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Free vibration of core wall structure coupled with connecting beams

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Abstract. In this paper, a core wall structure coupled with connecting beams is discretized and modeled as an equivalent thin-walled member with closed section, while the connecting beams between openings are replaced by an equivalent shear diaphragm. Then, a numerical method (finite member element method, FMEM) for dynamic analysis of the core wall structure is proposed. The numerical method combines the advantages of the FMEM and Vlasov's thin-walled beam theory and the effects of torsion, warping and, especially, the shearing strains in the middle surface of the walls are considered. The results presented in this paper are very promising compared with the ones obtained from finite element method.

Key words: thin-walled closed cross section; finite member element; warping; vibration.

1. Introduction

Core walls are frequently used in tall buildings to provide lateral resistance. Shear walls distributed around the structure and coupled with floors may constitute a core structure, or a hallow three-dimensional structure, such as an elevator shaft, act as a core wall. These core wall structures are often called upon to carry also torque, resulting from either designed or accidental eccentricity between the line of action of the lateral forces and the axis of rigidity of the structural system. Such structures are typically idealized as thin-walled members that exhibit significant out-plane warping due to torsion. The torsional response of the core wall structure lies between that of a cantilever beam with an open cross section, which is controlled by flexural and torsional couple, and that of a closed cross section that is controlled by shear-torsional couple, depending on the stiffness of the connecting beams. These two traditional procedures provide reliable results only for the extreme cases, e.g., either very flexible or very stiff connecting beams. However, they can not describe adequately the behaviour of core walls with moderately stiff connecting beams, where bending and shear in the core walls are of equal importance. Several methods were developed to deal with the torsional response of a core wall structure during the last two decades (Chan and Kuang 1988, Heidebrecht and Stafford-Smith 1973, Khan and Stafford-Smith 1975, Liauw 1978, Liauw and Luk 1980, Rosman 1969, Rutenberg and Tso 1975, Stafford-Smith and Taranath 1972, Sumer and Askar 1992, Tso and Biswas 1973) to investigate the behaviour of battened open section structures subjected to applied twisting moment. But a few attempts have been made to treat the dynamic problem on the basis of computer analysis. Urgent practical requirements have given rise in recent years to extensive investigations, both theoretical and experimental, of dynamic analysis of the thin-

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walled structural members. Some closed-form solutions for small amplitude vibrations of such members were published by Vlasov (1961) and Timoshenko *et al.* (1961). Useful explicit formulae to determine the lowest natural flexural and torsional frequencies for thin-walled members with open and/or closed cross section can also be found in other publications (Heidebrechi and Ravindora 1971, Robert 1987, and Weaver and Johnston 1987). However, the shear deformation in the middle surface of the walls can not be incorporated correctly in the methods as presented. Consequently, the results obtained from the above studies may not represent the true behaviour. No simple theory comparable to the Timoshenko or Vlasov theory has been developed to deal with shear lag (Gjelsvik 1991). Among the previous methods for dynamic analysis, the finite element method is widely used for the vibration analysis of structures including thin-walled structures (Gellin and Lee 1988, and Krajcinovic 1969). But, finite element analysis of such structures are quite expensive, even when planar formulation is adopted (Rutenberg and Eisenberger 1983), and unless very fine meshes are used, they are not likely to be more accurate than some of the continuum methods. In this study, connecting beams are replaced by equivalent continuum of one storey height. Thus, the structure resembles a closed section, a mono-symmetric core wall structure coupled with connecting beams at floor levels will be analyzed by using finite member element method (FMEM) in which the effects of torsion, warping and, especially, the shearing strain in the middle surface of the walls are taken into account. The FMEM, considered as a semi-analytical method, has proved to be an efficient method for the buckling analysis of thin-walled members with arbitrary cross sections (Wang 1999), in which the member is discretized in one direction only. By using a one-way geometric discretization, the FMEM reduces a two-dimensional problem to a one-dimensional problem. This discretization results in fewer equations with a small half-bandwidth and greater computational efficiency than the finite element method. The results based on the proposed method are compared with the solution of the core wall with symmetrical cross section, which is presented by Vasquez & Riddell (1980). In the paper, only one of two Vlasov' assumptions, i.e., "rigid cross section" is retained while no shearing strain in the middle surface of walls is abandoned.

2. Assumptions

The following formulation of the problem is based on several simplifying assumptions that appear to be aquite realistic: (1) The core wall is modelled as a prismatic thin-walled beam; (2) connecting beams can be replaced by a continuum with equivalent thickness; (3) the cross section along the height is constant; and (4) the floor slab has in-plane rigidity.

