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# Viscoplastic collapse of titanium alloy tubes under cyclic bending

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**Abstract.** This paper presents the experimetal result on the viscoplastic response and collapse of the titanium alloy tubes subjected to cyclic bending. Based on the capacity of the bending machine, three different curvature-rates were used to highlight the viscoplastic behavior of the titanium alloy tubes. The Curvature-controlled experiments were conducted by the curvature-ovalization measurement apparatus which was designed by Pan *et al.* (1998). It can be observed from experimental data that the higher the applied curvature-rate, the greater is the degree of hardening of titanium alloy tube. However, the higher the applied curvature-rate, the greater is the degree of ovalization of tube cross-section. Furthermore, due to the greater degree of the ovalization of tube cross-section for higher curvature-rates under cyclic bending, the number of cycles to produce buckling is correspondingly reduced. Finally, the theoretical formulation, proposed by Pan and Her (1998), was modified so that it can be used for simulating the relationship between the controlled curvature and the number of cycles to produce buckling for titanium alloy tubes under cyclic bending with different curvature-rates. The theoretical simulation was compared with the experimental test data. Good agreement between the experimental and theoretical results has been achieved.

Key words: titanium alloy tube; viscoplastic collapse; cyclic bending; curvature; ovalization.

## 1. Introduction

The circular tube components in a number of practical industrial applications, such as offshore structures, nuclear reactor components, earthquake resistant structures, transporting tubes of heat exchanger, etc., must be designed to resist cyclic bending. The major characteristic of the circular tube subjected to bending is the nonlinear behavior of ovalization of the tube cross-section. It is known that the magnitude of the tube ovalization increases when the bending moment increases. If the bending moment increases cyclically, the magnitude of tube cross-section causes a progressive reduction in its bending rigidity (accumulation of damage) which can ultimately result in buckling or fracture of the circular tube. Therefore, studies concerning the response and collapse of circular tubes subjected to cyclic bending are very important for many industrial applications.

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Experimentally or theoretically investigation on the elasto-plastic behavior of several metal tubes subjected to monotonic or cyclic bending with or without external pressure have been done by researchers (Kyriakides and Shaw 1982, Shaw and Kyriakides 1985, Kyriakides and Shaw 1987, Corona and Kyriakides 1988, Corona and Kyriakides 1991, Kyriakides and Ju 1992a, Kyriakides and Ju 1992b, Pan and Leu 1997). However, the investigation on the viscoplastic behavior of circular tubes subjected to monotonic or cyclic bending was only worked by Pan and his coworkers. Pan and Fan (1998) investigated the effect of prior curvature-rate at preloading stage (viscoplastic effect) on the subsequent creep (hold constant moment for a period of time) or relaxation (hold constant curvature for a period of time) behavior of 304 stainless steel tubes. It has been found that the curvature-rate at the preloading stage has a strong influence on the subsequent pure bending creep or relaxation. Furthermore, Pan and Her (1998) investigated the viscoplastic collapse of 304 stainless steel tubes subjected to cyclic bending. Three different curvature-rates were used for conducting the loading of cyclic bending. They discovered that the higher the applied curvature-rate, the greater is the degree of hardening of 304 stainless steel tube. However, the ovalization of tube cross-section increases when the applied curvature-rate increases. Pan and Hsu (1999) experimentally and theoretically studied the viscoplastic behavior of 304 stainless steel tubes subjected cyclic bending. The endochronic viscoplastic theory, which was proposed by Pan and Chern (1997), was used to investigate the viscoplastic behavior of the tubes under cyclic bending.

It has been shown from the experimental result that engineering alloys such as 304 and 316 stainless steels, high-strength titanium alloys, exhibit viscoplastic behavior (Krempl 1979, Kujawski and Krempl 1981, Iktgami and Ni-Itsu 1983). Although the viscoplastic response and collapse of 304 stainless steel have been done by Pan and Her (1998), the viscoplastic response and collapse of other engineering alloy tubes have not been investigated. The proposed theoretical formulation for the relationship between the controlled curvature and the number of cycles to produce buckling for 304 stainless steel tubes under cyclic bending with different curvature-rates should be verified for other engineering alloy tubes. Based on this recognition, we propose the following experimental and theoretical researches.

