubular bonded joints in (Pugno *et al.* 2000a).

The brittle collapse of the joint was investigated in (Pugno *et al.* 2001). The stability of brittle crack propagation and the size effects on mechanical collapse behaviour, as well as the ductile-brittle transition, were analyzed. Experimental measurements of failure loads under torsion for non-tubular bonded joints agree satisfactorily with the theoretical predictions.

In this article the dynamic behaviour of bonded joints, with particular reference to the non-tubular type, is analyzed. A parametric study illustrating the influence of several characteristic variables on the torsional natural frequencies is presented. Finally, a comparison between torsional natural frequencies predicted by the proposed procedure and FEM simulations has been performed, demonstrating that theoretical and numerical results agree satisfactorily.

2. Constitutive, compatibility and equilibrium equations in the bonding

Assuming linear elastic constitutive laws (Pugno *et al.* 2000a & 2000b) the equation of motion of the overlap (Fig. 1), in a dynamic regime, can be written as:
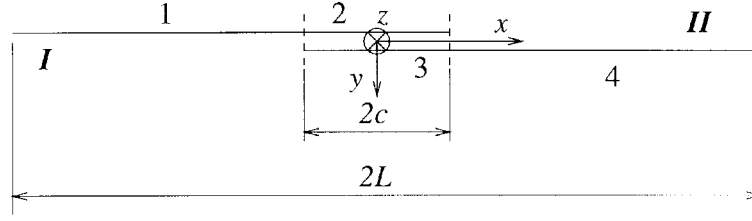
Fig. 1 Schematic view of the lap

$$K^*(\theta_I(x, t) - \theta_{II}(x, t)) = M_{I,x}(x, t) - (\rho I_p)_I \ddot{\theta}_I(x, t),$$ (1)

where K^* is the adhesive stiffness, given by:

$$K^* = \frac{E_a^* b^3}{12 h_a},$$

$$K^* = \frac{2\pi R^3 G_a}{h_a},$$ (2)

for a non-tubular and a tubular overlap respectively, with b the width of the cross section for the non-tubular joint, R the radius of the adhesive surface in the tubular one and h_a the thickness of the adhesive layer. E_a^* is:

$$E_a^* = \frac{1 - v_a}{(1 + v_a)(1 - 2 v_a)} E_a.$$ (3)

E_a and G_a are the longitudinal and shear moduli of elasticity of the adhesive and v_a its Poisson ratio. Furthermore, in Eq. (1) $\theta_I(x, t)$ and $\theta_{II}(x, t)$ are the rotations of the two connected beams, $(\rho I_p)_I$ the mass density times the polar moment of inertia of the first (I) beam, $M_I(x, t)$ its twisting moment and $(\ )_{,x}$ and $(\dot{\ })$ represent derivatives with respect to the longitudinal coordinate and time respectively.

The dynamic equilibrium equations for tubular and non-tubular joints respectively allow obtaining the corresponding predominant stress and strain fields in the adhesive (Pugno *et al.* 2000a and 2000b) according to:

$$\tau(x, t) = \frac{K^* \Delta\theta(x, t)}{2\pi R^2}, \quad \gamma(x, t) = \frac{\tau(x, t)}{G_a},$$ (4)

$$\sigma_y(x, z, t) = K^* \Delta\theta(x, t) \frac{12z}{b^3}, \quad \varepsilon_y(x, z, t) = \frac{\sigma_y(x, z, y)}{E_a^*},$$

where $\Delta\theta(x, t) = (\theta_I(x, t) - \theta_{II}(x, t))$.

By introducing

$$M_I(x, t) = (GI_t)_I \theta_{I,x}(x, t),$$ (5)

with $(GI_t)_I$ the shear elastic modulus times the factor of torsional rigidity (equal to I_p for tubular joints) of the first beam, Eqs. (4) provide the predominant stress and strain fields in the adhesive

and Eq. (5) the torsional moment, varying section by section, transmitted along the overlap provided that rotations θ_I and θ_{II} are known.

3. Evaluation of torsional natural frequencies

The equation of motion for torsional vibrations is derived in the case of the most common laps in practical applications: the non-tubular and the tubular lap; it is sufficient to use the proper value for K^* to obtain the desired determinantal equation leading to torsional natural frequencies.