3. Analytical model

Consider a core wall structure coupled with connecting beams which rests on a rigid foundation; as shown in Fig. 1, in which a and b are the width and depth of the core wall, respectively; d and l are the depth and length of a connecting beam, respectively; H and h are the height of the core wall and storey height, respectively, and t is the thickness of the core wall. In accordance with previous work, the battened cross section can be idealized as an equivalent closed section by replacing the connecting beams with an equivalent shear diaphragm (Khan and Stafford-Smith 1975). The model

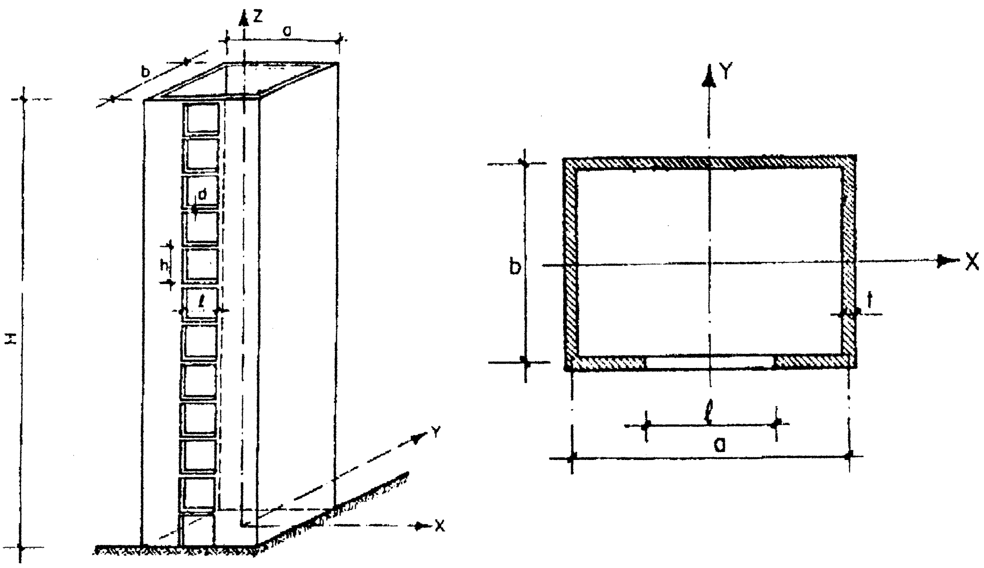

Fig. 1 Core wall structure and its plan

incorporated two effects not previously considered in the literature. These are the influence of side-wall distortion and continuous shear flow around the contour. Herein, the thickness of the equivalent shear diaphragm is determined, taking into account bending and shear deformation of the bracing beams, out-of-plane bending of the side walls, and continuous shear flow around the contour of the equivalent closed section. To simplify the initial analysis, the influence of the channel lip is neglected by assuming the span of the bracing beams as equal to the width of the core wall. If the thickness of core wall is constant, the thickness of the equivalent shear diaphragm can be given by Roberts and Achour (1989) as follows:

$$t_o = \frac{-C_1 + \sqrt{C_1^2 + 4C_2C_3}}{2C_2} \tag{1}$$

in which

$$C_1 = \frac{hG}{2a};\quad C_2 = \frac{C_1(a+2b)}{a};\quad C_3 = \frac{BW^2 + WB^2a^2}{(Ba^2+W)^2};\quad B = \frac{Etd^3}{2[a^3 + 3(1+v)ad^2]};$$

$$W = D\left[2(1-v)\frac{d}{b} + \frac{2\pi^2h^2b}{3(h-d)^3} + (1-v)\frac{(2d^2+h^2)}{(h-d)b}\right];$$

D is the flexural rigidity of a plate; E and G are Young's modulus and shear modulus, respectively; and v is Poisson's ratio. Having determined t_o, the section is idealized as an equivalent closed section, as shown in Fig. 2.

A possible approximate evaluation of the dynamic characteristics for a core wall with connecting beams may be derived by the thin-walled member with the equivalent closed section.