In this paper, the viscoplastic response and collapse of titanium alloy tubes under cyclic bending was studied. A four-point bending machine (Pan et al. 1998, Pan and Her 1998, Pan and Fan 1998, Pan and Hsu 1999) was used for conducting the cyclic bending tests. The curvature-ovalization measurement apparatus (COMA), designed by Pan et al. (1998), was used for controlling the curvature-rate during the cyclic bending tests. Based on the capability of the bending machine, three curvature-rates, 0.3, 0.03 and 0.003  $m^{-1}/sec$  (m<sup>-1</sup>s<sup>-1</sup>), were used to demonstrate the effect of viscoplastic response and collapse of titanium alloy tubes. The magnitudes of curvature and ovalization of the tube cross-section were measured by the COMA. The magnitude of bending moment was measured by the two load cells, mounted in the bending machine. It was observed from the experimental result that due to the hardening of the metal tube under higher curvaturerates, a larger moment is required to bend the tube. Furthermore, the ovalization of tube crosssection progressively increases with the number of cycles, when the tube is subjected to cyclic bending. In addition, the ovalization of tube cross-section increases when the applied curvature-rate increases. Due to the greater degree of ovalization for higher curvature-rates under cyclic bending, the number of cycles required to produce buckling is correspondingly reduced, as compared to that under slower curvature-rates.

Finally, the theoretical formulation, proposed by Pan and Her (1998), was modified so that it could be used for simulating the relationship between the controlled curvature and the number of

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Fig. 1 (a) A schematic drawing of the bending device, (b) A schematic drawing of the COMA

cycles to produce buckling for titanium alloy tubes under cyclic bending with different curvaturerates. It has been shown from the comparison between the theoretical simulation and the experimental data that the theoretical formulation can properly simulate the experimental result.

#### 2. Experiments

The experiments were conducted using a test facility, designed by Pan and his coworkers, consisting of a pure bending device and a curvature-ovalization measurement apparatus (COMA). The bending device was used for conductiong the cyclic bending tests and the COMA was used for measuring the variations in tube curvature and the ovalization of the tube cross-section. The bending facilities, tested material and experimental procedure are discussed in the following.

## 2.1 Bending facilities

Fig. 1(a) shows a schematic drawing of the bending device. It was designed as a four-point bending machine capable of applying bending and reverse bending (Pan et al. 1998, Pan and Her 1998, 1998 and Fan 1998, Pan and Hsu 1999). This bending device is similar to the bending devices reported by Kyriakides and coworkers (Kyriakides and Shaw 1982, Shaw and Kyriakides 1985, Kyriakides and Shaw 1987, Corona and Kyriakides 1988, Corona and Kyriakides 1991, Kyriakides and Ju 1992a). The bending device consists of two heavy assemblies resting on two beams. Heavy chains run around these sprockets and are connected to two hydraulic cyclinders and load cells forming a closed loop. Each tube tested is fitted with solid rod extensions. The rods are chamfered at the ends to reduce stress concentrations and to avoid premature buckling at the end of the test specimen. The test specimen assembly is engaged by the bending device through four rollers located on each sprocket. Bending is achieved by contracting either of the cyclinders, in the process causing the rotation of the sprockets. As the sprocket rotates, the test specimen assembly is loaded by a couple formed by concentrated loads from two of the rollers. During bending, the rolling contact between the test specimen assembly and the device guarantees the freedom of movement of the tube in the axial direction. Bending in the reverse sense is achieved by reversing the direction of the flow in the hydraulic circuit. The applied bending moment is directly proportional to the tension in the chain, which is monitored by two load cells in the chain loop.

The magnitudes of the tube curvature and the ovalization of the cross-section were measured by a special instrument, curvature-ovalization measurement apparatus (COMA), which was designed by Pan *et al.* (1998). Fig. 1(b) shows a schematic drawing of the COMA. The apparatus is a lightweight instrument which can be mounted close to the tube's mid-span during the test. By using magnetic detector (middle part of the COMA), it can measure the net change of the outside diameter of the tube cross-section. This measuring amount can be used to calculate the ovalization of tube cross-section. Simultaneously, it can measure variations in the tube curvature close to the mid-span from the inclinometer signals. Based on the fixed distance between the two side-inclinometers and the angle changes detected by the two side-inclinometers, the tube curvature can be obtained (see Fig. 1b). A detailed description of the COMA can be found in the previous work by Pan *et al.* (1998).

