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Advanced inelastic static (pushover) analysis for earthquake applications

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Abstract. Whereas the potential of static inelastic analysis methods is recognised in earthquake design and assessment, especially in contrast with elastic analysis under scaled forces, they have inherent shortcomings. In this paper, critical issues in the application of inelastic static (pushover) analysis are discussed and their effect on the obtained results appraised. Areas of possible developments that would render the method more applicable to the prediction of dynamic response are explored. New developments towards a fully adaptive pushover method accounting for spread of inelasticity, geometric nonlinearity, full multi-modal, spectral amplification and period elongation, within a framework of fibre modelling of materials, are discussed and preliminary results are given. These developments lead to static analysis results that are closer than ever to inelastic time-history analysis. It is concluded that there is great scope for improvements of this simple and powerful technique that would increase confidence in its employment as the primary tool for seismic analysis in practice.

Key words : seismic analysis; pushover; inelastic response.

1. Preamble

Notwithstanding the advances made in recent years in understanding the seismic behaviour of inelastic systems, design practice is still struggling with this subject. Perhaps two of the main reasons for this situation, the large gap between research and application, are (i) the limited or non-existent training given to civil engineers in advanced structural dynamics and inelastic behaviour and (ii) the difficulties encountered by design offices in utilising advanced software for inelastic dynamic analysis. Under the pressure of researchers, and a sub-class of professionals with vision and exceptional abilities, seismic codes do require, explicitly or implicitly, that sources of inelastic energy dissipation are *identified*, and their energy absorption capacity *quantified*, if elastic forces are to be reduced for design purposes. The required 'identification' and 'quantification' of sources of inelastic energy absorption necessarily call for inelastic analysis.

The complications and requirements for decisions in dynamic analysis are an order of magnitude higher than for static analysis. Whereas static modelling is concerned with stiffness and strength (with masses represented by point loads), dynamic analysis is sensitive to stiffness, strength and mass modelling and location. The issue of viscous, hysteretic and algorithmic damping (Bathe 1982) adds significantly to the complications. In recent years, it was shown that the relationship between ductility-driven hysteretic damping and equivalent viscous damping is significantly more

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complex than simple expressions are able to express (Priestley 1998, Borzi and Elnashai 2000). Also, the effect of algorithmic damping is not well understood, especially for inelastic systems. Indeed, an analysis of the particular inelastic structure under a specified input motion may yield force and acceleration results differing by 200% or more, with little or no effect on displacements by only changing the level of algorithmic damping through the dynamic time-integration operators (Broderick *et al*. 1992). The sum total of the above is that inelastic dynamic analysis is perhaps still beyond the realms of design office applications for small and medium enterprises thus a gap is created between seismic design code principles and design office capabilities. It is therefore concluded that efforts aimed at formalising, improving and verifying inelastic static analysis are at the heart of bridging the gap between the essence of modern seismic design and office practice. In this paper, the current practice in inelastic static (pushover) analysis is briefly reviewed, its pros and cons are discussed, new developments explained and future directions identified.

2. Application of inelastic static analysis in earthquake engineering

The conventional approach to seismic design force and deformation calculation is based on conducting elastic analysis under a set of forces factored by a response modification (force reduction) or ‘behaviour’ factor. The ensuing displacements are then amplified by another factor usually derived from the aforementioned force reduction factor. The accuracy of the calculated deformations is therefore highly sensitive to the selection of the force reduction factors as the internal actions. It is customary to include rather large material safety factors, hence the accurate assessment of applied actions is non-critical. On the contrary, inaccuracies in evaluating the deformations of a structure would invariably affect the damage inflicted on it, and under-estimating the deformations is verging on the disastrous. It follows that methods of inelastic analysis have a vital role to play in modern seismic design. Moreover, the current shift in design codes from a single all-embracing limit state, expressed as a single set of actions and a single verification scenario, to a multi-level design procedure complicates the situation even further. Indeed, if the current ‘reduce forces – amplified displacements’ approach is to be continued, there is a need for several force reduction factors and analogous displacement amplification factors, to mirror the various performance limit states. A clear alternative is use of inelastic analysis, where more realistic and reliable estimates of both force and deformations at various response stations may be readily obtained. It is clear that there will be growing dependence on some form of inelastic analysis in the immediate future, in order that the performance objectives defined in modern codes and regulations are demonstrably satisfied.

Several developments have lent weight to the application of inelastic analysis in seismic design and assessment. The most quoted of which is the capacity-spectrum approach, first proposed by Freeman *et al.* (1975). The procedure comprises applying a set of displacements or forces to the degrees of freedom of an idealised structural system and conducting inelastic analysis. The ensuing force-displacement curve is then plotted, with appropriate transformations for dimensional consistency, onto a plot representing both spectral displacements and accelerations of the design event (composite spectrum). The procedure is illustrated in Fig. 1 where the composite spectrum of the Duzce (Kocaeli, Turkey August 1999) record is shown. The composite spectra represent the *demand*, whilst the capacity curve represents the *supply.* If the supply satisfies the demand, within acceptable limits of deformations, then the structure is adequate, otherwise either the supply is

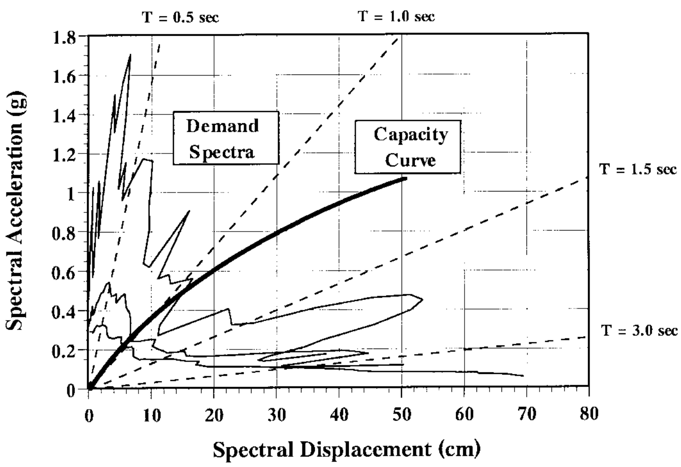

Fig. 1 Capacity-Spectrum method: Demand spectrum from Duzce (Kocaeli, Turkey 1999) earthquake, elastic and inelastic

increased (stiffness, strength and/or ductility), or the demand is reduced. Clearly, the demand can only be reduced as a consequence of changes (not necessarily increase) in the response characteristics of the structure. Changes in stiffness lead to period changes, hence a rotation about the origin in Fig. 1 to perhaps lower requirements, whilst changes in strength and/or ductility would lead to a shift to a lower composite spectrum for higher ductility (equivalent damping).

The capacity spectrum approach (in its original format, or as modified by Fajfar (1999) for inelastic spectra) is appealing, because it gives a visual representation of the supply-demand equation, suggests possible remedial action if the equation is not satisfied and easily incorporates several limit states, expressed as stations on the load-displacement curve of the structure. It, however, requires as a matter of necessity a realistic pushover curve for the structure that is representative of its true dynamic behaviour under the design event (s). Since this approach is the preferred design and assessment method (e.g. FEMA 1998), it follows that further development, refinement and verification of pushover analysis, as an acceptable alternative to inelastic dynamic analysis, are worthwhile research objectives.