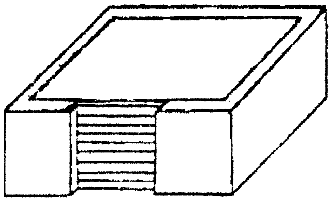

Fig. 2 Equivalent cross section

4. Energy equation of equivalent thin-walled member

The Hamilton principle in variation form for free vibration can be expressed as follows:

$$\delta\int_{t_1}^{t_2}(T-U)dt=0 \tag{2}$$

in which U is the strain energy induced in vibration; T is the corresponding kinetic energy. The integral interval can be chosen as

$$t_1=0;\ t_2=2\pi/\omega$$

in which ω is the circular frequency of vibration.

For a continuum, the strain energy is given by

$$U=\frac{1}{2}\int_v\sum\{E\varepsilon_i^2+G\gamma^2\}dv \tag{3}$$

in which ε_i and γ are the normal and the shear strains. Since the stress σ_s in the tangent direction and the stress σ_n in the normal direction of the central line of the cross section are much smaller than the longitudinal stress σ_z, both are neglcted generally, the linear solution is taken to be sufficient in the present analysis. The member element considered is basically one-dimensional along the z direction, the components of the strain are zero except for the following terms:

$$\varepsilon_z=\frac{\partial w}{\partial z}; \tag{4}$$

$$\gamma=2\rho_n\frac{d\theta}{dz}+\frac{1}{2}\left(\frac{\partial w}{\partial s}+\frac{\partial v_t}{\partial z}\right) \tag{5}$$

in which $w(s, z)$ is the longitudinal warping displacement along z direction, whose displacement distribution can be determined by the following Eq. (9); s is the curvilinear coordinate along the central line of the cross section; v_t is the displacement along the tangent direction of the central line at point s; θ is the twisting angle of the cross section; and ρ_n is the normal distance from the centre line. Substituting Eqs. (4) and (5) into Eq. (3) and neglecting higher order term, the expression for the strain energy U may be obtained (Wang and Li 1999)

$$U=\frac{1}{2}\int_0^{H_i}\left\{\int_{\Sigma_s}\left[E\left(\frac{\partial w}{\partial z}\right)^2+G\left(\frac{\partial w}{\partial s}+\frac{\partial v_t}{\partial z}\right)^2\right]t_o ds+GJ_d\left(\frac{d\theta}{dz}\right)^2\right\}dz \tag{6}$$

in which J_d is the St. Vanant's torsional constant; Σs is the length of whole cross-section; t_o is the equivalent thickness of wall when there is a row of holes in the wall. The kinetic energy is a general member element of length H_i can be given by the expression

$$T=\frac{1}{2}\int_0^{H_i}\int_{\Sigma_s}\rho_o\left[\left(\frac{\partial w}{\partial t}\right)^2+\left(\frac{\partial v_x}{\partial z}\right)^2+\left(\frac{\partial v_y}{\partial t}\right)^2\right]t_o ds dz \tag{7}$$

in which v_x and v_y is the displacement of any point of cross section in the x and y directions respectively; ρ_o is the density of material of the member. If the analysis does not take into account the shearing strains of the middle surface of walls,

$$\frac{\partial w}{\partial s}+\frac{\partial v_t}{\partial z}=0,$$

Eq. (6) can be reduced as

$$U=\frac{1}{2}\int_0^{H_i}\left\{\int_{\Sigma_s}E\left(\frac{\partial w}{\partial z}\right)^2 t_o ds+GJ_d\left(\frac{d\theta}{dz}\right)^2\right\}dz;$$

According to the "rigid cross section" assumption (Vlasov 1961), the tangent displacement can be expressed by the centroid displacement

$$v_t=[\eta_t]_{1\times 3}\{v_c\}_{3\times 1} \tag{8}$$

in which $[\eta_t]=[\cos\alpha\ \sin\alpha\ \rho]$; $\{v_c\}=[v_{cx}\ v_{cy}\ \theta]^T$; α is the angle between x axis and tangent of point s; $\rho(s)$ is the distance from c to the tangent of point s; v_{cx}, v_{cy} are x and y direction components of the centroid displacement v_c, respectively. The longitudinal warping displacement $w(s, z)$ of the whole cross section can be expressed as

$$w(s,z)=\sum_{i=1}^{n}\psi_i(s)w_i(z)=[\psi]_{1\times n}\{w\}_{n\times 1} \tag{9}$$

in which n denotes the total number of nodes which divided the whole cross section into subintervals; $\psi_i(s)$ are chosen coordinate function while $w_i(z)$ will be determined, and

$$[\psi]_{1\times n}=[\psi_{j1}(s),\ \psi_{j2}(s)\cdots\psi_{jn}(s)];\ \ \{w\}_{n\times 1}=[w_1(z),\ w_2(z)\cdots w_n(z)]^T.$$