In order to derive the equations, and due to the different field equations that rule the torsional vibrations in and outside the bonding region, it is necessary to divide both beams in two sections. As a consequence, sections 1 and 2 of the first beam indicate the region out and inside the bonding and, for the second beam, sections 3 and 4 indicate the region in and outside the bonding respectively. Moreover, the twisting angle of every region i is called $\theta_i(x, t)$ (Fig. 1).

The equation of free torsional vibrations for these four regions can be obtained manipulating Eqs. (1)−(5) and assuming that out of the adhesive, i.e., at sections 1 and 4, K^* is equal to zero:

$$\left((GI_t)_I \theta_1(x, t)\right)_{,x} - (\rho I_p)_I \ddot{\theta}_1(x, t) = 0,$$

$$\left((GI_t)_I \theta_2(x, t)\right)_{,x} - (\rho I_p)_I \ddot{\theta}_2(x, t) - K^*(\theta_2(x, t) - \theta_3(x, t)) = 0,$$

$$\left((GI_t)_{II} \theta_3(x, t)\right)_{,x} - (\rho I_p)_{II} \ddot{\theta}_3(x, t) - K^*(\theta_3(x, t) - \theta_2(x, t)) = 0,$$

$$\left((GI_t)_{II} \theta_4(x, t)\right)_{,x} - (\rho I_p)_{II} \ddot{\theta}_4(x, t) = 0, \tag{6}$$

where subscripts I and II are related to properties of the first and of the second beam respectively. It can be highlighted that in the first and the fourth equation, corresponding to regions outside the bonding, the equilibrium between elastic and inertial forces is being considered. In the second and third equations, corresponding to regions inside the bonding, the presence of the term related to the variation in twisting moment due to the bonding acting as a distributed elastic spring is evident.

The previous differential equations require appropriate boundary conditions to allow to evaluate natural frequencies of the bonded beams. In particular for every part of both beams it is necessary to consider two boundary conditions, one at the left and the other at the right end. By calling $2c$ the length of the lap and $2L$ the overall length of the joint having in the middle $x=0$ (Fig. 1), boundary conditions at the left and right end are:

$$\begin{aligned} \theta_{1,x}(x, t)\big|_{x = -L} &= 0, \\ \theta_{4,x}(x, t)\big|_{x = L} &= 0, \end{aligned} \tag{7}$$

for free and

$$\begin{aligned} \theta_1(-L, t) &= 0, \\ \theta_4(L, t) &= 0, \end{aligned} \tag{8}$$

for clamped ends. The other boundary conditions allow the continuity of the twisting angle and of its derivative, i.e., of the twisting moment, for both the first and the second beam:

$$\theta_1(-c, t) = \theta_2(-c, t), \ \theta_{1,x}(x, t)\big|_{x = -c} = \theta_{2,x}(x, t)\big|_{x = -c},$$

$$\theta_3(c, t) = \theta_4(c, t), \ \theta_{3,x}(x, t)\big|_{x = c} = \theta_{4,x}(x, t)\big|_{x = c}. \tag{9}$$

In correspondence of both the ends of the lap the twisting moment must be zero, i.e., the first derivative of the twisting angle must vanish:

$$\theta_{1,x}(x,t)\big|_{x=c} = 0, \ \theta_{3,x}(x,t)\big|_{x=-c} = 0. \tag{10}$$

By assuming that geometrical and material properties are equal for the two connected beams and that their cross section in constant along the axis, the four Eqs. (6) can be written as:

$$\ddot{\phi}_i(x,t) - \xi\phi_{i,xx}(x,t) + \zeta_i\phi_i(x,t) = 0, \tag{11}$$

where $\xi=(GI_t)/(\rho I_p)$ while $\phi_i(x,\ t)$ and ζ_i have the following values:

$$\begin{aligned}
\phi_1(x,t) &= \theta_1(x,t), \ \zeta_1 = 0, \\
\phi_2(x,t) &= \theta_2(x,t) - \theta_3(x,t), \ \zeta_2 = \frac{2K^*}{\rho I_p}, \\
\phi_3(x,t) &= \theta_2(x,t) + \theta_3(x,t), \ \zeta_3 = 0, \\
\phi_4(x,t) &= \theta_4(x,t), \ \zeta_4 = 0,
\end{aligned} \tag{12}$$