### 2.2 Material

The material used in this study is titanium alloy tube (ASTM B338 Grade 2), with chemical composition: N 0.01, C 0.01, H 0.001, Fe 0.005, O 0.09 and Ti remainder. The yield stress is 349 MPa, the tensil ultimate stress is 459 MPa and the percent elongation is 47%. The titanium alloy tube has the outside diameter D of 25.4 mm and wall thickness *t* of 0.7 mm (D/t = 36.3)

#### 2.3 Experimental procedure

The cyclic bending test was conducted by using the bending machine described in above Section. The test was a curvature-controlled cyclic bending test with curvature amplitudes varying from  $\pm 0.3$  to  $\pm 0.75$  m<sup>-1</sup>. The magnitudes of the curvature and curvature-rate were controlled by the COMA shown in Fig. 1(b). The ovalization of tube cross-section was also measured by the COMA. Tube were cyclically bent at three different curvature-rates ( $\dot{\kappa}$ ), 0.3, 0.03 and 0.003 m<sup>-1</sup>/sec (m<sup>-1</sup>s<sup>-1</sup>).



Fig. 2 Cyclic moment-curvature curves under three different curvature-rates of (a) 0.3, (b) 0.03 and (c)  $0.003 \text{ m}^{-1}\text{s}^{-1}$ 



Fig. 3 The corresponding cyclic ovalization ( $\Delta D/D$ )-curvature ( $\kappa$ ) curves under three different curvature-rates of (a) 0.3, (b) 0.03 and (c) 0.003 m<sup>-1</sup>s<sup>-1</sup>

#### 3. Experimental results

In this section, the experimental data of the viscoplastic response and viscoplastic collapse for titanium alloy tubes under cyclic bending are discussed.

#### 3.1 Viscoplastic response of titanium alloy tubes under cyclic bending

Fig. 2 presents a typical result of cyclic moment (M) - curvature ( $\kappa$ ) curves for titanium alloy tubes under three different curvature-rates. The magnitude of the cyclic curvature is  $\pm 0.6 \text{ m}^{-1}$ . It is observed from the moment-curvature curve that the titanium alloy tube is cyclically harden and gradually steady after a few cycles for symmetric curvature-controlled bending. The magnitudes of moment at the curvature of 0.6 m<sup>-1</sup> are 188, 195 and 200 N-m for the steady cycle under the curvature-rate of 0.3, 0.03 and 0.003 m<sup>-1</sup>s<sup>-1</sup>, respectively. It is shown from the moment-curvature curves that the faster the curvature-rate implies, the faster degree of hardening for titanium alloy tubes. Fig. 3 shows the ovalization of tube cross-section as a function of the applied curvature for three different curvature-rates. The ovalization is defined of  $\Delta D/D$  where D is the outside diameter and  $\Delta D(=D-D')$  is the net change in outsied diameter (Fig. 4). It is found that the magnitudes of ovalization at the curvature of 0.6 m<sup>-1</sup> are 0.002, 0.0043 and 0.0075 for the first cycle under the



$$\Delta \mathbf{D} = \mathbf{D}' - \mathbf{D}'$$

Fig. 4 The definition of the ovalization of tube cross-section



Fig. 5 Variation in ovalization ( $\Delta D/D$ ), at two extreme curvature values for each cycle, with the number of cycles (N) for a curvature magnitude of  $\pm 0.6 \text{ m}^{-1}$  under three different curvature-rates

curvature-rate of 0.3, 0.03 and 0.003 m<sup>-1</sup>s<sup>-1</sup>, respectively. The higher degree of the ovalizaton of tube cross-section can be noticed under higher curvature-rates. It can also be noted that the ovalization of tube cross-section increase in a ratcheting manner with the number of cycles. The accumulation of ovalization with the number of cycles for higher curvature-rate also increases more rapidly than that for slower curvature-rate.