5. Dynamic equations of thin-walled member

As to free vibration at a natural frequency of the thin-walled member, the motion at any point is simple, harmonic, and deflected shapes are independent of time, Eqs. (8) and (9) can be written as

$$v_t(z,t)=V_t(z)\sin(\omega t+\varepsilon)=[\eta_t]_{1\times 3}\{\bar{v}_c\}_{3\times 1}\sin(\omega t+\varepsilon) \tag{10}$$

$$w(s,z,t)=W(s,z)\sin(\omega t+\varepsilon)=[\psi]_{1\times n}\{\bar{w}\}_{n\times 1}\sin(\omega t+\varepsilon) \tag{11}$$

respectively, in which $V_t(z)$ and $W(s, z)$ are the functions of mode shape respectively; ε is the phase angle. Providing that the x and y axes are the principal axes passing through the centroid of a cross section, hence, v_x and v_y are related to v_{cx}, v_{cy} and θ by

$$\begin{cases} v_x(y,\ z,\ t)=(v_{cx}-y\theta)\sin(\omega t+\varepsilon), \\ v_y(x,\ z,\ t)=(v_{cy}+x\theta)\sin(\omega t+\varepsilon); \end{cases}$$

Substituting the above equation, Eqs. (10) and (11) into Eqs. (6) and (7) gives

$$U=\frac{1}{2}\int_0^{H_i}(E\{w'\}^T[A]\{w'\}+G\{w\}^T[B]\{w\}+2G\{w\}^T[C]\{v_c'\}$$
$$+G\{v_c'\}^T[D_t]\{v_c'\}+G\{\theta'\}^T J_d\{\theta'\})dz \tag{12}$$

$$T=\frac{1}{2}\int_0^{H_i}\rho_o\omega^2(\{w\}^T[A]\{w\}+\{v_c\}^T[\bar{D}]\{v_c\})dz \tag{13}$$

in which

$$[A]_{n\times n}=\int_{\Sigma_s}[\psi]^T[\psi]tds;\ \ [B]_{n\times n}\int_{\Sigma_s}[\psi']^T[\psi']tds;$$

$$[C]_{n\times 3}=\int_{\Sigma_s}[\psi']^T[\eta_t]tds;\ \ [D_t]_{3\times 3}\int_{\Sigma_s}[\eta_t]^T[\eta_t]tds;$$

$$[\bar{D}]_{3\times 3}\int_{\Sigma_s}\begin{bmatrix}1 & 0 & -y\\ 0 & 1 & x\\ -y & x & x^2+y^2\end{bmatrix}tds=\begin{bmatrix}A_o & 0 & 0\\ 0 & A_o & 0\\ 0 & 0 & I_x+I_y\end{bmatrix}$$

where A_o is the are of cross section of the thin-walled member; I_x and I_y are the second moments of inertia of the cross section with respect to the principal axis x and y respectively. By virtue of Hamilton's principle, the stationarity of the action integral yields the actual motion among all those which take place in the time interval between t_1 and t_2. This variational condition provides for Euler's equations and the natural boundary conditions, i.e., taking the first variation of the functional with respect to $\{w\}$ and $\{v_c\}$ in Eq. (2), and using some standard techniques such as integration by parts in the derivation, we have

$$E[A]_{n\times n}\{w''\}_{n\times 1}-G[B]_{n\times n}\{w\}_{n\times 1}-G[C]_{n\times 3}\{v_c'\}_{3\times 1}+\rho_o\omega^2[A]_{n\times n}\{w\}_{n\times 1}=\{0\}_{n\times 1}; \tag{14}$$

$$G[C]^T_{3\times n}\{w'\}_{n\times 1}+G[D]_{3\times 3}\{v_c''\}_{3\times 1}+\rho_o\omega^2[\bar{D}]_{3\times 3}\{v_c\}_{3\times 1}=\{0\}_{3\times 1}; \tag{15}$$

with the relevant boundary conditions

$$E[A]_{n\times n}\{w'\}_{n\times 1}=\{0\}_{n\times 1}; \tag{16}$$

$$[C]^T_{3\times n}\{w\}_{n\times 1}+[D]_{3\times 3}\{v_c'\}_{3\times 1}=\{0\}_{3\times 1}; \tag{17}$$

in which

$$[D]_{3\times3}=[D_t]_{3\times3}+J_d\begin{bmatrix}0 & & \\ & 0 & \\ & & 1\end{bmatrix};$$

Eqs. (14) and (15) are the governing differential equations for the dynamic analysis of a thin-walled member.