By applying the principle of separation of variables, the solution of Eq. (11) can be written as:

$$\phi_i(x,t) = \psi_i(x)\gamma_i(t), \tag{13}$$

so that Eq. (11) becomes:

$$\frac{\ddot{\gamma}_i(t)}{\gamma_i(t)} = \xi\frac{\psi_{i,xx}(x)}{\psi_i(x)} - \zeta_i = -\omega^2, \tag{14}$$

where ω must be a constant. As a result, $\psi_i(x)$ and $\gamma_i(t)$ are given by:

$$\begin{aligned}
\psi_i(x) &= A_i \sin(\lambda_i x) + B_i \cos(\lambda_i x), \\
\gamma_i(t) &= \sin(\omega t + \Phi),
\end{aligned} \tag{15}$$

where

$$\lambda_i^2 = \frac{\omega^2 - \zeta_i}{\xi}. \tag{16}$$

Moreover, according to (12) $\zeta_1=\zeta_3=\zeta_4=0$ so that $\lambda_1=\lambda_3=\lambda_4=\lambda$, while it is set $\lambda_2=\bar{\lambda}$ to simplify the notation.

By introducing expressions (15) into (12), it is possible to determine the corresponding expression for $\theta_i(x,\ t)$:

$$\theta_i(x,t) = \beta_i(x)\gamma_i(t), \tag{17}$$

where the four functions $\beta_i(x)$ are given by:

$$\beta_1(x) = A_1\sin(\lambda x) + B_1\cos(\lambda x),$$

$$\beta_2(x) = 1/2\left(A_2\sin(\bar{\lambda}x) + B_2\cos(\bar{\lambda}x) + A_3\sin(\lambda x) + B_3\cos(\lambda x)\right),$$

$$\beta_3(x) = 1/2\left(-A_2\sin(\bar{\lambda}x) - B_2\cos(\bar{\lambda}x) + A_3\sin(\lambda x) + B_3\cos(\lambda x)\right),$$

$$\beta_4(x) = A_4\sin(\lambda x) + B_4\cos(\lambda x). \tag{18}$$

Finally, by introducing (17) into boundary conditions (7)−(10), a set of eight algebrical equations depending on coefficients A_i and B_i is derived:

$$\begin{aligned}
\beta_{1,x}(x)\big|_{x=-L} &= 0, \quad \text{or} \quad \beta_1(-L) = 0,\\
\beta_{4,x}(x)\big|_{x=L} &= 0, \quad \text{or} \quad \beta_4(L) = 0,\\
\beta_1(-c) &= \beta_2(-c),\\
\beta_{1,x}(x)\big|_{x=-c} &= \beta_{2,x}(x)\big|_{x=-c},\\
\beta_{2,x}(x)\big|_{x=c} &= 0,\\
\beta_3(c) &= \beta_4(c),\\
\beta_{3,x}(x)\big|_{x=c} &= \beta_{4,x}(x)\big|_{x=c},\\
\beta_{3,x}(x)\big|_{x=-c} &= 0.
\end{aligned} \tag{19}$$

The first two equations of (19) can be rewritten as:

$$\begin{aligned}
A_1 &= -\tan(\lambda L + n_l\pi/2)B_1 = C_1 B_1,\\
A_4 &= \tan(\lambda L - n_r\pi/2)B_4 = C_4 B_4,
\end{aligned} \tag{20}$$

where n_l and n_r refer to the left and right end respectively and they are equal to 0 or 1 if the corresponding end is whether free or clamped. As a result, the entire system of algebraic Eqs. (19) can be rewritten as:

$$[M]\{X\} = \{0\}, \tag{21}$$

where matrix $[M]$ is

$$[M] = \begin{bmatrix}
\bar{S} & S & 2(C+C_1S) & -\bar{C} & -C & 0\\
-\bar{C}^* & -C & 2(S-C_1C) & -\bar{S}^* & -S & 0\\
\bar{C}^* & C & 0 & -\bar{S}^* & -S & 0\\
-\bar{S} & S & 0 & -\bar{C} & C & -2(C+C_4S)\\
-\bar{C}^* & C & 0 & \bar{S}^* & -S & 2(S-C_4C)\\
-\bar{C}^* & C & 0 & -\bar{S}^* & S & 0
\end{bmatrix} \tag{22}$$