3. Review of previous work – the state-of-analysis

It is instructive to review recent work on pushover analysis, as applied only in earthquake analysis of structures (the offshore industry also makes us of pushover analysis for stability and strength assessment of fixed oil production platforms). This is undertaken below in chronological order, followed by a more detailed account of selected papers of more pertinence to the current work.

The use of inelastic static analysis in earthquake engineering dates back to the work of Gulkan and Sozen (1974) or earlier, where a single degree of freedom system is derived to represent equivalently the multi-degree of freedom structure. The load-displacement curve of this substitute to the real structure is evaluated by either finite element analysis or hand calculation to obtain the initial and post-yield stiffness, the yield strength and the ultimate strength. Simplified inelastic analysis procedures for multi-degree of freedom systems have also been proposed by Saiidi and

Sozen (1981) and Fajfar and Fischinger (1988). Therefore, pushover analysis per se is not a recent development. However, this review is concerned with multi-degree of freedom inelastic analysis of complex structures, which is relatively recent.

There are several publications that review the advantages and disadvantages of pushover analysis, with varying degrees of success. Lawson, Vance and Krawinkler (1994) discuss in some detail the range of applicability, the expected realism for various structural systems and highlight the difficulties encountered. The latter study is both conceptual and applied, and is therefore reviewed in more detail at the end of this section. In the course of describing recent trends in seismic design, Krawinkler (1995) discusses pushover analysis as a prelude to capacity spectrum applications. The author mentions a contentious point, which is that in most cases the normalised displacement profile at a first estimate of the target displacement level is utilised for these (defining the shape vector) purposes'. This issue is discussed in subsequent sections of this paper, where it is shown that utilising the displacement profile may lead to gross errors in response calculations. Another important issue raised in the latter review is that 'it must be emphasised that the pushover analysis cannot disclose performance problems caused by changes in the inelastic dynamic characteristics due to higher mode effects'. This is true if the current (as in 1995) techniques are assessed, but is not an insurmountable problem, as discussed in the current work. Several cases of success of the pushover method were reported by Faella (1996), who also pointed towards difficulties with static-dynamic comparisons when the strong-motion input is rich in long period amplifications.

Attempts at improving the procedure have been made, with varying degrees of rigour and success. The simplest and most pragmatic of which is the work of Sasaki *et al.* (1996). This comprises running several pushover analyses under forcing vectors representing the various modes deemed to be excited in the dynamic response. If the individual pushover curves, converted to spectral displacement-spectral acceleration space using the dynamic characteristics of the individual modes, are plotted alongside the composite spectra, it becomes apparent which mode would be the cause of more damage and where is the damage likely to occur. The procedure is intuitive, and does indeed identify potential problems that conventional single mode pushover analysis fails to point out. It, however, falls short of the work of Bracci *et al.* (1997), which is the most recent in-depth study of pushover analysis, and is therefore reviewed in greater detail herein.

The pros and cons of the procedure were also discussed by Krawinkler and Seneviratna (1998). Amongst many other interesting comments, the authors stress that the most important shortcoming of the procedure is the definition and invariance of the applied load vector. The significance of defining the target displacement (required for evaluating structural adequacy under a specific earthquake) is considered secondary to the load vector definition and control. Whereas the authors report on successful pushover cases, they emphasise problem areas by discussing the response of a tall (20 storey) structure, and a structure with a full-height wall. In the former case, the errors are due to the omission of higher mode effects, whilst the latter demonstrates the difficulties encountered with effect of concentrated local demand (base of the wall) on force distribution. A short review by Tso and Moghadam (1998) concluded that fixed load patterns in pushover analysis are limiting, but newly proposed variable load patterns are not sufficiently verified as a superior option.

Kim and D'Amore (1999) set out to assess pushover analysis in comparison with inelastic time-history procedures. They concluded that not all analyses of the same structure under a set of distinct earthquake records are predicted by pushover analysis, a rather obvious conclusion that did not require inelastic dynamic analysis to prove. The interaction between the continuously-changing dynamic characteristics of the inelastic multi-degree of freedom structure with the various frequencies of a set of natural records cannot possibly be duplicated by a single pushover analysis under a predefined and fixed transverse load or displacement vector. But, again, this problem may have a solution.

An adaptive procedure is described in the paper by Bracci *et al.* (1997), and attributed to a previous publication by Reinhorn and Vladescu. This comprises starting the analysis assuming a certain force distribution, usually triangular. Loads imposed in subsequent increments are calculated from the instantaneous story resistance and the base shear in the previous step.

This procedure is applied in the context of defining the moment curvature relationship of the various members as an input parameter, and is intended, as reflected in Eq. (1), to capture the effect of local mechanisms. It does not account for higher mode contribution. The procedure was implemented in the dynamic analysis package IDARC (Kunnath *et al.* 1992) and demonstrated in the paper to give accurate results for the structure considered. However, numerical tests conducted by Lefort (2000) showed that the above procedure grossly under-estimate the strength, compared to inelastic dynamic analysis using IDARC, by up to 60% for a regular 10 storey structure. Peculiarly, conventional pushover with triangular or uniform load distribution gave results far superior to the above adaptive method. This issue is explored further in subsequent sections of the paper.

The aforementioned paper by Lawson *et al.* (1994) is recalled with a view to clarifying the existing obstacles towards refinement of the performance of static inelastic analysis. The authors state that the method 'has no theoretical background and will provide approximate information at best'. It is further explained that 'the issue of seismic design evaluation has little to do with accuracy, since no two earthquakes are alike'. This statement is accurate and revealing, since a procedure that takes the earthquake characteristics into account would clearly be very attractive. Notwithstanding the generalisations in the paper, four steel structures subjected to seven earthquakes were studied dynamically as well as analysed statically using DRAIN 2DX. The results gave very good correlation between static and dynamic response for the 2 storey structure, adequate correlation for the 5 storey case and completely unacceptable comparisons for the 10 and 15 storey building frames. It is surprising that the force distribution based on a square root of sum of squares (SRSS) including spectral ordinates resulted in exceptionally poor results. Estimates of normalised lateral displacements differed by more than 350% between static and dynamic analysis for the SRSS load distribution. The authors attributed this to the method leading to over-representation of higher modes. This is conceivably the reason for the low levels of structural strength observed when using IDARC with the fully adaptive procedure mentioned above.

The paper also discusses the important issue of the roof displacement at which assessment of the dynamic response is mapped by the static analysis. The procedure proposed by Qi and Moehle (1991) and Miranda (1991) to construct a SDOF system that may replace the MDOF in dynamic analysis is recalled as one option to evaluate the target top displacement. A simple form is also proposed, whereby the elastic displacement of the MDOF system is calculated from its fundamental period and the spectral ordinate corresponding to it. Comparisons quoted between the two methods vary by 18%, 6%, 5% and 6% for the 2, 5, 10 and 15 storey structures, respectively. However, there was no pattern as to which is consistently more or less conservative. In spite of the latter point, it seems that reasonable estimates of the target displacement are achievable, hence this issue is not discussed further in this paper.