6. Solution of dynamic equation

From the "rigid cross section" assumption, the displacements at one end of the member element are only the lateral displacements and twisting angle of the cross section and ***n*** longitudinal displacements of knots. In matrix form, the end displacements of the member element are expressed by a vector $\{W_E\}$ as:

$$\{W_E\}_{(2n+6)\times1}=[v_{1x},\ v_{1y},\ \theta_1,\ w_{11},\ w_{12}\cdots w_{1n},\ v_{2x},\ v_{2y},\ \theta_2,\ w_{21},\ w_{22}\cdots w_{2n}]^T \tag{18}$$

in which the first subscript 1 or 2 denote first or second end cross section; v_{1x} denotes the lateral displacement of the cross section in the x direction at first end, w_{11} denotes the longitudinal displacement of first knot at first end, etc. Similar to the finite element analysis, a finite member element can be established for the thin-walled structure analysis. Define a transformation as

$$\{w\}_{n\times1}=[a]_{n\times n}\{U\}_{n\times1} \tag{19}$$

in which

$$[a]=[\{a_1\},\ \{a_2\},\ \ldots,\ \{a_n\}];\ \ \{U\}=[U_1,\ U_2,\ \ldots,\ U_n]^T,$$

and $\{a_i\}$ is an eigenvector. From the studies (Wang and Li 1999), the following displacement vector can be expressed in terms of end displacements of member element:

$$\begin{Bmatrix} v_c \\ w \end{Bmatrix}_{\{n+3\}\times1}=\begin{pmatrix}[T_v(z)]_{3\times\{2n+6\}} \\ [a]_{n\times n}[T_w(z)]_{n\times\{2n+6\}}\end{pmatrix}[T_E]^{-1}_{\{2n+6\}\times\{2n+6\}}\{W_E\}_{\{2n+6\}\times1}; \tag{20}$$

in which

$$[T_v(z)]_{3\times\{2n+6\}}=\Big[-[D]^{-1}_{3\times3}[C]^T_{3\times n}[a]_{n\times n}\int_0^z[R_0]_{n\times n}dz[a]^T_{n\times n}[C]_{n\times3}+z\frac{G}{E}[I]_{3\times3},$$

$$[I]_{3\times3},\ -[D]^{-1}_{3\times3}[C]^T_{3\times n}[a]_{n\times n}\int_0^z[R]_{n\times2n}dz\Big];$$

$$[T_w(z)]_{n\times\{2n+6\}}=[[R_0]_{n\times n}[a]^T_{n\times n}[C]_{n\times3},\ [0]_{n\times3},\ [R]_{n\times2n}];$$

$$[T_E]_{\{2n+6\}\times\{2n+6\}}=\begin{bmatrix}[T_v(0)]_{3\times\{2n+6\}} \\ [a]_{n\times n}[T_w(0)]_{n\times\{2n+6\}} \\ [T_v(H_i)]_{3\times\{2n+6\}} \\ [a]_{n\times n}[T_w(H_i)]_{n\times\{2n+6\}}\end{bmatrix},$$

$$[R_o]_{n\times n}=\begin{bmatrix} \frac{z^2}{2} & & & & & & \\ & \frac{z^2}{2} & & & 0 & & \\ & & \frac{z^2}{2} & & & & \\ & & & -\frac{1}{\Lambda_4}\frac{E}{G} & & & \\ & & & & \cdot & & \\ & 0 & & & & \cdot & \\ & & & & & & \cdot \\ & & & & & & -\frac{1}{\Lambda_n}\frac{E}{G} \end{bmatrix}$$

$$[R]_{n\times 2n}=\begin{bmatrix} z,\ 1 & & & & & & & \\ & z,\ 1 & & & & & 0 & \\ & & z,\ 1 & & & & & \\ & & & e^{-\lambda_4\xi},\ e^{-\lambda_4\eta} & & & & \\ & & & & \cdot\ \cdot & & & \\ & & & & & \cdot\ \cdot & & \\ & & 0 & & & & \cdot\ \cdot & \\ & & & & & & & e^{-\lambda_n\xi},\ e^{-\lambda_n\eta} \end{bmatrix}$$

$$\eta=\frac{z}{H_i},\ \ \xi=1-\eta,\ \lambda_i=H_i\sqrt{\frac{G}{E}\Lambda_i}$$

and $[I]$ is an unit diagonal matrix; Λ_1, Λ_2, ..., Λ_n are the eigenvalues of the following eigenvalue equation:

$$\Lambda[A]_{n\times n}\{a\}_{n\times 1}-([B]_{n\times n}-[C]_{n\times 3}[D]^{-1}_{3\times 3}[C]^{T}_{3\times n})\{a\}_{n\times 1}=\{0\}_{n\times 1} \tag{21}$$