with

$$\begin{aligned}
\bar{S} &= \sin(\bar{\lambda}c), \quad \bar{S}^* = \lambda^*\sin(\bar{\lambda}c),\\
\bar{C} &= \cos(\bar{\lambda}c), \quad \bar{C}^* = \lambda^*\cos(\bar{\lambda}c),\\
S &= \sin(\lambda c), \quad C = \cos(\lambda c),
\end{aligned}$$

and $\lambda^*=\bar{\lambda}/\lambda$. Furthermore,

$$\{X\}^T = [A_2\ A_3\ B_1\ B_2\ B_3\ B_4].$$

In order to obtain a non-zero solution, it is necessary to find the eigenvalues ω_n so that:

$$\det([M(\omega_n)]) = 0, \tag{23}$$

i.e., the determinant of the matrix $[M(\omega)]$ is zero. Eigenvalues ω_n are the natural torsional circular frequencies of the bonded joint, with corresponding eigenvectors (or modeshapes) given by $\{X_n\}$.

4. Numerical examples

Numerical simulations have been performed to determine the trend of the fundamental natural frequency of torsional vibrations of clamped-clamped and clamped-free beams connected with a non-tubular joint with respect to some parameters of the junction.

The properties of the two equal aluminium beams connected with the single-lap joint are the following: Young's modulus of 7×10^{10} N/m^2, Poisson ratio equal to 0.31, material density of 2700 kg/m^3, width of the cross section b of 55 mm, height h of 3 mm, overall length of the joint $2L$ of 260 mm, moreover, the adhesive has a Young's modulus E_a of 2.9×10^9 N/m^2, Poisson ratio $v_a = 0.31$ and thickness h_a of 0.3 mm.

Figs. 2 and 3 are related to a clampled-free structure and show the frequency ratio f_1/f_{10} where f_1 is the fundamental torsional natural frequency of the joint while f_{10} is the first torsional natural frequency of a single clamped-free beam with the properties previously listed and length of 260 mm. Fig. 2 illustrates the trend of the frequency ratio with respect to the Young's modulus of the
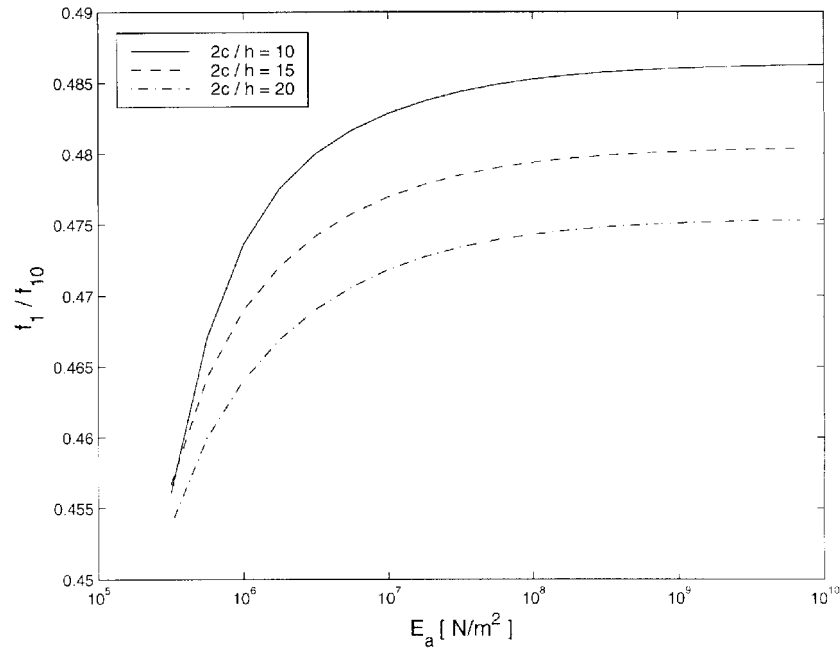