Three questions were posed by the authors, these are: (i) to what extent does pushover analysis simulate dynamic analysis? (ii) how sensitive are the results to characteristics of the ground motion and the structural model? and (iii) is roof displacement an adequate control parameter for assessment and at what level should comparisons be undertaken? The response to these three questions sums the state of development of the method and its potential to augment, or even replace, inelastic dynamic analysis. The three questions are used at the end of this paper to gauge the significance or otherwise of the developments presented in the current work.

The above review is non-exhaustive, and there are possibly other investigations aimed at identifying the relative merits of static inelastic analysis and/or developing improved techniques. This brief expose, nonetheless, gives a global picture of the state of development of pushover analysis, the problems addressed by various researchers and the degree of success or otherwise in dealing with the problems at hand. There is clearly room for further developments to address the following problems:

· Combining pushover, conventional or advanced, with fibre models where no prior assumptions are made on the behaviour of the member, and where the moment-curvature response is derived from the material characterisation.
· Fully adaptive pushover analysis that takes into account both the current level of local resistance and higher mode contributions.
· Inclusion in the updating process of the load vector a measure of relative spectral amplification corresponding to current periods of vibration.
· Investigating the most realistic and the most stable approaches for updating the applied actions shape vector in adaptive pushover.
· Potential for including more features in pushover analysis that renders it closer to time-domain inelastic dynamic analysis, such as earthquake duration and features peculiar to near-source earthquake records.

To develop the above ideas into an implementation framework, the conventional approach is first reviewed and sources of concern and possible improvements are considered and discussed, as below.

4. Conventional pushover analysis

4.1 Descritption

The procedure is an incremental-iterative solution of the static equilibrium equations. The forcing function is a set of displacements or forces that are necessarily kept constant during the analysis. During an increment the resistance of the structrue is evaluated from the internal equilibrium conditions and the sitffness matrix is updated under certain conditions dependent on the iterative scheme adopted. The out-of-balance forces are re-applied if they are deemed large until a convergence criterion is satified. At convergence, the stiffness matrix is necessarily updated and another increment of displacements or forces is applied. The solution proceeds either until a pre-defined limit state is reached, or the program fails to converge. It is presumed that the program has been sufficiently verified so that numerical, as opposed to structual, collapse is not operative. Three critical elements of the process are therefore worthy of consideration, namely the forcing function nature, distribution and magnitude. In other words, should displacements or forces be applied, what is their distribution, be it constant or variable and what value of applied action should be chosen at each load step if they are not held constant. These issues are dealt with below.

4.2 Force- versus displacement-control

Considering that the objective is to simulate the dynamic response of the structure, the question arises whether the displacements or the forces from a particular mode should be kept constant, noting that inelastic effects cause a deviation between the two options. It is interesting to visualise, as per Fig. 2, the possible effect of displacement- or force-distribution on the response (Elnashai 1997). Application of a constant displacement profile (a) lead to the concealment of the first storey soft response apparent in the constant force plot (b). Consequently, all storey drift-storey shear results are affected. The next question is, which is closer to dynamic response proper. Conceptually, dynamic analysis is inertia force-driven hence the constant force seems more appropriate, but storey forces in a dynamic analysis, even when one mode is dominant, do not exhibit a constant multiplier. It is, however, clear that fixing the displacement distribution may give seriously misleading results. Difficulties arise in conducting stable inelastic analysis under force control though. It is therefore necessary to have the ability to:

a. prescribe a set of forces to be used as the basis for application of the storey forces in the incremental solution,
b. control one displacement, say the roof node,
c. calculate the force corresponding to the constrained applied displacement at the roof node,
d. impose forces at other storey levels in compliance with the fixed ratios specified in (a) above.

This procedure resolves an important problem, that of applying forces but controlling displacements, and it is the procedure utilised in the software package INDYAS (Elnashai *et al.* 2000) used for most of the analysis presented in subsequent sections and the package ADAPTIC (Izzuddin and Elnashai 1989). By adopting this approach, local effects may be identified in most cases, provided that the force distribution used represents the dynamic response at its maximum demand situation. The above procedure is the main reason why the conventionally-used force

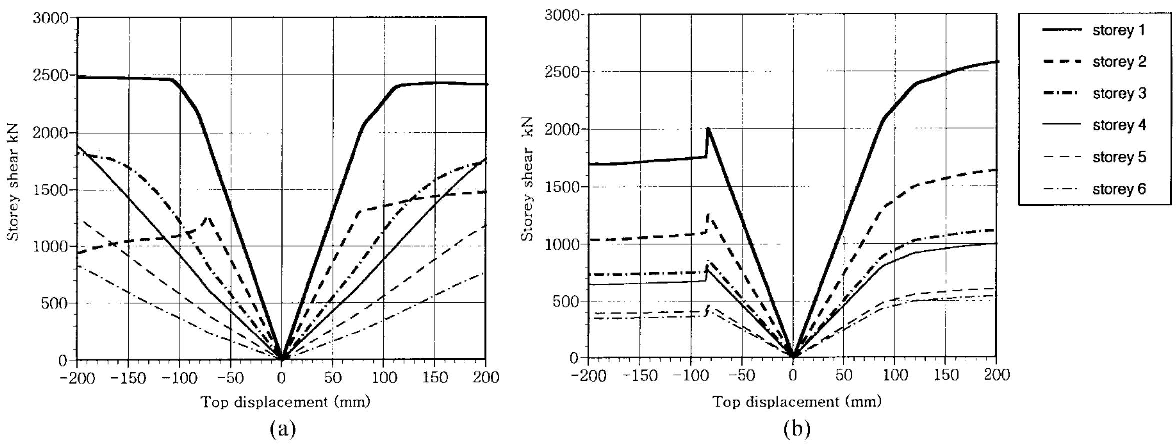

Fig. 2 Storey drift versus storey shear in pushover analysis of 6 story asymmetric frame under constant displacement (a) and constant force (b) contributions

distributions (code-based, uniform or multi-modal) yield less dispersed results than those reported in the literature (e.g. Lawson *et al.* 1994), as demonstrated in Fig. 9.

4.3 Contribution of higher modes

Whereas the problem has been identified long ago, few studies have investigated the effect of higher modes in pushover analysis. Two of the references cited in Section 3 have discussed this, employing a single structure with fixed characteristics. In a recent study, twelve RC structures designed for different ground acceleration and with different levels of ductile detailing (Carvalho and Coelho 1998, where details of the structures are given) were studied, employing 8 earthquake ground motion records of different frequency contents (Mwafy and Elnashai 2000). A sample of the results is shown in Fig. 3 where a polynomial fit of 97 dynamic analyses is shown and Fig. 4 where