# 3.2 Viscoplastic collapse of titanium alloy tubes under cyclic bending

Fig. 5 demonstrates the variation in ovalization, at two extreme curvature values for each cycle, with the number of cycles for a constant cyclic curvature of  $\pm 0.6 \text{ m}^{-1}$ , but under three different curvature-rates. It can be seen that the ovalization increases faster for higher curvature-rates. Due to the rapid increase in tube ovalization for higher controlled curvature-rate, the number of cycles required for buckling to occur is reduced. In addition, the maximum ovalization reached in these



Fig. 6 Ovalization at buckling  $(\Delta D/D)_b$  versus the controlled curvature ( $\kappa_c$ ) for all tested titanium alloy tubes



Fig. 7 Controlled curvature ( $\kappa_c$ ) versus the number of cycles to produce buckling ( $N_b$ ) curve

three cases is approximately the same. Fig. 6 demonstrates the ovalization at buckling  $(\Delta D/D)_b$  versus the magnitude of controlled curvature ( $\kappa_c$ ) for all tested titanium alloy tubes. It can be found that the ovalization at buckling are approximately the same  $(\Delta D/D)_b \cong 0.078$ ). This phenomenon is different from the experimental result of 304 stainless steel tubes tested by Pan and Her (1998). The magnitude of controlled curvature ( $\kappa_c$ ) versus the number of cycles to produce buckling ( $N_b$ ) for three different controlled curvatures-rates is shown in Fig. 7, wherein the magnitudes of cyclic curvatures were varied from  $\pm 0.3$  to  $\pm 0.75$  m<sup>-1</sup>. As indicated in this figure, the curves move upward when a higher curvature-rate is applied.



Fig. 8 Controlled curvature ( $\kappa_c$ ) versus the number of cycles to produce buckling ( $N_b$ ) curve (log-log scale)

## 4. Discussion

The experimental result of Fig. 7 plotted on log-log scale is shown in Fig. 8. Three straight solid lines in this figure, determined by the least-square method, denote three different levels of curvature-rate. For a certain controlled curvature-rate, the experimental data of  $\kappa_c$  versus  $N_b$  on log-log scale fall on a straight line. Similar phenomenon can be seen from the experimental data of 304 stainless steel tubes tested by Pan and Her (1998). A formulation for the relationship between the magnitudes of controlled curvature ( $\kappa_c$ ) and the number of cycles to produce buckling ( $N_b$ ) was proposed by them to be

$$\kappa_c = B(N_b)^{-\alpha} \tag{1}$$

where B is a function of the controlled curvature-rate, and can be expressed as

$$B = B_o + \beta \left[ \log \frac{\kappa_c}{\kappa_o} \right]^2 \tag{2}$$

where  $B_o$  is a material parameter for the lowest controlled curvature-rate and  $\kappa_o$  is the lowest controlled curvature-rate,  $\kappa_c$  is the other controlled curvature-rate and  $\beta$  is the material parameter. Material parameters  $B_o$  and  $\beta$  are related to the material properties and the D/t ratio. For circular tubes of 304 stainless steel with D/t ratio of 50, the magnitudes of  $B_o$  and  $\alpha$  were determined to be 0.357 m<sup>-1</sup> and 0.118, respectively, by letting  $\kappa_c = \kappa_o$ . Based on the variation of  $\kappa_c - N_b$  curves for different controlled curvature-rates, the value of  $\beta$  was found to be -0.017 in their study (Pan and Her 1998).

In this study, Eqs. (1) and (2) were used for describing the experimental result of titanium alloy tubes. However, the description was not satisfactory. Therefore, the Eq. (2) was modified so that it can be used for describing the experimental result of titanium alloy tubes subjected to cyclic bending with different curvature-rates. Thus, Eq. (2) was proposed to be

$$B = B_o + \beta \left[ \log \frac{\dot{\kappa}_c}{\dot{\kappa}_o} \right]^n \tag{3}$$

where *n* is the material parameter. For circular tubes of 304 stainless steel with D/t ratio of 50, the magnitudes of *n* is equal 2. For titanium alloy tubes with D/t ratio of 36.3, the magnitudes of  $B_o$  and  $\alpha$  were determined to be 1.5529 m<sup>-1</sup> and 0.191, respectively, by letting  $\dot{\kappa}_c = \dot{\kappa}_o$ . Based on the variation of  $\kappa_c -N_b$  curves for different controlled curvature-rates, the values of  $\beta$  and *n* were found to be -0.1049 and 0.9504, respectively. The correlated results based on Eqs. (1) and (3) are shown in Fig. 8 in dashed lines. Good agreement between the experimental and theoretical results has been achieved.

#### 5. Conclusions

In this paper, the viscoplastic response and collapse of titanium alloy tubes subjected to cyclic bending were investigated. Based on the experimental theoretical results, the following important conclusions can be drawn:

(1) It can been seen from the moment-curvature curve that the titanium alloy tube is cyclically harden and gradually steady after a few cycles under symmetric curvature-controlled bending. However, the faster the curvature-rate implies, the faster degree of hardening for titanium alloy tubes.