Fig. 2 Trend of the fundamental frequency ratio vs Young's modulus of the adhesive (clamped-free joint, h fixed to 3 mm)
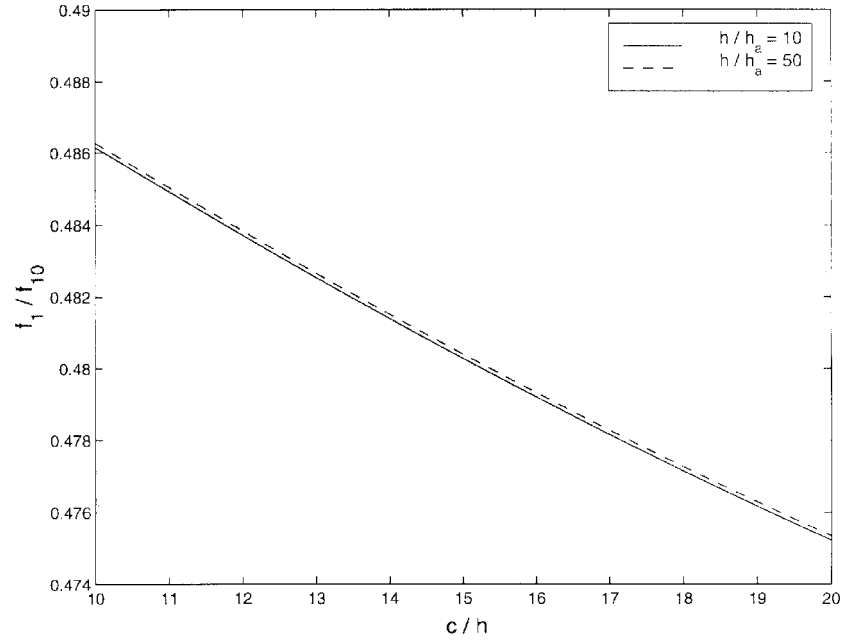
Fig. 3 Trend of the fundamental frequency ratio vs length of the lap (clamped-free joint, *h* fixed to 3 mm)
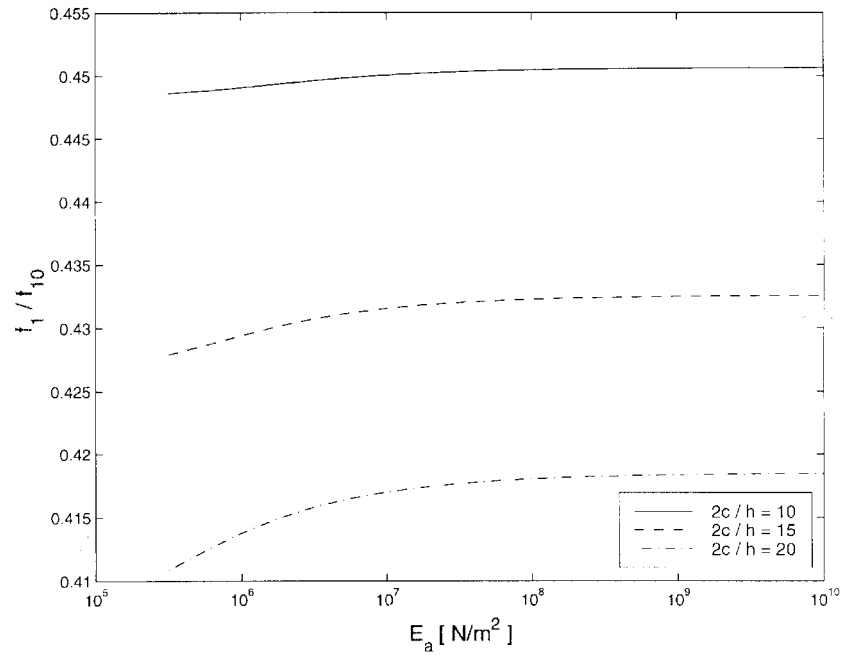

Fig. 4 Trend of the fundamental frequency ratio vs Young's modulus of the adhesive (clamped-clamped joint, *h* fixed to 3 mm)