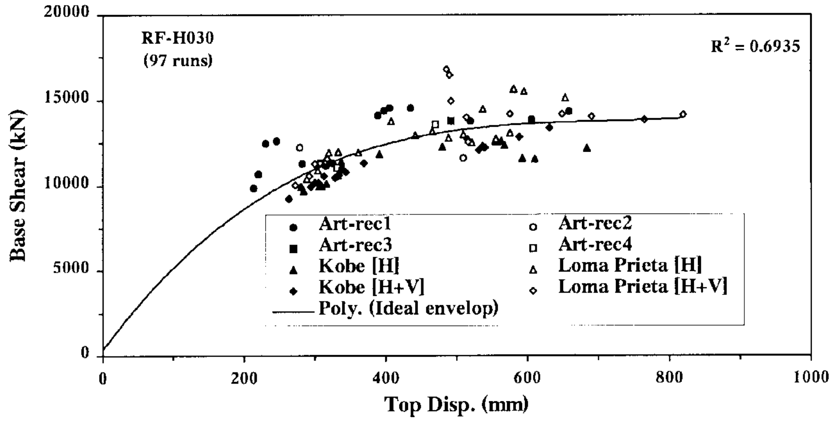

Fig. 3 Analysis of mutli-stroey RC frame. Ninty seven inelastic dynamic analysis results using 8 earthquake records at different intensities are fitted with a polynomial for comparison with static pushover results (Mwafy and Elnashai 2000)

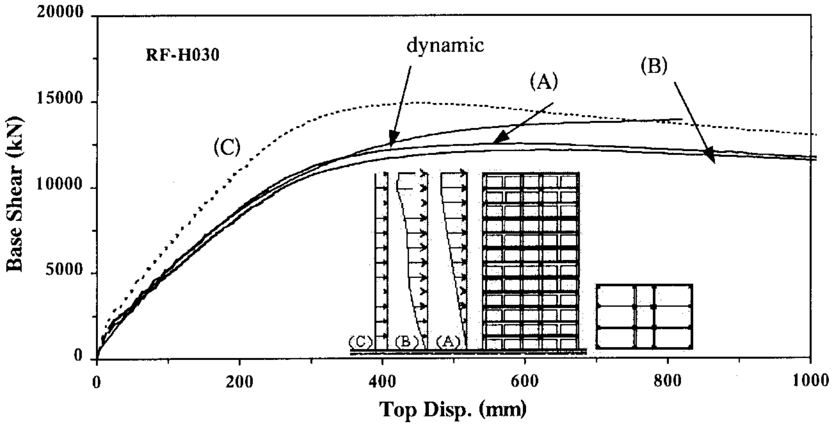

Fig. 4 Comparison between dynamic best-fit and pushover curves for a number of force distributions (same structure as in Fig. 3)

pushover results for uniform, triangular (code) and multi-modal force distributions are compared to the polynomial fit of the dynamic analysis. It is clear from the latter figure that no one distribution is capable of representing the dynamic response throughout the full displacement range. In the same study, it was observed that conventional pushover is particularly poor in predicting the response of frame-wall structures, an observation also reported by Lawson *et al.* (1994). This is probably due to the significant period shift and change of force distribution upon yielding of the wall base. Framed structures will have a more gradual period elongation and force re-distribution.

The coincidence of the dynamic curve and the uniformly distributed forces solution (at large deformations) may indicate that heavy damage has been inflicted on the structure at a specific storey thus favouring a pattern of behaviour akin to a SODF structure. It is also possible that the dynamic curve is affected by the characteristics of the input motion (although four of the eight records are spectrum-compatible, hence are not vastly different). In which case, the coincidence of the two curves is merely fortuitous. That there is practically no difference between the single mode and the multi-modal curves shown in Fig. 4 is noteworthy. For regular structures of moderate periods (less than 1.0-1.2 sec) subjected to earthquake records of a normal type (with high amplification in the usual range of about 0.1 to 0.5 sec, decreasing for longer periods), conventional pushover is adequate when a rigorous procedure for applying forces and controlling one displacement is employed.

The discussion above points towards the necessity of further verification and development of inelastic static analysis procedures. This would lead to more confidence in this potentially powerful yet simple procedure that provides well for seismic design for more than one performance target.

5. Advanced pushover analysis

5.1 Discussion of force distribution and updating

Even if analysis software is able to deal with continuous variation of the applied load vector, it is by no means clear how to update the forces and increment them to provide realistic and stable results. Bracci *et al.* (1997) used the following expression to describe the force variation during pushover analysis, which is considered a form of incremental updating of the force vector:

$$\Delta F_i^{j+1} = V^j\left(\frac{F_i^j}{V_b^j} - \frac{F_i^{j-1}}{V_b^{j-1}}\right) + \Delta V_b^{j+1}\left(\frac{F_i^j}{V_b^j}\right) \tag{1}$$

Where

i : story number

j : increment number

V_b : base shear

ΔV_b : increment of base shear

The author and his co-workers have investigated this issue recently. The storey force distribution given in Fig. 5 is obtained by adding an increment of force to that existing from a previous step to arrive at the new distribution. The precise expression for this operation is described in Section 5.2.

On the other hand, the storey force distribution of Fig. 6 is based on obtaining the current storey shear entirely from a new set of forces accounting for the current state of resistance distribution.

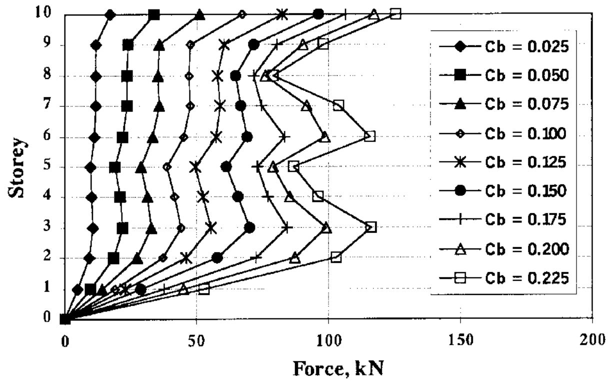

Fig. 5 Step-wise force distribution for a ten storey RC frame with updating the forces using increments of new force distribution added to exisiting forces (cb is the load factor by which the resultant applied force is scaled)

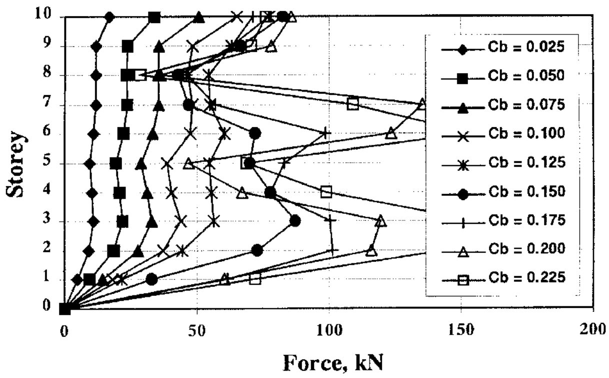

Fig. 6 Step-wise force distribution for the same structure as in Figure 5 but calculated from the total force at each step (cb as above)

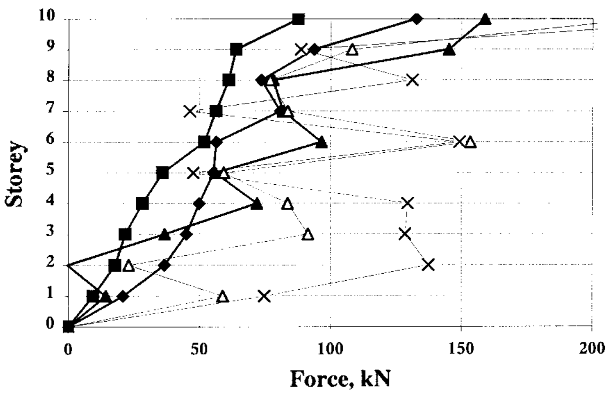

Fig. 7 Snap-shot of force distribution during dynamic analysis of the same structure as in Fig. 5. Legend as in Figs. 6 and 7 (different lines for values of cb, the load factor)