Substituting Eq. (20) into Eqs. (14) and (15) yields the following standard formula of finite member element stiffness equation for dynamic analysis:

$$([K]+\omega^2[M])_{(2n+6)\times(2n+6)}\{W_E\}_{(2n+6)\times 1}=\{0\}_{(2n+6)\times 1} \tag{22}$$

in which $[K]$ is the linear strain stiffness matrix, following the standard procedures of the finite element method, the stiffness matrix $[K]$ of the member element can be obtained; $[M]$ is referred to as the nonlinear general mass matrix and can be derived from the loading potential energy and written by

$$\begin{aligned}[M]_{\{2n+6\}\times\{2n+6\}}=&([T_E]^{-1}_{\{2n+6\}\times\{2n+6\}})\{\int_0^H \rho([T_v(z)]^{T}_{\{2n+6\}\times 3}[\overline{D}]_{3\times 3}[T_v(z)]_{3\times\{2n+6\}}\\ &+[T_w]^{T}_{\{2n+6\}\times n}[a]^{T}_{n\times n}[a]_{n\times n}[T_w]_{n\times\{2n+6\}})dz\}[T_E]^{-1}_{\{2n+6\}\times\{2n+6\}};\end{aligned} \tag{23}$$

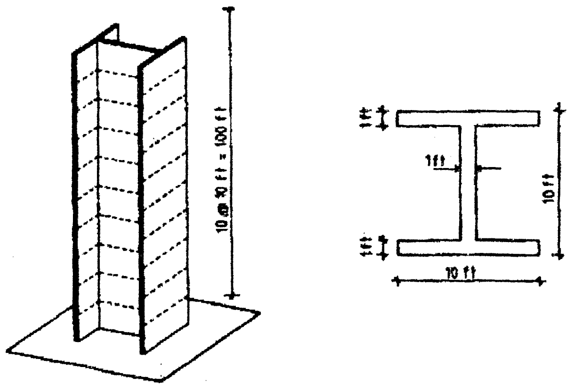

Fig. 3 Isolated core-wall example

7. Examples of dynamic behaviour of core walls

In order to illustrate the application of the proposed method two examples of torsional dynamic behaviour of core wall structure are presented.

Example 1: Consider first the case of the simple ten-story structure shown in Fig. 3. For comparison with the results obtained by Vasquez and Riddell (1980), same data and US customary units are used. In the example, E=5.04×10^8 kip/ft^2; G=E/2.3.

Table 1 shows the torsional periods of vibration of the structure, considering the effects of torsion, warping and, especially, the shearing strains in the middle surface of the walls. For easiness, the value of story rotational inertia is chosen so as to give a fundamental period of one second. Table 1 shows a comparison of the results obtained from different methods, in which the percentage errors (in parentheses) are in reference to the present solution. It can be seen that the results are in close agreement between the numerical method by Vasquez and Riddell (1980) and the proposed method except only the high modes.

Example 2: In this second example, a central core wall identical to that of the first structure is connected to a set of four girders at each story level. The beams provide warping restraint to the core wall without further altering the torsional behaviour of the building. The torsional periods of

Table 1 Periods in seconds for modes of structure of Fig. 3

Mode	Finite element method	Proposed method
1	1.0000	1.0010 (100%)
2	0.2713	0.2602 (95.9%)
3	0.1208	0.1157 (95.8%)
4	0.0690	0.0669 (97.0%)
5	0.0461	0.0450 (97.6%)
6	0.0343	0.0336 (97.1%)
7	0.0276	0.0262 (96.0%)
8	0.0236	0.0215 (91.1%)
9	0.0212	0.0181 (85.4%)
10	0.0199	0.0157 (78.9%)