adhesive, showing that even for very stiff adhesives the frequency ratio is lower than 0.5. Moreover, Fig. 3 shows that the frequency ratio decreases with increase in the length of the lap with a trend
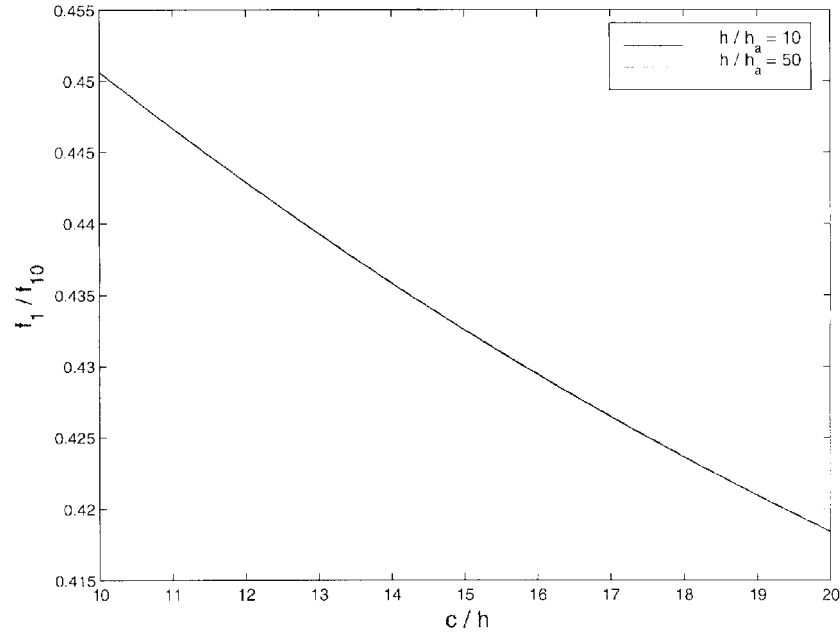
Fig. 5 Trend of the fundamental frequency ratio vs length of the lap (clamped-clamped joint, *h* fixed to 3 mm)

Table 1 Comparison with NASTRAN results for a clamped-free joint

E_a [N/mm^2]	f_1 (NASTRAN) [Hz]	f_1 (present work) [Hz]
270	333.3	323.0
2700	339.6	323.2

that is almost linear.

Figs. 4 and 5 show the frequency ratio for a clamped-clamped joint versus both the Young's modulus of the adhesive and the length of the lap. It is clear that the frequency ratio is both lower than 0.5, as in the previous case, and less sensitive to variations of the Young's modulus of the adhesive with respect to a clamped-free joint. Furthermore, the trend of the frequency ratio versus the length of the lap is similar to that shown in Fig. 2 even if it is more sensitive to the length of the lap with respect to the clamped-free joint.

In Figs. 3 and 5 the effect of the thickness of the adhesive on the fundamental torsional natural frequency of the joint is shown. In particular, it is evident that the frequency ratio has a very limited sensitivity to variations in the thickness of the adhesive and it is almost zero for the clamped-clamped joint. Moreover, in Figs. 2 and 4 the effect of the lap length 2*c* is shown: in general it is evident that by increasing the length of the lap the fundamental torsional natural frequency is reduced, mainly due to the increase of the overall mass of the joint.

Finally, results obtained by applying the procedure described in this article have been compared to corresponding results derived by using a finite element model of the joint and the NASTRAN finite element code. The joint has been discretized through brick elements and the comparison has been performed by varying the Young's modulus of the adhesive. Fundamental torsional natural

frequencies for a joint with the previous properties and $2c = 20$ mm are shown in Table 1, demonstrating that corresponding results are in good agreement.

5. Conclusions

In this article torsional natural frequencies of non-tubular joints have been evaluated by determining the roots of a determinantal equation. The latter has been written by deriving the equation of motion of the joint for torsional vibrations and by imposing boundary conditions allowing the continuity of both the twisting angle and of the twisting moment at every section of the entire structure. The presence of the lap has been introduced in the equation of motion as a 'distributed spring' whose stiffness has been obtained in a previous article by analysing the state of stress of the adhesive.

Some numerical simulations have been performed for a simple-lap joint and a given geometry so as to illustrate the variation of the fundamental torsional frequency of the joint with respect to changes of both the Young's modulus and thickness of the adhesive and of the length of the lap. A comparison between natural frequencies predicted by using the approach proposed in this article and corresponding results obtained by using NASTRAN permitted to validate this procedure.

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References

Adams, R.D., and Peppiatt, N.A. (1977), "Stress analysis of adhesive bonded tubular lap joints", *J. Adhesion,* **9**, 1-18.

Alwar, R.S., and Nagaraja, Y.R. (1976), "Viscoelastic analysis of an adhesive tubular joint", *J. Adhesion*, **8**, 79-92.

Carpinteri, A. (1997), *Structural Mechanics-A Unified Approach*, E & FN Spon.

Chen, D., and Cheng, S. (1992a), "Torsional stresses in tubular lap joints with tapered adherends", *J. Eng. Mech.,* ASCE, **118**, 1962-1973.