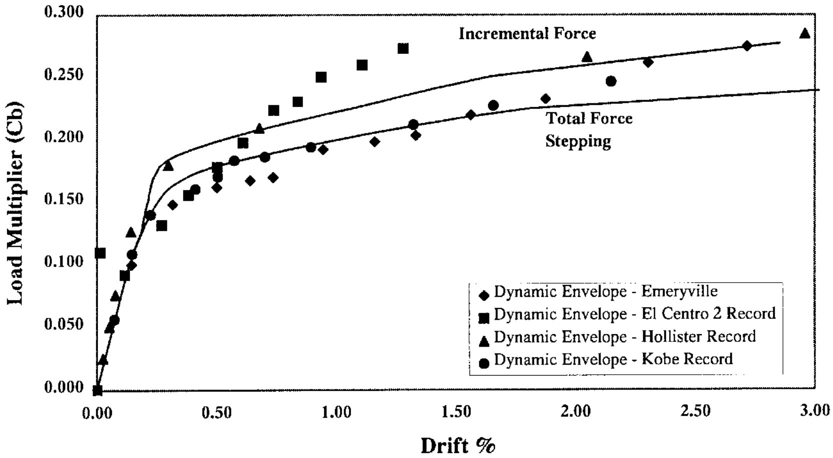

Fig. 8 Comparison of pushover curves for different force stepping methods with dynamic analysis results (using IDARC, no adaptive solution employed)

The two figures can therefore be viewed as incremental and total updating of the force distribution. Two issues are reflected in these plots, as follows:

· The force distribution near collapse is completely different to that at the start of the analysis. It is therefore emphasised that the fixed force distribution format is wholly inadequate to describe the behaviour of this structure.
· The two distributions are very different, and would yield different results, especially on the local level. Whereas the total approach is probably more accurate, it is doubtful that general-purpose analysis packages are capable of accommodating such severe and abrupt changes in the applied actions.

It is not obvious at this stage which of the two is more realistic, but it is clear that the distribution in Fig. 5 is superior to a fixed format and is likely to enable a stable solution to progress up to structural collapse. To assess the accuracy, as opposed to the stability, of the two extreme cases, the force distribution during dynamic analysis using IDARC, shown in Fig. 7 is examined (Lefort 2000). Also, the pushover curves without using the internal adaptive facility of IDARC but rather by including in the input data series of force distributions representative of the 'incremental-Fig. 5' and the 'total-Fig. 6' cases, respectively, are studied. These are shown in Fig. 8. Comparison of the three force distributions confirm that, qualitatively, the total force stepping option gives a better representation of the dynamic force distribution, with the exception of the top levels at large amplitudes, where a whipping response is observed in the dynamic case. This is a function of the characteristics of the input motion (El Centro), a full account of which is beyond the scope of this paper.

Several important observations emanate from Fig. 8. Firstly, up to a drift of about 2% the total force stepping (as per Fig. 6) gives results very close to the dynamic pushover curve (obtained from successively scaling the earthquake input and running inelastic dynamic analysis. Scaling in this case is simply by acceleration, whereas scaling of the records for the analysis results shown in Fig. 3 is by velocity spectrum intensity; Mwafy and Elnashai 2000). Above 2% drift, the results of the manually-adaptive IDARC analysis is unreliable, because the scaling procedure is inaccurate in this zone due to difficulties with obtaining eigensolutions for low levels of global stiffness. Incremental force stepping seems to over-estimate the strength of the structure, possibly due to having a high point of application of the resultant transverse force. The results from the El Centro analysis show that features of the strong-motion record whereby a single large pulse (so-called fling, or near-source effect) are capable of defeating the static pushover analysis. This issue is beyond the scope of this work, but is worthy of further discussion. It is reasonable to state herein that the total force stepping option results in more realistic and accurate solutions. However, if the analysis package is incapable of accommodating such abrupt changes in applied forces, the incremental option is still far superior to a fixed force approach.

5.2 The role of spectral shapes

The expectation that one single inelastic analysis will provide design or assessment information

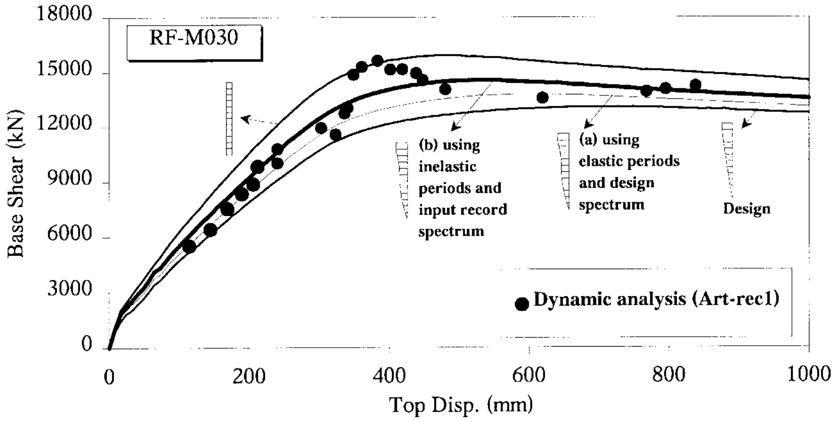

Fig. 9 Comparison of pushover curves for eight storey RC frame with and without spectrum scaling and different force distribution for spectrum-compatible artificial record

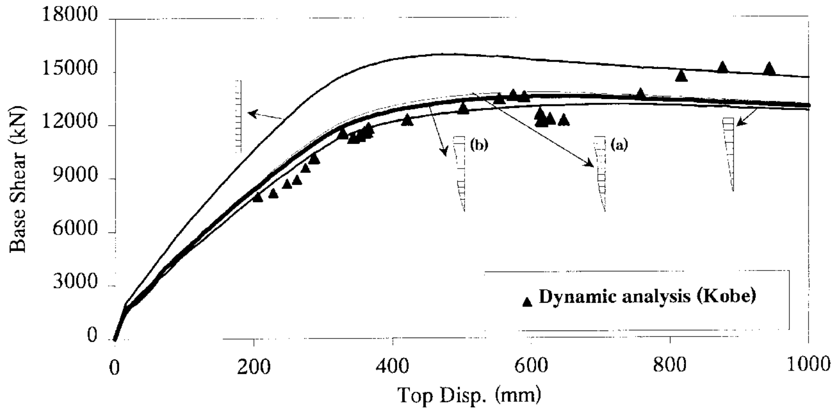

Fig. 10 Comparison of pushover curves for eight storey RC frame with and without spectrum scaling and different force distribution for Kobe (Kobe University) record