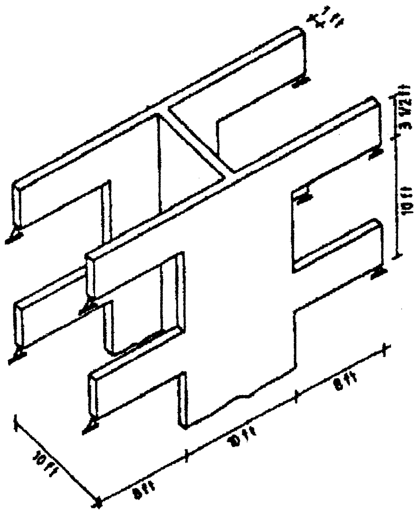

Fig. 4 Core wall structure coupled with connecting beams

vibration of the structure are presented in Table 2 and compared with the results by Vasquez and Riddell (1980). The same value of the story rotational inertia of the first example is used.

Comparing the results presented in this paper with the ones obtained by another numerical method in this table, the observation can be made that the higher modes periods are not in good agreement. The difference increases towards the higher modes and the results from the proposed model in this paper are smaller than that given by Vasquez and Riddell (1980). In fact, the difference between the results lies mainly in the analyzed structure of Fig. 4 is idealized as an equivalent thin-walled closed cross section, while the connecting beams between openings are replaced by an equivalent shear diaphragm in this paper, which has bigger shear-torsional stiffness.

8. Conclusions

The core wall structure is discretized and modeled as an equivalent thin-walled closed section,

Table 2 Periods in seconds for modes of structure of Fig. 4

Mode	Finite element method	Proposed method
1	0.5041	0.5039
2	0.1594	0.1410
3	0.0879	0.0691
4	0.0574	0.0434
5	0.0415	0.0311
6	0.0323	0.0241
7	0.0267	0.0195
8	0.0232	0.0164
9	0.0210	0.0140
10	0.0199	0.0123

while the connecting beams between openings are replaced by an equivalent shear diaphragm. A more rational approach for the dynamic analysis of core wall structure coupled with connecting beams is presented in this paper. By employing finite member element method (FMEM), and by considering the shear strains of the middle surface of walls, numerical examples that were given in this paper verify the simplicity and feasibility of the proposed method. The results of the analysis are compared with the results from the finite element method. An acceptable solution to the problem has been obtained. However, using the proposed procedure a great saving in computer time and data preparation effort are achieved when compared with the finite element procedure.

References

Chan, H.C. and Kuang, J.S. (1988), "Torsional behaviour of braced thin-walled open sections", *J., Thin-Walled Structures*, **6**, 343-354.

Gellin, S. and Lee, G.C. (1988), "Finite element analysis of thin-walled structures: Finite element available for the analysis of non-curved thin-walled structures", *Elsevier Applied Science*, London and New York, 1-30.

Gjelsvik, A. (1991), "Analogy-beam methods for determining shear-lag effects", *J. of Engrg. Mech., ASCE*, **117**(7), 1575-1594.

Heidebrecht, A.C. and Ravindora, K.R. (1971), "Frequency analysis of thin-walled shear walls", *J. of Engrg. Mech., ASCE*, **97**, 239-252.

Heidebrecht, A.C. and Stafford-Smith, B. (1973), "Approximate analysis of open section shear wall subjected to torsional loading", *J. Struct. Div. ASCE*, **99**(ST12), 2355-2373.

Khan, M.A. and Stafford-Smith, B. (1975), "Restraining action of bracing in thin-walled open section beams", *Proc. Instn Civ., Engrs*, Part 2, **59**, 67-78.

Krajcinovic, D. (1969), "A consisted discrete element technique for thin-walled assemblages", *Int. J. of Solid and Structures*, **5**, 649-662.

Liauw, T.C. (1978), "Torsion of multi-story spacial core walls", *Proc. Instn Civ, Engrs*, Part 2, **65**, 601-609.

Liauw, T.C. and Luk, W.K. (1980), "Torsion of core walls of nonuniform section", *J. Struct. Div. ASCE*, **107**(9), 1921-1931.

Roberts, T.M. (1987), "Free vibrations of thin-walled bars", *J. of Engrg. Mech., ASCE*, **113**, 1584-1593.

Roberts, T.M. and Achour, B. (1989), "Torsion and bending of braced thin-walled open section", *J. of Struct. Engrg, ASCE* **116**(1), 1-12.

Rosman, R. (1969), "Torsional of perforated concrete shafts", *J. Struct. Div., ASCE* **95**(ST5), 991-1010.

Rutenberg, A.V. and Eisenberger, M. (1983), "Torsion of tube structures: Planar formulation", *J., Computer and Structures*, **17**, 257-260.