(2) The ovalization of tube cross-section increases in a ratcheting manner with the number of cycles. The accumulation of ovalization with the number of cycles for higher curvature-rate also increases more rapidly than that for slower curvature-rate.

(3) Although the controlled curvature and curvature-rate are different for tested titanium alloy tubes, the ovalizations at buckling are approximately the same ( $\Delta D/D \approx 0.078$ ). This phenomenon is different from the experimental result of 304 stainless steel tubes tested by Pan and Her (1998).

(4) The relationship between controlled curvature magnitude ( $\kappa_c$ ) and the number of cycles to produce buckling ( $N_b$ ) under different curvature-rates, proposed by Pan and Her (1998), was modified (Eq. 3) so that it can be used for constructing the same relationship for the titanium alloy tubes. It has been shown that the theoretical simulation can properly describe the experimental data.

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#### References

Corona, E. and Kyriakides, S. (1988). "On the collapse of inelastic tubes under combined bending and pressure", *Int. J. Solids Struct.*, **24**(5), 505-553.

Corona, E. and Kyriakides, S. (1991), "An experimental investigation of the degradation and buckling of circular tubes under cyclic bending and external pressure", *Thin-Walled Struct.*, **12**, 229-263.

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- Ikegami, K. and Ni-Itsu, Y. (1983), "Experimental evaluation of the interaction effect between plastic and creep deformation", *Plasticity Today Symposium*, Udine, Italy, June, 27-30.
- Krempl, E. (1979), "An experimental study of room-temperature rate-sensitivity, creep and relaxation of AISI type 304 stainless steel", J. Mech. Phy. Solids, 27, 363-375.
- Kujawski, D. and Krempl, E. (1981), "The rate(time)-dependent behavior of Ti-7Al-2Cb-1Ta titanium alloy at room temperature under quasi-static monotonic and cyclic loading", ASME J. Appl. Mech., 48, 55-63.
- Kyriakides, S. and Ju, G.T. (1992a), "Bifurcation and localization instabilities in cylindrical shells under bending. I: Experiments", *Int. J. Solids Struct.*, **29**(9), 1117-1142.
- Kyriakides, S. and Ju. G.T. (1992b), "Bifurcation and localization instabilities in cylindrical shells under bending. II: Prediction", *Int. J. Solids Struct.*, **29**(9), 1143-1171.
- Kyriakies, S. and Shaw, P.K. (1982), "Response and stability of elastoplastic circular pipes under combined bending and external pressure", *Int. J. Solids Struct.*, **18**(11) 957-973.
- Kyriakides, S. and Shaw, P.K. (1987), "Inelastic buckling of tubes under cyclic loads", ASME J. Press. Vessel Technol., 109, 169-178.
- Pan, W.F. and Chern, C.H. (1997), "Endochronic description for viscoplastic behavior of materials under multiaxial loading", Int. J. Solids Struct., 34(17), 2131-2160.
- Pan, W.F. and Fan, C.H. (1998), "An experimental study on the effect of curvature-rate at preloading stage on subsequent creep or relaxation of thin-walled tubes under pure bending", *JSME Int. Journal, Series A*, **41**(4), 525-531.
- Pan, W.F. and Her, Y.S. (1998), "Viscoplastic collapse of thin-walled tubes under cyclic bending", ASME J. Engng. Mat. Technol., 120, 287-290.
- Pan, W.F. and Hsu, C.M. (1999), "Viscoplastic analysis of thin-walled tubes under cyclic bending", Int. J. Struct. Engng. Mech., 7(5), 457-471.
- Pan, W.F. and Leu, K.T. (1997), "Endochronic analysis for viscoplastic collapse of a thin-walled tube under combined bending and external pressure", *JSME Int. Journal, Series A*. **40**(2), 189-199.
- Pan, W.F., Wang, T.R. and Hsu, C.M. (1998), "A curvature-ovalization measurement apparatus for circular tubes under cyclic bending", *Experimental Mechanics*, 38(2), 99-102.
- Shaw, P.K. and Kyriakides, S. (1985), "Inelastic analysis of thin-walled tubes under cyclic bending", Int. J. Solids Struct., 21(11), 1073-1100.