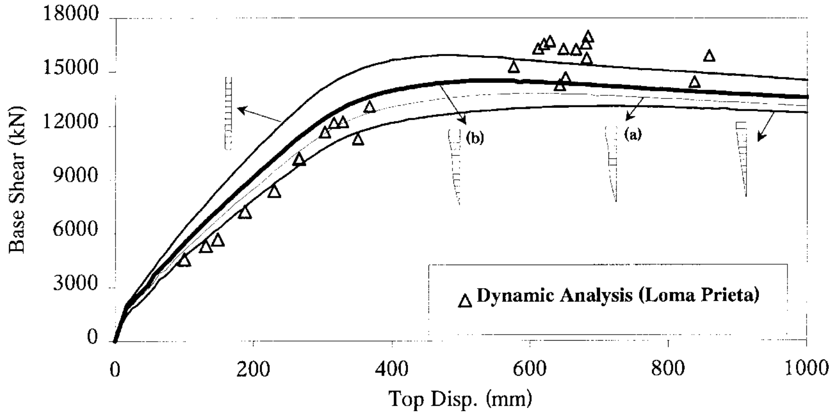

Fig. 11 Comparison of pushover curves for ten storey RC frame with and without spectrum scaling and different force distribution for Loma Prieta (Sylmar Hospital) record

for a structure under all earthquake input records regardless of their frequency content is rather unrealistic. The idea of including in the incremental force updating process a measure of spectral shape is therefore a natural extension worthy of consideration. This approach can be employed in programs that do not necessarily have a specific facility for spectral scaling. Preliminary trials using the analysis package ADAPTIC (Izzuddin and Elnashai 1992) have given promising results, as shown in Figs. 9, 10 and 11. The frame referred to is one of the structures described by Carvalho and Coelho (1998).

The plots have been obtained by deriving a fixed load distribution using a scaling procedure for the first two modes utilising the spectral ordinates corresponding to the three elastic periods (as in curves marked ‘a’) or cracked member periods (as in curves marked ‘b’). The latter was estimated from Fourier analysis of the dynamic response. In case (a), the design spectrum was used to scale the modal contributions, whilst in case (b) the actual earthquake spectrum (Artificial, Kobe, Loma Prieta) was employed. The process is depicted graphically in Fig. 12, noting that the artificial record

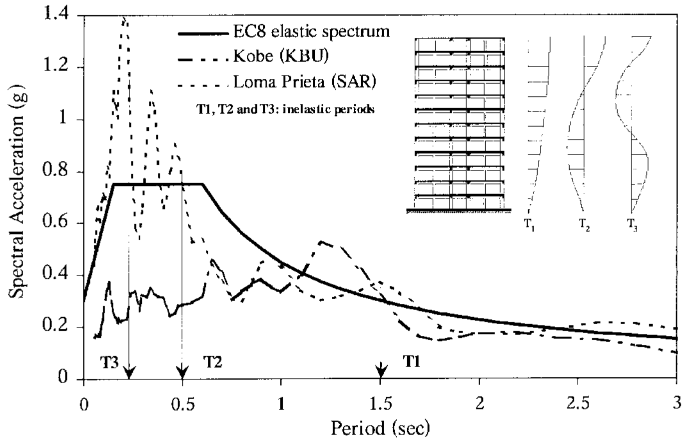

Fig. 12 Scaling of force distrubtion using spectral ordinates for three modes

was compatible with the EC8 spectrum.

The shown results indicate that using spectrum scaling improves the comparison between static and dynamic inelastic analysis results. This improvement varies between marginal (e.g. Fig. 10) and significant (e.g. Fig. 9). The two curves for spectrum-scaled forces are much closer to the dynamic response points than either of the two others. Moreover, for the range of displacement response corresponding to moderate-to-heavy damage, use of inelastic periods gives better estimates of the dynamic response of the structure. It is reiterated that the inelastic periods are calculated from discrete Fourier amplitude spectra of the inelastic dynamic response. The uniform distribution of forces leads to, as expected, higher resistance due to the lower point at which the resultant force is applied. In Fig. 9, for example, there is no difference between the design and record spectra, since the record is spectrum-compatible. Hence the difference in results is due to difference in spectrum ordinates corresponding to elastic and inelastic periods. This indicates that the simple extension to pushover analysis using existing software packages (IDARC, DRAIN etc) by scaling forces spectrally using estimates of cracked stiffness would improve the results in comparison with dynamic analysis.

Whereas this simple idea leads to improved results, it is still open for interpretation, in terms of the procedure of scaling the force vector. In Section 4.3, two distinct force distributions during pushover analysis were given, as employed in IDARC analysis (but not using the internal adaptive procedure). These were derived from the expressions:

$$C_{S_{a,i}} = \frac{W_i \sqrt{\sum_{k=1}^{m} \left[\phi_{ik} \Gamma_k \frac{S_{a_k}}{\sum_{k=1}^{m} S_{a_k}} \right]^2}}{\sum_{l=1}^{n} W_1 \sqrt{\sum_{k=1}^{m} \left[\phi_{lk} \Gamma_k \frac{S_{a_k}}{\sum_{k=1}^{m} S_{a_k}} \right]^2}} \qquad (2)$$

$$F_{i,j} = C_{S_a,i} \Delta V_b - F_{i,j-1} \qquad \text{for force distribution in Fig. 5} \qquad (3)$$

$$F_{i,j} = C_{S_a,i} V_b \qquad \text{for force distribution in Fig. 6} \qquad (4)$$

Where

i : storey number
j : increment number
m : number of modes
n : number of stories
ϕ : mode shape vector
Γ : mass participation factor
S_a : spectral acceleration ordinate
V_b : base shear
ΔV_b : increment of base shear
k, l : counters

On the other hand, the results shown in Figs. 9, 10 and 11 use direct scaling of the modal force vectors by the corresponding spectral ordinate. Other alternatives exist and this is still an area of further investigation (Papanikolaou 2000, Antoniou 2002).

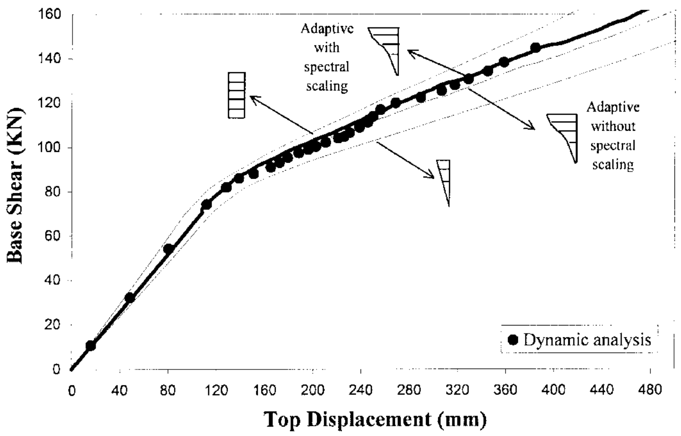

Fig. 13 INDYAS results for a 6 DOF idealised inelastic regular structure. Shown comparison between conventional and fully adaptive analysis with and without spectrum scaling under the Loma Prieta (Corralitos, Santa Cruz) earthquake of 17 October 1989