Rutenberg, A.V. and Tso, W.K. (1975), "Torsional analysis of perforated core structure", *J., Struct. Div. ASCE,* **101**(ST3), 539-549.

Stafford-Smith, B. and Taranath, B.S. (1972), "The analysis of tall core-supported structure subjected to torsion", *Instn Civ, Engrs*, **53**, 173-187.

Sumer, O. and Askar, G. (1992), "Analysis of a mono-symmetric core wall structure coupled with connecting beams", *J., Thin-Walled Structures*, **14**, 193-208.

Timoshenko, S.P. and Gere, J.M. (1961), *Theory of Elastic Stability*, 2nd Ed, McGraw-Hill Book Co., New York, N.Y.

Tso, W.K. and Biswas, J.K. (1973), "General analysis of nonplanar coupled shear wall", *J., Struct. Div. ASCE,* **99**(ST3), 365-551.

Vlasov, V.Z. (1961), *Thin-Walled Elastic Beams*, 2nd Ed, Israel Program for Scientific Translations, Jerusalem, Israel.

Vasquez, J. and Riddell, R. (1980), "Thin-walled core element for multistorey buildings", *J. of Struct. Engrg. ASCE,* **110**(ST5), 1021-1034.

Wang, Quanfeng and Li, W.Y. (1999), "Spatial stability of thin-walled eccentric compressive members", *J. of*

Engrg. Mechanics, ASCE, **125**(2), 197-205.
Wang, Quanfeng (1999), "Vibration of thin-walled members with shear lag using finite member element method", *J., Communications in Numerical Methods in Engineering*, **15**, 273-282.
Weaver, W. and Johnston, P.R. (1987), *Structural Dynamics by Finite Elements*, Prentice Hall, Inc., Englewood Cliffs.

Notation

The following symbols are used in this paper:

A_o : the area of cross section of the thin-walled member
a, b : the width and depth of the core wall, respectively
$\{a_i\}$: an eigenvector
c : the centroid of the cross section
D : the flexural rigidity of a plate
d : the depth and legnth of a connecting beam
E : the Young's modulus
G : the shear modulus
H_i : the longitudinal length of the member element
H : the height of the core wall
h : the storey height of the core wall
$[I]$: an unit diagonal matrix
I_x, I_y : the second moment of inertia of the cross section with respect to the principal axis x and y
J_d : the St. Venant's torsional constant
$[K]$: the linear strain stiffness matrix
$[M\}$: the nonlinear general mass matrix
l : the length of a connecting beam
n : an arbitrary chosen number of knots
s : the curvilinear coordinate along the central line of the cross section
Σs : the length of whole cross-section
T : the corresponding kinetic energy
t : the thickness of the core wall
t_o : the thickness of the equivalent shear diaphragm
U : the strain energy induced in vibration
$V_i(z)$: the functions of mode shape
v_{1x} : the lateral displacement of the cross section in x direction at first end
v_{cx}, v_{cy} : x and y direction components of the centroid displacement of v_c
v_c : the centroid displacement of v_c
v_t : the displacement along the tangent of the central line at point s
v_x, v_y : the displacement of any point of cross section in the x and y directions
$(\partial v_x/\partial z)^2$, $(\partial v_y/\partial z)^2$: the longitudinal strains caused by transverse displacements v_x and v_y
$W(s,z)$: the functions of mode shape
w_{11} : the longitudinal displacement of first knot at first end
$\{W_E\}$: the end displacements of the member element
w_i : the real knot displacement in the segment
$w(s, z)$: the longitudinal displacement along z direction
x, y axes : the principal axis and pass through the centroid c of the cross section
z axis : the longitudinal axis
α : the angle between x axis and tangent of point s
γ : the shear strains
ε_i : the normal strains
ε : the phase angle

θ : the twisting angle of the cross section

$\eta = \frac{z}{H}$

$\lambda_i = H\sqrt{\frac{G}{E}\Lambda_i}$

ν : Poisson's ratio
ξ : $1-\eta$
$\rho(s)$: the distance from c to the tangent of point s
ρ_n : the normal distance from the centre line
ρ_o : the density of material of the member
σ_s : the stress in the tangent direction of the central line of the cross section; much
σ_n : the stress σ_n is the normal direction of the central line of the cross section
σ_z : the longitudinal stress
ω : the circular frequency of vibration
$\psi_i(s)$: chosen coordinate function