Chen, D., and Cheng, S. (1992b), "Torsional stress in tubular lap joints", *Int. J. Solids and Structures,* **29**, 845-853.

Choi, J.H., and Lee, D.G. (1944), "The torque transmission capabilities of the adhesively-bonded tubular single lap joint and the double lap joint", *J. Adhesion,* **44**, 197-212.

Chon, C.T. (1982), "Analysis of tubular lap joint in torsion", *J. Composite Materials*, **16**, 268-284.

Gent, A.N., and Yeoh, O.H. (1982), "Failure loads for model adhesive joints subjected to tension, compression or torsion", *J. Materials Science,* **17**, 1713-1722.

Goland, M., and Reissner, E. (1944), "The stresses in cemented joints"*, J. Applied Mech*., **11**, 17-27.

Graves, S.R., and Adams, D.F. (1981), "Analysis of a bonded joint in a composite tube subjected to torsion", *J. Composite Materials*, **15**, 211-224.

Hipol, P.J. (1984), "Analysis and optimization of a tubular lap joint subjected to torsion", *J. Composite Materials*, **18**, 298-311.

Kim, K.S., Kim, W.T., Lee, D.G., and Jun, E.J. (1992), "Optimal tubular adhesive-bonded lap joint of the carbon-fiber epoxy composite shaft", *Composite Structures*, **21**, 163-176.

Ko, T.C., Lin, C.C., and Chu, R.C. (1995), "Vibration of bonded laminated lap-joint plates using adhesive interface elements", *J. Sound and Vibration*, **4**, 567-583.

Lee, S.J., and Lee, D.G. (1992), "Development of a failure model for the adhesively bonded tubular single lap joint", *J. Adhesion,* **40**, 1-14.

Lee, S.J., and Lee, D.G. (1995), "An iterative solution for the torque transmission capability of adhesively-bonded tubular single lap joints with nonlinear shear properties", *J. Adhesion*, **53**, 217-227.

Lubkin, J.L., and Reissner, E. (1956), "Stress distribution and design data for adhesive lap joints between circular tubes", *Trans.* ASME, **78**, 1213-1221.

Medri, G. (1977), "Il calcolo delle tensioni nell'adesivo in giunti tra tubi sollecitati da momento torcente", *Ingegneria Meccanica,* **7/8**, 247-251.

Medri, G. (1988), "Viscoelastic analysis of adhesive bonded lap joints between tubes under torsion", *J. of Vibration Acoustics Stress and Reliability in Design-Transactions of the* ASME, **110**, 384-388.

Nayebhashemi, H., Rossettos, J.N., and Melo, A.P. (1997), "Multiaxial fatigue life evaluation of tubular adhesively bonded joints", *Int. J. Adhesion and Adhesives,* **17**, 55-63.

Pugno, N. (1998), "Non-tubular bonded joint under torsion", Ph.D. Thesis, Dept. of Structural Engineering, Politecnico di Torino, Torino, Italy.

Pugno, N. (1999), "Optimizing a non-tubular adhesive bonded joint for uniform torsional strength", *Int. J. Materials & Product Technology*, **14**, 476-487.

Pugno, N., and Surace G. (2000a), "Tubular bonded joint under torsion: analysis and optimization for uniform torsional strength", *J. Strain Analysis for Engineering Design*, **35**(6).

Pugno, N., and Surace, G. (2000b), "Non tubular bonded joint under torsion: Theory and numerical validation", *Structural Engineering and Mechanics*, **10**(2), 125-138.

Pugno N., and Carpinteri A. (2001), "Strength, stability and size effects in the brittle behaviour of bonded joints under torsion: theory and experimental assessment", To be published on *Fatigue & Fracture of Engineering Materials and Structures.*

Rao, M.D., and Zhou, H. (1994), "Vibration and damping of a bonded tubular lap joint", *J. Sound and Vibration,* **178**, 577-590.

Reedy, E.D., and Guess, T.R. (1993), "Composite-to-metal tubular lap joints strength and fatigue resistance", *Int. J. Fracture,* **63**, 351-367.

Treloar, L.R.G. (1975)*, The Physics of Rubber Elasticity*, Clarendon Press Oxford, 3rd Edition.

Zhou, H.M., and Rao, M.D. (1993), "Viscoelastic analysis of bonded tubular joints under torsion", *Int. J. Solids and Structures*, **30**, 2199-2211.