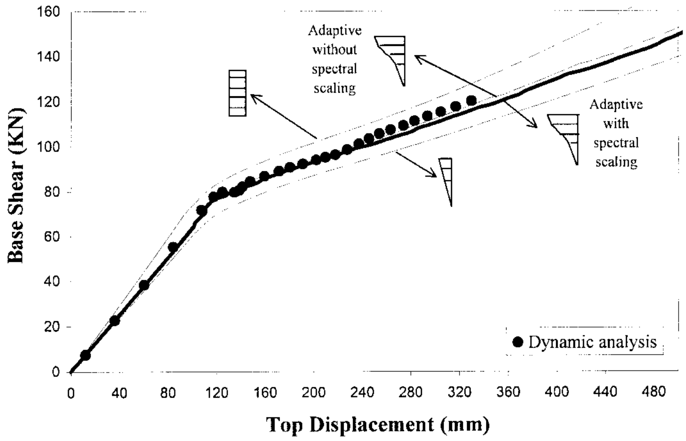

Fig. 14 INDYAS results for an idealised inelastic 6 DOF structure with a soft storey (2nd floor). Shown are comparisons between conventional and fully adaptive analysis under the Loma Prieta (Emeryville, California) earthquake of 17 October 1989

5.3 Current status and further development

The outcome of the above discussion and assessment of results lead to the development of a fully adaptive pushover facility in the analysis program INDYAS (Elnashai *et al.* 2000), featuring the following:

- Fibre modelling of all sections for RC and steel analysis, utilising advanced formulations for steel plasticity and concrete inelasticity including the effect of confinement and cyclical degradation.
- Geometric nonlinearity taken into account, in order that flexible elastic structures can be analysed effectively.

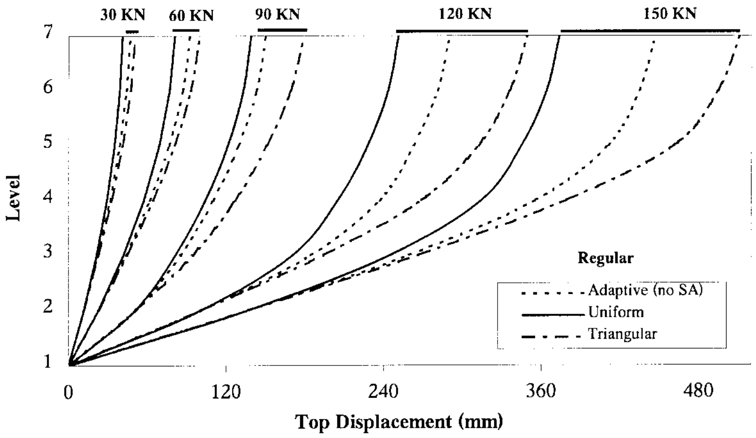

Fig. 15 Storey drift distribution for three types of analysis (regular structure)

- At pre-assigned, or at all, deformation levels, during pushover analysis, the instantaneous nonlinear and inelastic stiffness matrix and the mass matrix are used to calculate new eigensolutions.
- The new eigenvectors are used to update the applied load vector according to user-defined options such as absolute sum, square root of sum of squares or complete quadratic combination of modal contributions.
- User defined options to scale the individual contributions above by a given spectrum, with the knowledge of the new eigenvalues. There is also the possibility of feeding in a time-history and INDYAS would evaluate the spectrum and use it for scaling purposes.
- The analysis is kicked-off with an assumed force distribution and force control, with an automatic transition to displacement control to cross limit points.

In Fig. 13 an idealised yielding structure is used to test the new developments. The yielding is characterised on the stress-strain level and the structure represents a six storey shear frame subjected to the Corralitos (Santa Cruz, California) and Emeryville (Oakland, California) records from the Loma Prieta earthquake of 17 October 1989. Fig. 14 gives results for the same structure but with a soft second storey. The comparisons show the accuracy of the fully adaptive procedure, compared to fixed mode pushover analysis. Both adaptive solutions, with and without spectrum scaling, are close to the dynamic analysis results. It is also evident that other fixed force distributions perform better for regular structures. The results collectively also underplay the role of spectrum scaling. This is because the idealised structures are all of short-to-medium periods, hence the important effect of higher mode amplification (especially under the Emeryville record) is not significant. The differences between fixed and adaptive analysis is even more striking on the storey drift level. This is indicated in Fig. 15 where the structure profile is shown at different levels of base shear. Considerable errors in deformation response would result from using a fixed triangular distribution of forces, in contrast to the adpative solution in INDYAS. Finally, it is noteworthy that whereas the nonlinear dynamic analysis failed to converge, even with the re-taking of non-convergent steps as implemented in INDYAS, whilst the static analysis continued without problems.

The analyses required 500 re-evaluations of the inelastic-nonlinear eigenvalues and vectors. The running time for the fully adpative solution for this simple structure was 51 seconds compared to 44 seconds for the fixed load vector analysis, an increase of 14%. This would be very much reduced if the number of re-calculation of eigenvalue problems is restricted to stations of significance in the response of the structure. This will not lead to a reduction in accuracy since the milestones of response alteration would be perhaps in the range of 10-15, not 500.

Whereas the above developments open a whole range of possibilities for investigation and take inelastic static analysis closer to dynamic analysis than before, there are still major problems to resolve. These range from definitive guidelines on how to update force vectors, as highlighted in previous sections of this paper, to issues of phase distribution and duration of input motion and possibilities of accounting for this in some way in static analysis. Also, if the pushover analysis is performed for the purpose of application of the capacity spectrum method, some interesting, and hitherto unresolved, questions arise. For example, the transformation of the load-displacement curve to the spectral acceleration-spectral displacement space necessarily utilises the dynamic characteristics of a substitute structure, vibrating in one mode. But, with the fully adaptive solution, the force distribution is derived from several ever-changing modes. Hence, it is inappropriate to use the simple transformation proposed in the literature (e.g. Sasaki *et al.* 1996). Moreover, when considering the level of damping to be applied to the demand curve, it is not clear which modal damping parameter would be more appropriate, since the capacity curve is not a single mode response representation. In addition to the above, the alternative of applying an adaptive displacement profile has not been investigated. Whereas displacement-controlled conventional pushover analysis has the disadvantage of forcing the structure into a response mode, an adaptive displacement profile is in principle superior to applying forces, and also potentially more stable. The above is only a sample of outstanding problems. However, the static pushover approach is sufficiently developed to be applied with confidence in most cases, and especially so with the proposed adaptive refinement for irregular and long-period structures. Notwithstanding, experimental verification of adaptive pushover has not been undertake and concerted effort in this direction are called for.

6. Conclusions

In this paper, the significance of pushover analysis as an alternative to inelastic dynamic analysis in seismic design and assessment has been discussed. A brief review of the available literature indicated that whereas important advancements have been made in the past decade or so, there are many fundamental issues still outstanding and even basic ingredients of the approach are not tackled. In particular, the effect of higher mode contributions and the continuous change in the resistance distribution of the structure are identified as elements of uncertainty in the results obtained from pushover analysis. Early results have been given and discussed. This leads to the conclusion that including a measure of spectral amplification in the procedure would liberate it from the restriction of providing one pushover curve to fit all dynamic analyses under a variety of earthquake ground motion records. However, the differences observed in the examples studied above are not spectacular. On the whole, results obtained so far show promise and provide a much closer fit to inelastic dynamic results than existing attempts. Finally, a fully adpative and versatile pushover procedure recently developed in the computer package INDYAS is described and sample results presented and discussed.

It is useful to recall the three questions posed in the paper by Lawson *et al.* (1994), namely: (i) to what extent does pushover analysis simulate dynamic analysis? (ii) how sensitive are the results to characteristics of the ground motion and the structural model? and (iii) is roof displacement an adequate control parameter for assessment and at what level should comparisons be undertaken? In the light of the work presented above using the program INDYAS, it is reasonable to respond to the first two questions as follows:

i) If the continuous change in resistance distribution and differing levels of modal contributions are taken into account, pushover analysis leads to results that are very close indeed to dynamic analysis, provided the duration of the record is neither exceptionally long nor short.
ii) The results could be very sensitive to both earthquake and structure characteristics, but both are taken into account by the proposed fully adaptive pushover method.

The issue of target displacement is beyond the scope of this paper, but for the time being it does not seem to be critical, based on the observations of Lawson *et al.* (1994), which are admittedly based on limited data.

It is clear that the developed advanced pushover procedures provide professionals with a real alternative to inelastic dynamic analysis, which requires more specialised knowledge than static analysis. It is also emphasised that fixed force distribution analyses may give eronious results and are potentially unconservative, concerns that are demonstrably removed by emplying adaptive pushover techniques. Much development is still needed, coupled with further analytical verification and ultimately experimental confirmation.

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References

Antoniou, S. (2002), "Pushover analysis for seismic design and assessment of RC structures", PhD thesis, Engineering Seismology and Earthquake Engineering Section, Imperial College, London, UK (to be submitted).

Bathe, K-J. (1982), *Finite Element Procedures in Engineering Analysis*, Prentice Hall.

Bracci, J.M., Kunnath, S.K., and Reinhorn, A.M. (1997), "Seismic performance and retrofit evaluation of RC structures", ST Division, ASCE, **123**(1), 3-10.

Broderick, B.M., Elnashai, A.S., and Izzuddin, B.A. (1992), "Observations on the effect of numerical dissipation on the nonlinear dynamic response of structural systems", *J. Eng. Struct.*, **16**(1), 51-62.

Borzi, B., and Elnashai, A.S. (2000), "Assessment of inelastic response of buildings using force and displacement-based approaches", *Structural Design of Tall Buildings*, **9**(4), 251-77.

Carvalho, E.C., and Coelho, E. (Eds.) (1997), "Numerical investigations on the seismic response of RC frames designed to EC8", ECOEST-PREC8 Rep. no. 7, September.

Elnashai, A.S. (1997), "Seismic assessment of steel structures designed with NEHRP capacity reduction factors", Nuclear Electric Final Report Reference BB/G/40179, UK.

Elnashai, A.S., Pinho, R., and Antoniou, S. (2000), "INDYAS, a program for **IN**elastic **DY**namic **A**nalysis of **S**tructures", Engineering Seismology and Earthquake Engineering Report No ESEE 2/2000.

Faella, G. (1996), "Evaluation of RC structures seismic response by means of nonlinear static pushover analyses", *11th World Conf. on Earthq. Eng.*, Acapulco, Mexico, Paper no. 1146, 8.

Fajfar, P. (1999), "Capacity spectrum method based on inelastic demand spectra", *Earthq. Eng. and Struct. Dyn.*, **28**, 979-93.

Fajfar, P., and Fischinger, M. (1988), "N2 Method for nonlinear seismic analysis of regular structures", *Proc. of 9th World Conf. on Earthq. Eng.*, Tokyo-Kyoto, Japan, **5**, 111-116.

FEMA (1998), NEHRP Guidelines for Seismic Rehabilitation of Buildings, FEMA 273, October.

Freeman, S.A., Nicoletti, J.P., and Tyrell, J.V. (1975), "Evaluation of existing buildings for seismic risk - A case study of Puget Sound Naval Shipyard Bremerton, Washington", *Proc. of the United States Nat. Conf. on Earthq. Eng*., Berkeley, 113-22.

Gulkan, P., and Sozen, M.A. (1974), "Inelastic response of RC structures to earthquake motion", ACI, **71**, 609-14.

Izzuddin, B.A., and Elnashai, A.S. (1989), "ADAPTIC - a program for the adaptive static and dynamic analysis of frames structures", Engineering Seismology and Earthquake Engineering Report No. ESEE 7/89.

Kim, S., and D'Amore, E. (1999), "Push-over analysis procedures in earthquake engineering", *Earthq. Spectra*, **15**(3), August, 417-434.

Krawinkler, H., and Seneviratna, G.D. (1998), "Pros and cons of pushover analysis of seismic performance evaluation", *Eng. Struct.*, **20**(4-6), 452-64.

Kunnath, S.K., Reinhorn, A.M., and Lobo, R.F. (1992), "IDARC Version 3.0 - a program for inelastic damage analysis of reinforced concrete structures", Tech. Rep. no. NCEER-92-0022, Nat. Centre for Earthq. Eng. Res., State University of New York at Buffalo.

Lawson, R.S., Vance, V., and Krawinkler, H. (1994), "Nonlinear static pushover analysis, why, when and how?", *Proc. of 5th US Nat. Conf. on Earthq. Eng.*, Chicago, **1**, 283-292.

Lefort, T. (2000), "Advanced pushover analysis of RC multi-storey buildings", MSc dissertation, Engineering Seismology and Earthquake Engineering Section, Imperial College, London, UK.

Miranda, E. (1991), "Seismic evaluation and upgrading of existing buildings", PhD thesis, University of California, Berkeley.

Mwafy, A.A., and Elnashai, A.S. (2001), "Static pushover versus dynamic collapse analysis of RC frames", *Eng. Struct.*, **23**, 407-424.

Papanikolaou, V. (2000), "Developement and verification of adaptive pushover analysis procedures", MSc dissertation, Engineering Seismology and Earthquake Engineering Section, Impeial College, London, UK.

Priestly, J.M.N. (1998), "Displacement-based appraoches to rational limit state design of new structures", *Proc. of 11th Europ. Conf. on Earthq. Eng*., September, 317-335.

Qi, X., and Moehle, J.P. (1991), "Displacement design approach for RC structures subjected to earthquakes", Report UCB/EERC-91/02, Earthq. Eng. Res. Ctr., University of California, Berkeley, January.

Saiidi, M., and Sozen, M.A. (1981), "Simple nonlinear analysis of RC structures", ASCE, Struct. Div., **107**(ST5), 937-951.

Sasaki, K.K, Freeman, S.A., and Paret, T.F. (1996), "Multi-modal pushover procedure (MMP) - A method to identify the effects of higher modes in a pushover analysis", *Proc. of 6th US Nat. Conf. on Earthq. Eng*., EERI, Oakland California, 12 pp.

Tso W.K., and Moghadam A.S. (1998), "Pushover procedure for seismic analysis of buildings", *Progress in Struct. Eng. and Mater.*, **1**(3), 337-344.